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HIERARCHICAL TIME-SCALE STRUCTURE IN THE DEPENDENCE OF RAINFALL DISTRIBUTION ON TOPOGRAPHY

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SYNOPSIS

Temporal and spatial variations of rainfall distribution over mountainous regions are very complicated particularly in Japan. However, if special attention is focused on the hierarchical time-scale structure in the dependence of rainfall distribution on topography, it is possible that the properties of rainfall distribution may become clear when the accumulation time scale is greater than a boundary scale. In other words, it is possible to device a formula to explain the properties by using topographic features. Through the analysis of data observed by weather radars, which are located in the southern Kyushu region and in the Kinki region of Japan, it was found that rainfall distribution over mountainous regions has a hierarchical time-scale structure. Moreover, this study attempts to determine the characteristics of the hierarchical time-scale structure by investigating how the properties of rainfall distribution differ with the types of accumulated rainfall events.

1. INTRODUCTION

It is very difficult to determine the properties of rainfall distribution in Japan, because the influence of topography on rainfall is so significant. Moreover, due to some physical restrictions, it is not possible to set up rain gauges at a large number of locations in such regions. As a result, rainfall distribution structures in mountainous regions are not yet obvious.

One of the common aspects of past studies on rainfall distribution is that the accumulation time scale of analyzed rainfall distribution is relatively short scale, that is, the time scale of one or two rainfall events. Oki et al.(1), for example, explained clearly the relation between topographic effects on accumulated rainfall distribution and its temporal-spatial scale. They concluded that the topographic effect becomes very significant when the accumulation time scale is a little longer than the temporal-spatial scale of an individual meteorological disturbance (i.e., in the time scale of one rainfall event or daily rainfall). We agree with this conclusion. However, when our attention is focused on the rainfall distribution in such an accumulation time scale, it becomes difficult to determine the properties of rainfall distribution since the topographic effect on rainfall varies significantly with each meteorological disturbance.

On the other hand, Nakakita et al.(2) suggested that the relation between rainfall distribution and topography varies with the meteorological disturbance. However, it is possible to find a universal relation between them when special attention is focused on the topographic effects over an even longer accumulation time scale. Therefore, it is necessary to take such a long time scale into consideration that covers the scale of plural meteorological disturbances for the purpose of determining the universal properties of rainfall distribution.

In past studies, the data obtained from ground-based rain gauges that are set at discrete locations are used as rainfall data. However, there is a limit to reproducing rainfall distributions in mountainous regions by using such data, which does not have adequate observation density in mountainous regions.

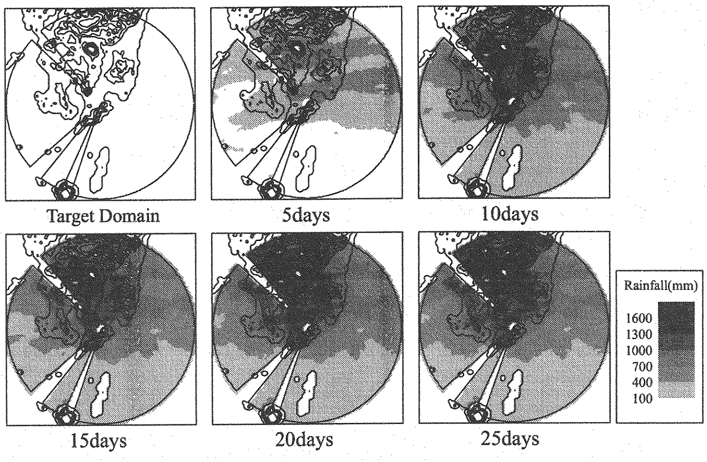


Fig. 1 Accumulated rainfall distributions at various time scales (southern Kyushu region, from August 1,1993).

