

## DIFFUSIONAL MASS TRANSFER ACROSS THE SEDIMENT-WATER INTERFACE FOR TURBULENT FLOW IN A RECTANGULAR TRENCH

by

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### SYNOPSIS

The effects of the flow velocity and the trench depth on diffusional mass transfer for turbulent flow in a rectangular trench are investigated experimentally and a model is presented for evaluation of the release flux of dissolved substances from the bottom sediment to flowing water. The release flux of dissolved substances increases as the mean flow velocity increases in almost all experiments. It is also found that the smaller the trench depth, the larger the release flux. These results suggest that the release flux increases when the exchange of water between the layer just above the sediment-water interface and its overlying layer is encouraged by an increase in mean flow velocity or by an increase in disturbance caused by a decrease in the trench depth.

### INTRODUCTION

Dissolved oxygen (DO) consumption by muddy bottom sediment and the release of nutrients from the sediment are important factors for water quality in rivers, reservoirs and estuaries. In order to control the aquatic environment, it is necessary to know the mechanism of diffusional mass transfer between the bottom sediment and the water column just above the sediment because this mechanism is closely related to such problems.

Although there have been many studies on DO consumption by the sediment and the release of nutrients from the sediment performed from biological and chemical viewpoints, some recent reports have also investigated the effect of water flow on diffusional mass transfer [1]. The authors have conducted a fundamental study for modeling the process of dissolved substance release from the sediment to flowing water [2].

Most of the preceding reports were based on a smooth flat bed; however, the real bottom has undulations due to variations in the bed materials, presence of gates, wear and other various factors. Dredging a channel or a harbor can also make additional undulations. Diffusional mass transfer from the sediment toward the overlying water which contains a lot of organic compounds, nutrients and some chemical materials entrained from concavities has a harmful influence on ecosystems. Although the turbulent flow structures in a rectangular trench have been accurately measured and analyzed by numerical simulations [3], [4], their effect on the water quality has not been studied until now. This paper reports a fundamental study on the effect of the water flow velocity just above a rectangular trench and the effect of the aspect ratio of the trench on diffusional mass transfer from the bottom sediment to flowing water for turbulent flow in the trench.

Laboratory experiments using kaolinite as the sediment and methylene blue as the dissolved substance were performed in order to investigate the effects of the flow velocity and the depth of the trench on the release flux of dissolved substances in a trench flow. A model has also been presented by formulating the process of dissolved substance release with flow pattern changes due to the variation in the trench depth taken into account.

## FORMULATING THE DIFFUSIONAL MASS TRANSFER FOR TURBULENT FLOW IN A RECTANGULAR TRENCH

### Turbulent flow structures for a rectangular trench

The patterns of open channel flows in a rectangular trench depend on the aspect ratio of the trench. A study on diffusional mass transfer was performed for turbulent flows as shown in Figs. 1 and 2. Figure 1 shows a quite complex open channel flow which contains the backward-step and forward-step. The separated vortices are generated at the forward-step, which flow down to the reattachment point so that the reverse flow zone appears. When the trench depth increases, the flow pattern changes from a reattaching type to a cavity type as shown in Fig. 2.

### Process of dissolved substance release for turbulent flow in a rectangular trench

It is known from experiments with smooth beds that the release flux of dissolved substances from the bottom sediment to flowing water varies depending on the flow velocity and the physical properties of the sediment, such as the water content and the amount of dissolved substance contained in the sediment [2]. That is, the release flux is dominated by both the diffusional mass transfer at the sediment-water interface and by the mass transfer accompanied by adsorption-desorption between the solute and the particles inside the sediment. Taking the characteristics of diffusional mass transfer from the sediment toward the overlying water into consideration, the process of dissolved substance release for trench flow is investigated in this study as follows:

The flows shown in Figs. 1 and 2 have different patterns due to the differences in the aspect ratio of the trench and the flow velocity just above the trench. The vertical flux of solute traveling from the sediment-water interface into the water column depends on both the frequency of the water exchange between the layer just above the interface and its overlying layer, and the difference in the solute concentration between the interface and bulk water. The release flux  $J$  for turbulent flow in a rectangular trench is determined by the turbulent structures just above the sediment surface and by the mass transfer accompanied by adsorption-desorption inside the sediment.

