

A NUMERICAL MODEL FOR PREDICTION OF DISSOLVED OXYGEN VARIATION  
DURING SEDIMENT FLUSHING

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SYNOPSIS

A mathematical model was developed to predict the variations of dissolved oxygen (DO) in reservoirs and the downstream of the dams during sediment flushing. The model was proposed based on the finding that the decrease of DO during sediment flushing was caused by oxygenation of Fe(II) that had been accumulated in the bottom sediment.

To quantitatively estimate the influence of sedimentation period on DO decrease, the model describing the transport and transform process of DO, Fe(II) and Fe(III) was incorporated into an one-dimensional sediment dynamic model. The present model was validated by comparing the calculated results with the measured data collected at several stations downstream of Dashidaira dam on the Kurobe River during sediment flushing operations in 1994 and 1999.

1. INTRODUCTION

Flushing out the deposited sediments through the flushing outlet is an effective method of maintaining the storage capacity of reservoirs (1). During sediment flushing, however, the decrease of dissolved oxygen (DO) concentration in the releasing flow may have a negative effect on the evasion action of the downstream aquatic organisms such as fish, if deposited sediments have been accumulated long-term and large amounts of reduced materials have been stored in reservoirs (2). Therefore, the prediction of DO variation is very important from the viewpoint of environmental conservation of rivers, before operation of flushing is undertaken. In order to determine a suitable flushing frequency and evaluate the effects of sediment flushing on the downstream in advance, a

mathematical model, which enables prediction of the DO variation in consideration of the deposit situation and sedimentation period, was developed. This model is based on observations in the Dashidaira Reservoir, whose total storage capacity of 9.01 million m<sup>3</sup> and drainage area of 461.18 km<sup>2</sup>, on the middle stream of the Kurobe River (3). The concept and validity of the numerical model are described in this study.

## 2. BASIC IDEA

The test sediment flushing at the Dashidaira dam was executed to collect basic data for the environmental impact assessment in February 1994. According to the former investigation results, the decrease of DO concentration during flushing was caused by the oxidation reaction of reduced materials, such as Fe<sup>2+</sup>, Mn<sup>2+</sup>, S<sup>2-</sup>, in the sediment deposit. Among these Fe(II) played a predominant role. Therefore, it is necessary to predict the deposition rate and variation of the amount of DO consumption materials, in order to deal with the problem of DO decrease, because sediment was deposited continuously between the flushing events. For consideration of the deposit situation and period, parameters related to water qualities such as DO and Fe (II) were incorporated into the one-dimensional sediment dynamic model(4) to solve the DO variation both in the reservoir and in the downstream river channel during flushing.

The calculation domain was divided into the reservoir and the downstream river channel as shown in Figure 1. The reaeration and dissolved oxygen consumption by oxidation of Fe(II), that cause the DO variation, were modeled for the reservoir. For the river channel, besides the above processes, the exchange of the pore water in the gravel layer with the flowing water above the gravel was taken into account.

Consumption of DO is expressed as follows.



Because this reaction is promoted when Fe (II) concentration is high, it is necessary to solve the time series change of Fe (II) concentration to evaluate the consumption of DO. Moreover, the time series variation of Fe (III) should be solved similarly since Fe (II) is generated by the reduction of Fe (III) in the sediments.

In addition, it is assumed that all iron in the calculation domain exists either in the form of Fe (II) or Fe (III), and that concentration variation in the oxidation reaction or the reduction reaction is equal to those of Fe (II) or Fe (III).

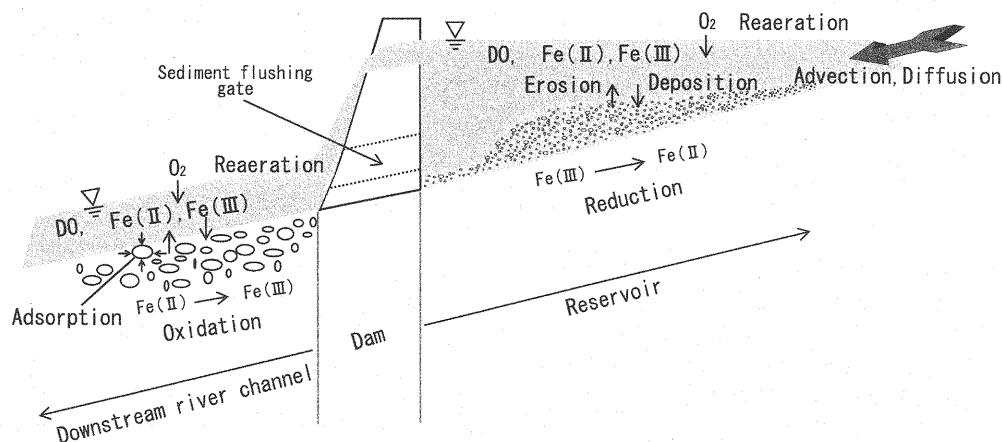


Fig. 1 Schematic description of calculation domain

### 3. GOVERNING EQUATIONS OF DO VARIATION

Assuming that the reaction rates of DO and Fe (II) in a very short time are proportional to each concentration at the time, then the following equations are obtained,

