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NUMERICAL ANALYSIS OF THE OVERTURNING OF DENSITY STRATIFICATION IN LAKE KOYAMA USING A ONE-DIMENSIONAL MODEL

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SYNOPSIS

Numerical models to predict the change of thermal and salinity profiles are very important for water quality management in a brackish lake. A one-dimensional simulation model which is applicable to a brackish lake is developed, and computed results are compared with observed data in Lake Koyama. Using the model, the influence of thermal and salinity profiles on the characteristics of the overturning of density stratification is investigated analytically. The frequency of overturning of density stratification based on the results of numerical simulations under various stratified conditions is also discussed.

INTRODUCTION

The occurrence of density stratification by vertical distributions of water temperature and salinity often causes a lack of dissolved oxygen in the bottom layer of a brackish lake, which leads to a deterioration in water quality. Therefore, simulation models to predict the change in thermal and salinity structures should be developed. Thanks to powerful computational resources, numerical models developed recently have changed from one-dimensional to three-dimensional. However, three-dimensional models are not necessarily easy to apply to a lake study. Consequently, in this paper a one-dimensional model is examined on Lake Koyama.

Lake Koyama is located in the eastern part of Tottori Prefecture, Japan, and is connected to the Japan Sea via the Koyama River(see Fig. 1). It is approximately 4 km long and 2.5 km wide (water-surface area is 6.1 km^2) and has a maximum depth of 6.5 m and an approximate mean water depth of 2.8 m. Its volume is 1.9x10^7 m^3 and average residence time is three months. Seawater intrusion is controlled by a gate installed in the middle of the Koyama River and salinity is maintained for some hundreds ppm through the year. The annual average of water quality is reported as COD of about 6 mg/ℓ, total-nitrogen of about 0.8 mg/ℓ, and total-phosphorus of about 0.07 mg/ℓ (7). These data show that this lake is significantly eutrophicated. To purify the lake, dredging has been done to remove bed materials and new sewerage works are being constructed around the lake. However, water quality has not improved so far. One reason for this

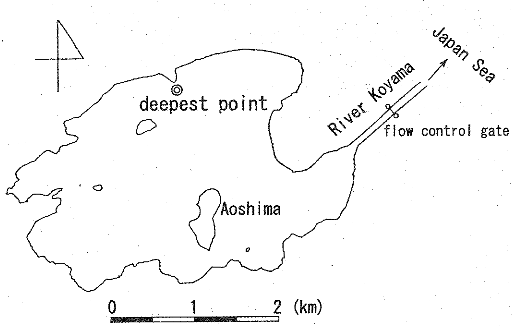

Fig. 1 Lake Koyama

is stratification. Because once the water forms into stratified layers, dissolved oxygen in the bottom layer can easily be consumed and releasing nutrient salts from the bottom make water quality worse. Thus it is important to know the overturning condition of a lake.

Preceding this study, the destratification of the Lake Koyama was examined in terms of two parameters, i.e., the Richardson number (*Ri*) and the Wedderburn number (*We*) (1). Both have been found to be good parameters to evaluate the overturning of stratification. However, the time-dependent changes in stratified structures cannot be predicted using these parameters. In other words, the time required for destratification could not be known without an unsteady process analysis. Consequently, in order to discuss the wind speed and blowing duration required for overturning a formed stratification, a one-dimensional heat and salinity transportation model is examined.

MODEL DESCRIPTION

Basic Equations for One-dimensional Model

The model used here is almost the same as that proposed by Takatsu *et al.* (5). However, special attention is paid to the evaluation of vertical eddy diffusivity, which depends on a vertical profile of the flow velocity and density of water. Transportation equations of heat and salinity, respectively, are:

$$\frac{\partial T}{\partial t} = \frac{1}{A(z)}\frac{\partial}{\partial z}\left\{A(z)Kz(z,t)\frac{\partial T}{\partial z}\right\} - \frac{1}{\rho C_p A(z)}\frac{\partial}{\partial z}\{A(z)q(z)\} \qquad (1)$$

$$\frac{\partial S}{\partial t} = \frac{1}{A(z)}\frac{\partial}{\partial z}\left\{A(z)Kz(z,t)\frac{\partial S}{\partial z}\right\} \qquad (2)$$

where T = temperature of water; t= time; z= the distance measured downward from the surface; $A(z)$= horizontal water area at water depth z; Kz= vertical eddy diffusivity; ρ = density (calculated by Knudsen's formula with water temperature and salinity); C_p = specific heat; q = internal heat flux; and S = salinity. Based on Dake and Harleman's equation, q is expressed as follows.

