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NUMERICAL CALCULATION OF BED DEFORMATION IN MULTIPLE AND BRAIDED BAR STREAM

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SYNOPSIS

Two types of numerical models are tested to predict the flow and bed deformation of a braided bar channel. The first model is the classical two-dimensional quasi-steady flow model with upwind differential numerical scheme. The second model is a two-dimensional unsteady flow model with a CIP numerical scheme. The results of the model tests compared favorably with the movable bed experiments. The models were tested in a very wide channel to predict the development of multiple bars or braided bed configuration. These tests revealed that only the second model with a CIP scheme performed well. Furthermore, findings indicate that the second model can be a useful tool to predict the formation of a braided stream in alluvial rivers.

INTRODUCTION

Predicting riverbed configuration and channel geometry are an important matters of river engineering. This branche of engineering aims to improve river environments and ecosystems. A quantitative prediction is more important in respect to the meso-scale river form. Research on the meso-scale bed forms in alluvial rivers started with the pioneering works of Kinoshita[1], followed by many experimental, theoretical analyse and numerical studies. Most of the early theoretical research was based on the linear stability analysis (Kuroki and Kishi[2], Blondeaux and Seminara[3]), and regime criteria of meso-scale bars have been proposed. A finite amplitude form of alternate bars in a straight channel and a mutual relationship between channel geometry and characteristics of free and forced bars were investigated by non-linear analysis (Shimizu et al.[4]).

Most of these studies are related to single-low bars. The formation of meandering streams was studied by Michiue et al.[5], however, they did not investigate the formation of braided channels. In fact, the research on multi-low bars is still of a developing stage. Multi-low bars are often observed in rapid stream rivers on alluvial fans. They are abundant when active sediment transportation is entailed. Also, braided rivers can often be found in developing countries where river improvement works are not in progress (Enggrob and Tjerry[6]). Therefore, it is important to study braided streams from the viewpoint of development in these countries. Fig. 1 shows a satellite photograph of the Mekong River on the borders of Thailand and Cambodia. The goal of this research is to develop a model to analyze the flow and bed deformation in braided stream quantitatively.

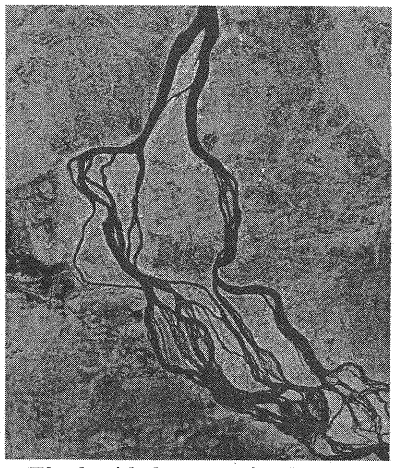

Fig. 1 The braided stream in the Mekong River (courtesy of Dr.N.Izumi - A.I.T.)

In these models, the most typical methods of flow calculation are coupled with two-dimensional shallow water equations or quasi-three-dimensional flow equations that contain the velocity distribution in the water depth direction[4],[7]. Some of them assume quasi-steady state and others calculate fully the unsteady flow state, but all use the continuity equation of bed material to calculate unsteady bed elevation changes. Various combinations of numerical schemes and coordinate systems are proposed.

In this study, the performances of two types of numerical models are compared; (I) The classical two-dimensional quasi-steady flow model with upwind differential numerical scheme[8] and (II) the two-dimensional unsteady flow model with CIP numerical scheme[9],[10]. The validity of these numerical models has already been investigated in curved or straight channels with single-low bars. However, they have not been tested under conditions where multiple bars or braided bed configuration are dominant. The results of these models test compared favorably with the movable bed experiments of multiple bars conducted by Kinoshita[1]. Moreover, the models were tested to predict the development of a braided bed configuration in a very wide channel.

MODEL EQUATIONS AND NUMERICAL METHOD

The outlines of two models are explained as follows.