$$\frac{\partial}{\partial t} = -K_{SF} \cdot C \quad (2)$$

$$\frac{\partial C_{F2}}{\partial t} = -K_F \cdot C_{F2} \quad (3)$$

Where,  $K_{SF}$  = consumption rate constant of DO (1/s),  $C$  = DO concentration (mg/l),  $K_F$  = coefficient of oxidation reaction rate of Fe (II) (1/s), and  $C_{F2}$  = concentration of Fe (II) (mg/l). The following equation can be obtained by the introduction of Eqs. (2) and (3) into Eqs. (1).

$$K_F = 4 \frac{W_{Fe} C}{W_{O_2} C_{F2}} K_{SF} \quad (4)$$

Where,  $W_{Fe}$  = atomic weight of iron (55.8) and  $W_{O_2}$  = molecular weight of oxygen (32). The correlation between  $K_{SF}$  and  $C_{F2}$  is described later. The equations that describe the DO variation are given as follows.

#### (1) Concentration equation of DO, Fe (II) and Fe (III) in the reservoir

DO variation in the reservoir is solved by the following equation

$$\frac{\partial (C \cdot A)}{\partial t} + \frac{\partial (u \cdot C \cdot A)}{\partial x} = \frac{\partial}{\partial x} \left( AD_x \frac{\partial C}{\partial x} \right) + K_r \cdot A (C_s - C) - K_{SF} \cdot A \cdot C + q \cdot C_{in} \quad (5)$$

Where,  $A$  = sectional area of flow ( $m^2$ ),  $u$  = sectional average velocity (m/s),  $D_x$  = dispersion coefficient ( $m^2/s$ ),  $C_s$  = saturation concentration of DO (mg/l),  $K_r$  = reaeration coefficient (1/s),  $q$  = unit inflow of tributary ( $m^3/s/m$ ), and  $C_{in}$  = DO concentration of tributary inflow (mg/l).

Fe (II) and Fe (III) concentrations at any place at each time are governed by the following one dimensional diffusion equations.

$$\frac{\partial (C_{F2} \cdot A)}{\partial t} + \frac{\partial (u \cdot C_{F2} \cdot A)}{\partial x} = \frac{\partial}{\partial x} \left( AD_x \frac{\partial C_{F2}}{\partial x} \right) - K_F \cdot A \cdot C_{F2} + \left( \beta R_{F2} + (1 - \beta) \cdot 10^6 R_{W2} \right) \cdot \sigma_s \cdot B_b \cdot DE \quad (6)$$

$$\frac{\partial (C_{F3} \cdot A)}{\partial t} + \frac{\partial (u \cdot C_{F3} \cdot A)}{\partial x} = \frac{\partial}{\partial x} \left( AD_x \frac{\partial C_{F3}}{\partial x} \right) + K_F \cdot A \cdot C_{F2} + \left( \beta R_{F3} + (1 - \beta) \cdot 10^6 \cdot R_{W3} \right) \cdot \sigma_s \cdot B_b \cdot DE \quad (7)$$

Where,  $R_{F2}$ ,  $R_{F3}$  = contents of Fe (II) and Fe (III) at the riverbed surface (mg/kg),  $R_{W2}$  = ratio of concentrations of Fe (II) in water (mg/l) and suspended solid (SS) (mg/l),  $R_{W3}$  = ratio of concentrations of Fe (III) in water (mg/l) and suspended solid (SS)

concentration (mg/l),  $\beta$  = constant (0 for deposition and 1 for erosion),  $\sigma_s$  = unit weight of sand ( $=2.65 \text{ g/cm}^3$ ),  $B_b$  = width of riverbed (m) and DE = erosion or deposition velocity of sand with particle size of 0.2mm or less (m/s, + for erosion).

## (2) Equations of Fe (II) and Fe (III) contents in sediments of the reservoir

The variations of Fe (II) and Fe (III) contents are described by the reduction reaction and the transport due to the riverbed variation, however, the precipitation and release are negligible. That is

$$\frac{\partial R_{F2}}{\partial t} = K_{\text{red}} \cdot R_{F3} + \Delta R_{ZF2} \quad (8)$$

$$\frac{\partial R_{F3}}{\partial t} = -K_{\text{red}} \cdot R_{F2} + \Delta R_{ZF3} \quad (9)$$

Where,  $K_{\text{red}}$  = Fe (II) generation rate constant by reduction (1/s),  $\Delta R_{ZF2}$ ,  $\Delta R_{ZF3}$  = transport rate of Fe (II) and Fe (III) according to riverbed variation, respectively (mg/kg·s).