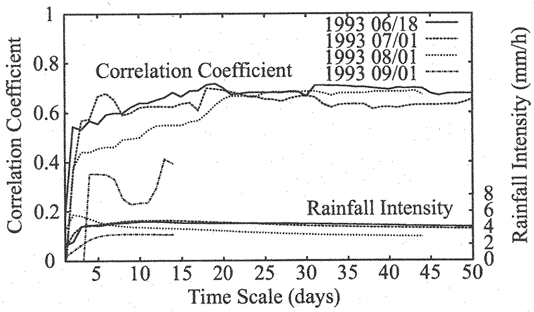


Fig. 2 Correlation coefficient between T-days-accumulated rainfall $R_T(x, y)$ and topographic elevation $h(x, y)$ (southern Kyushu region).

During our research, we examined closely the hierarchical time-scale structure in the dependence of rainfall distribution on topography as one of the properties of rainfall distribution. We tried to determine the universal properties of the dependence on topography, and to make clear rainfall distribution structure through the analysis of rainfall data observed by weather radars.

Our research analyzed the data observed by the Kunimiyama Radar, which is located in the southern Kyushu region, and the Miyama Radar, which is located in the Kinki region of Japan. Both radars are owned by the Japanese Ministry of Construction. They observe the inside of a circular range with a radius of 120 km. The spatial resolution of the observed data is 3 km. The data is converted to an 80 by 80 x-y grid data set. The temporal resolution is 5 minutes. At this point, we had to investigate the problem of ground clutters. In terms of the radar data in this study, ground clutters were removed by using the substruction method for the Kunimiyama Radar, and the MTI method for the Miyama Radar. Therefore, we consider that they pose no problem for this study. However, there are obvious influences at some grids of ground clutters and shadow, which is the area behind mountains, and thus these grids were excluded from the target domain of this study. A digital elevation data set with a spatial resolution of 3 km is used for the data of topographic elevations. The data set is produced by averaging the digital elevation model (DEM) data set with a resolution of 1 km.

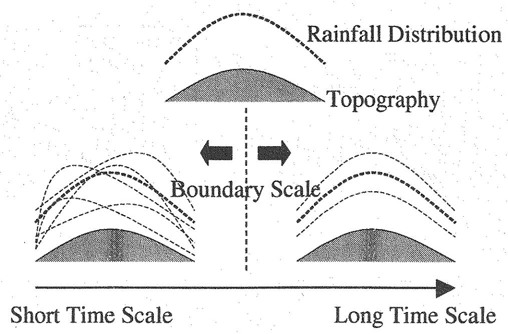


Fig. 3 Schematic figure of the hierarchical time-scale structure of rainfall distribution.

2. HIERARCHICAL TIME-SCALE STRUCTURE OF RAINFALL DISTRIBUTION

Spatial rainfall distributions at various accumulation time scales of between 5 and 25 days are shown in **Fig. 1**. This figure shows that the relation between rainfall distribution and topographic elevation becomes clear as the accumulation time scale of rainfall increases. Furthermore, it shows that the relation does not vary so significantly in the accumulation time scale longer than approxmately 20 days, and appears to converge into a constant relation as the time scale increases.

The correlation coefficient (C.C.), between T-days-accumulated rainfall distribution for each point $R_T(x, y)$ and topographic elevation $h(x, y)$, is then calculated while changing the accumulation time scale. The results are shown in **Fig. 2**. This figure also shows that the correlation coefficient increases as the accumulation time scale increases, and that the value becomes almost constant after the time scale reaches approximately a month.

These results indicate that universal relations between rainfall distribution and topography can be found only if special attention is focused on the properties of long-time-scale rainfall distributions (e.g., one-week or one-month rainfall distribution). The reason is that predominant topographic effects can be found in them, unlike the relation between short-time-scale rainfall distribution (e.g., one-event rainfall distribution) and topography, which varies in a number of ways. This concept is represented in a schematic form in **Fig. 3**.

In this study, this feature of rainfall distribution is called "hierarchical time-scale structure in the dependence of rainfall distribution on topography". By drawing attention to this, we attempted to devise a formula to explain the dependence of rainfall distribution on topography, which has been difficult in past studies. In other words, we attempted to determine rainfall distribution structures in various time scales, including short and long time scales. This paper describe these analyses of the hierarchical time-scale structure of rainfall distribution.

