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IMPACTS OF BAROMETRIC CHANGES ON A PUMPING TEST

By

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SYNOPSIS

In this study during the pumping tests, the impact of barometric changes on pressure fluctuation in wells at confined aquifers was analyzed with stochastic approaches. We derive the analytical solution of the dimensionless power spectrum of the head fluctuation at the pumping well under the barometric changes. After examining the analytical solution, the head fluctuation was found to decrease with an increase in the dimensionless frequency (α) of the barometric changes. Finally, we analyze the procedures to identify the hydraulic parameters of a confined aquifer with the pumping test under the influence of the barometric changes.

INTRODUCTION

Over the past thirty-years, many articles concerning groundwater research (2, 3, 4, 11) have focused on the fluctuation behavior of the hydraulic head caused by various factors in a confined aquifer. In general, the primary factors that caused the fluctuation of the head were the adjacent aquifers of nearby rivers, the ebb and rise of tides, earthquakes, and barometric changes. In the inland area, this phenomenon was far more significant where the aquifer was affected by barometric changes. The changes of the hydraulic head in wells that reach the aquifers, particularly, were closely related to barometric changes. This phenomenon could be easily seen when we checked the hydraulic-head records in wells after pumping test. In addition, there were still other factors that controlled the fluctuations and characteristics of the head, such as the radius of the well, the depth of the aquifer, the storage coefficient, the transmissivity, the hydraulic conductivity of saturated and unsaturated aquifers, etc. Hence, the above-mentioned factors must be taken into consideration when the fluctuation of the hydraulic head is discussed.

In the past, the study of groundwater was mostly carried out by the pumping test and using the Theis method to solve the hydraulic parameters. Nevertheless, this method requires a lot of maneuvering of a curve line to fit available graphic data, was too complicated and tended to be very subjective. In 1940, Jacob (8) published an article dealing with the fluctuation of the hydraulic head in wells influenced by barometric changes. Jacob (8) assumed that the aquifer was perfectly confined when its hydraulic conductivity was high and the radius of the well was negligible. As a result, the changes of the head fluctuation in the well were affected by the barometric changes, which were equaled to the changes due to the distortion of a confined aquifer without drainage. Under the conditions mentioned above, the relationship between the barometric pressure and the hydraulic head could be shown in a simple-linear parameter, otherwise known as the barometric efficiency. Thenceforth, many researchers, such as Peck (9), Turk (13), and Weeks (14), who had different points of view and approaches, started to study the fluctuation behavior of the hydraulic head of wells in aquifers.

Weeks (14) conducted research on the change of the hydraulic head caused by the barometric pressure. He assumed that the changes of the head were equaled to that of the mean pressure in a saturated aquifer. In this case, the value of the lateral hydraulic conductivity was high. The result demonstrated that the hydraulic conductivity of the unsaturated zone had a considerable influence on the fluctuation of the head of a saturated aquifer.

Cooper et al. (3) carried out research on the fluctuation of the hydraulic head and noticed two occurrences that influenced the fluctuations of groundwater. Whenever the aquifer increased in size and/or the water entered the aquifer vertically, the groundwater fluctuated. Bredehoeft (2) and Hsieh et al. (7) incorporated this idea into their research soon after the publication of this finding. They concluded that the ebbs and flows of the water caused the fluctuation of the hydraulic head in an aquifer, therefore, changing the size of the aquifer. Moreover, the difference of the amplitude of the ebbs and flows was thought to result in different phases of the head fluctuation. Therefore, the hydraulic parameters could be obtained from the data observed and from harmonic analysis.

In 1974, Gelhar (4) published a work on the stochastic approaches, which splits the hydraulic head into the mean head and head perturbation by using the small perturbation method. Then he transformed the perturbation into the frequency domain by using the method of the Fourier-Stieltjes Transform. Afterwards, with the spectral theory, a spectrum of the head fluctuation was determined. Finally, given the spectrum of the hydraulic head, he investigated the spectrum in one-dimensional linear Dupuit aquifers and two-dimensional Laplace aquifers. The result showed that the propagation of the hydraulic head had increased with higher frequencies and had decreased with greater dimensionless distances.

Nevertheless, not all aquifers are completely confined, and the value of hydraulic conductivity varies with locations. In addition, the radius of a well also affects the fluctuation of the hydraulic head in a well. The fluctuation effect of the hydraulic head, therefore, is not entirely induced by the barometric pressure.

In 1988, Rojstaczer (11) made a study of the fluctuation effect of the hydraulic head by the barometric pressure within an unconfined aquifer. Rojstaczer (11) surmised that the air current in the unsaturated zone and the water current in the unconfined aquifer would influence the hydraulic behavior of groundwater. When the barometric pressure changed very slowly, the barometric variation would cause the fluctuation of the hydraulic head in the well by traveling through the air current in the unsaturated zone first, then traveling through the water current in the unconfined aquifer and, finally, entering the well. In this case, it was found that the aquifer had partial drainage. On the other hand, when the barometric pressure changed dramatically and fluctuated the hydraulic head, it was found that the aquifer had no drainage. Furthermore, when the radius of the well was wide or the lateral hydraulic conductivity was low, the effect of the barometric pressure on the hydraulic head would be negligible.