## (3) Equations of DO, Fe (II) and Fe (III) concentrations in the downstream river channel

The concentration of DO is governed by the following equation:

$$\begin{aligned} & \frac{\partial (C \cdot A)}{\partial t} + \frac{\partial (u \cdot C \cdot A)}{\partial x} \\ &= \frac{\partial}{\partial x} \left( AD_x \frac{\partial C}{\partial x} \right) + K_r \cdot A (C_s - C) - K_{SF} \cdot A \cdot C \\ &+ q \cdot C_{in} + \lambda_b \cdot B_b \cdot V_e (C_b - C) \end{aligned} \quad (10)$$

Where,  $\lambda_b$  = porosity of the river bed,  $V_e$  = exchange velocity between the pore water in the gravel layer and the flowing water above the gravel (m/s), and  $C_b$  = DO concentration in the pore water (mg/l).

The variations of Fe (II) and Fe (III) concentrations are represented by the decrease of Fe (II) and generation of Fe (III) due to the advection, diffusion and oxidation reaction and the exchange with the pore water in the riverbed gravel layer.

$$\begin{aligned} & \frac{\partial (C_{F2} \cdot A)}{\partial t} + \frac{\partial (u \cdot C_{F2} \cdot A)}{\partial x} \\ &= \frac{\partial}{\partial x} \left( AD_x \frac{\partial C_{F2}}{\partial x} \right) - K_F \cdot A \cdot C_{F2} + \lambda_b \cdot B_b \cdot V_e (C_{F2b} - C_{F2}) \end{aligned} \quad (11)$$

$$\begin{aligned} & \frac{\partial (C_{F3} \cdot A)}{\partial t} + \frac{\partial (u \cdot C_{F3} \cdot A)}{\partial x} \\ &= \frac{\partial}{\partial x} \left( AD_x \frac{\partial C_{F3}}{\partial x} \right) + K_F \cdot A \cdot C_{F2} + \lambda_b \cdot B_b \cdot V_e (C_{F3b} - C_{F3}) \end{aligned} \quad (12)$$

Where,  $C_{F2b}$ ,  $C_{F3b}$  = Fe (II) and Fe (III) concentrations (mg/l) in the pore water, respectively.

(4) Equations of DO, Fe (II) and Fe (III) concentrations in the pore water of gravel river bed

DO in the pore water is consumed by Fe (II) in both the pore water and the surface of the riverbed material, It can be expressed as

$$\frac{\partial C_b}{\partial t} = -\frac{V_e}{\alpha d} (C_b - C) - K_{SF} \cdot C_b \quad (13)$$

$$\frac{\partial C_{F2b}}{\partial t} = -\frac{V_e}{\alpha d} (C_{F2b} - C_F) - K_{Fb} \cdot C_{F2b} - K_F \cdot C_{F2b} \quad (14)$$

$$\frac{\partial C_{F3b}}{\partial t} = -\frac{V_e}{\alpha d} (C_{F3b} - C_{F3}) - K_{Fb} \cdot C_{F3b} + K_F \cdot C_{F2b} \quad (15)$$

$$\frac{\partial C_{F2d}}{\partial t} = K_{Fb} \cdot C_{F2b} - K_F \cdot C_{F2d} \quad (16)$$

$$\frac{\partial C_{F3d}}{\partial t} = K_{Fb} \cdot C_{F3b} + K_F \cdot C_{F2d} \quad (17)$$

Where,  $\alpha$ = coefficient,  $d$ =gravel particle size (m),  $\alpha d$ = thickness of power water layer (0.5m),  $K_{Fb}$  = adsorption rate of suspended particle to riverbed (1/s),  $C_{F2d}$ ,  $C_{F3d}$  = Fe (II) and Fe (III) concentrations on the surface of the riverbed material, respectively (mg/l), and  $K_F$  is given as follows:

$$K_F = 4 \frac{W_{Fe} \cdot C_b}{W_{O_2} (C_{F2b} + C_{F2d})} K_{SF} \quad (18)$$

#### 4. FORMULATION OF MAIN PARAMETERS

The main parameters used in this study are described below.

##### (1) Reaeration coefficient

The reaeration coefficient is determined by the diffusion coefficient of oxygen in water and various factors that affect turbulence near the surface (5). Many expressions have been proposed for modeling the reaeration coefficient. Among them, the formulae developed by Dobbins (6) and Murakami (7) are widely used (see Table1).