· Model(I) : The quasi-steady flow model with 1st order upwind differential numerical scheme

The classical method composed of the 2-D depth averaged momentum and continuity equations is firstly tested as a numerical model. The algebraic equations of continuity and momentum equations are calculated with the SIMPLE method, in which the convection terms are calculated by a first order upwind differential scheme. Flow velocity and depth are calculated iteratively until a steady state solution is obtained. The bed load transport rate is evaluated from the local bed shear stress obtained from the flow quantities in the steady state solution, and the temporal changes in the bed elevation are calculated based on the continuity equation of bed material. The discretized continuity equation of the bed material is calculated by means of a first order upwind differential scheme in a downstream direction, and by means of a central differential scheme in a transverse direction. Details of this scheme are explained by Shimizu[8].

· Model(II) : The unsteady flow model with CIP numerical scheme

The two-dimensional, unsteady, incompressible flow field is calculated by using continuity and momentum equations. To solve momentum equations, they are separated into two phases of advection and non-advection. The CIP method is adopted for the advection phase, while the SOR method is used for the non-advection phase. The CIP method is originally proposed by Yabe et al.[9] and modified for the calculation of open channel flow by Nakayama et al.[10]. Time dependent change of bed elevation is computed by using central finite differences. This is also explained in detail by Shimizu et al.[11].

In the two models mentioned above, they have the following equations in common.

(a) Bed shear stress in momentum equations is expressed by Manning's equation. The coefficient of kinematic viscosity is assumed to the function of local water depth which is given by the following equation.

$$\nu_t = \frac{\kappa}{6} u_* h \tag{1}$$

in which h is water depth, κ is the von Karman constant and u_* is shear velocity, calculated by

$$u_*^2 = \frac{gn^2(u^2+v^2)}{h^{1/3}} \tag{2}$$

in which u and v are depth averaged velocity components in downstream and transverse directions, g is acceleration of gravity, and n is Manning's roughness coefficient.

(b) Bed load sediment transport rates per unit width in downstream and transverse directions are calculated by Meyer Peter-Müller's formula and Hasegawa's formula[12], respectively.

$$\frac{q_{bx}}{\sqrt{sgd^3}} = 8(\tau_* - \tau_{*c})^{3/2} \tag{3}$$

$$q_{by} = q_{bx}\left(\frac{v}{u} - N_*\frac{h}{r_*} - \sqrt{\frac{\tau_{*c}}{\nu_s \nu_k \tau_*}}\frac{\partial \eta}{\partial y}\right) \tag{4}$$

in which s and d are submerged specific gravity and representative diameter of bed material, respectively, τ_* is non-dimensional bed shear stress[$=u_*^2/(sgd)$], and τ_{*c} is non-dimensional critical shear stress calculated by Iwagaki's formula. ν_s and ν_k are static and kinetic friction coefficient of non-cohesive sediment particles, respectively. η is bed elevation. The second term on the right hand side of Eq. 4 acts as additional transverse sediment load when the secondary flows develop. The streamline is curved because of the development of a three-dimensional bed configuration. A constant value of 7.0 is used for N_* according to Engelund[13], and r_* is expressed by the radius of curvature of a streamline, and calculated by the following equation proposed by Shimizu and Itakura[14].

$$\frac{1}{r_*} = \frac{1}{(u^2+v^2)^{3/2}}\left\{u\left(u\frac{\partial v}{\partial x} - v\frac{\partial u}{\partial x}\right) + v\left(u\frac{\partial v}{\partial y} - v\frac{\partial u}{\partial y}\right)\right\} \tag{5}$$

in which x and y are coordinates in downstream and transverse direction, respectively.

CALCULATION CONDITIONS

The two models are tested under identical conditions in which the flow and bed deformation are classified into multiple bars condition. Calculations are conducted under the same conditions as in Kinoshita's experiment[1]. In the experiment, a platform with a movable slope of 10m in length and 1.2m in width is used. The channel slope is 1/53.3, mean water depth is 1.79cm. Discharge of 15.4l/sec is chosen. The channel bed is covered with uniform grain size sand of 2.33mm diameter. Fig. 2 shows the results of Kinoshita's[1] experiment. Scale-shaped bars, that is, multiple bars with half-wavelengths of about 3 m are formed. In this picture, however the wave-height is not clear, and it is estimated to be about the scale of the water depth.