3. BOUNDARY SCALES OF THE HIERARCHICAL TIME-SCALE STRUCTURE

3.1 The Relations Between Boundary Scales and Types of Rainfall Events

It is necessary to know the time scales which are the boundary of the hierarchical structure in order to determine exactly its properties. It is also important to determine whether it is a universal feature of rainfall that the boundary scale is approximately 20 days as shown in **Fig. 2**. Therefore, researches conducted to determine the smallest accumulation time scale T_{min} at which the correlation coefficient between rainfall distribution and topographic elevation become high enough. However, if there is a long period with no rain or with only a little rain in a accumulation period, the smallest time scale T_{min} is influenced and increases. It is necessary to remove the influence by excluding such dry periods for the purpose of the analysis of the time scale T_{min}. Therefore, an imaginary time series of rainfall intensity was produced only with the data of which regional average is over 1 mm/h, and was used to examine the smallest time scale T_{min} by changing the start time of accumulation on the imaginary time series.

In order to exclude the dry periods without breaking the continuity of rainfall in a duration of rainfall, the value of rainfall intensity of 1 mm/h is chosen as a limit value. Excluding the dry periods also provides

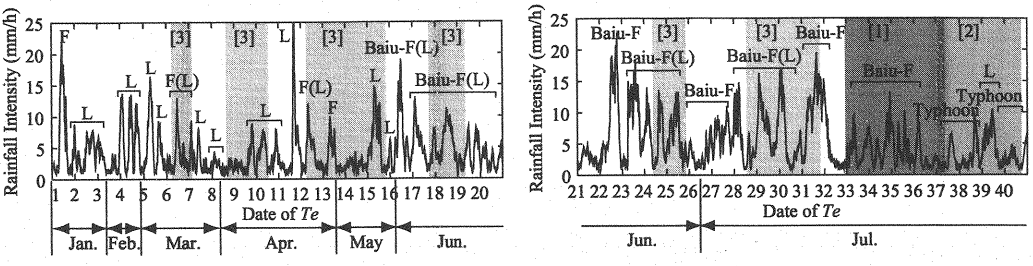


Fig. 4 Imaginary time series of regional average rainfall intensity (southern Kyushu region, 1993).

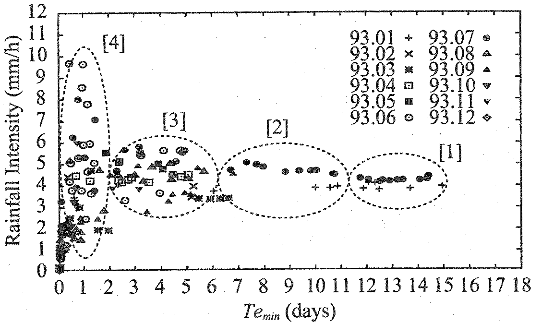


Fig. 5 The smallest time scale Te_{min} and regional average rainfall intensity.

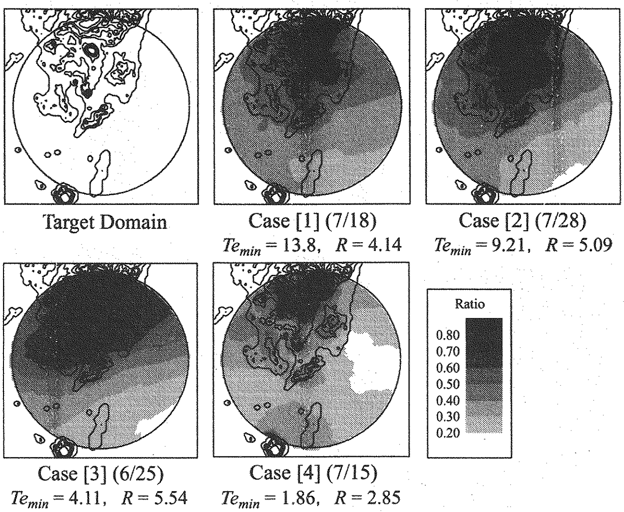
an advantage in that ground clutters in the dry periods can be removed. This imaginary time series of rainfall intensity is represented in **Fig. 4**. In this study, the time scale on the imaginary time series is called the effective rainfall time and is expressed as Te (days). At the same time, the smallest time scale T_{min} on the imaginary time series is expressed as Te_{min} (days).