##### (2) Consumption rate constant of DO

The consumption rate constant of DO,  $K_{SF}$ , is determined by means of experiments. The samples were taken from the surface mud 60-80m upstream of the dam in October 1996.  $K_{SF}$  is calculated by relation of DO variation with time in a batch experiment, and the following equation is obtained by the correlation of  $K_{SF}$  with concentration of Fe (II),  $C_{F2}$ , as shown in Figure 2.

$$K_{SF} = 6.0 \times 10^{-6} \times C_{F2}^{1.5} \quad (19)$$

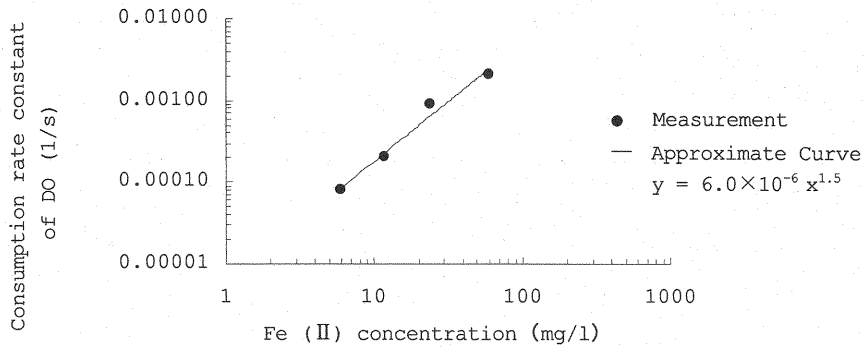


Fig. 2 Relationship of DO consumption rate constant with Fe (II) concentration

### (3) Generation rate constant of Fe (II)

Generally, the reduction reaction in the bottom sediments progresses gradually and has a vertical distribution. Therefore, the generation situations of Fe (II) are different in each layer of the bottom sediments with different chemical properties. However, the model would be extremely complex if all the quantitative variations of influential factors were pursued one by one. In addition, the vertical distribution of ORP was not clear from the investigation results of the bottom mud at Dashidaira dam in 1997. Therefore, vertical distribution of the anaerobic property in the bottom sediment is not considered in the model. The generation rate constant of Fe (II) is assumed to be a function of mud temperature alone, and can be as follows (Arrhenius Formula)

$$K_{\text{red}} = A_a e^{-E_a/RT} \quad (20)$$

Where,  $A_a$  = constant (1/s),  $E_a$  = activation energy (J/mol),  $R$  = gas constant (8.3145 J/mol/K), and  $T$  = absolute temperature (K).  $E_a = 1.16 \times 10^5$  (J/mol) and  $A_a = 1.42 \times 10^{17}$  (1/s) are obtained by the experiment on anaerobic property of the bottom sediment by using the surface mud which was taken from 60-80m upstream of the dam in 1997.

### (4) Solution of variations of Fe (II) and Fe (III) during riverbed evolution

Former measurements of the bottom sediments in the Dashidaira reservoir show a clear correlation between the iron content and the particle diameter. The contents of Fe (II) and Fe (III) increase as particle size of the sediments decreases. Therefore, it is necessary to evaluate the size-dependent variations of Fe (II) and Fe (III) in the sediments. As the grain size varies temporally and vertically according to erosion or deposition events, the variations of Fe (II) and Fe (III) should be calculated by dividing the layer in the same manner as used for particle size distribution analysis in the dynamic model of sediment(8) (see Figure 3). The variations of Fe (II) and Fe (III) during riverbed variation can be calculated by the following equations:

For erosion:

$$\sum_k E_{\text{Fe}2k} = \sum_k (R_{\text{S}2k} \times \sigma_s \times E_{\text{S}k}) \quad (21)$$

Where,  $E_{\text{Fe}2k}$  = flux of Fe (II) from riverbed to water during the erosion of the particle size of  $k$  (g/m<sup>2</sup>/s),  $R_{\text{S}2k}$  = content rate of Fe (II) for particle size of  $k$  in the riverbed



a) Calculation conditions for test sediment flushing

Model calculations for test sediment flushing in 1994 were performed to determine the exchange velocity in the downstream river channel. The calculation conditions shown in Table 1. The flushing was carried out under free flow after decrease of the water level, while the inflow of  $40\text{m}^3/\text{s}$  from the Kurobe dam in the upstream was kept constant. The free flow condition was maintained for an hour from 8:00 to 9:00 on February 28, as shown in Fig. 4. The content of Fe (II) in the initial riverbed was assumed by the correlation with 50% particle size for the sediment of the particle size of 0.2mm or less (See Figure 5). The samples were taken between the two flushings in 1996 and 1997. In order to evaluate the sensitivity of the reaeration coefficient, 5 cases shown in Table 1 were performed. The values of  $K_r$  in the Cases1-3 were in the range of the representative values for the rapid stream river (1.0-10.0/day) (10).