$$q(z) = (1 - \beta)Q_0 \exp(-\eta z) \qquad (3)$$

where β = the ratio of absorption for short wave radiation near the water surface; Q_0 = net solar radiation; and η = extinction coefficient of internal heat flux. Considering that the transparency in summer is less than 50 cm in Lake Koyama and the result of validation, η is referred to as 2.0 and β is 0, respectively. In numerical calculation, the basin is divided into 30 layers each 20 cm thick.

Boundary Conditions

For boundary condition at the surface (z=0), a series of equations that Murakami *et al.* devised is used (3).

The total amount of heat exchange is as follows.

$$Q_0 = Q_S+Q_L+Q_E+Q_h \qquad (4)$$

where Q_S= effective solar radiation; Q_L= effective longwave radiation; Q_E= flux of latent heat due to evaporation from the surface; and Q_h= sensible heat flux. They are calculated using the following equations. The direction from atmosphere to water surface is defined to be positive in all heat fluxes.

$$Q_S = (1 - r)I \qquad (5)$$

$$Q_L = -\varepsilon\sigma\theta_S^{\,4}(0.39 - 0.058e_A^{\,0.5})(1 - k_C C^2) - 4\sigma\theta_S^{\,3}(\theta_S - \theta_A) \qquad (6)$$

$$Q_E = -L\rho E \tag{7}$$

$$Q_h = -RQ_E \tag{8}$$

where r= albedo for the water surface (=0.04); I= solar radiation (w/m^2); ε=0.96; σ= Stefan-Boltzmann's constant (=5.67×10^{-8} w/m^2/K^4); θ_S= water temperature at the surface (K); θ_A= atmospheric temperature (K); e_A = water vapor pressure near the water surface (hPa); k_c=0.65; L= latent heat per unit mass (w/m^2); E= evaporation rate (m/s); C = cloud index (C = 1.0 when the sky is completely covered with cloud and C = 0.0 for a clear sky); and R= Bowen ratio. R, L and E are given by the following equations.

$$R = 0.66(\theta_s - \theta_A)/(e_s - e_A) \tag{9}$$

$$L = 2.5 \times 10^6 - 2400 \times \theta_S \tag{10}$$

$$E = 1.2 \times U_{10}(e_s - e_A) \times 10^{-5} \tag{11}$$

where e_s= saturation vapor pressure for θ_S; U_{10} = wind speed measured at a height of 10 m (data obtained at Tottori airport about 1 km north of Lake Koyama).

The boundary condition at the bottom (z=H) is as follows.

$$\partial q / \partial z = 0 \tag{12}$$

Vertical Eddy Diffusivity K_Z

As shown in Eq. 13, eddy diffusivity is considered as a function of the Richardson number (Eq. 14).

$$K_z = A_v \cdot f(Ri) \tag{13}$$

$$Ri = g/\rho \cdot d\rho/(du/dz)^2 \tag{14}$$

where A_v = eddy diffusivity under neutral stratification; $f(Ri)$=stratification function with a Richardson number variable; u = flow velocity; $d\rho$ = the difference in density between layers. The following formula is used for a stratification function (2).

$$f(Ri) = (1 + 10/3 \cdot Ri)^{-1.5} \tag{15}$$

Moreover, the following equations which give the different flow velocity profiles for a two-layered lake are used (8).

