Journal of Hydroscience and Hydraulic Engineering
Vol. 19, No.1 May, 2001, 33-42

# FREQUENCY ANALYSIS ON CONCURRENCE OF STORM SURGE AND RAINFALL FOR URBAN RIVER SYSTEM AND ITS APPLICATION TO PLANNING FOR STORM DRAINAGE

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## SYNOPSIS

This paper discusses the concurrent characteristics of the maximum storm surge and rainfall during a typhoon by analyzing the records of hourly rainfall, tide level and storm surge caused by typhoons. Using the bivariate exponential distribution theory, the joint distribution function of storm surge and rainfall is obtained analytically. The diagram of return-period is shown in order to evaluate past main typhoon events. We also investigate how to determine the design rainfall depth necessary in planning the drainage for a tidal river in case that the tide gates are closed against the storm surges and the precipitated water has to be artificially drained to the bay by pumping, etc.

## INTRODUCTION

In Japan, most cities have developed on low-lying flat land near the sea and are accordingly exposed to the hazards from storm surges attacking the coastlines whenever typhoons pass. In tidal rivers, there is also the possibility that flooding may occur due to heavy rain right after or before the storm surge attacks. The high water level resulting from the concurrence of storm surge and rainfall is one of the major concerns in flood control. Therefore, it is fundamentally important for planning and management of such river systems to consider the concurrent characteristics of the two phenomena.

The Osaka district has suffered from storm surge disasters many times in the past, due to the geographical properties of the district lying at the back of Osaka Bay and extending to the foot of Mt. Ikoma on the alluvial plain just above the sea level (Figure 1). Three major storm surges associated with typhoons have caused extensive damage to this district in the past 60 years. The maximum storm surges observed during Typhoons Muroto, Jane and the second Muroto were 2.92 m, 2.37 m and 2.22 m, respectively. They caused great damage to the Osaka area; 80,000 to 160,000 houses were submerged and 2,000 to 21,000 people were killed or injured each time (Table 1).

**Table 1 Heavy typhoons that hit Osaka district**

| Typhoon | Muroto | Jane | 2nd Muroto | 7916 |
|---|---|---|---|---|
| Date (D/M/Y) | 21/09/1934 | 03/09/1950 | 16/09/1961 | 01/10/1979 |
| Lowest atmospheric pressure (hPa) | 954.4 | 970.3 | 937.3 | 971.4 |
| Maximum storm surge (m) | 2.92 | 2.37 | 2.22 | 1.07 |
| Highest tide level (OP + m) | 4.19 | 3.85 | 4.12 | 3.04 |
| Maximum hourly rainfall intensity (mm/hr) | 6.8 | 19.8 | 12.2 | 64.5 |
| Total rainfall depth (mm) | 19.5 | 64.7 | 44.2 | 114.5 |
| Cause of damage | Storm surge | Storm surge | Storm surge | Runoff |
| Inundated area (ha) | 4,921 | 5,625 | 3,100 | 1,035 |
| No. of houses inundated | 166,720* | 80,464 | 126,980 | 27,736 |
| Deaths and injuries | 17,898* | 21,465 | 2,165 | 0 |

1. Rainfall and tide level: Derived from records of the Osaka Meteorological Observatory.
2. OP: Datum sea level for the Osaka district. Recent 5-year average of mean sea level of Osaka Bay is OP +1.43 m.
3. Figures marked with *: Data for Osaka Prefecture. Figures for Typhoon 7916 are those in the Neya River Basin.