Table 1 Calculation conditions for test sediment flushing

Item		Calculation Conditions
Period		7:00 on Feb. 27~24:00 on Feb. 28, 1994
Reaeration coefficient		CASE1 : $K_r=1.0\text{day}^{-1}$ CASE2 : $K_r=5.0\text{day}^{-1}$ CASE3 : $K_r=10.0\text{day}^{-1}$ CASE4 : Dobbins Formula CASE5 : Murakami Formula
Discharge		Actual discharge during above-mentioned period
Downstream water level of downstream river channel		Average tidal level
Initial condition of riverbed	Elevation of riverbed	Configuration in February 1994 (estimated from simulation results for the reservoir based on the riverbed dynamic model(4))
	Grain size Distribution	Grain size estimated for February 1994
	Content of Fe(II)	the correlation with 50% particle size(Figure 4)

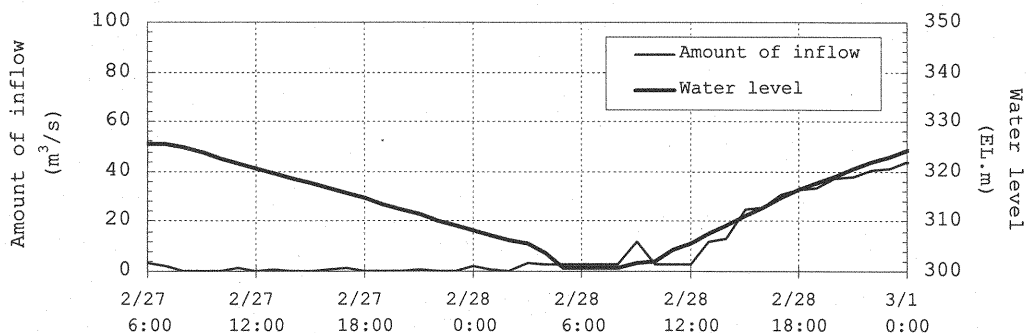
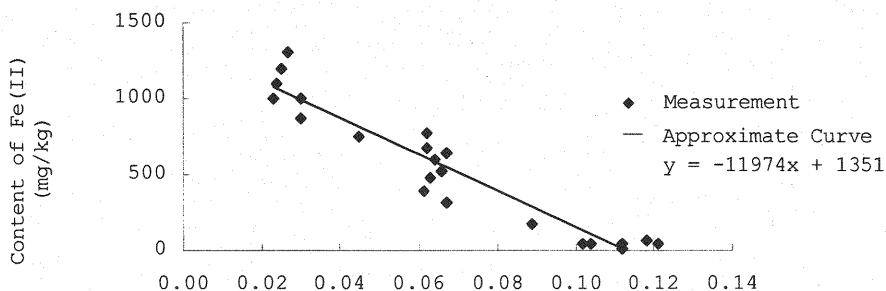


Fig.4 Actual measurements of water level and inflow amount at the Dashidaira dam in February 1994





50% particle size for the sediment of the particle size  $\leq 0.2\text{mm}$  (mm)

Fig. 5 Relationship of Fe(II) content with 50% particle size

### b) Calculation results

Figures 6 and 7 show the measurement data and the calculated results of the longitudinal variation of DO in the downstream river channel and variations at two points. The exchange velocity and adsorption rate are determined as given in Equation (24).

$$\frac{V_{ex}}{11\pi} = 4.33 \times 10^{-5}; \quad K_{Fb} = 0.005 \text{ (1/s)} \quad (24)$$

Profiles of DO at 12:00 in Figure 6 reveal that DO begins to decrease in the reservoir, reaches the lowest at 7km-10km downstream of the dam, and recovers gradually afterwards. DO values in the vicinities of 2.5km and 5km downstream of the dam increase rapidly because of the larger inflows from the tributaries at the two points, as compared to small discharge from the dam during flushing. It can be found from Figure 7 that calculations reproduced well the timings and magnitudes of minimum DO at different points. Although the reaeration coefficient has little effects on the consumption and recovery processes of DO, Case 3 is most suitable, if the minimum DO is considered. Complete depletion of DO observed in the vicinity of Point A is thought to have been caused by a relatively small flushing discharge and a long term sedimentation (three years had passed from the last flushing). After the test flushing of 1994, such phenomenon has not occurred, because of a shorter deposition period and an increase in flushing discharge (flushing was performed during flood period).