Later, Rojstaczer (11) conducted research on the relationships between the hydraulic head fluctuation in the aquifer and the elasticity of an aquifer and the characteristics of fluid in an aquifer under barometric pressure. His methods on the research divided the fluctuation mechanism of the hydraulic head into two parts, the barometric efficiency and phase shift. He used a solution in terms of the complex function for the head fluctuation behavior and investigated how these two mechanisms related to the frequency domain due to the impact that the frequency had on them. The result showed that the head fluctuation behavior could be divided into three levels of frequency: low, medium and high. Among these three levels, the behavior of the head fluctuation in the low frequency was related to the diffusivity of the confined aquifer and the unsaturated zone, except for the permeability of the aquifer. The behavior of the head fluctuation in the medium frequency was related to the elasticity of the aquifer, but not to its fluidity. As for the high frequency, there was a close relationship with the permeability of the aquifer, but not with the diffusivity of the confined aquifer and the unsaturated zone.

Since Rojstaczer (11) investigated the head fluctuation behavior by using the complex function, which is a rather complex concept, we found that it was necessary to do the work again by using the stochastic approaches. The aim of this study is to analyze the behavior of the head fluctuation of the aquifer under barometric changes. We will perform the spectrum of the head fluctuation induced by the barometric changes. In this way, the behavior of the head fluctuation can be understood more clearly by examining the head spectrum.

DERIVATION OF THEORY

The purpose of this paper is to analyze the groundwater-flow problems in a two-dimensional infinite confined aquifer (Fig. 1) and to study the hydraulic behavior of the head pressure fluctuation in a well affected by barometric changes.

Two-dimensional Aquifer

When a well in the confined aquifer is drained steadily by a fixed amount of volume, the affected scope will enlarge with time. When the groundwater is pumped out of the aquifer, the groundwater pressure will decrease and result in an unsteady flow. The unsteady flow equation is used for a two-dimensional aquifer (12). The equation is as follows:

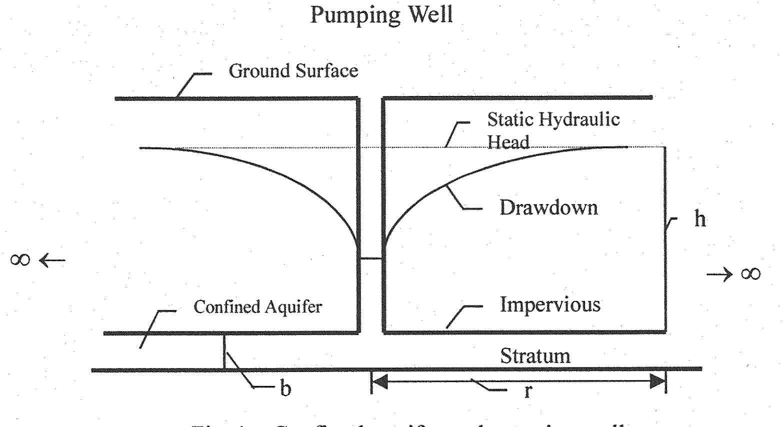

Fig. 1 Confined aquifer and pumping well

$$\frac{\partial^2 h}{\partial r^2}+\frac{1}{r}\frac{\partial h}{\partial r}=\frac{S}{T}\frac{\partial h}{\partial t} \qquad (1)$$

where h (r,t) = the hydraulic head of a confined aquifer; r = the distance between the observation point and pumping well; S = the storage coefficient; T = transmissivity (=Kb); K = the hydraulic conductivity of the confined aquifer; and b = the thickness of the confined aquifer.

If we proceed with pumping (assuming there are no pumping changes on the pumping system) under the barometric changes, it will cause fluctuation of the head pressure in the aquifer. In other words, the pumping volume will be unsteady. Under these circumstances, it is possible that the pumping test may not efficiently determine the transmissivity of the aquifer.

DERIVATION OF THE STOCHASTIC APPROACH

For the disturbance of the hydraulic head (h'), which is assumed only to be disturbed by barometric changes, we make use of the concept of the small perturbation, which is shown as follows:

$$h=\overline{h}+h' \qquad (2)$$

$$E\left[h'\right]=0 \qquad (3)$$

where $\overline{h}$ is defined as

$$\overline{h}=E[h]=\langle h\rangle=\int_{-\infty}^{\infty} h(r,t;t_w)f(t_w)dt_w$$

in which $f(t_w)$= the probability density function of h(t,r) distribution at time t; and E[h] as well as <h> = the expectations of h. The h' = the disturbance of hydraulic head (h), which is induced by the barometric changes. Therefore,

$$E[h']=E[h] \quad E[\overline{h}]=\langle h'\rangle=\int_{-\infty}^{\infty} h'(r,t;t_w)f(t_w)dt=0$$

By inserting Eqs. 2 and 3 into Eq. 1, we obtain

$$\frac{\partial\left(\frac{\partial\overline{h}+\partial h'}{\partial r}\right)}{\partial r}+\frac{1}{r}\left(\frac{\partial\overline{h}+\partial h'}{\partial r}\right)=\frac{S}{T}\left(\frac{\partial\overline{h}+\partial h'}{\partial t}\right) \qquad (4)$$

Eq. 4 can be rewritten as

$$\frac{\partial}{\partial r}\left(\frac{\partial\overline{h}}{\partial r}\right)+\frac{\partial}{\partial r}\left(\frac{\partial h'}{\partial r}\right)+\frac{1}{r}\frac{\partial\overline{h}}{\partial r}+\frac{1}{r}\frac{\partial h'}{\partial r}=\frac{S}{T}\frac{\partial\overline{h}}{\partial t}+\frac{S}{T}\frac{\partial h'}{\partial t} \qquad (5)$$