Therefore, in order to evaluate the release flux, it is essential to clarify the interaction between the turbulent structures just above the sediment surface and the mass transfer accompanied by

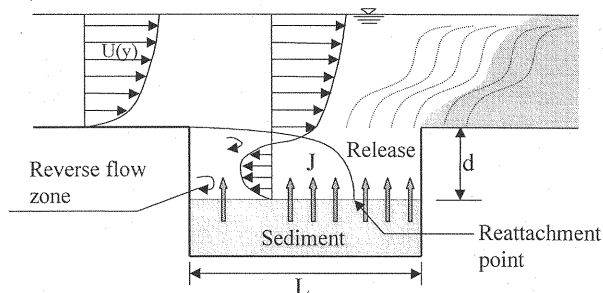


Fig. 1 Reattaching type flow pattern and diffusional mass transfer for turbulent flow in a rectangular trench.

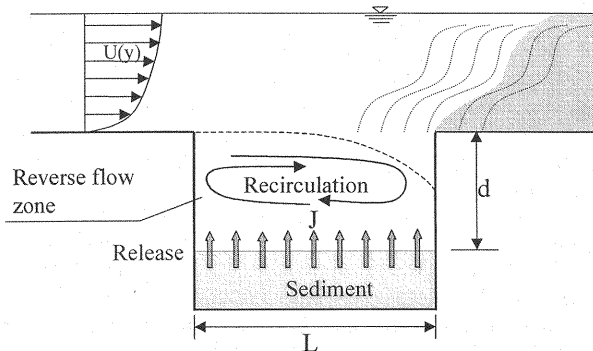


Fig. 2 Cavity type flow pattern and diffusional mass transfer for turbulent flow in a rectangular trench.

adsorption-desorption inside the sediment, in addition to understanding the turbulent structures in the rectangular trench. That is, we need to know how the flow patterns change with variations in the aspect ratio of the trench and the flow velocity just above the trench, and how the flow pattern changes influence the solute concentration at the interface. It is also necessary to formulate the dependence of the mass transfer in the sediment on the physical properties of the sediment.

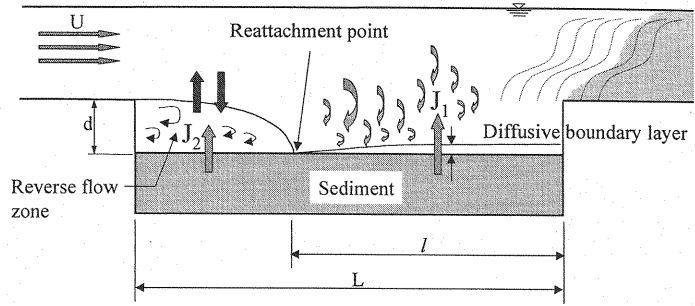


Fig. 3 Schematic view of the process of dissolved substance release in a reattaching type flow.

#### Formulating the diffusional mass transfer for trench flow

In this study the process of dissolved substance release in a trench flow is analyzed based on the relationship between the flow patterns (shown in Figs. 1 and 2) and the diffusional mass transfer mentioned above. The assumed conditions are a steady state, no chemical reactions, no microorganism metabolism and no other phenomena except for physical mass transfer.

In shallow trenches, a diffusive boundary layer develops immediately behind the reattachment point as shown in Fig. 3; in which, the process of dissolved substance release can be regarded to be the same as that for smooth beds. As the trench depth increases, the flow pattern changes from a reattaching type to a cavity type, and the release flux is dominated by the exchange of water between the cavity zone and the overlying layer. This process of variation in the diffusional mass transfer depending on the trench depth needs to be formulated in order to evaluate the release flux.