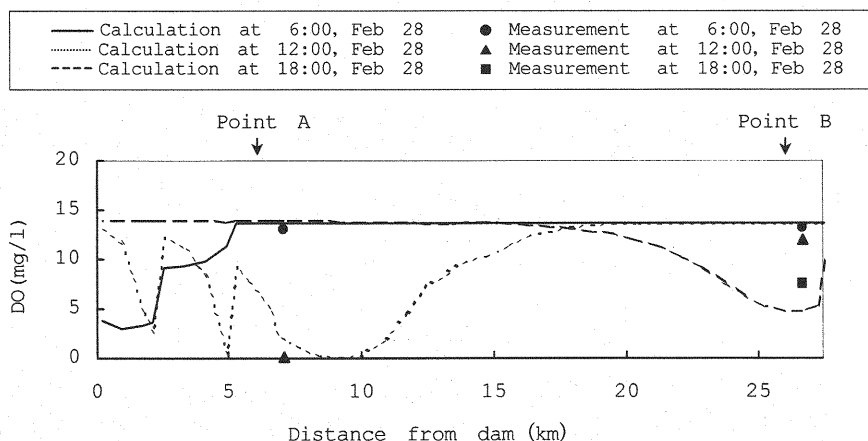


Fig. 6 Longitudinal variation of DO in the downstream river channel (CASE3)

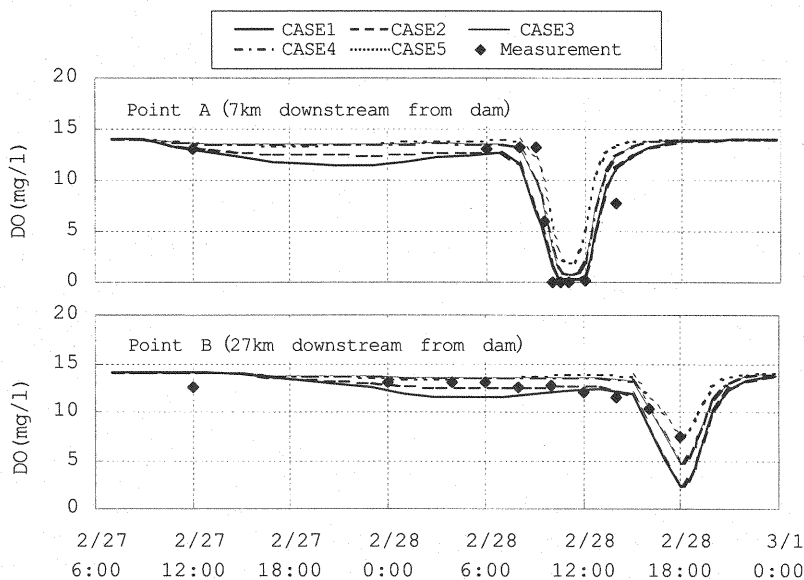


Fig. 7 Time series variation of DO in the downstream river channel (measurements and calculated results of test flushing)

## (2) Simulation of sediment flushing in September 1999

### a) Calculation conditions

The model was validated by means of simulation with calibrated parameters. The calculation conditions for the flushing in September 1999 are listed in Table 2. The flushing conditions such as discharges are different from those of the test sediment flushing because the present method of flushing was revised to minimize impacts on the environment. Flushing was performed under natural inflow conditions at the declining phase of the flood (see Table3).

Table 2 Calculation conditions for flushing in September 1999

Item		Calculation conditions
Period		1:00, Sep 14, 1999 ~ 24:00, Sep18, 1999
Particle size		CASE A: 50%particle size 0.09mm CASE B: 50%particle size 0.10mm
Discharge		Actual discharge during flushing
Downstream water level of downstream river channel		Average tidal level
Initial condition of riverbed	Elevation of riverbed	Configuration in September 1999 (estimated from simulation results for the reservoir based on the riverbed dynamic model(4))
	Grain size distribution	Grain size estimated for September 1999
	Content of Fe(II)	Correlation with 50% particle size (Figure 4)

Table 3 Comparison of flushing methods

	Flushing season	Flushing discharge
Test flushing	Late February	Controlled discharge of 40m <sup>3</sup> /s from the Kurobe dam
Present flushing	June~ August During flood event	Declining phase of natural flooding