We can take the expectation of Eq. 5 to produce the following:

$$\frac{\partial}{\partial r}\left(\frac{\partial\langle\bar{h}\rangle}{\partial r}\right)+\frac{\partial}{\partial r}\left(\frac{\partial\langle h'\rangle}{\partial r}\right)+\frac{1}{r}\frac{\partial\langle\bar{h}\rangle}{\partial r}+\frac{1}{r}\frac{\partial\langle h'\rangle}{\partial r}=\frac{S}{T}\frac{\partial\langle\bar{h}\rangle}{\partial t}+\frac{S}{T}\frac{\partial\langle h'\rangle}{\partial t}$$

(6)

In addition, $\langle\bar{h}\rangle=\bar{h}$ and $\langle h'\rangle=0$. Then, we obtain

$$\frac{\partial}{\partial r}\left(\frac{\partial\bar{h}}{\partial r}\right)+\frac{1}{r}\frac{\partial\bar{h}}{\partial r}=\frac{S}{T}\frac{\partial\bar{h}}{\partial t} \quad (7)$$

By subtracting Eq. 7 from Eq. 5, we obtain the second-order partial differential equation of h':

$$\frac{\partial}{\partial r}\left(\frac{\partial h'}{\partial r}\right)+\frac{1}{r}\frac{\partial h'}{\partial r}=\frac{S}{T}\frac{\partial h'}{\partial t} \quad (8)$$

Since h' is the fluctuation of the hydraulic head in an aquifer, which is induced by the barometric change, the Fourier-Stieltjes representative can be applied to h' as

$$h'(t,r)=\int_{-\infty}^{\infty}\exp(i\omega t)dZ_h(\omega,r)$$

where $dZ_h(\omega,r)$ = the Fourier amplitude of $h'(t,r)$ (Gelhar, (4)); $i=\sqrt{-1}$ = an imaginary unit; ω= the frequency of the disturbance of the hydraulic head which is induced by the same frequency of the barometric changes. Then, Eq. 8 can be rewritten as

$$\int_{-\infty}^{\infty}\exp(i\omega t)[d''Z_h+\frac{1}{r}d'Z_h-\frac{S}{T}(i\omega)dZ_h]=0 \quad (9)$$

From Eq. 9, since the term of the integration of $\exp(i\omega t)$ does not equal zero, we obtain

$$\int_{-\infty}^{\infty}\exp(i\omega t)d\omega \neq 0 \quad (10)$$

$$r^2 d''Z_h + rd'Z_h - \frac{S}{T}r^2(i\omega)dZ_h = 0 \quad (11)$$

Eq. 11 is a standard second-order Bessel differential equation. The solution of Eq. 11 (Wylie, (15)) can be expressed as

$$dZ_h = C_1 I_0\left(\sqrt{\frac{Si\omega}{T}}r\right)+C_2 K_0\left(\sqrt{\frac{Si\omega}{T}}r\right) \quad (12)$$

where C_1 and C_2 are unknown, which need to be determined by two boundary conditions. The first boundary condition is that the fluctuation of the hydraulic head is stationary at the farther distance from the well:

$$dZ_h(r,\omega) = 0, \text{for } r = R \to \infty \tag{13}$$

By inserting Eq. 13 into Eq. 12, we obtain

$$C_1 = 0 \tag{14}$$

The second boundary condition is the relationship between the fluctuation amplitudes of the hydraulic head and of the pumping discharge at the well:

$$\frac{d}{dr}(dZ_h)\bigg|_{r=r_w} = \frac{dZ_Q}{2\pi rT}\bigg|_{r=r_w} \tag{15}$$

where dZ_Q = the Fourier amplitude of the fluctuation of the pumping discharge, and r_w = the radius of the pumping well, which is measured from the centerline of the well to the casing of the well. The condition of Eq. 15 is applied to Eq. 12. Then, we obtain the unknown C_2:

$$C_2 = -\frac{dZ_Q}{2\pi r_w T} \times \sqrt{\frac{T}{Si\omega}} \times \frac{1}{K_1(\sqrt{\frac{Si\omega}{T}}r_w)} \tag{16}$$

Therefore, Eq. 12 can be rewritten as

$$dZ_h = F(\omega, r)dZ_Q \tag{17}$$

where

$$F(\omega, r) = -\frac{1}{2\pi r_w T} \times \sqrt{\frac{T}{Si\omega}} \times \frac{K_0(\sqrt{\frac{Si\omega}{T}}r)}{K_1(\sqrt{\frac{Si\omega}{T}}r_w)} \tag{18}$$

By applying the definition of the spectrum theory of the hydraulic head and the pumping discharge (Gelhar, (4)), we obtain

$$S_{hh} = E[dZ_h(\omega)dZ_h^*(\omega)] \tag{19}$$

$$S_{QQ} = E[dZ_Q(\omega)dZ_Q^*(\omega)] \tag{20}$$

where dZ_h^* and dZ_Q^* = the conjugates of dZ_h and dZ_Q respectively; and S_{hh} and S_{QQ} = the spectral density function of the hydraulic head and pumping discharge respectively. We obtain the expected value by conjugating Eq. 17 and by multiplying the conjugated form of Eq. 17 by Eq. 17:

$$S_{hh} = FF^*S_{QQ} \tag{21}$$

where F^* = the conjugated function of F, S_{hh} = the spectral density function of the hydraulic head at radius r, S_{QQ} = the spectral density function of the pumping discharge at the well. From Eq. 21, we can derive a spectral density function of the hydraulic head expressed in terms of the spectral density function of the pumping discharge:

$$S_{hh} = FF^{*}S_{QQ} = \frac{S_{QQ}}{4\pi^2 r_w^2 S\omega T} \times \frac{K_0(\sqrt{\frac{Si\omega}{T}}r)K_0^{*}(\sqrt{\frac{Si\omega}{T}}r)}{K_1(\sqrt{\frac{Si\omega}{T}}r_w)K_1^{*}(\sqrt{\frac{Si\omega}{T}}r_w)} \tag{22}$$

where $K_0\left(\sqrt{\frac{S\omega i}{T}}r\right)$ and $K_1\left(\sqrt{\frac{S\omega i}{T}}r_w\right)$ = the modified Bessel function of the second kind of order 0 and 1, respectively; and $K_0^{*}\left(\sqrt{\frac{S\omega i}{T}}r\right)$ and $K_1^{*}\left(\sqrt{\frac{S\omega i}{T}}r_w\right)$ = the conjugate of the modified Bessel function of the second kind of order 0 and 1, respectively.