Figure 3 shows that the release flux  $J$  is related to both the diffusive flux  $J_1$  in the turbulent flow in the boundary layer and the diffusive flux  $J_2$  in the cavity flow. Therefore the release flux can be expressed as follows:

$$J = \frac{l}{L} J_1 + \frac{L-l}{L} J_2 \quad (1)$$

where  $L$ : the length of the trench in the flow direction, and  $l$ : the length scale of the turbulent boundary layer.

In the above equation,  $J_1$  and  $J_2$  can be expressed as follows from the analysis for the smooth bed [2] and penetration theory [5], respectively:

$$J_1 = \frac{3\sqrt{6}}{8\pi} n Sc^{-\frac{2}{3}} \sqrt{f} U_b (C_w - C_b) \quad (2)$$

$$J_2 = \alpha \sqrt{\frac{D}{T}} (C_w - C_b) \quad (3)$$

where  $C_w$ : solute concentration at the interface,  $Sc (= \nu/D)$ : Schmidt number,  $f (= 8\tau_0/\rho U_b^2)$ : friction factor,  $\nu$ : kinematic viscosity,  $D$ : molecular diffusivity,  $\tau_0$ : boundary shear stress,  $\rho$ : fluid density,  $n (= 0.1)$  and  $\alpha$ : numerical constant, and  $T$ : detention time scale of the water body in contact with the sediment surface in the cavity. In the above equation the value of  $U_b$  was assumed to be the mean velocity at the inlet section, and the bulk solute concentration  $C_b$  to be the mean value at the outlet section. Equation (3) has been derived from the assumption that the water body in contact with the sediment surface in the cavity is replaced by the overlying layer water after stagnating for a while, which results in traveling of the dissolved substance.

We expressed the detention time scale  $T$  by using both  $U_b$  and the trench depth  $d$  as follows:

$$T \propto \frac{d}{U_b} \quad (4)$$

Thus, Eq. (3) can be rewritten as follows:

$$J_2 = \alpha \sqrt{\frac{DU_b}{d}} (C_w - C_b) \quad (5)$$

The length scale of the turbulent boundary layer  $l$  in Eq. (1) should be dependent on the aspect ratio of the trench, decreasing as  $d$  increases and increasing as  $L$  increases. We assumed  $l$  to be the function of the aspect ratio of the trench  $d/L$  as follows:

$$l = Le^{-\beta \frac{d}{L}} \quad (6)$$

where  $\beta$  numerical constant.

Finally, the following formula can be obtained with respect to the release flux from Eqs. (2), (5) and (6):

$$J = \left\{ \frac{3\sqrt{6}}{8\pi} nSc^{\frac{2}{3}} \sqrt{f} U_b e^{-\beta \frac{d}{L}} + \alpha \sqrt{\frac{DU_b}{d}} (1 - e^{-\beta \frac{d}{L}}) \right\} (C_w - C_b) \quad (7)$$

In the above formula, the numerical constant  $\beta$  is used to express the flow pattern changes. We decided the value for  $\beta$  from the previous experimental results [3] as follows.

In the previous experiment using a 20 cm wide tilting flume to investigate the turbulent flow structures in a rectangular trench ( $L = 20$  cm), it was found that the flow pattern changed from the reattaching type to the cavity type at a trench depth of 4 cm. Figure 4 shows the relationship between  $l/L$  ( $L = 20$  cm) and  $d$  calculated by Eq. (6) for  $\beta = 5, 10$  and  $20$ . In Fig. 4, the region above the uppermost curve indicates the cavity flow zone, and the region below the lowermost curve indicates the turbulent boundary layer zone. The length of the turbulent boundary layer  $l$  can be regarded as zero when the trench depth is larger than 4 cm and the numerical constant  $\beta = 20$ . Hence, we assumed the numerical constant  $\beta$  to be 20.