$$u_u = \frac{\tau_{wind} \cdot h}{A_{V1}} \cdot \left[\left(1+\frac{z}{h}\right)+\frac{1}{4n}\cdot\frac{H-h}{h}\frac{6+3/n\cdot(H-h)/h}{4+3/n\cdot(H-h)/h}\left\{\frac{1}{2}\left(1-\frac{z^2}{h^2}\right)+\frac{H-h}{h}\right\}\right] \tag{16}$$

$$u_d = \frac{\tau_{wind}(H-h)}{nA_{V1}} \cdot \frac{2}{4+3/n\cdot(H-h)/h}\left\{\frac{3}{4}\cdot\frac{H^2-z^2}{(H-h)^2} - \frac{3}{2}\cdot\frac{h(H+z)}{(H-h)^2} - \frac{H+z}{H-h}\right\} \tag{17}$$

$$A_{V1} = 0.0022\sqrt{C_f}\,hU_{10}\left\{1 + \frac{1}{4n}\frac{H-h}{h} - \frac{6+3/n\cdot(H-h)/h}{4+3/n\cdot(H-h)/h}\left(\frac{1}{2}+\frac{1}{4n}\frac{H-h}{h}\right)\right\} \tag{18}$$

$$\tau_{wind} = \rho_a \cdot C_f \cdot U_{10}^2 \tag{19}$$

$$C_f = (1.0+0.07U_{10})\times10^{-3} \tag{20}$$

where A_{V1} = eddy diffusivity of the upper layer at a time of neutral stratification; n = the ratio of eddy diffusivity of the upper layer to the lower layer (=A_{v1}/A_{v2}); C_f= drag coefficient; and ρ_a = air density. The coefficient of n is set to 4 with reference to Yogoshi *et al* (8). The eddy diffusivity *Kz* is estimated by Eqs. 13 - 18.

Verification of the Model under Stable Stratification

The observational data from September 3 to 8, 1997 when a stable density stratification was formed are used to verify the model. During this period, the water temperature and salinity were measured every 20 cm at the deepest point in the lake almost every three hours. The data measured on September 16 (after the overturning of the stratification) are used as well.

As shown in Fig. 2, during the peak observation period (from September 3 to 8), the wind was weak (10 m/s at most) and average wind speed was approximately 4 m/s. Fig. 3 shows that solar radiation exceeded 500 w/m^2 in the daytime except on September 5 and 6. Thus a stable stratification was developed. In addition, the seawater that flowed into the lake in the middle of August was still piled up, so the double density structure composed of thermal and salinity stratification was developed (the salinity of the upper layer was about 130 ppm and that of the bottom layer was approximately 3500 ppm on September 3). After the peak observation period, a typhoon approached to the lake from September 12 to 15 with winds from the northeast exceeding 10 m/s (see Fig. 2). As a result, thermal and salinity stratification was found to be uniformly overturned on September 16.

The simulation started at 18:00 on September 3 for initial conditions and ended at 18:00 on September 8. A high-density layer of high salinity existed in 80 cm of the bottom layer at the start. The results of observation and calculation are shown in Fig. 4 (a) - (d). Comparing Fig. 4 (a) and (c) which show the change in water temperature, it is found that the calculation is in good agreement with the observation. Next, from comparing Fig. 4 (b) with (d), it is easy to see that the calculation reproduces the gradually decreasing trend of high salinity in the lowest layer, though the salinity diffuses slower in calculation than in observation. The difference between them is considered due to the fact that the flow velocity profile estimated by Eq. 17 could not fully express the velocity in the bottom layer, which is affected by bottom shear stress. Additional simulation was maintained from 18:00 on September 8 to 15:00

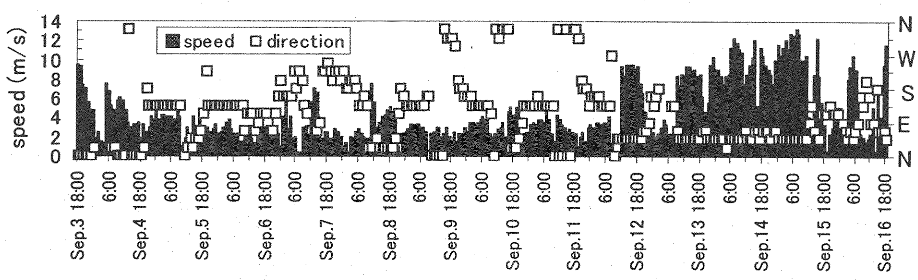

Fig. 2 Wind data at Tottori airport (September 3-16,1997)