A more detailed examination of Eq. 22 can be applied to the hydraulic-parameters (T and S) identification. This procedure will be examined in the following section.

PARAMETER IDENTIFICATION PROCEDURES AND DISCUSSION

In this study, we present two methods for identifying the hydraulic parameters (T and S) of an aquifer with the analytical solution, Eq. 22.

Method 1. To Identify the Transmissivity (T) and Storativity (S)

To analyze Eq. 22, we chose a dimensionless formula to perform the characteristics of the equation. First, the dimensionless formula we chose from Eq. 22 was

$$\alpha = \sqrt{\frac{S\omega}{T}}r \tag{23}$$

Let's call α the dimensionless frequency at radius r. The second dimensionless group chosen was

$$\frac{S_{hh}T^2}{S_{QQ}}$$

Therefore, the dimensionless spectral density function of the hydraulic head can be expressed as

$$\frac{S_{hh}T^2}{S_{QQ}} = \frac{1}{4\pi^2 r_w^2 \frac{S\omega}{T}} \frac{K_0\left(\sqrt{\frac{S\omega i}{T}}r\right) K_0\left(\sqrt{\frac{S\omega i}{T}}r\right)}{K_1\left(\sqrt{\frac{S\omega i}{T}}r_w\right) K_1\left(\sqrt{\frac{S\omega i}{T}}r_w\right)} = \frac{1}{4\pi^2 \alpha_w^2} \frac{K_0(\alpha\sqrt{i})K_0(\alpha\sqrt{i})}{K_1(\alpha_w\sqrt{i})K_1(\alpha_w\sqrt{i})} \tag{24}$$

where $\alpha_w = \sqrt{\frac{S\omega}{T}}r_w$ is the dimensionless frequency at radius r_w.

In general in the case of the same aquifer, since the radius r means that the distance from the centerline to an interested point that is outside the well, the r is always larger than the r_w. In other words, in the case of the same aquifer, the dimensionless frequency at radius r, α, is always larger than the one at r_w, α_w, except for when $r = r_w$, then $\alpha = \alpha_w$. Therefore, we can determine the type curve for the changes of the hydraulic parameters. When using this method it is only necessary to measure the hydraulic head and discharge variations at the pumping well, the radius r of the numerator in Eq. 24 is equal to r_w. Therefore, the dimensionless frequency of α in Eq. 24 equals α_w. Moreover, the power spectrum of the hydraulic head (S_{hh}) of Eq. 24 can be written as S_{HH}.

Fig. 2 shows the dimensionless spectral density function of the hydraulic head, $\frac{S_{HH}T^2}{S_{QQ}}$, versus the dimensionless frequency, $\alpha_w = \sqrt{\frac{S\omega}{T}}r_w$. It indicates that the dimensionless spectral density function of the hydraulic head decreases while the dimensionless frequency increases. This is due to the fluctuation of the hydraulic head induced by the barometric changes in a well propagates and decreases in a confined aquifer. Finally, the fluctuation diminishes when it reaches the infinite boundary of the aquifer. However, an asymptotic behavior of the dimensionless

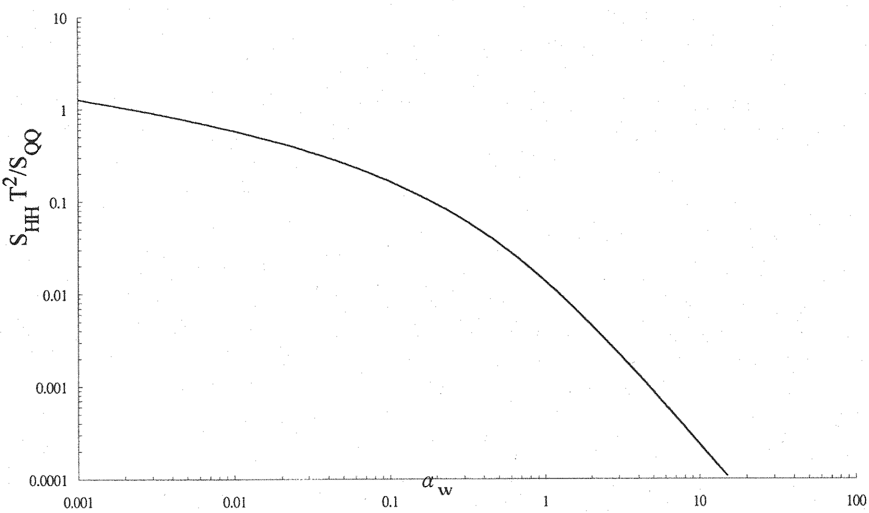

Fig. 2 Type curve of dimensionless spectrum of hydraulic head ($S_{HH}T^2/S_{QQ}$)

spectral density function $\left(\frac{S_{HH}T^2}{S_{QQ}} \to 0 \right)$ can be found when the dimensionless frequency $\alpha > 3$.