## EXPERIMENTAL WORK

### Experiment apparatus

Laboratory experiments to investigate the dissolved substance release from the sediment to the turbulent flow in a rectangular trench were performed in an acrylic open channel of 700 cm long, 20 cm wide and 20 cm deep. As shown in Fig. 5, a rectangular trench of 20 cm long and 10 cm deep was installed at about 5 m from the upper end of the channel. A sediment bed made of kaolinite and methylene blue solution which was adjusted to a specified water content was placed on the bottom of the trench. Water samples were collected at the lower end of the channel with the flow conditions varied, and the absorbance was measured using a spectrophotometer. The concentration of methylene blue  $C_{obs}$  in each sample was calculated from a calibration curve which had been predetermined for the concentration

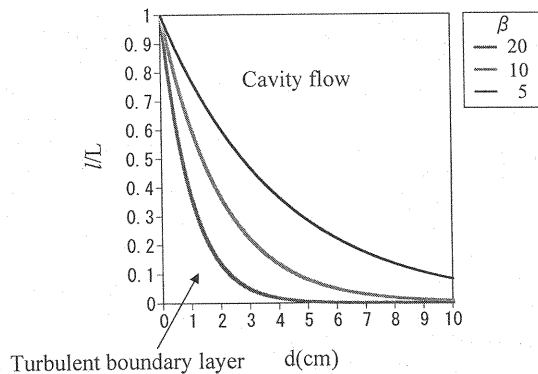


Fig. 4 Relationship between  $l/L$  and  $d$ .

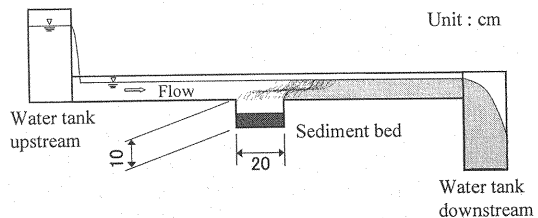


Fig. 5 Schematic view of the flow system.

against the absorbance. With the calculated value of  $C_{\text{obs}}$ , the release flux of dissolved substance  $J$  was obtained from the following equation.

$$J = QC_{\text{obs}}/A \quad (8)$$

where  $Q$ : flow discharge, and  $A$ : area of sediment bed ( $= 0.04 \text{ m}^2$ ).

Although the release flux is prone to be affected by both the turbulent flow structures just above the interface and by the physical properties of the sediment, such as the water content and the amount of dissolved substance contained in the sediment, the present study placed a special focus on the effect of the turbulent flow structures on the release flux. For this purpose, the water content and the initial methylene blue concentration in the sediment bed were fixed ( $w \approx 150\%$ ; and  $C_i = 15000 \text{ mg/l}$ ), and the trench depth  $d$  was varied from 1 to 8 cm by changing the amount of sediment material placed into the trench. In order to investigate the effect of the flow velocity just above the trench on the release flux, the value of the release flux for a given flow velocity was measured in each experiment.

#### Characteristics of dissolved substance release for trench flow

Figure 6 shows the dependence of the release flux  $J$  on the mean velocity at the inlet section  $U$  for each trench depth. In Fig. 6, plots for smaller trench depths ( $d = 1$  and  $2 \text{ cm}$ ) appeared in the upper region, and those for slightly larger trench depths ( $d = 3$  and  $4 \text{ cm}$ ) appeared almost in parallel but shifted slightly downward. The release flux increased as the flow velocity increased for each trench depth of 1, 2, 3 and 4 cm.

On the other hand, the plots for the trench depth of 5 cm appeared in the extremely lower region, with the slope of the release flux against the mean velocity being smaller than that for smaller trench depths ( $d = 1, 2, 3$  and  $4 \text{ cm}$ ). The same tendency was seen in the plots for a trench depth of 6 cm. When the trench depth was further increased to 7 or 8 cm, the plots did not shift downward any more but suggested a smaller dependency of the release flux on the flow velocity with the increase in the trench depth.