Though the channel length was set to be 10 m, to assume a longer calculation section, the uniform condition was used for the velocity, depth, sediment transport and bed elevation equally at the upper and lower reaches. Manning's roughness coefficient for the moveable channel bed was determined based on a formula proposed by Kishi and Kuroki[15] for flat beds. Numbers of computational grids in the downstream and transverse directions were 61 and 21, respectively. After making provisional computations, the time steps for computation of bed

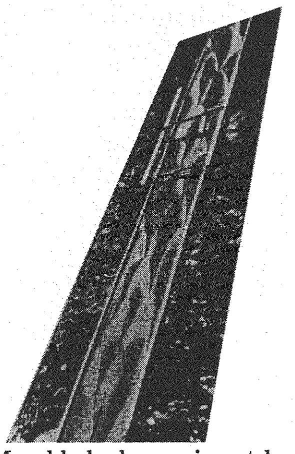

Fig. 2 Movable bed experiment by Kinoshita[1]

deformations were set to be 60 seconds in Model(I), and 0.02 seconds in Model(II). Initial bed form was set to be flat with a small disturbance of 1/10 water depth near sidewall. Flow and bed elevation were calculated for 3 hours in both Models.

CALCULATED RESULTS

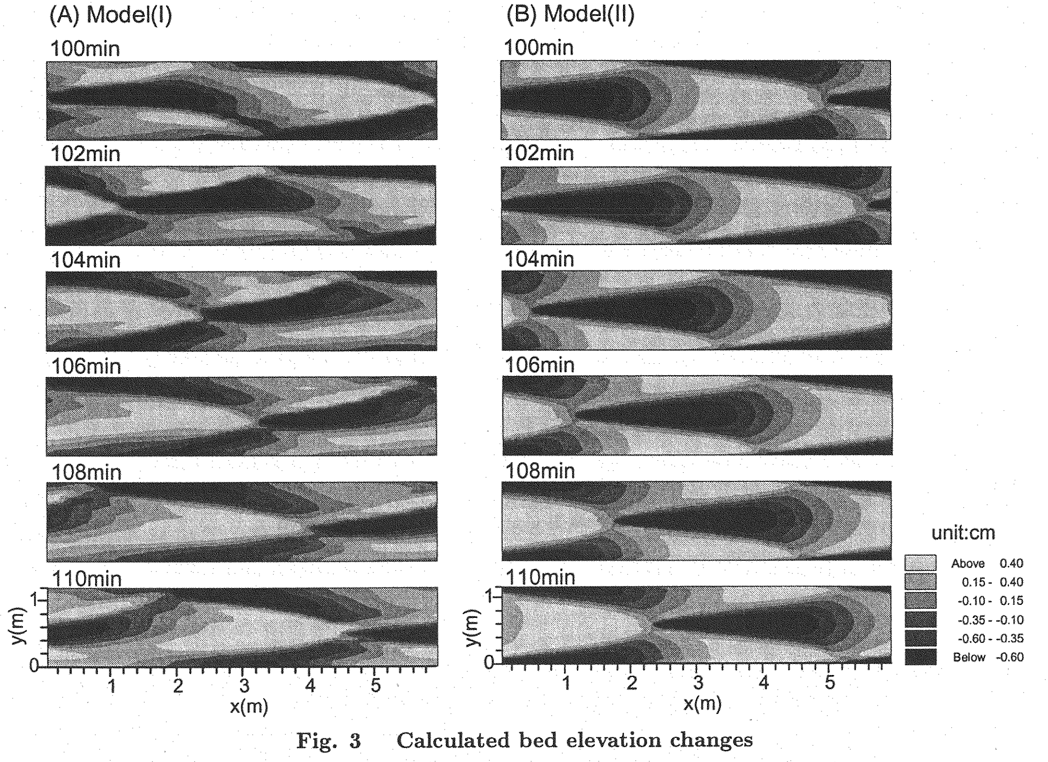

Fig. 3 Calculated bed elevation changes

In each method, the initial disturbance disappeared, and a detailed oblique stripe-shaped pattern appeared. Multi-low bars appeared after several minutes. The migration of multiple bars was observed in the downstream direction with a constant velocity. Calculated results are compared with each other after 100 minutes when the equilibrium state could be obtained. (A) and (B) of Fig. 3 show the bed elevation contours calculated by Model(I) and Model(II), respectively. Values show the deviation from initial bed (unit:cm). The white part is higher and the black part is lower. Each result shows multiple bars in which both parts near the sidewall were scoured when the center part of channel was deposited. Each calculated shape and wavelength of sandbar correspond to the result of the experiment by Kinoshita[1] well. Calculated bar height was about 1.0cm, which also agreed with the experiment. Fig. 3 shows that shapes of the sandbar form are not clearly produced by Model(I), but they are relatively clear and almost symmetric in the results of Model(II). As described in the previous section, the calculation of Model(II) is performed iteratively by the relaxation method until the water flows become steady in each time step. In Model(I), the iteration times are short in the early time stage, but when the channel bed becomes complex, the times for iteration becomes longer. Finally it becomes impossible to obtain a steady state solution (continuity equation at all calculation points is not satisfied). Therefore, the