In practice, we can measure the hydraulic head and pumping discharge at the pumping well during a pumping test (as shown in Fig. 3). The variation of the hydraulic head with and without barometric changes is shown in Fig. 4. It was found that the variation of the hydraulic head with the barometric changes (a black line in Fig. 4) fluctuates around the variation of the hydraulic head without the barometric changes (a dashed line in Fig. 4). Therefore, we can use the variation of the hydraulic head with barometric changes to minus the one without the barometric changes. Finally, we obtain the difference between the hydraulic head with and without the barometric changes. The difference of the hydraulic head is known as the fluctuation of the hydraulic head due to the barometric changes. The conceptual plot of the fluctuation of the hydraulic head is shown in Fig. 5. Since the fluctuation of the hydraulic head is due to the barometric changes, the natural frequency of the fluctuation of the hydraulic head is equal to the natural frequency of the barometric changes.

Similarly, we can plot the variation of the pumping discharge with and without the barometric changes as shown in Fig. 6. Taking the difference between the pumping discharge variation with and without barometric changes is called the fluctuation of the pumping discharge due to the barometric changes as shown in Fig. 7. Since the fluctuation of the pumping discharge is due to the barometric changes, the natural frequency of the pumping discharge fluctuation is related to the natural frequency of the barometric changes. In theory, the natural frequency of the hydraulic head fluctuation is equal to that of the pumping discharge fluctuation. When the fluctuations of the hydraulic head and pumping discharge under the barometric changes are determined, we evaluate the spectral density of the fluctuations.

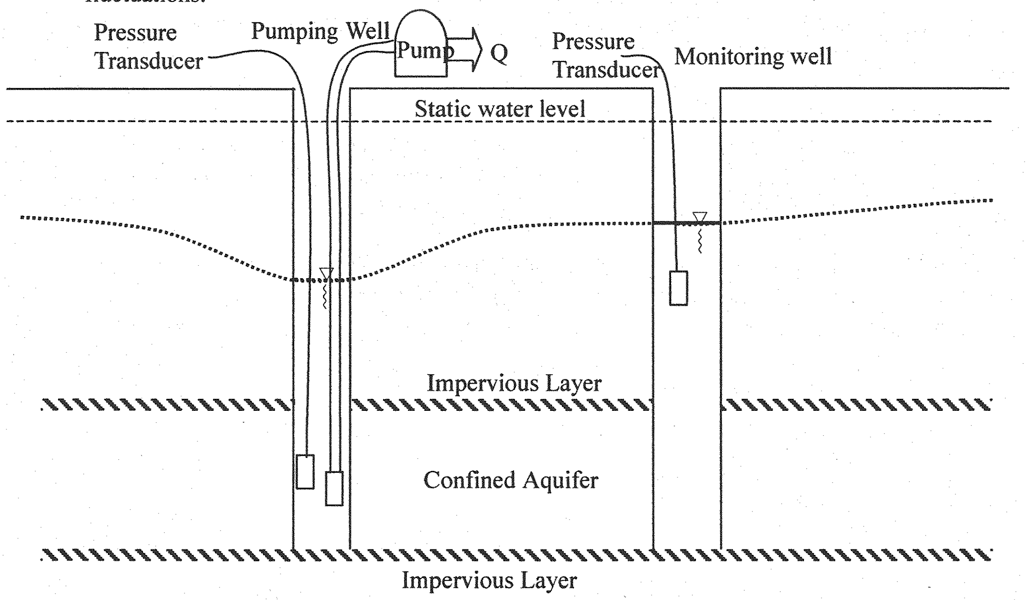

Fig. 3 Schematic diagram of pumping system for confined aquifer

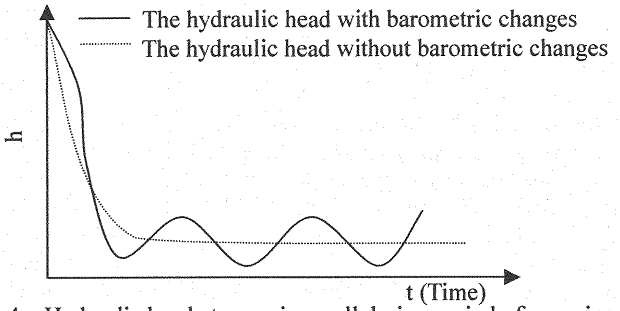

Fig. 4 Hydraulic head at pumping well during period of pumping test with and without barometric changes

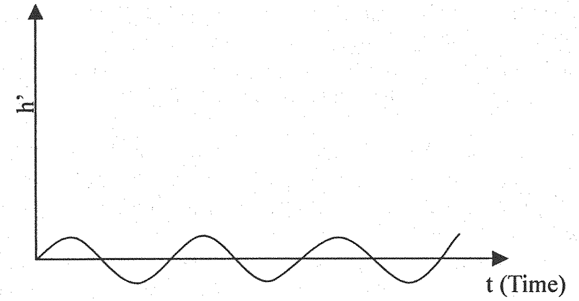

Fig. 5 Fluctuation of hydraulic head at pumping well during period of pumping test

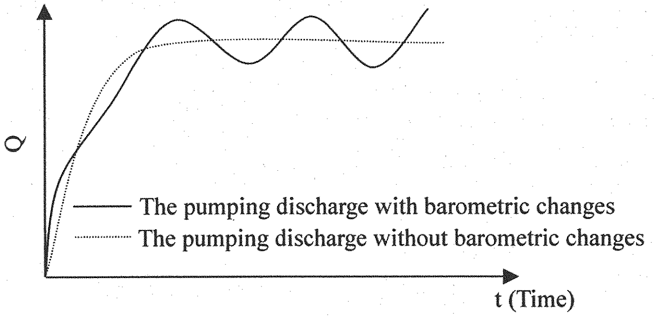

Fig. 6 Pumping discharge of pumping well during period of pumping test with and without barometric changes

