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OPTIMIZATION OF FIXED AND OPERATING COSTS OF WELLS FOR GROUNDWATER REMEDIATION USING OPTIMAL CONTROL AND GENETIC ALGORITHMS

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SYNOPSIS

Owing to difficulties in considering fixed and time-varying operating costs at the same time, the optimization of groundwater remediation is a challenging task. The difficulties are caused by the combinatorial nature of assigning discrete well locations and the large computational burden required for time-varying operating costs. This study presents a novel algorithm that integrates Genetic Algorithm (GA) and Constrained Differential Dynamic Programming (CDDP) to solve the two interesting problems of groundwater remediation problem. GA can easily incorporate the fixed costs associated with the installation of wells. CDDP is used to handle the problem of time-varying operating costs. A case study that incorporates fixed and time-varying operating costs is presented to demonstrate the effectiveness of the proposed algorithm. Simulation results indicate that the fixed costs can significantly influence the number and locations of wells, and that a notable total cost saving can be realized by applying the novel algorithm.

INTRODUCTION

In groundwater remediation problems, the pump-and-treat (P&T) method is a practical technology for large plume problems, and no alternative technology has yet been proven superior (1). The feasibility of coupling optimization techniques with groundwater flow and transport simulation to design P&T systems has been extensively studied (8,1,2,22,18,13,20,15). Chang et al. (4) employed an optimal control method, called the Successive Approximation Linear Quadratic Regulator (SALQR), to design a pumping system for the remediation of contaminated aquifers. None of the other methods is designed for time-varying optimization and none has been demonstrated to be more efficient than SALQR for solving time-varying problems (14). Although SALQR is superior in dealing with the time-varying problems, it fails to cope with the problems of fixed costs.

The P&T system design is important because well locations and pumping rates can markedly affect system performance. Generally, the decision variables involve determining the values of pumping rates from extraction wells and selecting the locations of wells. Owing to the discontinuous nature of well location selection, mathematical programming is often simplified by neglecting the fixed costs of well installation. The optimal network normally consists of those wells whose final optimized pumping rates are nonzero. However, this simplification can lead to designs that rely on numerous wells pumping at small rates over long periods (13). Recently, researchers have investigated various methodologies for incorporating these fixed costs. McKinney and Lin (12,13) used a genetic algorithm (GA) and mixed-integer nonlinear programming (MINLP) methods to solve groundwater management problems comprised of both fixed and operating costs. Meanwhile, Zheng and Wang (23) integrated tabu search and linear programming to solve the design of groundwater remediation with an objective function that also involved the fixed costs and operating costs. However, their model only considered groundwater flow as a steady state. Finally, Watkins and McKinney (21) applied generalized Benders decomposition (GBD) and outer approximation (OA) to water resource problems involving cost functions with both discrete and nonlinear terms. Although the two algorithms are effective ways of solving the mixed integer nonlinear programming, the computational bottleneck creates difficulties for large MINLP problems.

Related investigations have demonstrated that dynamic policies are more cost-effective than the best static policies because pumping policies are allowed to change as the contaminant plume moves (4,5). Owing to dynamic optimal control algorithms requiring a separable objective function for each stage t, they face difficulties in solving a problem with an objective function that contains fixed costs. Culver and Shoemaker (7) used QNDDP in groundwater reclamation with treatment capital cost assumed to be linearly related to extraction rate. However, their investigation did not incorporate the fixed costs of well installation. Meanwhile, Huang and Mayer (9) used GA to search for the optimal pumping rates and the discrete space of well locations in dynamic groundwater remediation management. Their results showed that the moving-well model is less expensive than solutions obtained using a comparable fixed-well model. Because of the properties of GA, their model requires considerable computational effort to obtain the optimal solution. Aly and Peralta (3) used the L_∞ norm as a global measure of aquifer contamination instead of the traditional control locations for contaminant concentrations, and compared the performance of GA and MINLP. Although their model can be applied to a dynamic system and involves fixed costs and operating costs, the multi-period planning problem must be approximated by a series of single-period problems. Therefore, their approach is not a fully dynamic optimization method.

As far as we know, no investigation has simultaneously considered the fixed costs of well installation and operating costs of time-varying pumping rates. The genetic algorithm is attractive because it does not require the differentiability of the objective function. Hence, the genetic algorithm can easily incorporate the fixed costs associated with the groundwater remediation problem. However, applying this technique to solve time-varying policies could dramatically increase the computational resources required. Therefore, this study proposes a novel approach for resolving this optimization problem by effectively combining Genetic Algorithm (GA) with Constrained Differential Dynamic Programming (CDDP).