Variations in the characteristics of diffusional mass transfer with respect to the trench depth are caused by the flow pattern changes. The flow pattern is a reattaching type (Fig. 1) for  $d = 1, 2$  and  $3 \text{ cm}$ , which changes to a cavity type (Fig. 2) at  $4 \text{ cm}$  depth [3]. When the trench depth is  $5 \text{ cm}$  or larger, the flow pattern is a complete cavity type. The release flux of dissolved substances is determined by the frequency of the exchange of water between the layer just above the interface and its overlying layer. In shallower trenches, the release flux increased as exchanges are encouraged with the increase in the mean flow velocity. In deeper trenches ( $d \geq 5 \text{ cm}$ ), however, the increase in the mean flow velocity increased the velocity of recirculation but did not encourage exchanges, resulting in almost no increase in the release flux.

These results suggest that the deeper the trench, the smaller the effect of the mean flow velocity on the exchange of water between the layer just above the interface and its overlying layer, leading to a lower dependency of the release flux on the mean flow velocity.

#### ESTIMATING THE RELEASE FLUX

Equations to estimate the release flux taking the flow pattern changes into account

The authors formulate the non-dimensional release flux, i.e., Sherwood number  $Sh$ , as a

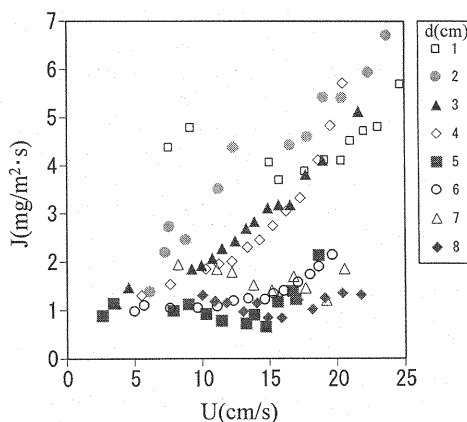


Fig. 6 Relationship between the dissolved substance release flux and the mean velocity at the inlet section.

function of the Reynolds number  $Re$ , the Schmidt number  $Sc$  and  $d/L$  from Eq. (7) as follows:

$$Sh = \frac{3\sqrt{6}}{8\pi} nm Sc^{\frac{1}{3}} Re^{\frac{7}{8}} e^{-\beta \frac{d}{L}} + \alpha \sqrt{Re Sc \frac{R_H}{d}} (1 - e^{-\beta \frac{d}{L}}) \quad (9)$$

where

$$Sh = \frac{h_D R_H}{D} = \frac{J}{C_{-\infty} - C_b} \cdot \frac{R_H}{D} \quad (10)$$

$$Re = \frac{U R_H}{\nu} \quad (11)$$

In the above equation,  $h_D$ : mass transfer coefficient,  $R_H$ : hydraulic radius, and  $m$ : numerical constant. The friction factor  $f$  is given by Blasius' formula as follows:

$$f = 0.3164 \cdot (4Re)^{-1/4} \quad (12)$$

Hence, the numerical constant  $m$  in Eq. (9) is 0.473.

In Eq. (10),  $C_{-\infty}$  is the solute concentration in pore water and can be calculated from Eq. (13) using the initial methylene blue concentration  $C_i$  and the water content of the sediment  $w$  [2].

$$C_{-\infty} = \frac{-g(C_i, w) + \sqrt{\{g(C_i, w)\}^2 + 4.22C_i}}{4.22} \quad (13)$$

where

$$g(C_i, w) = 1 + \frac{1.07 \times 10^6}{w} - 2.11C_i \quad (14)$$

Since Eq. (7) contains the solute concentration at the interface  $C_w$ , we need to know the value of  $C_w$  for estimating the release flux. However,  $C_w$  should be given as a function of the turbulent flow structure just above the sediment and the physical properties of the sediment.