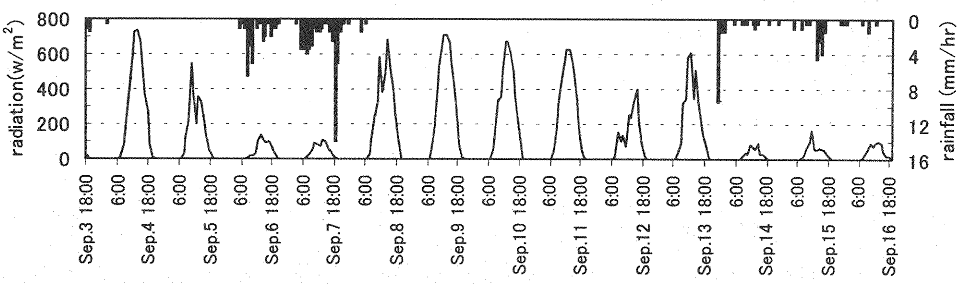

Fig. 3 Solar radiation and rainfall observed near Lake Koyama (September 3-16,1997)

on September 16. Fig. 4 (e) and (f) show the profiles of water temperature and salinity at the final stage. The calculation shows the uniform profiles of temperature and salinity on September 16 as reflected in observed data.

Although a simple one-dimensional model is used in this study, its applicability was confirmed to be excellent.