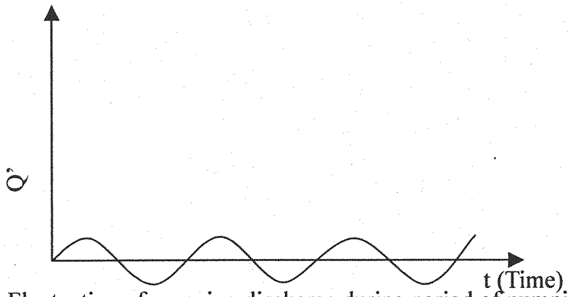

Fig. 7 Fluctuation of pumping discharge during period of pumping test

In fact, there are many methods to determine the spectral density function of a random process (such as the hydraulic head fluctuations, the pumping discharge fluctuations, etc.). These methods are discussed in many papers (1, 6, 10). Therefore, in this study, we did not introduce them again.

In order to estimate the spectral density function of the hydraulic head fluctuation (S_{HH}), we use the power spectrum estimation by using the fast Fourier transform by Press et al., (10). Likewise, we determine the spectral density function of the pumping discharge fluctuation (S_{QQ}) by using the same method as the one used to obtain the hydraulic head fluctuation. Then, we evaluate $\frac{S_{HH}}{S_{QQ}}$ by dividing the spectral density function of the hydraulic head fluctuation by the one of the pumping discharge fluctuation. Finally, we depict $\frac{S_{HH}}{S_{QQ}}$ versus $\omega^{\frac{1}{2}}$ as shown in Fig. 8.

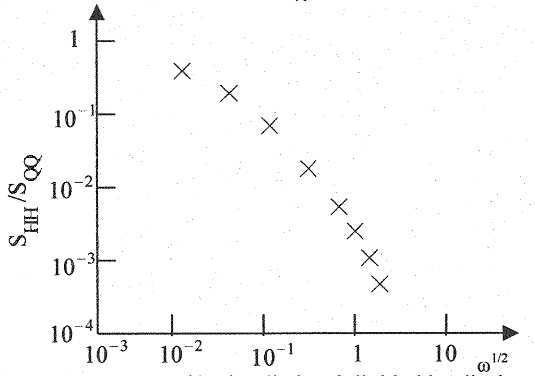

Fig. 8 Estimated spectrum of hydraulic head divided by discharge (S_{HH}/S_{QQ})

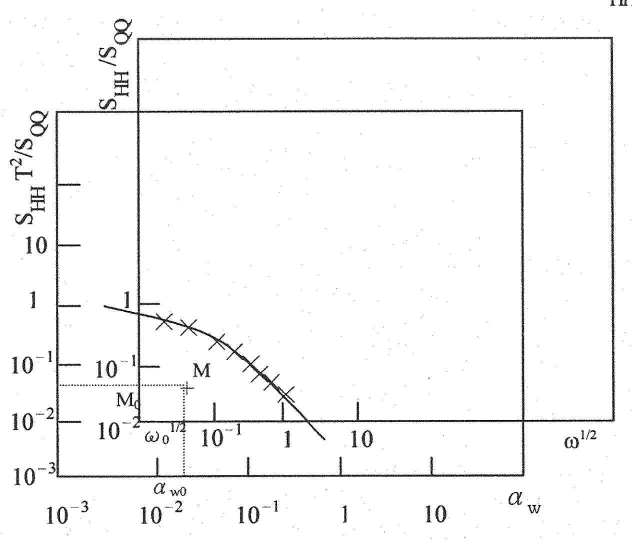

Fig. 9 Pumping test interpolation using Method 1

Now, we overlap Fig. 8 onto Fig. 2 for the curve on Fig. 8 to match the type curve on Fig. 2 as shown in Fig. 9, but the axes have to be kept in parallel to each other. When the two graphs are put on top of each other and matched, identification is immediate. An arbitrary point M of the plane is then chosen (not necessarily on the curves) and its coordinates are expressed according to the two systems

$$M = \begin{cases} \left(\dfrac{S_{HH}}{S_{QQ}}\right)_0 = M_0 \\ \dfrac{1}{\omega_0^2} \end{cases} \quad \text{and} \quad \begin{matrix} \left(\dfrac{S_{HH}T^2}{S_{QQ}}\right)_0 = M_0' \\ \left(\sqrt{\dfrac{\omega S}{T}}r_w\right)_0 = \alpha_{w0} \end{matrix} \tag{25}$$

By the definition, we can have

$$M_0 T^2 = \left(\frac{S_{HH}}{S_{QQ}}\right)_0 T^2 = \left(\frac{S_{HH}T^2}{S_{QQ}}\right)_0 = M_0' \tag{26}$$

$$\alpha_{w_0} = \sqrt{\frac{\omega_0 S}{T}}r_w \tag{27}$$

from which we get

$$T = \left[\frac{M_0'}{M_0}\right]^{\frac{1}{2}} \tag{28}$$

$$S = \frac{T\alpha_{w_0}^{\ 2}}{\omega_0 r_w^{\ 2}} \tag{29}$$

The last two identities are the transmissivity (T) and storativity (S) of the confined aquifer, respectively.

Method 2. To Identify Diffusivity $\left(\frac{T}{S}\right)$

In groundwater research, there is a hydraulic parameter—diffusivity—that has frequently been identified by groundwater researchers (5). Therefore, in this study, we propose a procedure to identify the parameter with the model that we developed.