total computation time is relatively long in Model(II). On the other hand, as the unsteady equation is used in Model(II), it is possible to calculate the flow and bed deformation continuously by means of constant time steps. When the computations are advanced with a fixed time step, the total computational time in Model(I) is shorter than that in Model(I) as a result. Although it is not easy to make a detailed comparison between calculated and experimental data, Method(II) seems be able to obtain smooth bed forms. This may be due to the fact that the precision of flow calculated by Model(II) was better than that calculated by Model(I).

CALCULATIONS OF BRAIDED STREAM

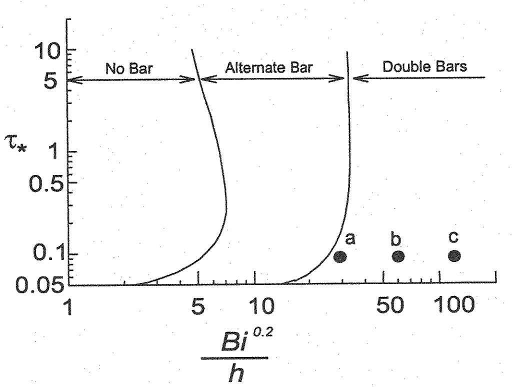

Fig. 4 Hydraulic conditions of calculation on Regime criteria of meso-scale bar

The above calculation condition is equivalent to the condition of the point (a) of a diagram proposed by Kuroki and Kishi[2] in Fig. 4. In Fig. 4, τ_* is non-dimensional bed shear stress, B is channel width, i is channel slope and h is mean water depth. Point (a) is plotted in a double bars regime, however, it is very close to the boundary between the alternate bar and the double bars regime. In this section, the model is applied to a very wide channel to predict the development of the multiple bar or the braided bed configuration. The second model with CIP scheme that performed well in the previous calculation was adopted. The same mean water depth, channel slope and bed material are used. Only channel width is expanded to double and four times (Discharges are increased to double and four times, as well). The numbers of computational grids in the downstream and transverse directions increased to double and four times. These conditions are equivalent to those of the point (b) and (c) in the Fig. 4, respectively.

Fig. 5 shows the calculated bed elevation contours from 100 to 108 minutes in the calculation when channel width is expanded to double. Fig. 6 shows the calculated bed elevation contours and velocity vectors from 100 to 110 minutes in the calculation of four times channel width. These figures reveak that the mode of bars increases in proportion with the increase of the channel width. And velocity vectors in Fig. 6 express the

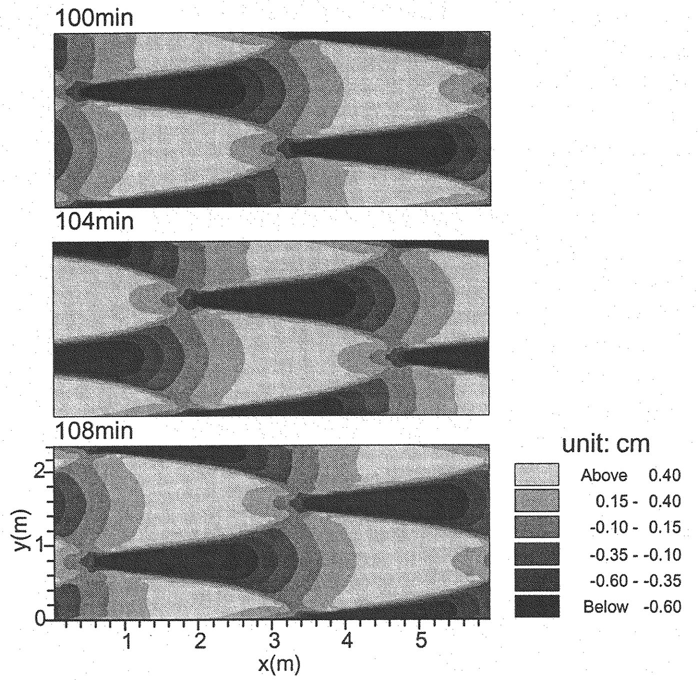

Fig. 5 Calculated bed elevation changes when channel width is expanded to double

dispersion of flow at the top of the bars and the concentration of flow at the bottom side of the bars.