Since the focus in this study was placed on the relationship between the diffusional mass transfer and the turbulent flow in a rectangular trench,  $C_w$  was assumed to be equivalent to  $C_{-\infty}$  and was calculated from Eq. (13) using  $C_i$  and  $w$ , and the values of  $Sh$  estimated by Eq. (9) were compared with the experimental results to check the availability of Eq. (9) to the evaluation of the release flux for turbulent flow in a rectangular trench.

The concentration  $C_b$  was approximated by  $C_{obs}$  measured for each sample, and  $Sh$  was calculated from Eq. (10) using the hydraulic radius  $R_H$  and the molecular diffusivity  $D$ .

Figure 7 shows the relation curves of  $Sh$  against  $Re$  calculated by Eq. (9) with the numerical constant  $\alpha$  given as 0.01, together with the plots of the experimental values. The Schmidt number  $Sc$  was assumed to be constant at 3500 in the calculation, although it varied from 2800 to 3900 in the experiments.

The relation curves appear almost parallel to each other, shifting downward as the trench depth increases. The slopes of the relation curves agreed fairly well with those obtained from the plots of the experimental values when the trench depth was larger than 4 cm, suggesting that Eq. (3) which takes the exchange of the water body in contact with the sediment surface into account successfully expresses the process of dissolved substance release for a cavity flow.

However, the slopes of the plots of experimental values are larger than those of the calculated curves when the trench depth is 4 cm or smaller, suggesting that

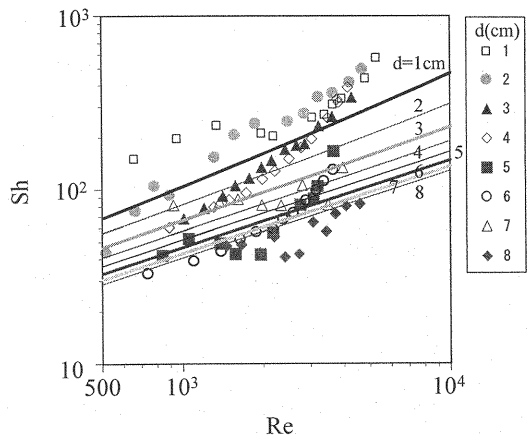


Fig. 7 Comparison between the observed and calculated results for  $Sh$  vs.  $Re$ .

Eq. (9) failed in reproducing the experimental results. Suspecting the flow pattern changes as the cause of the change in the slope of the experimental values at the trench depth of 4 cm, we tried to improve the model by further examination as follows.

Evaluating the effect of reattachment on the release flux and improving the model

It is known that the heat transfer coefficient reaches its maximum near a reattachment point in a heat transfer in a backward-step flow, one of the typical flows with separation and reattachment, especially when the Prandtl number is large [6]. This tendency for heat transfer is found to be more remarkable as the Prandtl number increases.

The fact described above allowed us to predict that the mass transfer coefficient would also reach its maximum level near reattachment points in diffusional mass transfers with larger Schmidt numbers. Since the significant changes in the experimental values of  $Sh$  and the slopes of their plots at a trench depth of 4 cm are considered to be caused by the higher mass transfer rates due to reattachment, we formulate the mass flux due to reattachment  $J'$  as a function of the bulk flow velocity  $U_b$  and the difference between the solute concentration at the sediment-water interface and the bulk water  $C_w - C_b$ .

$$J' = \gamma U_b (C_w - C_b) \quad (d/L < 1/6) \quad (15,a)$$

$$J' = 0 \quad (d/L > 1/6) \quad (15,b)$$

where  $\gamma$ : numerical constant. Since the reattachment point  $x_r$  is known to be  $x_r/d = 6.0-7.5$  [3], we decide that the flow is a reattaching type when  $d/L < 1/6$  and a cavity type when  $d/L > 1/6$ . Consequently, the release flux can be expressed as follows.