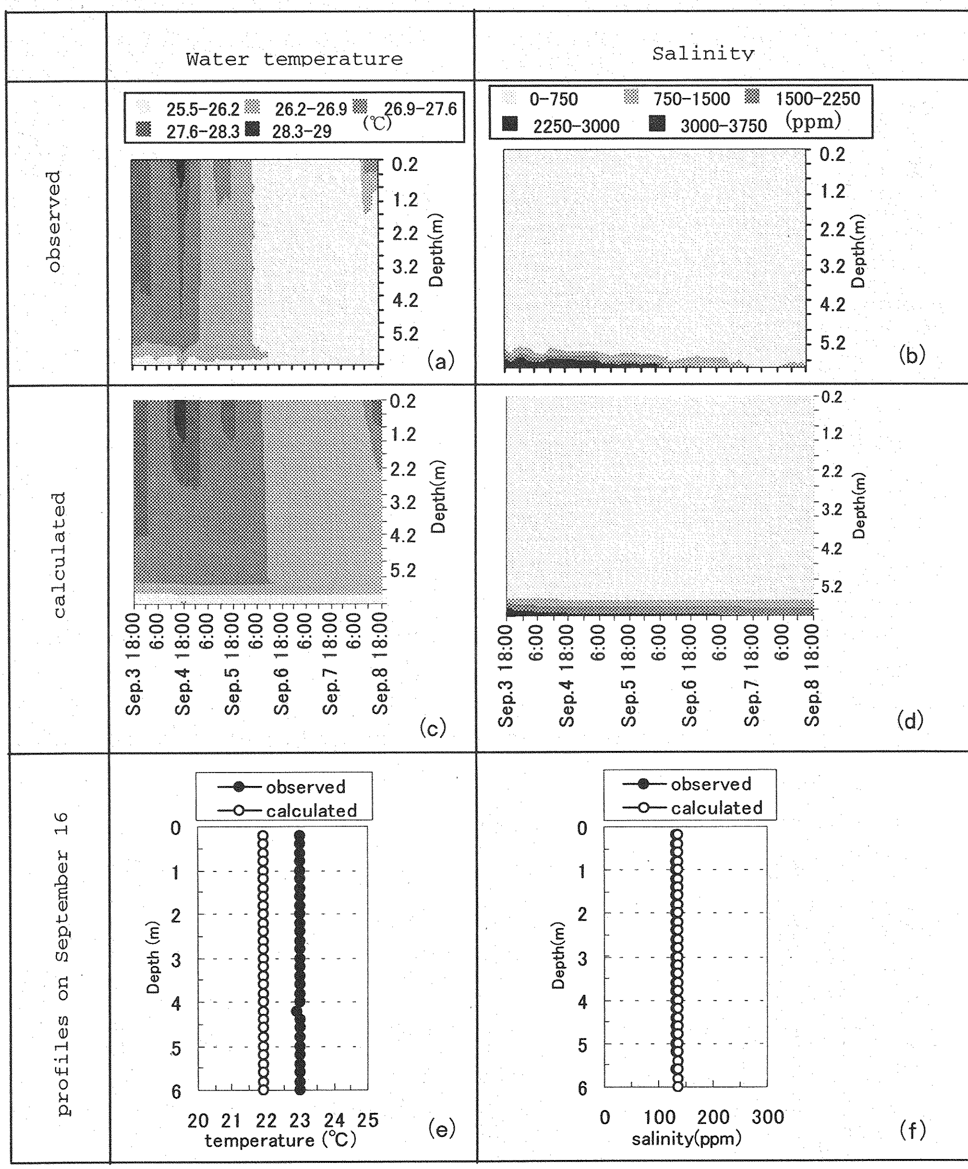

Fig. 4 Comparison of water temperature and salinity profiles between observed and calculated

Overturning of Density Stratification under Constant Wind Speed

To examine the characteristics of the overturning of density stratification under constant wind speed, a one-dimensional mathematical model developed here is applied. For simplicity, the exchange of heat flux at the water surface is not taken into consideration. As to the initial conditions of stratification, the thickness of the upper layer is set at 4 m, and the maximum thermal difference is set at 5 ℃ for thermal stratification based on data from the observation. For salinity stratification, the initial thickness of the upper layer is set at 4 m and 5.2 m, and the salinity difference ranges between 0 and 10000 ppm, while the salinity of the upper layer is kept constant at 500 ppm. These conditions of calculation are listed in Table 1. Besides initial conditions, this table shows the wind speed calculated on the condition that Ri < aspect ratio (condition 1) and We = 3 (condition 2) are also shown. Here note that Ri and We used here are defined as follows, respectively.

・Richardson number (Ri): $Ri = \Delta\rho g h_u / (\rho_u u_*^2)$ (21)

・Wedderburn number (We): $We = Ri \times 2h_u / L^*$ (22)

where $\Delta\rho$ is the density difference between the upper and lower layer, h_u is the thickness of the upper layer, ρ_u is the density of the upper layer, $u_*^2 = C_f \rho_a U_{10}^2 / \rho_0$, ρ_0 is water density at the surface, and L^* is a representative length of the lake in the direction of the predominant winds (here the value L=2.94 km for an idealized diameter is used).

Spiegel and Imberger showed that in condition 1, significant mixing occurs in all layers and the basin will soon become homogenous (4). In condition 2, Thompson and Imberger showed that the interface may setup, and that the bottom layer upwelling takes place mainly via the shear production mechanism (6). In estimating Ri and We, the depth where the maximum density gradient appears is treated as the bottom of the upper layer.

In numerical calculations, the density stratification is considered to be completely mixed when the density difference between the 1st layer (surface) and the 30th layer (bottom) becomes less than 0.5 kg/m^3.

Table 1 Conditions of profile for water temperature and salinity

Case	water temperature (℃)		salinity (ppm)		thickness of upper layer (m)		wind speed for overturning the stratification (m/s)	
	upper layer	lower layer	upper layer	lower layer	water temp.	salinity	condition 1	condition 2
1	30	25	500	500	4	-	7.3	4.2
2	30	25	500	1500	4	5.2	10.3	5.9
3	30	25	500	3500	4	5.2	14.5	8.4
4	30	25	500	10500	4	5.2	24.0	13.8
5	27	27	500	1500	-	5.2	7.2	4.2
6	27	27	500	3500	-	5.2	12.5	7.2
7	27	27	500	10500	-	5.2	22.8	13.1
8	27	27	500	1500	-	4	6.3	3.7
9	27	27	500	3500	-	4	10.9	6.3
10	27	27	500	10500	-	4	20.0	11.5