FORMULATION OF THE MANAGEMENT MODEL

The management model attempts to minimize the total cost of remediation, composed of the fixed costs of well installation and operating costs of the pumping and treatment system. The problem can be formulated as

$$\min_{\substack{I \subset \Omega \\ u_t^i,\, i\in I,\, t=1,\ldots,N}} \quad J(I,u_t(I)) = \sum_{i\in I}\{a_1 y^i(I) + \sum_{t=1}^{N}[a_2 u_t^i(I) + a_3 u_t^i(I)(L_*^i(I) - h_{t+1}^i(I))]\} \tag{1}$$

subject to

$$\{x_{t+1}\} = T(x_t, u_t(I), t), \quad t = 1,2,\ldots,N, \quad I \subset \Omega \tag{2}$$

$$c_{N,j} \le c_{\max}, \quad j \in \Phi \tag{3}$$

$$\sum_{i\in I} u_t^i \le u_{total}, \quad t = 1,2,\ldots,N \tag{4}$$

$$u_{\min}^i \le u_t^i(I) \le u_{\max}^i, \quad t = 1,2,\ldots,N, \quad I \subset \Omega, \quad i \in I \tag{5}$$

where Ω is an index set that defines all candidate well locations in the aquifer; and I is a potential network alternative (design) and is a subset of Ω. The upper index i denotes a well in the network design (I). $J(\cdot)$ represents total cost of I ; $x_t = [h_t : c_t]^T \in R^{(n_h+n_c)\times 1}$ are the state continuous variables representing heads (h_t) and concentrations (c_t), n_h and n_c denote total number of hydraulic heads and concentrations, respectively; $u_t(I) \in R^{m\times 1}$ represent the vector of control variables whose dimension depends on I , m is the number of control variables; $T(x_t, u_t(I), t)$ represents the transition equation; Φ is the set of observation wells; a_1, a_2, and a_3 are factors used to convert the well installation cost, treatment cost, and operating cost, respectively, into monetary values ($); $L_*(I) \in R^{m\times 1}$ are the distance from the ground surface to the lower datum of the aquifer for wells; $h_{t+1}(I)$ denote hydraulic head for nodes at time $t+1$; $y(I)$ are the depth of wells, and u_{total} represent the maximum allowable total pumping rates from all extraction wells. Equation (5) specifies the capacity constraints for each well. The transition equation, T , in (2) is solved with ISOQUAD (Pinder, 1978), a finite element groundwater flow and transport model for a confined two-dimensional aquifer. The transport model includes changes in head due to pumping as well as changes in the contaminant concentration owing to advection, diffusion, dispersion, and linear equilibrium sorption.

The first component in Eqn. (1) refers to the costs of well installation, and are incurred if a well is installed for pumping. The costs of well installation are a discrete operation and require the use of binary variables in the optimization model. The second component in Eqn. (1) expresses the operating costs, involving pumping and treatment costs. These costs are continuous functions of the state and control variables and are separable functions for each stage t. Therefore, the groundwater remediation model defined by Eqns. (1) to (5) is a mixed integer time-varying optimization problem. Because of the discrete nature of the installation cost, the problem, defined by Eqns. (1) to (5), is difficult to solve by using CDDP alone (4,5). Meanwhile, the near global optimization techniques, such as simulated annealing (17), genetic algorithm (9), or tabu search (23), do not require the objective function to be continuous, convex, or differentiable. Hence, these techniques have the potential to solve an optimization problem containing fixed costs. However, applying these techniques to solve time-varying policies could dramatically increase the computational resources required (7,23). Therefore, the above techniques are inappropriate for time-varying optimization.

INTEGRATION OF GA AND CDDP

This investigation integrates GA and CDDP (GCDDP) to solve the problem defined by Eqns. (1) to (5). In this integrated approach, GA, a near global optimization algorithm, is used to locate the optimal well sites, while CDDP is employed to calculate the optimal pumping rates. Figure (1) illustrates the procedure of the algorithm. In this figure, the algorithm is a GA with CDDP embedded to compute the optimal operation costs for a potential network alternative (represented by a chromosome). The total cost for each network alternative (chromosome) is the sum of the optimal operation costs and its fixed costs. A higher total cost implies a lower fitness value for a chromosome. In this investigation, time-varying pumping rates are measured while evaluating the optimal operation costs by using CDDP. These procedures can be clarified by using the following step by step procedure:

Step 0: Initialization

Encode the network alternatives as chromosomes and randomly generate an initial population. A conventional encoding scheme in GA is binary encoding, which a highly efficient means of resolving network design problems. Hence, this study uses a binary indicator to represent the status of the well installation on a candidate site, thus on a chromosome, represented by a binary string, defines a network alternative. Each bit in a chromosome is associated with a candidate site, and the length of the chromosome is equal to the total number of candidate sites available for well installation. If the value of a bit equals one, the associated candidate site will install a well, otherwise the value of a bit equals zero and the associated candidate site will not install a well.

To demonstrate the operation of chromosome encoding, a hypothetical, homogeneous, isotropic confined aquifer with dimensions of 600 m by 1200 m serves as an example. Figure 2 presents the finite element mesh, the associated boundary conditions for both hydraulic head and contaminant concentration, the location of candidate sites for extraction wells, and the location of observation wells. There are ninety-one finite element nodes, as well as twenty-four candidate well sites, and seventeen observation wells.