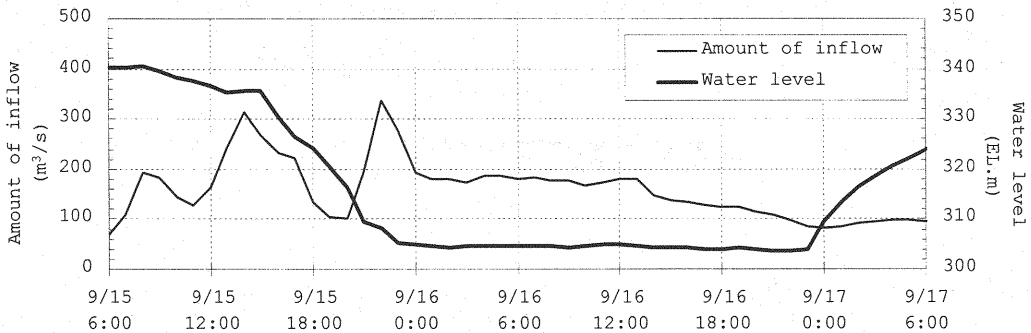


Fig.8 Actual measurements of water level and inflow amount at the Dashidaira dam in September 1999

The free flow condition was reached between 21:00 on Sep.15 and 21:00 on Sep.16, 1999, as shown in Fig.8. Initial conditions of the elevation and grain size distribution of the riverbed were estimated from the results of the previous calculations both for variation of the riverbed configuration in the reservoir and for the suspended sediment of outflow from the dam during flushing. According to a separate investigation of the bottom sediment, 50% particle size of sediment of 0.2mm or less was set as 0.09mm for Case A and 0.10mm for Case B, respectively.

#### b) Calculation results

Simulated results of the longitudinal variation of DO in the downstream river channel and time variation of DO at the two points are shown in Figures 9 and 10, together with the measurement data. The calculated values of the minimum DO for Case A are slightly lower than the measurements. The relationship between 50% particle size and the maximum oxygen deficit is shown in Figure 11.

The above findings provide evidence that this model can reproduce sediment flushing for the cases with different discharge and sediment accumulation period. This model is appropriate for the reproduction of decline and recovery of DO in the downstream river channels during flushing, although the determination and adjustments of some parameters need to be improved.

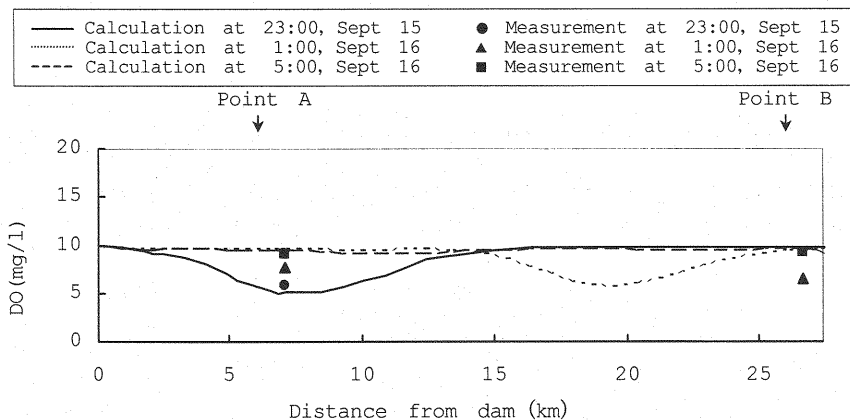


Fig. 9 Longitudinal variation of DO in the downstream river channel (CASE A)

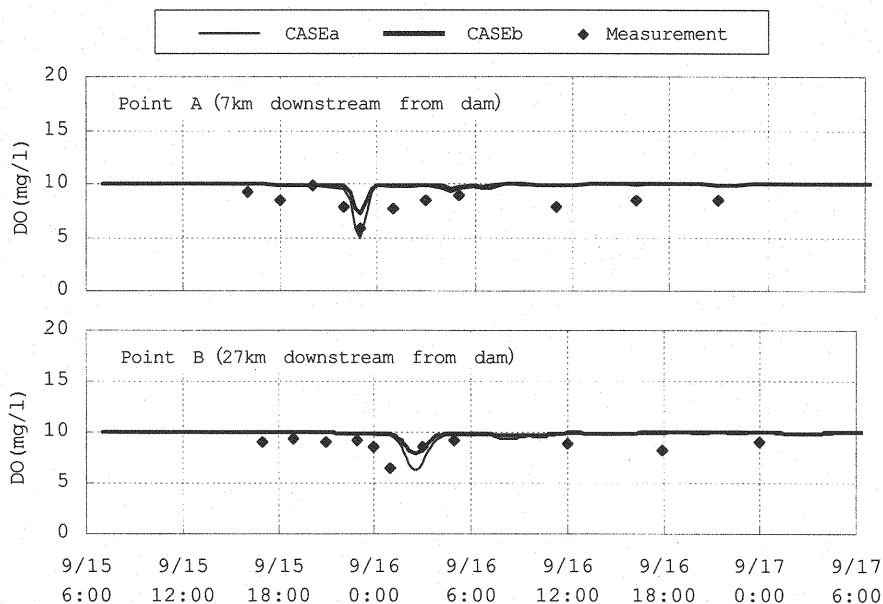


Fig. 10 Time series variation of DO in the downstream river channel (measurements and calculated results of flushing in September 1999)