Following a similar concept to Method 1, we construct a type curve for identifying diffusivity. We use the same dimensionless formula, Eq. 24, as is used in Method 1. The second dimensionless group is

$$\frac{S_{hh}}{S_{HH}}$$

where S_{HH} = the spectral density function of the hydraulic head at the pumping well; and S_{hh} = the spectral density function of the hydraulic head at the well with a distance r from the pumping well. The theoretical dimensionless spectral density function of the hydraulic head can be written as

$$\frac{S_{hh}}{S_{HH}} = \frac{K_0\left(\sqrt{\frac{Si\omega}{T}}r\right)K_0^*\left(\sqrt{\frac{Si\omega}{T}}r\right)}{K_o\left(\sqrt{\frac{Si\omega}{T}}r_w\right)K_0^*\left(\sqrt{\frac{Si\omega}{T}}r_w\right)} \tag{30}$$

Similar to Fig. 2, we can plot a dimensionless spectral density function of the hydraulic head as shown in Fig. 10. The dimensionless spectral density function of the hydraulic head, as shown in Fig. 10, decreases with each increase of the dimensionless frequency. Since the impact of the barometric changes on the hydraulic head at the pumping well during a pumping test propagates radically, and dies out at the greatest distance (r enlarges) from the pumping well, the spectral density function of the hydraulic head at the greatest distance diminishes.

In practice, we need two wells: one is the pumping well and the other is a monitoring well (as shown in Fig. 3). We use the pumping well for doing the pumping test. The monitoring well is used for monitoring the groundwater hydraulic head at the monitoring well. In this method, we only need to use the hydraulic head measurements at the pumping well and to use the monitoring well during the pumping test. The measurements of the hydraulic head at the two wells with and without barometric changes are depicted in Fig. 11. In Fig. 11, the black lines represent the hydraulic head at the pumping well and monitoring well with barometric changes. The dashed lines represent the hydraulic head at the two wells without barometric changes.

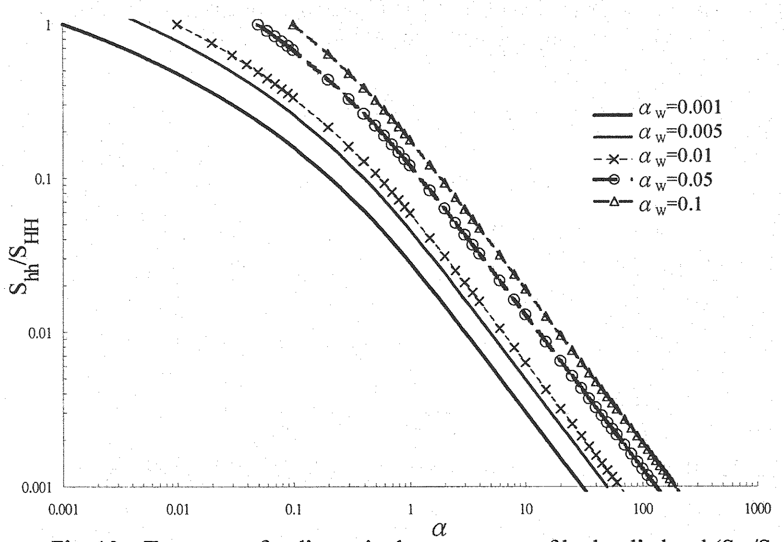

Fig. 10 Type curve for dimensionless spectrum of hydraulic head (S_{hh}/S_{HH})

Following the same procedure as Method 1, we take the difference between the hydraulic head at the pumping well with and without the barometric head, which is known as the fluctuation of the hydraulic head at the pumping well. Again, we determine the difference between the hydraulic head at the monitoring well with and without the barometric head. This difference is known as the fluctuation of the hydraulic head at the monitoring well. After both of the fluctuations of the hydraulic head at the pumping well and the monitoring well are determined, we then evaluate the spectral density functions of the hydraulic head at the pumping well and monitoring well, S_{HH} and S_{hh}, respectively. Then, we evaluate $\frac{S_{hh}}{S_{HH}}$ by dividing the spectral density function of the hydraulic head at the monitoring well by the one at the pumping well. Finally, we depict $\frac{S_{hh}}{S_{HH}}$ versus $\omega^{1/2}$ as shown in Fig. 12.

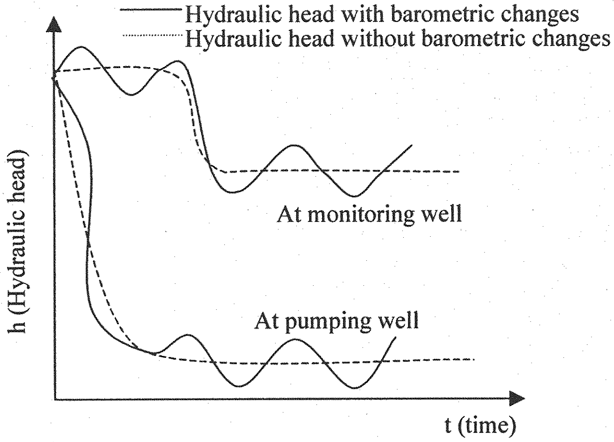

Fig. 11 Hydraulic head variations at pumping and monitoring wells during period of pumping test with and without barometric changes