Because the hydraulic head, the initial concentration, locations of observation wells and candidate sites for pumping wells are symmetrical in Fig. 2, this study assumes that the optimal network is also symmetrical. Base on this assumption, the combination of network configurations will decrease in GA and the computational effort will decrease. The chromosome contains sixteen bits, where the first eight bits represent the sites along the centerline and the last eight bits represent candidate sites in the upper region. When a bit among the last eight bits has a value, it represents two wells placed symmetrically to the centerline. The chromosome in Fig. 3 represents a network design and selects only four wells. These wells are located at nodes 32, 67, 45 and 47. Since the well selection is binary, encoding and decoding the chromosome is straightforward.

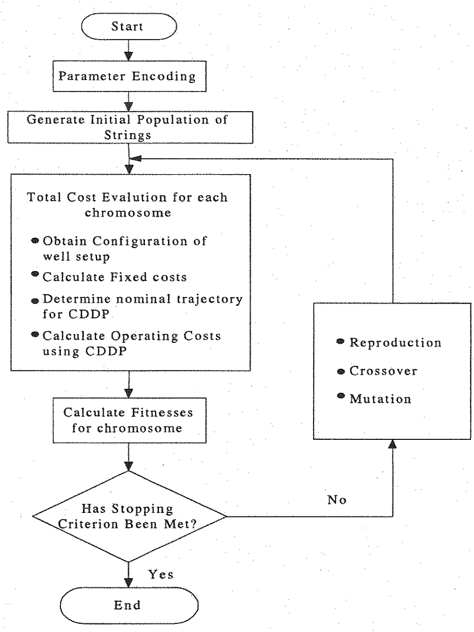


Figure 1. Flowchart of GCDDP groundwater remediation model

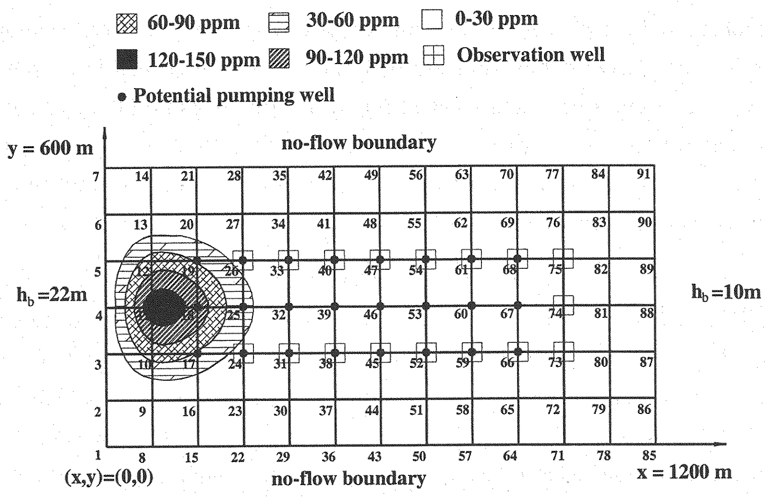


Figure 2. Finite element mesh, Boundary Conditions, Initial Plume, and Locations of Numbered Observation and Potential Extraction Wells for all Runs of the Groundwater Reclamation Example

0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0

Figure 3. Chromosome representation

Step 1: Evaluation of the total cost and fitness value of each chromosome.

For each chromosome, the well locations are selected and the fixed costs in Eqn. (1) are ready for evaluation before calculating the optimal operating costs. The sum of the fixed costs and associated optimal operation costs is the optimal total cost of each chromosome. The chromosome can be represented as a binary string in the form $o_i = x_1, x_2, ..., x_8, x_9, ..., x_{16}$, where o_i denotes a chromosome in the population. Each digit x_l has a value of 1 or 0. The number of wells for the chromosome can be calculated as follows:

$$N_{well_i} = \sum_{i=1}^{8} x_i + 2\sum_{9}^{16} x_i \qquad (6)$$

For each chromosome, remediation design attempts to minimize the operating costs. The optimization model can then be rewritten as follows:

$$\min_{u_t^i, i\in I, t=1,...,T} J(I, u_t(I)) = \sum_{i\in I} \left\{ \sum_{t=1}^{N} \{a_2 u_t^i(I) + a_3 u_t^i [L_*^i(I) - h_{t+1}^i(I)]\} \right\} + C \qquad (7)$$

subject to

Eqns. (2), (3), (4), (5) (8)

where C is a constant representing the fixed costs. Because C is a constant, it does not affect the determination of the operating costs. Therefore, the CDDP can be used to solve the model defined in Eqns (7) – (8).

The CDDP used herein is a modification of SALQR (4). Using a penalty function to incorporate the water quality and extraction constraints (Eqns. (3)-(5) into (7)), SALQR solves the optimization problem as an unconstrained problem. This study adopts the penalty function to resolve the water quality constraints that are represented in Eqn. 3. The Quadratic Programming (11) is applied at each stage in the backward and forward sweep of CDDP to handle the control constraints in Eqns. (4) and (5). The penalty function that is used in this study has the following form (10):

$$p_i(f_i) = \xi_i \qquad \xi_i \leq 1$$

$$p_i(f_i) = a\xi_i^2 + b\xi_i^{1/2} + c \qquad \xi_i > 1$$

with

$$\xi_i = (w_i^2 f_i^2 + \varepsilon_i^2)^{1/2} + w_i f_i$$

where w_i is the weighting coefficient of the ith constraint, ε_i is a shape parameter of the hyperbolic function ξ_i, and a, b, and c are constant coefficients. Chang et al. (4) demonstrated that this hyperbolic penalty function, ξ_i, is numerically efficient; it was later used by Culver and Shoemaker (5,6,7) as well as Mansfield and Shoemaker (15). In all cases, weights on the penalty function increased until an optimal solution that did not significantly violate the constraints was found.