The smallest time scale Te_{min} at which the correlation coefficient between Te-days-average rainfall distribution $R_{Te}(x, y)$ and topographic elevation $h(x, y)$ becomes over 0.73 is calculated for the imaginary time series. The result is represented in **Fig. 5**. In this figure, the marks of plots represent the month of the start time of accumulation. In this analysis, however, rainfall distribution and topographic elevation were smoothed by using the moving average method (i.e., approximately 7 × 7 grids where a target point is on the center are averaged) in order to make it clearer that the hierarchical time-scale structure exists. As a result, the correlation coefficient increased by approximately 0.1 on the whole. The correlation coefficient value of 0.73 was chosen as the most suitable value to distinguish the boundary time scales of the hierarchical structure. **Fig. 5** shows that the smallest time scale Te_{min} varies from less than 1 day to around 15 days.

The plots in **Fig. 5** are then classified into 4 cases according to the time scale, and the case numbers are labeled. In **Fig. 4**, the case numbers are also labeled according to the time when accumulation of each case begins. The types of rainfall events on a large scale that are estimated with weather maps are shown in **Fig. 4**. In the figure, L expresses a low pressure system, F expresses a frontal system and F(L) expresses a low pressure system with a front.

These figures show that Case-[1], where the time scale Te_{min} is the longest, corresponds to the case where accumulation of rainfall began during the rainy season with Baiu rainfall in July. Case-[2], where the time scale Te_{min} is the second longest, corresponds to the case where accumulation of rainfall began during the period with a series of typhoons in late July. In the same way, Case-[3] corresponds to the period with low pressure systems. These findings demonstrate that the properties of the relation between rainfall distribution and topographic elevation on a large scale varies greatly according to the types of rainfall events (e.g., rainfall with a low pressure system or a frontal system).

Our attention is focused on the various types of rainfall events on a small scale, for example stratiform



Target Domain

Case [1] (7/18) $Te_{min} = 13.8$, $R = 4.14$

Case [2] (7/28) $Te_{min} = 9.21$, $R = 5.09$

Case [3] (6/25) $Te_{min} = 4.11$, $R = 5.54$

Case [4] (7/15) $Te_{min} = 1.86$, $R = 2.85$

Fig. 6 The length of a rainfall period where rainfall over 1 mm/h is observed at each point, which is expressed as the ratio toward Te_{min} (days) (R : regional average rainfall intensity (mm/h), southern Kyushu region, 1993).

rainfall or convective rainfall, in the following section.

3.2 Spatial Distribution of the Length of a Rainfall Period

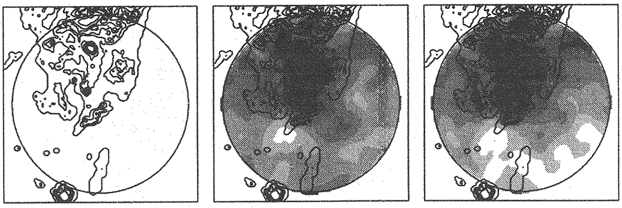
We consider that there are two factors that determine the dependence of rainfall distribution on topography. One is the dependence of rainfall intensity, and the other is the dependence of the length of a rainfall period. Therefore, it is necessary to distinguish the two factors when we investigate the effect on rainfall distributions of topography.