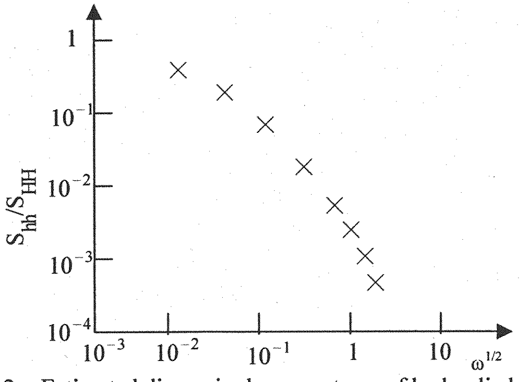

Fig. 12 Estimated dimensionless spectrum of hydraulic head (S_{hh}/S_{HH})

Now, we overlap Fig. 12 onto Fig. 10 for the curve on Fig. 12 to match one of the type curves on Fig. 10 as shown in Fig. 13, but the vertical axes have to match-up and the horizontal axes have to be parallel to each other. When the two graphs are put on top of each other and matched, identification is immediate. An arbitrary point N on Fig. 13 is then chosen, not necessarily on the curves, and its coordinates are expressed according to the system

$$N = \begin{cases} \left(\dfrac{S_{hh}}{S_{HH}}\right)_0 = N_0 \\ \omega_0^{1/2} \end{cases} \quad \text{and} \quad \begin{cases} \left(\dfrac{S_{hh}}{S_{HH}}\right) = N_0 \\ \left(\sqrt{\dfrac{\omega S}{T}} r\right)_0 = \alpha_0 \end{cases} \tag{31}$$

By the definition, we can have

$$\alpha_0 = \sqrt{\frac{\omega_0 S}{T}} r \tag{32}$$

Since the parameter r in Eq. 32 denotes the distance of the monitoring well from the pumping well, the r is measurable in practice. Let us assume that the r is equal to a known value r_M. Replacing the r in Eq. 32 with r_M yields

$$\frac{T}{S} = \left(\frac{\omega_0 r_M^{\ 2}}{\alpha_0^{\ 2}}\right) \tag{33}$$

Eq. 33 is the diffusivity $\left(\dfrac{T}{S}\right)$ of the confined aquifer of the pumping system.

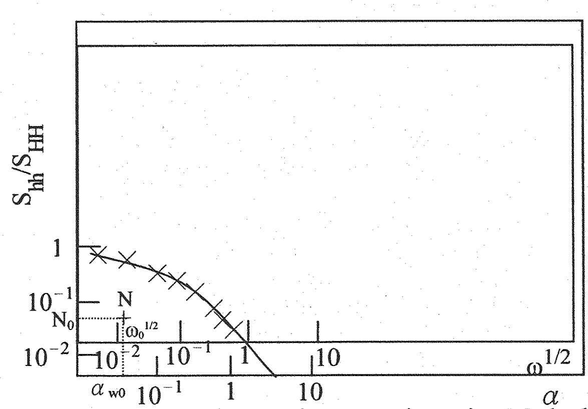

Fig. 13 Pumping test interpretation using Method 2

CONCLUSION

The findings of our research indicate the following: the analytical solution of the spectrum of the hydraulic-head fluctuation revealed that when the dimensionless frequency α increased, the value of the spectral density function of the hydraulic head in the aquifer decreased as far as the fluctuation of the hydraulic head radically propagated to the greatest distance from the pumping well.

We also presented two methods for identifying the hydraulic parameter of the confined aquifer of the pumping system. The first method only involves the use of a pumping well with a pressure transducer and pump for the measurement of the fluctuations of the hydraulic head and pumping discharge at the pumping well. By this means, we were able to identify the storativity (S) and transmissivity (T) of the confined aquifer by using the procedure we introduced in this study. In the second method, we needed one pumping well and a monitoring well at the site with the pressure transducers for both wells. After the fluctuations of the hydraulic head at the pumping and monitoring wells were measured, we were able to identify the diffusivity of the confined aquifer of the pumping system by using the procedure proposed in this study. However, in the future, we intend to investigate the real cases of the pumping test in order to elaborate on the findings discussed in this paper.

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APPENDIX – NOTATION

The following symbols are used in the paper:

b	= the thickness of the confined aquifer;
dZ_h^*, dZ_Q^*	= conjugates of dZ_h and dZ_Q, respectively;
dZ_h	= Fourier amplitude of $h'(t,r)$;
dZ_Q	= Fourier amplitude of fluctuation of pumping discharge;
E[h], <h>	= expectations of h;
$f(t_w)$	= probability density function of h(t,r) distribution at time t;
h	= hydraulic head;
h'	= disturbance of hydraulic head (h) induced by barometric changes;
$i=\sqrt{-1}$	= an imaginary unit;
K	= the hydraulic conductivity of confined aquifer;
$K_0\left(\sqrt{\frac{S\omega i}{T}}r\right)$, $K_1\left(\sqrt{\frac{S\omega i}{T}}r_w\right)$	= the modified Bessel function of the second kind of order 0 and 1, respectively;
$K_0^*\left(\sqrt{\frac{S\omega i}{T}}r\right)$, $K_1^*\left(\sqrt{\frac{S\omega i}{T}}r_w\right)$	= conjugate of the modified Bessel function of the second kind of order 0 and 1, respectively;
r	= the distance between the observation point and pumping well or distance of monitoring well from pumping well;
r_w	= radius of the pumping well;
S	= the storage coefficient;
S_{hh}	= spectral density function of hydraulic head at radius r;
S_{HH}	= spectral density function of the hydraulic head at pumping well;
S_{QQ}	= spectral density function of pumping discharge at well;
t	= time;

T	= transmissivity (=Kb);
α, α_ω	= dimensionless frequency of barometric changes at radius r and at radius r_w, respectively; and
ω	= frequency of the disturbance of hydraulic head.

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