The GCDDP requires recalculation of the problem defined by Eqns. (7)-(8), dramatically increasing the total computational effort. If the computational effort in CDDP can be decreased, the total CPU requirement will decrease significantly. Therefore, this study applies the sparsity structure of the derivative equations on state transition, developed by Mansfield, et al. (14), to reduce the computational effort of the CDDP. Using the sparsity structure, the CPU time falls to about 12% of an algorithm that neglects sparsity. Furthermore, each CDDP calculation requires an initial nominal policy to get started. Therefore, a systematic procedure to obtain an initial nominal policy associated with each chromosome is required. This study uses a "do-nothing" policy (all zero pumping rates) as an initial nominal policy for all the chromosomes.

Step 2: Reproduction

This study carries out reproduction by tournament selection (19). The selection mechanism plays a prominent role in driving the search towards superior individuals and maintaining high genotypic diversity in the population. In each tournament selection, a group of individuals are randomly chosen from the population, and the fittest individual is selected for reproduction. This procedure is repeated until the number of chromosomes required for crossover is fulfilled. Therefore, strings with above average objective function will have an above average probability of being selected as parents. The algorithm can converge to a set of chromosomes with high fitness values.

Step 3: Crossover

Crossover involves randomly coupling the newly reproduced strings, with each pair of strings partially exchanging information. Crossover aims to exchange gene information so as to produce new offspring strings that preserve the best material from both parent strings. Generally, the crossover is performed with a certain probability (p_{cross}) to ensure it is performed on most of the population. Herein, one point crossover is selected, as shown in Fig. 4, where p_{cross} ranges from 0.8 – 1.0.

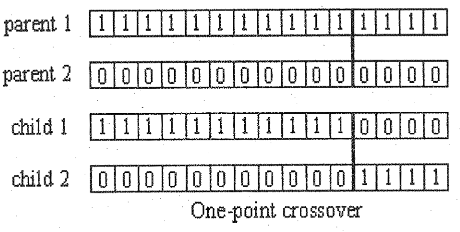


Figure 4. Crossover operator

Step 4: Mutation

Mutation restores lost or unexplored genetic material to the population, preventing the GA from prematurely converging to a local minimum. A mutation probability (p_{mutat}) is specified so that individual genes can be mutated randomly. The value of p_{mutat} normally ranges from 0.01 – 0.05. Before implementing a mutation, a random number with a uniform distribution is generated. If this number is smaller than the mutation probability, the mutation is performed; otherwise, it is skipped. Notably, mutation changes a specific gene ($0 \rightarrow 1$ or $1 \rightarrow 0$) according to the specified probability in the offspring string that is produced by the crossover operation. An example of mutation is displayed in Fig. 5, and is displayed as the block is changed from 0 in the old string to 1 in the new string.

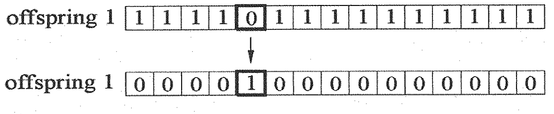


Figure 5. Mutation operator

Step 5: Termination

After steps 1 to 4, a new population is formed. The new population necessitates evaluating the total cost of the groundwater remediation problem as in step 1. Meanwhile, total cost evaluation is used to calculate the fitness and to assess the stopping criterion. The stopping criterion is based on the change of either objective function value (total cost) or optimized parameters. If the best design does not improve over a preselected number of generations (roughly 10), or the maximum number of generations is exceeded the algorithm will terminate, otherwise, the cycle repeats itself (another generation).

NUMERICAL RESULTS

A groundwater reclamation test problem, which is a modification of the example from Chang et al. (4) and Culver and Shoemaker (7), is adopted to verify the effectiveness of the methodology discussed in the preceding section. Figure 2 displays the aquifer. The hydraulic head distribution prior to pumping is assumed to be steady, the initial peak concentration within the aquifer is 150 mg/L, and the water quality goal at the end of five years must be less than or equal to 0.5 mg/L at all the observation wells. The total pumpage for all the wells at each stage must be less than 2.0 $m^3/\sec$, and the maximum and minimum capacities of each well must be 0.5 and 0 $m^3/\sec$, respectively. The simulation period between each stage in the management model is 91.25 days. Table 1 lists the properties of the aquifer. The crossover and mutation probabilities in the GA algorithm are 0.8 and 0.01, respectively.