We define a rainfall period as a period where a rainfall over 1 mm/h is observed. And its length is calculated at each point in the target domain to determine which is the main factor of the variation of the time scale Te_{min} according to the types of rainfall events. The results are represented in **Fig. 6**. Each figure in **Fig. 6** corresponds to an example from each case displayed in **Fig. 5**. The contours in these figures express the ratio between the length of a rainfall period at each point and the smallest time scale Te_{min} of each case for standardizing the values of the length.

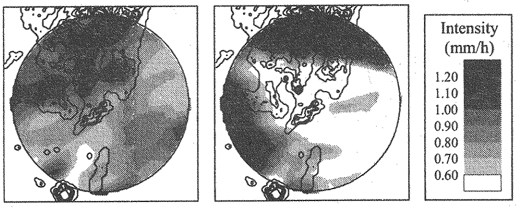
These figures demonstrate the following. In Case-[1] corresponding to the case of rainfall with frontal systems, the time scale Te_{min} is the longest and the values of the contours are relatively small on the whole. The spatial bias of the values toward mountainous regions is also small in this case. Case-[2], corresponding to the case of rainfall with typhoons, appears to have a larger bias toward mountainous regions than Case-[1]. And Case-[3], which had rainfall with low pressure systems, appears to have an even larger bias than Case-[2]. These results indicate that the bias toward mountainous regions becomes larger as the time scale Te_{min} becomes smaller. In Case-[4], where the time scale Te_{min} is the smallest, the relation between the contours and the types of rainfall events is not apparent. It is, however, considered that the time scale Te_{min} is the smallest since a rainfall greatly depending on topography occurred by accident.

Generally speaking, a rainfall with a frontal system includes both a small region with a convective rainfall and a large region with a stratiform rainfall. Thus, it can be said that Case-[1] corresponds to the case of a stratiform rainfall. On the other hand, Case-[2] and Case-[3] can be said to correspond to the case of a convective rainfall, which is influenced only slightly by atmospheric conditions on the synoptic scale in contrast to stratiform rainfalls. Therefore, **Fig. 6** shows that the accumulation time scale, which was necessary until the topographic effect became predominant, is longer in the case of a stratiform rainfall than in the case of a convective rainfall.

Furthermore, in the case of stratiform rainfalls, the dependence of the length of a rainfall period on topography is considered in general to be relatively small, since the rainfall frequency is influenced by atmospheric



Target Domain Case [1] (7/18) Case [2] (7/28)



Case [3] (6/25) Case [4] (7/15)

Fig. 7 The distribution of rainfall intensity where rainfall over 1 mm/h is observed at each point (southern Kyushu region, 1993).

conditions on the synoptic scale. In the case of convective rainfalls, the dependence is considered to be large, since the frequency of generation of convective cells is significantly influenced by topography, which can become a trigger for generating an ascending current. The results shown in **Fig. 6** are consistent in these respects.

We turn our attention to the investigation of the dependence of rainfall intensity on topography. The average rainfall intensity during rainfall periods at each point is calculated. The results in **Fig. 7** lead to the following findings. The dependence of rainfall intensity does not differ so greatly with the types of rainfall events. Moreover, the degree of dependence on topography is smaller than that of the rainfall frequency. The dependence of rainfall intensity in the case of a convective rainfall is a little greater than in the case of a stratiform rainfall.

The following conclusions can be drawn from our investigations. In the case of a convective rainfall, the rainfall frequency greatly depends on topography, and thus the dependence of the rainfall intensity on topography can be said to be relatively small. The high dependence of the rainfall frequency is the main reason why the correlation coefficient between rainfall distribution and topography becomes high enough to be over 0.73 in a relatively short accumulation time scale. In the case of a stratiform rainfall, the rainfall frequency and the rainfall intensity depend on topography to almost the same extent. However, they depend on topography much less than the rainfall frequency in the case of a convective rainfall does. Therefore, it is necessary to accumulate the dependences of them for a long period until topographic effects become apparent. In other words, the smallest time scale Te_{min} differs greatly, mainly because the dependence features of the rainfall frequency on topography differ with the types of the accumulated rainfall events.