$$Sh = \frac{3\sqrt{6}}{8\pi} nmSc^{\frac{1}{3}} Re^{\frac{7}{8}} e^{-\beta \frac{d}{L}} + \alpha \sqrt{ReSc} \frac{R_H}{d} (1 - e^{-\beta \frac{d}{L}}) + \gamma ReSc \quad (d/L < 1/6) \quad (16,a)$$

$$Sh = \frac{3\sqrt{6}}{8\pi} nmSc^{\frac{1}{3}} Re^{\frac{7}{8}} e^{-\beta \frac{d}{L}} + \alpha \sqrt{ReSc} \frac{R_H}{d} (1 - e^{-\beta \frac{d}{L}}) \quad (d/L > 1/6) \quad (16,b)$$

The relation curves of  $Sh$  against  $Re$  estimated by Eq. (16) with the numerical constant  $\gamma$  given as  $10^{-5}$  are shown in Fig. 8 together with the experimental values. This chart demonstrates that our model gives a qualitative description of the tendency of the experimental values in which the slopes of the plots change abruptly at a trench depth of 4 cm due to the flow pattern changes.

Although the diffusional mass transfer for turbulent flow in a rectangular trench was simplified during the process of formulation of Eq. (16) in this study, the characteristics of the diffusional mass transfer near the reattachment point have not been made clear. In order to estimate the release flux accurately, our model needs to be further improved through detailed investigations.

## CONCLUSION

Laboratory experiments using kaolinite as the bottom sediment and methylene blue as the dissolved substance were performed to investigate the diffusional mass transfer from the sediment to flowing water for turbulent flow in a rectangular trench. It was found that the release flux increases as the flow velocity increases irrespective of the trench depth, and that

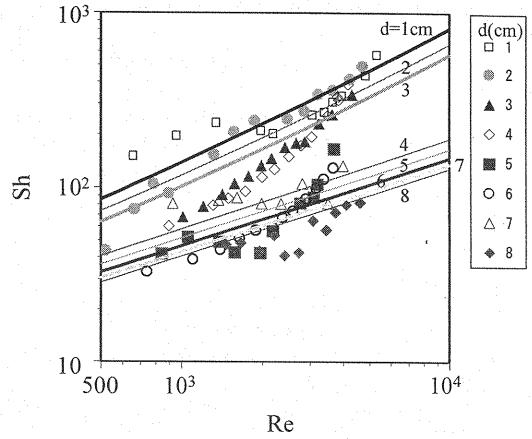


Fig. 8 Comparison between the experimental results and the estimates by Eq. (16) for  $Sh$  vs.  $Re$ .

the characteristics of the diffusional mass transfer vary depending on the flow pattern, i.e., a reattaching type or a cavity type. The release flux was larger for reattaching flows than for cavity flows, with a reduced dependency on the mean flow velocity when the trench depth decreased.

In this study the focus was placed on the relationship between the diffusional mass transfer and the patterns of turbulent flow in a rectangular trench. Future research should include examination of the effects of the water content and other physical properties of the sediment on the release flux, thereby improving the presented model with these various factors formulated in.

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## APPENDIX-NOTATION

The following symbols are used in this paper :

$J$	= release flux;
$C$	= solute concentration;
$\tau_0$	= boundary shear stress;
$U$	= mean velocity;
$\rho$	= fluid density;
$D$	= molecular diffusivity;
$\nu$	= kinematic viscosity;
$C_w$	= solute concentration at the sediment-water interface;
$C_b$	= bulk solute concentration;
$Sc$	= Schmidt number;
$f$	= friction factor;
$l$	= length of trench in the flow direction;
$L$	= length scale of turbulent boundary layer;
$d$	= trench depth;
$Q$	= flow discharge;



$A$	= area of sediment bed;
$w$	= water content of the sediment;
$C_{\text{obs}}$	= methylene blue concentration in each sample;
$C_i$	= initial methylene blue concentration;
$C_{\infty}$	= solute concentration in pore water;
$R_H$	= hydraulic radius;
$h_D$	= mass transfer coefficient;
$Sh$	= Sherwood number; and
$Re$	= Reynolds number.

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