TABLE 1. Aquifer properties of the example application

Parameter	Value
Hydraulic conductivity	$4.31\times10^{-4}\,\mathrm{m/s}$
Longitudinal dispersivity	70 m
Transverse dispersivity	3 m
Diffusion coefficient	$1\times10^{-7} m^2/s$
Storage coefficient	0.001
Porosity	0.2
Sorption partitioning coefficient	$0.245\ cm^3/g$
Media bulk density	$2.12\ g/cm^3$
Aquifer thickness, b	$10\ m$
L_*	120 m

Case without fixed costs

The problem considering operating costs only is solved independently by both CDDP and GCDDP to illustrate the advantages of GCDDP. This case contains twenty-four potential installation sites for pumping wells that remove the contaminant plume as depicted in Fig. 2. This case attempts to minimize the operating costs of the pump and treatment system. The cost function of the CDDP algorithm is expressed as follows:

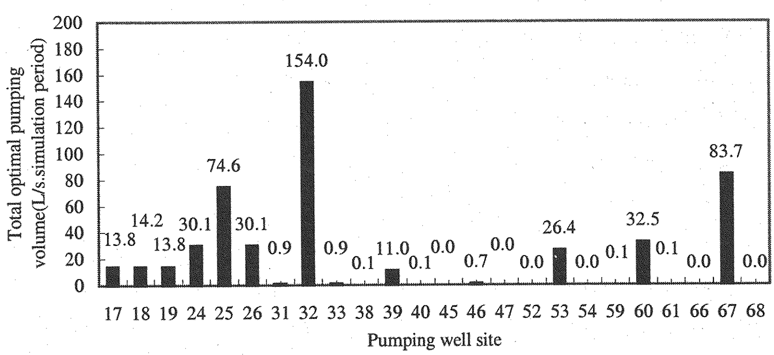
$$\min_{u_t, t=1,\ldots,T} J(u_t) = \sum_{t=1}^{N} \{a_2 l^T u_t + a_3 u_t [L_* - h_{t+1}]\} \tag{9}$$

where a_2 and a_3 are constants as provided in Table 2. l^T is an m row vector with value of 1's. Figures 6, 7, and 8 illustrate the solutions of these cases. Figures 6 and 7 display the optimal total pumping volume of each well obtained by CDDP and GCDDP for the total planning horizon. According to Fig. 6, although eighteen wells have pumpage, seven of these have a very small total pumping volume (less than 1 ($L/s \cdot simulation\ period$). The possibility of obtaining a network design with many wells which have small pumping rates is the disadvantage of applying a gradient-base algorithm such as CDDP or other nonlinear programming on the groundwater remediation planning as indicated by McKinney and Lin (13).

TABLE 2. The values of the Cost Function Coefficient in the Example Problem

Coefficient	Value
a_1	\$ 0 m^{-1} to \$ 240 m^{-1}
a_2	\$ 40000/($m^3 / s \cdot simulation\ period$)
a_3	\$ 1000/($m^3 / s \cdot m \cdot simulation\ period$)

Figure 6. The optimal total pumping volumes of each well solved by CDDP.



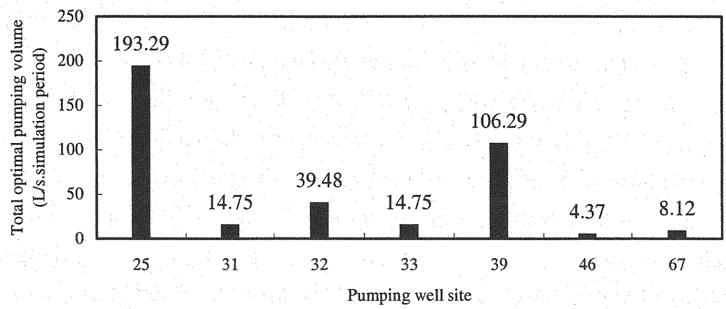


Figure 7. The optimal total pumping volumes of each well solved by GCDDP.

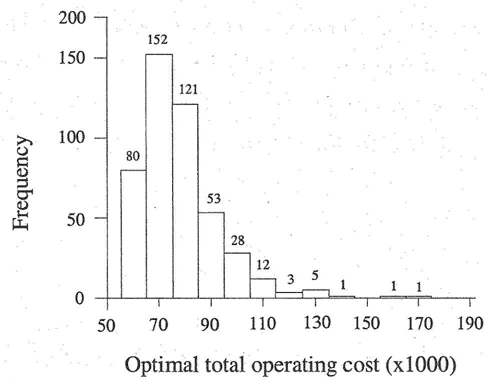


Figure 8. Histogram of the optimal operation cost for the 457 chromosomes in GCDDP (without fixed cost)