As mentioned above, it is necessary to take the types of rainfall events into account in order to determine the hierarchical time-scale structure of rainfall distribution in detail in the future. Therefore, we must create a way of distinguishing the types of rainfall events by using three-dimensional information from weather radars to investigate the properties of hierarchical time-scale structure of rainfall.

4. INDICES FOR THE EXPRESSION OF THE HIERARCHICAL STRUCTURE

4.1 Properties of Rainfall Distribution in the Kinki Region of Japan

This section shows the investigation on the properties of rainfall distribution in the Kinki region of Japan. As is the same in **Fig. 1**, spatial rainfall distributions with various accumulation periods are shown in **Fig. 8**. However, in the case of **Fig. 8**, the accumulation time scale is expressed with the effective rainfall time Te. That is, only rainfall with an intensity over a given threshold value is subject to accumulation. In this figure, 0.3 mm/h is chosen as a threshold value. This figure shows that the relation between rainfall distribution and

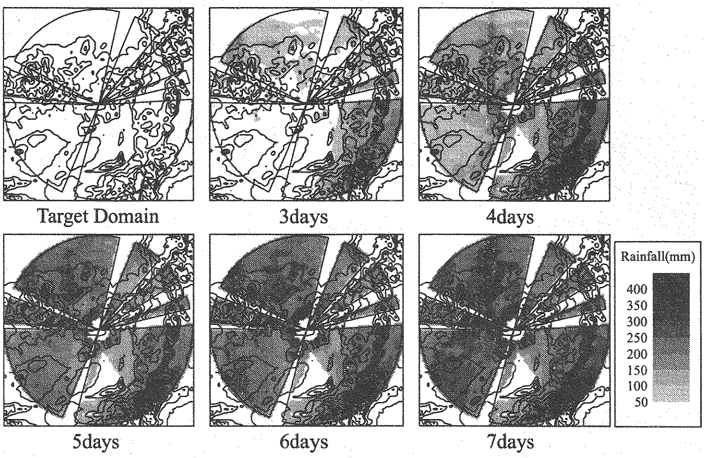


Fig. 8 Accumulated rainfall distributions at each time scale *Te* (Kinki region, from September 1, 1998).

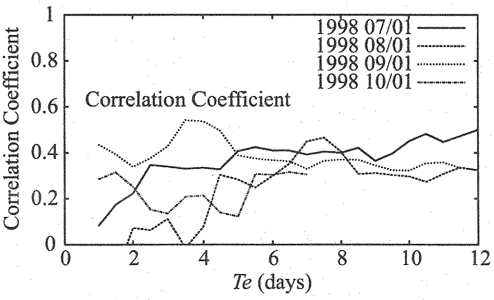


Fig. 9 Correlation coefficient between *Te*-days-accumulated rainfall $R_{Te}(x,y)$ and topographic elevation $h(x,y)$ (Kinki region).

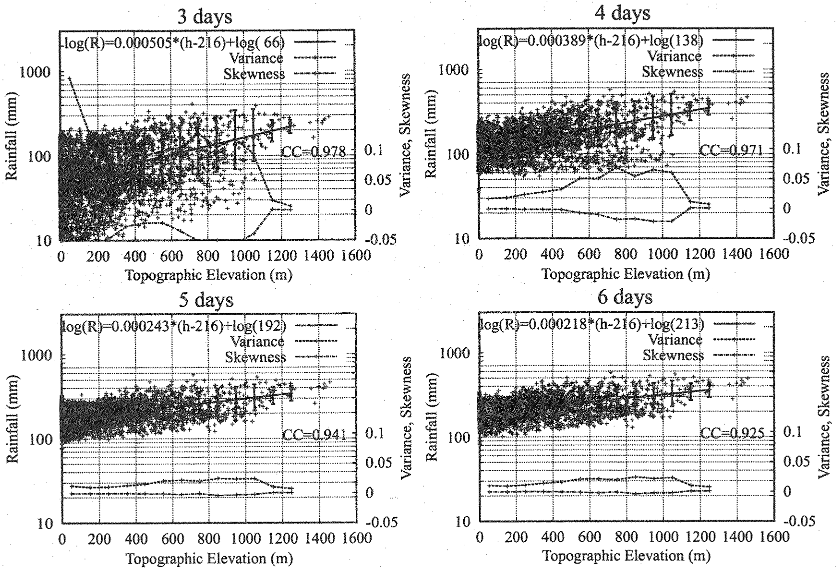
topographic elevation becomes apparent as the accumulation time *Te* increases, as is the same feature of rainfall distribution in the southern Kyushu (**Fig. 1**). Such a tendency is so outstanding around the Kii Mountains, which is located in the south of the Kinki region. However, unlike the southern Kyushu region, there are some regions shown in **Fig. 8** where the dependence of rainfall on topography is not apparent, for example the coastal regions of the Sea of Japan.