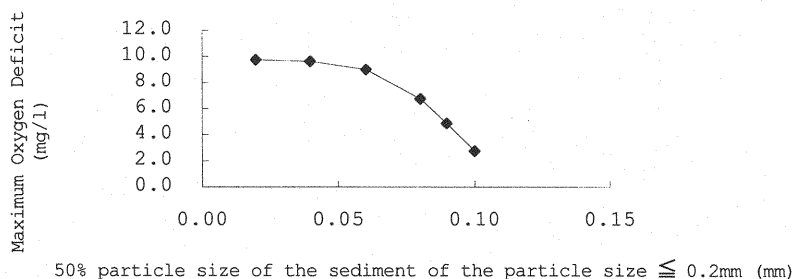


Fig. 11 Relationship of maximum decrease of DO with 50% particle size

## 6. CONCLUSIONS

The present model predicts well the decrease and recovery of DO for different flushing methods. Therefore, the model provides an effective tool for assessing the environmental impact of flushing on the downstream. However, further research is necessary to expand the applicability of the present model.

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## APPENDIX - NOTATION

The following symbols are used in this paper:

$d$	= gravel particle size (m);
$q$	= unit inflow of tributary ( $\text{m}^3/\text{s}/\text{m}$ );
$u$	= sectional average velocity (m/s);
$u_*$	= Friction velocity;
$A$	= sectional area of flow ( $\text{m}^2$ );
$A_a$	= constant (1/s);
$B_b$	= width of riverbed (m) and $DE$ = erosion or deposition velocity of sand with particle size of 0.2mm or less (m/s, + for erosion);
$C$	= DO concentration (mg/l);
$C_b$	= DO concentration in the pore water (mg/l);
$C_{Fe}$	= concentration of Fe (II) (mg/l);
$C_{Fe2b}, C_{Fe3b}$	= Fe (II) and Fe (III) concentrations (mg/l) in the pore water, respectively;
$C_{Fe2d}, C_{Fe3d}$	= Fe (II) and Fe (III) concentrations on the surface of the riverbed material, respectively (mg/l);
$C_{in}$	= DO concentration of tributary inflow (mg/l);
$C_s$	= saturation concentration of DO (mg/l);
$D_{Fe2k}$	= flux of Fe (II) during deposition of particle size of $k$ ( $\text{g}/\text{m}^2/\text{s}$ );
$D_{sk}$	= deposition velocity of particle size of $k$ (m/s);
$D_x$	= dispersion coefficient ( $\text{m}^2/\text{s}$ );
$E_a$	= activation energy (J/mol);
$E_{Fe2k}$	= flux of Fe (II) from riverbed to water during erosion of particle size of $k$ ( $\text{g}/\text{m}^2/\text{s}$ );
$E_{sk}$	= erosive velocity of particle size of $k$ (m/s);
$K_F$	= coefficient of oxidation reaction rate of Fe (II) (1/s);
$K_{fb}$	= adsorption rate of suspended particle to riverbed (1/s);
$K_r$	= reaeration coefficient (1/s);
$K_{red}$	= Fe (II) generation rate constant by reduction (1/s);
$K_{SF}$	= consumption rate constant of DO (1/s);
$R$	= gas constant (8.3145 J/mol/K);
$R_{Fe2}, R_{Fe3}$	= contents of Fe (II) and Fe (III) in riverbed surface (mg/kg);
$R_{S2k}$	= content rate of Fe (II) for particle size of $k$ in the riverbed surface (exchange layer) (mg/kg);
$R_{w2}, R_{w3}$	= ratio of concentrations of Fe (II) and Fe (III) in water (mg/l) and suspended solid (SS), respectively (mg/l);
$T$	= absolute temperature (K);
$V_e$	= exchange velocity between the pore water in the gravel layer and the flowing water above the gravel (m/s);
$V_{ex}$	= Exchange velocity;
$W_{Fe}$	= atomic weight of iron (55.8);
$W_{O2}$	= molecular weight of oxygen (32);
$\alpha$	= coefficient;
$\beta$	= constant (0 for deposition and 1 for erosion);
$\lambda_b$	= porosity of river bed;
$\sigma_s$	= unit weight of sand ( $\text{g}/\text{cm}^3$ ); and
$\Delta R_{2F2}, \Delta R_{3F3}$	= transport rate of Fe (II) and Fe (III) according to riverbed variation, respectively ( $\text{mg}/\text{kg}\cdot\text{s}$ ).