TABLE 3. Optimal solutions without fixed cost

Algorithms	No. of wells	Total operating cost
CDDP	18	71938
GCDDP	7	56341

The same problem is solved again by GCDDP. The algorithm converges after sixteen generations and the number of chromosomes in each generation is one hundred twenty. Table 3 provides evidence that both the total optimal operation cost and well numbers obtained by GCDDP are less than that determined by CDDP. The results can be explained as follows: the design of twenty-four potential pumping wells should have the minimum operation costs since it has the largest degree of freedom to manipulate the pumping rates. However, a single run of CDDP can only derive the local optimal solution since groundwater remediation problems are nonlinear non-convex problems. On the other hand, each chromosome (network alternative) in the GCDDP algorithm requires a CDDP computation. Hence, a multiplicity of CDDP solutions are produced and compared before the GCDDP algorithm is finished. Although each CDDP solution may only generate the local optimum, some of the local solutions may be less expensive than that of the previous case with twenty-four potential wells and solved by CDDP only. Therefore, the GCDDP can derive a better solution than the CDDP because the GA procedure within the GCDDP selects an optimal design among the provided alternatives. Figure 8 illustrates the histogram of the optimal operating costs for 457 distinct chromosomes within the sixteen generations. Nearly 60% of optimal total operating costs are approximately \$65,000 to \$85,000. A small percentage of optimal total operating costs are between \$55,000 to \$65,000, which is less than the optimal total operating cost of the previous twenty-four potential wells design. Thus, the proposed GCDDP algorithm can attain a better solution than a CDDP algorithm even for a non-fixed cost remediation problem.

Figures 9 and 10 illustrate the optimal network design and distribution of the pollutant concentration at the end of the planning period with respect to the CDDP and GCDDP solutions. Only eleven wells in Fig. 9 have a total pumping volume more than 1 ($L/s \cdot \textit{simulation period}$). The concentration distribution in the two figures is very similar, and both are within the specified water quality standard.

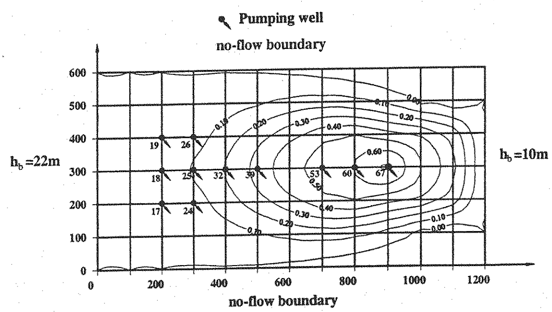


Figure 9. The optimal number of wells and concentration distribution for CDDP (only 11 wells are presented).

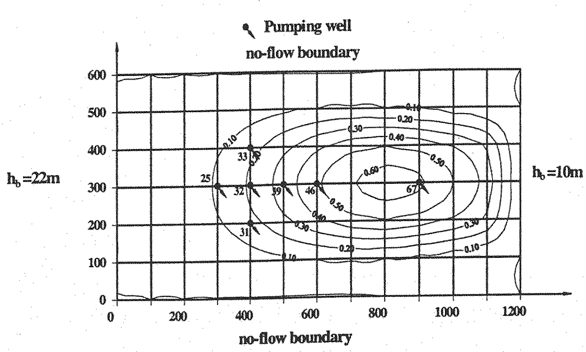


Figure 10. The optimal number of wells and concentration distribution by GCDDP (without fixed cost)

Cases with fixed costs

This subsection deals with several cases with distinct unit fixed costs to investigate the impact of fixed costs. All the cases are solved by GCDDP and the objective function includes both the fixed and operating costs. The coefficients of the cost function are listed in Table 2, while Table 4 summarizes the number of strings (chromosomes, network alternatives), CPU times and generations calculated in each case. Each generation contains one hundred twenty chromosomes (binary strings) in all cases. The case where a_1 equals zero has been described in the previous subsection. Table 5 summarizes the results of various unit fixed costs including the optimal number of wells, optimal total operating cost, minimum and maximum total pumping volume of the wells for the planning horizon. Table 5 includes the previous case with no fixed costs to facilitate the comparison. The varying unit fixed cost case in Table 5 has varying fixed unit costs for each well, whereas the fixed unit costs of the other cases are the same for all wells. Table 5 reveals that the number of wells with various fixed unit costs decreased when the fixed unit cost is increased from 0 m^{-1} to \$ 240 m^{-1}. Alternatively, the operating cost is increased. The optimal design requires only one well when the unit fixed cost is \$ 240 m^{-1}. The minimum total pumping volume for the case with no fixed cost is only 4.37 ($L/s \cdot simulation\ period$) which is far less than that of other cases. This confirmed that an optimal design tends to have wells pumping at small rates if the fixed cost is not considered. Figure 11 depicts the optimal concentration distribution and locations of well setup with a fixed unit cost of \$ 120 m^{-1}.

Previously, the unit fixed cost (a_l) and hydraulic conductivity was assumed constant in the study area. However, this is unlikely to be true due to typical heterogeneous geological conditions. Therefore, in this study, the value of a_l is spatially varied to simulate the consequence of geological heterogeneity. The location of the wells in an optimal network should be diverse from that in which a constant unit fixed cost is assumed. Figure 12 presents two geological zones and their associated fixed unit costs as well as the optimal concentration distribution, number and location of wells. For the previous cases illustrated in Figs. 9, 10 and 11, the designed pumping wells are concentrated in the west region of the aquifer because the higher hydraulic head in that region requires less pumping. Nevertheless, when the unit fixed cost in the east region is lower than the west region, the well sites are relocated to the east region as illustrated in Fig. 12. Owing to the boundary condition, the well placed in the east region require a larger operating cost than that placed in the west region. Therefore, the total operating costs for the case in Fig. 12 are larger than those for the case in Fig. 11. Comparing Figs. 11 and 12 reveals that the high fixed costs may compensate for the low pumping cost.