The correlation coefficient between *Te*-days-accumulated rainfall distribution and topographic elevation is calculated, as it was for **Fig. 2**, and the result is shown in **Fig. 9**. When compared to the results in **Fig. 2**, it is found that the features of fluctuation are different from those of the southern Kyushu region. That is, the correlation coefficient does not increase with rainfall accumulation and also does not converge into a constant value.

As a result, it is necessary to use another index instead of correlation coefficient in order to analyze the hierarchical time-scale structure of rainfall distribution in the Kinki region. In the next section, we introduce a new index.

4.2 Analysis of the Hierarchical Structure through Classification of Topographic Elevation

The figures in **Fig. 10** represent the relation between topographic elevation, on the horizontal axis, and *Te*-days-accumulated rainfall, on the vertical axis. In these figures, the values of all points in the target domain are plotted. Each figure represents a case with accumulation time scales *Te* of between 3 days and 6 days.



Te = 3 days : September 1~September 21 *Te* = 4 days : September 1~September 24
Te = 5 days : September 1~September 25 *Te* = 6 days : September 1~September 27

Fig. 10 Accumulated rainfall and topographic elevation at each accumulation time scale *Te* (Kinki region, from September 1, 1998).

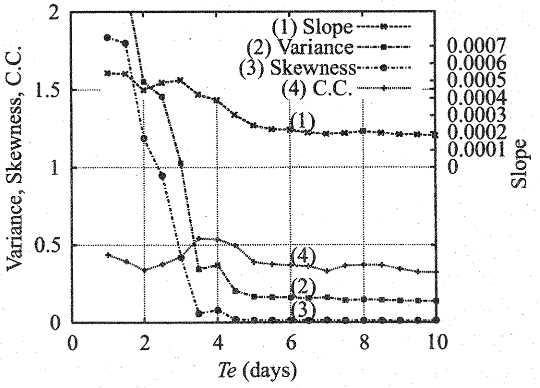
In every case, the accumulated rainfall tends to increase on the whole as the topographic elevation increases. Although such a qualitative tendency has already been reported in other studies which used ground-based rain-gauge data, it is still difficult to devise a formula to explain such properties of rainfall distribution.

Here, the topographic elevation is classified at 100 m intervals, and then a regression line is made by using the spatial averages of accumulated rainfall for each class. The results are shown in **Fig. 10**. These results indicate that it may be possible to determine the dependence of rainfall distribution on topography, since the correlation coefficient between the spatial averages of accumulated rainfall and topographic elevation for each class indicates a high value, which is over 0.9. In addition, in each class, the variance and skewness of accumulated rainfall on a logarithmic axis $\log(R)$ from the regression line are calculated. The results are also shown in **Fig. 10**. These results indicate that the variance of accumulated rainfall in each class decreases as the accumulation time increases, and converges into a constant value. In other words, the relation between rainfall distribution and topographic elevation converges into a constant relation as rainfall accumulation time increases. This is the nature of the hierarchical time-scale structure of rainfall distribution.