To observe the response of bed form against changes in discharge (water depth), calculation was continued with half discharge of initial one after 100 minutes in Fig. 6 as the initial bed condition. Result shown in Fig. 7, indicate that bed form reacted immediately and is changed into braided bed configuration.

The same calculation was made with the model(I), however, a stable solution could not be obtained.

CONCLUSIONS

In this study, two types of numerical models were tested to predict flow and bed deformation of a multiple bars channel or a braided bar channel. Findungs reveal that the Model(II) proposed in this study can be a very useful tool to predict the formation of a multiple bar stream and a braided stream. This study also indicates that the Model(II) can predict phenomenon such as the bed form reaction to the change of discharge. However, the results presented in this study are only qualitative ones. Therefore, comparisons with the experimental data are required in a further study. We expect that this model willbe able to predict the real river-scale channel deformation by coupling with the bank erosion model (Nagata et al.[16], Shimizu et al.[17]).

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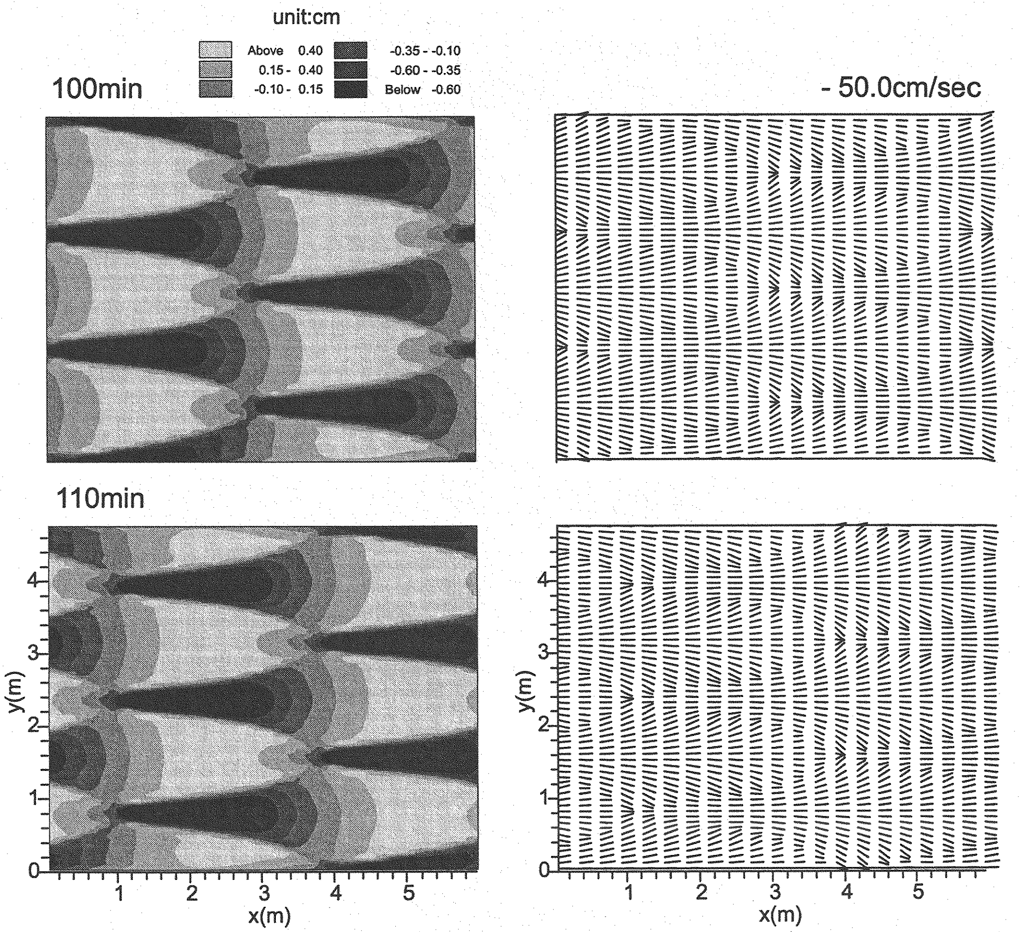