Table 4 Computational summaries for GCDDP

Fixed unit costs	Total strings	Total Generations	CPU time (sec)
\$ 0 (m^{-1})	1920	16	108668
\$ 120 (m^{-1})	2040	17	121484
\$ 240 (m^{-1})	2040	17	114339
Varying fixed unit cost	2040	17	111545

Runs were implemented on a PC with AMD Athlon™ 750 MHZ CPU.

TABLE 5. Optimal solutions for the GCDDP

Fixed unit costs	No. of well	Total operating cost (\$)	Min	Max
\$ 0 (m^{-1})	7	56341	4.37	193.29
\$ 120 (m^{-1})	2	59909	179.15	218.92
\$ 240 (m^{-1})	1	75690	502.59	502.59
varying fixed unit cost	2	65230	25.63	406.10

Min: Minimum total pumping volume ($L/s \cdot simulation\ \ period$)

Max: Maximum total pumping volume ($L/s \cdot simulation\ \ period$)

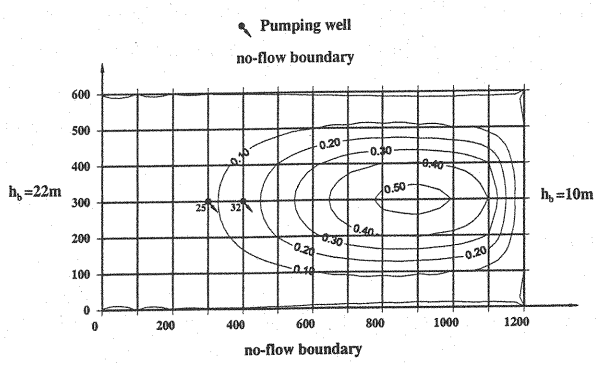


Figure 11. The optimal concentration distribution for GCDDP with fixed unit cost is \$ 120 m^{-1}

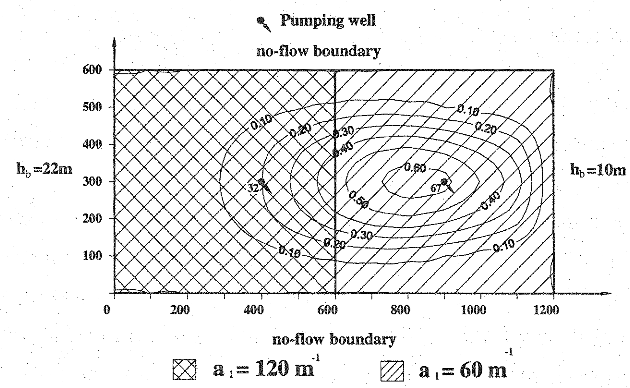


Figure 12. The optimal number of wells and concentration distribution for GCDDP with varying fixed costs.

Total cost comparison

This subsection compares the total cost between the optimal design of CDDP and that of GCDDP to demonstrate the advantage of applying GCDDP, and to compare the fixed costs in the design process. The CDDP determines the optimal network based only on the operating costs. Table 6 summarizes the total cost of the network designs. The total cost of the CDDP design can be estimated by adding the calculated operating costs with the fixed costs and the fixed costs can be estimated by multiplying the well depth by the unit fixed cost. The number of wells in the CDDP designs remains the same for varying fixed unit costs since this method does not consider the fixed cost. On the contrary, the number of wells in the GCDDP designs varies according to the unit fixed cost.

Table 6: Total cost comparison with fixed costs and no fixed costs in an optimization model

Coefficient a_l	\$ 120.0 m^{-1}	\$ 240 m^{-1}
Total cost for the network designed by CDDP (18 wells)	331138 (18 wells)	590338 (18 wells)
Total cost for the networks designed by GCDDP	88709 (2 wells)	104499 (1 well)
Ratio of difference (%)	73.21%	82.30%

Table 6 provides evidence that the total cost of the network designed by CDDP is 82.30 % more than that designed by GCDDP, when the value of coefficient a_1 is \$240.0 m^{-1}. Therefore, a significant total cost saving can be achieved by applying the novel GCDDP algorithm and by considering the fixed costs in the design process.

Other computational issues

Some computational issues are discussed in this subsection since each chromosome in the GCDDP algorithm requires a CDDP computation. Figure 13 depicts the distribution of the iteration numbers in CDDP for all 457 chromosome evaluated in the case with no fixed cost. Although a total of 1920 chromosomes must be evaluated for the case with no fixed cost as indicated in Table 4, only 457 of them are different. 77% of the 457 chromosomes have CDDP iterations less than 100 and the average iteration number is 66. The CDDP calculates the optimal operating cost of each chromosome. Figure 14 reveals more about the CDDP convergence rates within the GCDDP algorithm by presenting the evolution of the total operating cost (the progress of the CDDP computations) with respect to the four selected chromosomes that are the optimal designs of the four cases in Table 5. Figure 14 demonstrates that the proposed CDDP algorithm is computationally efficient for the remediation problem since the four CDDP's computations are all converging within 50 iterations. A highly efficient CDDP algorithm increases significantly the computational power of the GCDDP since a GCCDP calculation requires many CDDP computations.