A relation between the wind-blowing duration and a critical wind speed for overturning density stratification is shown in Fig. 5 (a) and (b). Fig. 5 (a) shows that wind speed required for destratification becomes large or wind-blowing duration becomes longer with increasing salinity difference. For example, wind needs to be more than 12 m/s for overturning a salinity stratification of 3000 ppm (case 3) within 10 hours, and needs to be more than 16 m/s for overturning a salinity stratification of 10000 ppm (case 4).

The density difference corresponding to a thermal difference of 5 ℃ is approximately equivalent to a salinity difference of 1100 ppm. Therefore in a brackish lake, a salinity difference much more predominantly affects stratification than a thermal difference does. Moreover, when a constant wind speed equivalent to destratification condition 1 (see Table 1) continues, it requires about 30 hours for case 1, 10 hours for case 2, 5 hours for case 3 and 4 hours for case 4 to overturn the stratification. Compared with the conditions shown by Spiegel *et al.* (4), the model used here may estimate a longer wind blowing duration or a lower wind speed required for overturning the stratification when the difference between the upper and lower density is small, or when the strength of stratification is weak. However, this model estimates that stratification is overturned in a few hours to a day for the wind speed that Spiegel *et al.* (4) examined. Therefore, it is considered that Fig. 5 (a) provides an appropriate result.

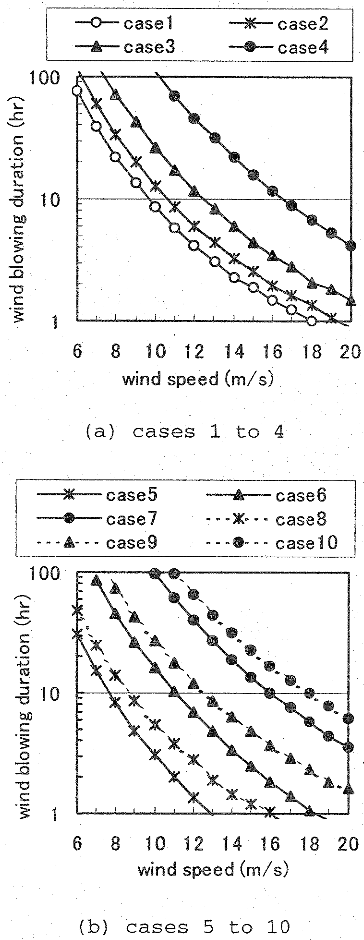

Fig. 5 Relation between wind speed and blowing duration for destratification

To see the influence of the thickness of a high salinity layer, the results of cases 5 - 10 are examined. In these cases a thermal difference is lacking, and the salinity difference ranges between 1000 and 10000 ppm. Cases 5 - 7 have an 80-cm thickness of the high salinity layer and cases 8 - 10 have one of 2 m. Fig. 5 (b) shows that the wind speed required for overturning stratification becomes approximately 2 m/s higher if the thickness of the high salinity layer increases from 80 cm to 2 m. Therefore, the influence of the thickness of the high-density layer is small compared with the salinity difference of the stratification in Lake Koyama.

Overturning of Density Stratification According to Past Wind Records

To examine the frequency for overturning of a formed density stratification, the past data set of wind records at Tottori airport were used. Since stratification can be more of a problem in summer than in winter, the meteorological data from July to September are used, focusing on summer.