Fig. 6 Calculated flow and bed elevation changes when channel width is expanded to four times

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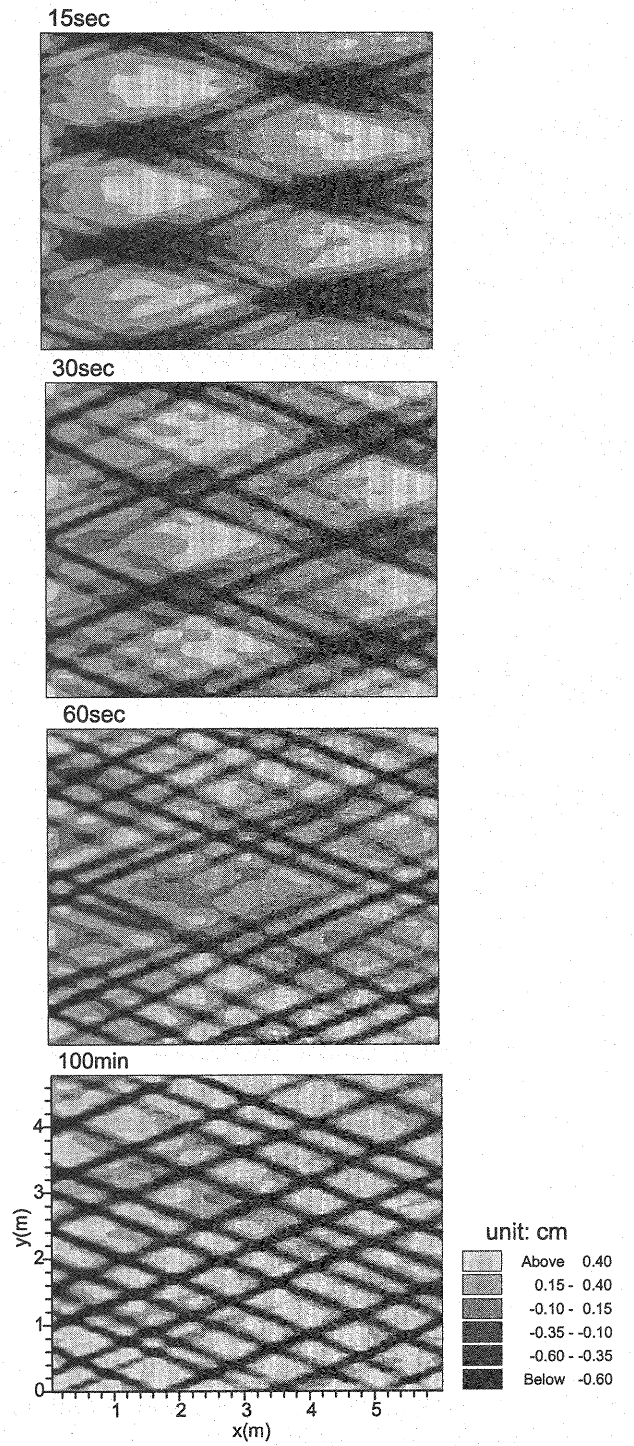

Fig. 7 Calculated bed elevation changes when water depth (discharge) was decreased to half

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APPENDIX – NOTATION

The following symbols are used in this paper:

ν_t	=	kinematic viscosity;
κ	=	von Karman constant;
u_*	=	shear velocity;
h	=	water depth;
g	=	acceleration due to gravity;
n	=	Manning's roughness coefficient;
u, v	=	velocity components in x and y direction;
q_{bx}, q_{by}	=	bed load sediment transport rate per unit width in x and y direction;
s	=	specific gravity of bed material;
d	=	diameter of bed material;
τ_*	=	non-dimensional bed shear stress;
τ_{*c}	=	non-dimensional critical bed shear stress;
N_*	=	constant according to Engelund;
r_*	=	radius of curvature of a streamline;
ν_s, ν_t	=	static and kinetic friction coefficient of non-cohesive sediment particles;
η	=	bed elevation; and
x, y	=	downstream and transverse direction

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