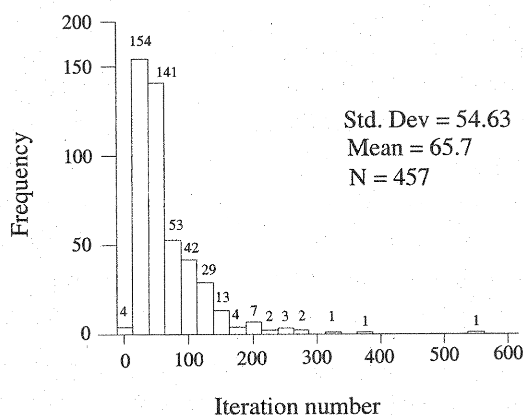


Figure 13. The distribution of the numbers of required iterations for CDDP of all 457 chromosomes.

The GCDDP computation generates one superlative design (chromosome) among the population (120 chromosomes in this study) for each generation and the best design is improved from generation to generation. Figure 15 demonstrates the evolution of the best design versus generation for the case when a_1 = \$ 120 m^{-1}. Figure 15 also illustrates the change in the value of the objective function and the number of wells for the best chromosome in each generation. Although the operating costs increase during the 3rd and 4th generation, the total costs always decrease in each generation and the solutions converged after the 7th generation.

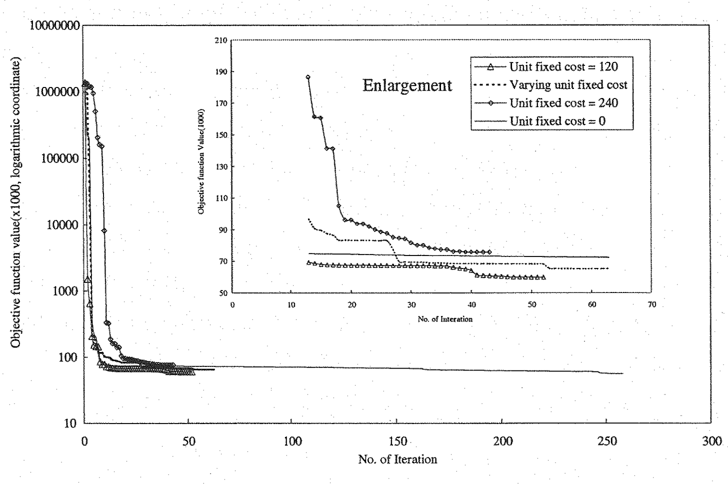


Figure 14. The operating costs of the optimal chromosome of the four examples as a function of the iteration.

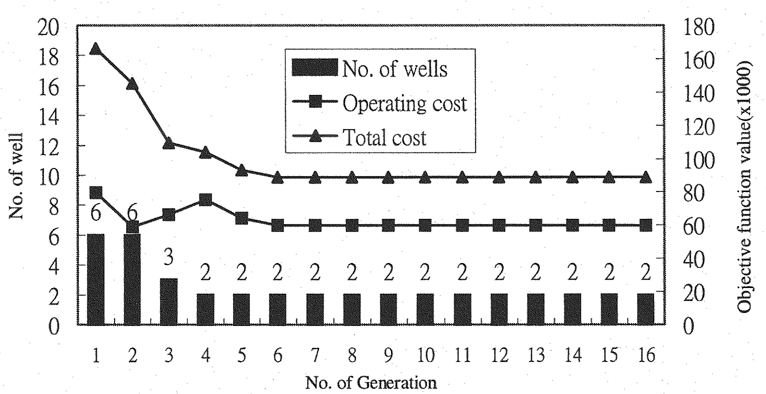


Figure 15. The objective function values and the number of wells versus the number of generations.

CONCLUSION

A GCDDP (an integration of GA with CDDP) groundwater remediation planning model was developed to minimize the total cost of a pump-and-treat aquifer remediation system. Although the total cost including the fixed and operating costs should be the objective function of a groundwater remediation problem, previous studies have not considered this since the problem contains discrete nature of the fixed cost and dynamic characteristics of the operating cost.

The proposed GCDDP algorithm calculates the minimum total cost while simultaneously considering the fixed and time-varying operating costs. A numerical study based on a homogeneous, isotropic confined aquifer revealed several salient facts. A CDDP algorithm consistently designs a remediation plan with many wells pumping at small rates since it only considers the operating cost. However, the GCDDP algorithm can overcome the problem by considering the fixed cost in the design process. The total cost of a CDDP design can be significantly higher than that of a GCDDP design for a high unit fixed cost. Several case studies also indicated that the fixed cost can significantly influence the number of wells and the locations of the optimal remediation design. Thus, the GCDDP algorithm is a feasible groundwater remediation planning method. In conclusion, the novel GCDDP algorithm can consider the fixed cost, which is a significant factor in the process of groundwater remediation planning, and in the design process of providing a more realistic solution.

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