The variation of the properties of rainfall distribution described above is shown in **Fig. 11**, where the horizontal axis represents a accumulation time scale *Te*. Some indices are shown in the figure. We should focus our attention on (1) the slope of the regression line, and (2) the variance of rainfall distribution from the regression line, which is represented with mean square errors.

This figure shows that the variance decreases as the accumulation time scale *Te* increases, and then converges into an almost constant value after the *Te* increases to be over about 5 days. This phenomenon indicates the existence of a hierarchical time-scale structure and that 5 days of *Te* is one of the boundary time scales. Furthermore, the slope of the regression line also converges into an almost constant value at the point of about 5 days of *Te*. The point is that the slope of the regression line converges into an almost constant value, and the variance also converges into a constant value. These findings indicate that it is possible to devise a formula to explain the dependence of rainfall distribution on topography by using such statistic values as one of indexes, only if the accumulation time is over a certain value. The reason for this is that the regression line represents a expected value of accumulated rainfall distribution and, therefore, forms the basis of this analysis.

The accumulation time scale of 5 days of *Te* is equal to approximately 25 days in real time, which correspond to the boundary time scale of 20 days found in the analysis in the southern Kyushu region. Moreover,



(1) Slope of the regression line (2) Variance from the regression line (3) Skewness around the regression line (4) Correlation coefficient between *Te*-days-accumulated rainfall distribution and topographic elevation

Fig. 11 Variation of the properties of rainfall distribution (Kinki region, from September 1, 1998).

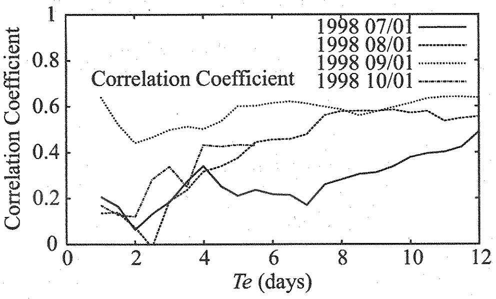


Fig. 12 Correlation coefficient between *Te*-days-accumulated rainfall distribution $R_{Te}(x, y)$ and topographic elevation $h(x, y)$ (only around the Kii Mountains, 1998).

according to **Fig. 5**, the period during September, 1998 nearly corresponds to the period of Case-[3], where 2 days $< Te_{min} <$ 6 days, since a stationary autumnal rain front was observed many times during the period. Therefore, the time scale shown above, 5 days of *Te*, is considered reasonable.

4.3 Analysis of the Hierarchical Structure through Limitation of the Target Domain

As mentioned above, the properties of rainfall distribution in the Kinki region have different features from those in the southern Kyushu region. That is, the correlation coefficient in the Kinki region does not increase so much as the accumulation time increases. One of the reasons for this is that the whole Kinki region does not belong to a simple climatic division. The correlation coefficient was then calculated as in **Fig. 9**, by limiting the target domain to around the Kii Mountains. The result is shown in **Fig. 12**. This figure indicates that the value increases by about 0.2 and the features of the hierarchical time-scale structure appear more clearly as compared to **Fig. 9**. In other words, by limiting the target domain to a region which belongs to a simple climatic division, the hierarchical time-scale structure can be determined by using the correlation coefficient between rainfall distribution and topographic elevation even in the Kinki region.

5. CONCLUSION

In this paper, "the hierarchical time-scale structure in the dependence of rainfall distribution on topography" is investigated for the purpose of determining the properties of rainfall distribution. We found the following findings.

(1) The existence of a hierarchical time-scale structure of rainfall distribution was demonstrated clearly.

(2) Two indices were presented for the purpose of determining the hierarchical structure. One is the correlation coefficient between rainfall distribution and topographic elevation, and the other is the statistic values of the properties of rainfall distribution, for example the variance from the regression line.

(3) It was demonstrated that the properties of the hierarchical time-scale structure varied according to the types of accumulated rainfall events.

(4) The dependence of rainfall distribution on topographic elevation was determined by means of classifying the topographic elevation and investigating the relation between the averages of accumulated rainfall and the topographic elevation in each class.

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