The wind records for calculation were selected for when wind speeds exceeded 10 m/s from 1991 to 1998. One record of wind data is defined as follows: a wind event starts when the wind speed falls below 3 m/s before exceeding 10 m/s, and ends when the velocity drops below 3 m/s after exceeding 10 m/s. A total of 79 data sets were

Table 2 Frequency of overturning of density stratification

case	91	92	93	94	95	96	97	98	total (/79)
1	5/11	9/14	3/6	4/9	3/9	7/14	5/11	1/5	37
2	3/11	7/14	3/6	3/9	2/9	5/14	4/11	1/5	28
3	2/11	2/14	2/6	1/9	1/9	2/14	2/11	1/5	13
4	1/11	0/14	0/6	1/9	0/9	1/14	0/11	0/5	3
5	9/11	14/14	6/6	7/9	8/9	8/14	9/11	3/5	64
6	3/11	7/14	2/6	1/9	1/9	4/14	4/11	1/5	46
7	1/11	0/14	1/6	1/9	0/9	1/14	1/11	0/5	5
8	9/11	14/14	6/6	7/9	8/9	8/14	9/11	3/5	64
9	3/11	7/14	2/6	1/9	1/9	4/14	4/11	1/5	46
10	1/11	0/14	1/6	1/9	0/9	1/14	1/11	0/5	5

obtained using these criteria. On average the wind speed exceeded 10 m/s 10 times from July to September, i.e., about 3 times per month.

After calculating based on 79 sets of wind data under the given stratification conditions shown in Table 1, we examined whether or not the stratification is overturned. Table 2 shows that overturning frequency for each case and in each year. As shown in Table 2, at a salinity difference of 1000 ppm (cases 5 and 8), the stratification will be largely overturned when the wind speed exceeds 10 m/s. However, if a thermal difference of 5 ℃ occurs, it would be overturned at a frequency of 1.5 times per month (=37 times/ 8 years /3 months), or, once every 20 days. In practice, thermal stratification becomes weaker at night, so it would presumably not last for long. In addition, in case 3, which is similar to a condition observed in September 1997, a stratification will be overturned about 0.5 times per month (=13/8/3). This means that a stratification with a 3000 ppm salinity difference may be maintained for 2 months. Therefore, if a density stratification due to salinity is produced, the stratification may persist for a long time, and the water quality will be deteriorate accordingly.

CONCLUSIONS

This study examines the characteristics of the overturning of density stratification in a shallow brackish lake using a one-dimensional numerical simulation model. The results obtained in the study are as follows.

If a vertical eddy diffusivity is carefully evaluated, the one-dimensional numerical model developed here can predict hourly changes in the water temperature and salinity in a shallow brackish lake.

The influence of the density difference due to salinity on the overturning of stratification is very significant, and the thickness of the high salinity layer has little effect on the density structure in Lake Koyama.

The numerical simulation results using past wind records show that if the stratification at a salinity difference of 3000 ppm occurs, the density stratification will persist for more than two weeks and water quality will thereby deteriorate.

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APPENDIX - NOTATION

The following symbols are used in this paper:

$A(z)$ = horizontal water area at water depth z;
A_{v1}, A_{v2} = eddy diffusivity of upper and lower layer under neutral stratification;
C = cloud index;
C_f = drag coefficient;
e_A = water vapor pressure near the water surface;
e_s = saturation vapor pressure for θ_S;
E = evaporation rate;
H = maximum water depth of lake;
I = solar radiation;
K_z = vertical eddy diffusivity;
L = latent heat per unit mass;
L^* = representative length of lake in the direction of the predominant winds;
n = ratio of eddy diffusivity for upper to lower layer ($=A_{v1}/A_{v2}$);
q = internal heat flux;
Q_0 = net solar radiation;
Q_S = effective solar radiation;
Q_L = effective longwave radiation;
Q_E = flux of latent heat due to evaporation from the surface;
Q_h = sensible heat flux;
r = albedo for the water surface;
R = Bowen ratio;
S = salinity;
t = time;
T = temperature of water;
u, u_u, u_d = flow velocity, and velocity for upper and lower layer;
U_{10} = wind speed measured at a height of 10 m;
z = distance measured downward from the surface;
ρ = water density;
ρ_a = air density;
ρ_u = density of upper layer;
ρ_0 = water density at the surface;
$d\rho$ = difference in density between layers;
$\Delta\rho$ = density difference between upper and lower layer;
η = extinction coefficient of internal heat flux;
β = ratio of absorption for short wave radiation near the water surface; and
θ_s, θ_A = water temperature at the surface and atmospheric temperature.

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