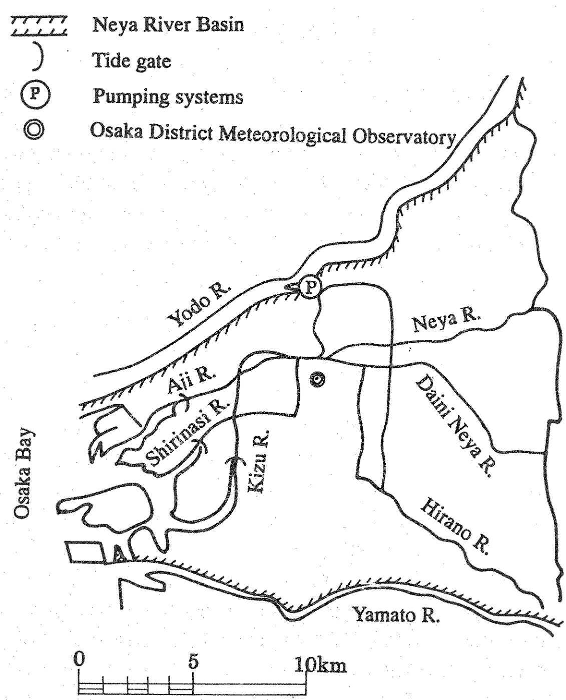



**Fig. 1 Channels of the Neya River System**

The Neya River runs through the center of Osaka Prefecture. To prevent storm surges running up the tidal reach, arch-shaped tide gates spanning 57 m were built in 1970 across each of three branches of the river. The specifications of these gates are based on the storm surge control plan with the design high tide level of OP (Osaka Peil) +5.2 m and the design maximum storm surge of 3.0 m. In this system, flood water is to be pumped out of the Neya River to the Yodo River and stored within the river channels while the gates are closed against storm surges. For the Neya River system, the design rainfall depth adopted for storm drainage has been 19.8 mm/hr which was recorded during Typhoon Jane, because it caused a storm surge of larger than 2.0 m and the highest rainfall (maximum rainfall depth: 19.8 mm/hr and 41.7 mm/4hrs). But this design rainfall of 19.8 mm/hr proved to be underestimated when Osaka was hit by the 16th typhoon in 1979 with a maximum hourly rainfall of 64.5 mm/hr while the gates were closed. This typhoon gave rise to the need for a quantitative re-evaluation of the concurrent characteristics of maximum storm surge and rainfall.

This paper discusses the concurrent characteristics of maximum storm surge and rainfall during a typhoon by analyzing the hourly records of rainfall, tide level and storm surge during individual typhoons that hit the Osaka district over a 45-year period (1950–1994), using bivariate exponential distribution.

## CONCURRENCE OF STORM SURGE AND RAINFALL

### (1) Hydrological Data

This paper discusses the concurrent characteristics of maximum storm surge and rainfall by using the bivariate probability distribution theory. After choosing 120 typhoons which caused storm surges of larger than 25 cm at the Osaka Port over a 45-year period (1950-1994), the authors analyzed the hourly records of rainfall, tide level and maximum storm surge for these typhoons. Maximum surge values were obtained by deducting the astronomical tide from the tide levels measured at the Osaka Port Tide Observation Center. Rainfall values were those observed at the Osaka District Meteorological Observatory.

### (2) Bivariate Exponential Distribution

Here, applicable bivariate probability distributions are the bivariate (logarithmic) normal distribution and the bivariate Gamma distribution (bivariate exponential distribution if the shape parameter is unity). In many cases, the former is suitable for processing annual maximum series, while the latter is suitable for partial duration series. It is not always the case that the typhoon which brought the maximum rainfall in a year was also the cause of maximum storm surge in that year. Therefore, in analyzing the concurrent characteristics of the maximum storm surge and the maximum rainfall, the data should be of the partial duration series. For this reason, we have chosen bivariate exponential distribution as the suitable method.

This bivariate exponential distribution is further classified into two types: the special case of the bivariate Gamma distribution proposed by Izawa, and the bivariate extension of exponential distribution proposed by Freund. Characteristics of the distributions have already been discussed by Nagao and Hashino, so only the conclusions are given below.

#### a) Bivariate exponential distribution by Izawa

In the hydrological area, Gamma distribution with the shape parameter set to unity, or exponential distribution for a partial duration series, is often used for probability and stochastic analysis. In bivariate analyses, bivariate exponential distribution may be suitable for use as a special case of bivariate Gamma distribution with the shape parameter set to unity. Izawa expressed the joint probability density function, Marginal probability density functions and the shape parameters of this bivariate exponential distribution as follows:

**Joint probability density function**

$$f(x,y)=\frac{\beta_1\beta_2}{1-\rho}\exp\left(-\frac{\beta_1 x}{1-\rho}-\frac{\beta_2 y}{1-\rho}\right)\times I_0\left(\frac{2\sqrt{\rho}}{1-\rho}\sqrt{\beta_1\beta_2 xy}\right) \tag{1}$$

where $I_0$ : the modified Bessel function of the first kind of degree zero

**Marginal probability density functions**

$$f(x) = \beta_1 \exp(-\beta_1 x) \quad , \quad f(y) = \beta_2 \exp(-\beta_2 y) \tag{2}$$

**Shape parameter**

$$\beta_1 = \frac{1}{\bar{x}} \quad , \quad \beta_2 = \frac{1}{\bar{y}} \quad , \quad \rho = \frac{\overline{xy}}{\bar{x} \cdot \bar{y}} - 1 \tag{3}$$

where $\bar{x}$ :average of $x$, $\bar{y}$ :average of $y$, $\overline{xy}$ :average of $x \cdot y$

**b) Bivariate exponential distribution by Freund**

Freund proposed a bivariate distribution expressed only by an exponential distribution function for a life span test model composed of two elements. The joint probability density function, the marginal probability density functions and the shape parameters are shown as follows:

**Joint probability density function**

$$f(x,y) = \begin{cases} a_1 b_2 \exp\left\{-b_2 y - (a_1 + b_1 - b_2)x\right\} & \text{for } 0 < x \leqq y \\ \\ b_1 a_2 \exp\left\{-a_2 x - (a_1 + b_1 - a_2)y\right\} & \text{for } 0 < y < x \end{cases} \tag{4}$$

where $a_1, a_2, b_1, b_2$ :shape parameter

**Marginal probability density function of y**

$$f(y) = \begin{cases} \dfrac{a_1 b_2}{a_1 + b_1 - b_2} \exp(-b_2 y) + \dfrac{(a_1 + b_1)(b_1 - b_2)}{a_1 + b_1 - b_2} \exp\left\{-(a_1 + b_1)y\right\} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for } a_1 + b_1 \neq b_2 \\ \\ (b_1 + a_1 b_1 y)\exp(-b_2 y) \qquad\qquad\qquad \text{for } a_1 + b_1 = b_2 \end{cases} \tag{5}$$

To express the marginal probability density function of $x$, we replaced $y$ with $x$, $b_2$ with $a_2$, and exchanged $a_1$ with $b_1$ in Eq. (5).

**Shape parameters**

$$\frac{1}{a_1} = \frac{1}{N_1}\left\{\sum_{j=1}^{N1} x_j^1 + \sum_{j=1}^{N2} y_j^2\right\} \quad , \quad \frac{1}{b_1} = \frac{N_1}{N_2} \cdot \frac{1}{a_1}$$

$$\frac{1}{a_2} = \frac{1}{N_2}\sum_{j=1}^{N2}(x_j^2 - y_j^2) \quad , \quad \frac{1}{b_2} = \frac{1}{N_1}\sum_{j=1}^{N1}(y_j^1 - x_j^1) \tag{6}$$

where, $(x_j^1, y_j^1)$ and $(x_j^2, y_j^2)$ indicate the $(x, y)$ data that fit conditions $x \leqq y$ and $x > y$, respectively, and $N_1$ and $N_2$ indicate the number of each data.

**c) Comparison of Izawa's and of Freund's distributions**

In order to compare the characteristics of Izawa's and Freund's distributions, the relationship of shape parameters of both distributions were considered. When the parameters in Eq. (4) are put as Eq. (7), the marginal distributions of $x$ and $cy$ become identical.

$$a_1 = b_1 = a \quad , \quad a_2 = b_2 = b \quad , \quad y = cy \tag{7}$$

Thus, the joint excess probability density function is given as follows:

$$f(x,y)=\begin{cases} ab\exp\{-bcy-(2a-b)x\} & \text{for } 0<x\leqq cy \\ ab\exp\{-bx-(2a-b)cy\} & \text{for } 0<cy<x \end{cases} \tag{8}$$

The marginal probability density function is expressed as follows:

$$f(x)=\int_0^\infty f(x,y)dy \quad , \quad f(y)=\int_0^\infty f(x,y)dx \tag{9}$$

When $2a-b\neq 0$,

$$f(x)=\frac{ab}{2a-b}\exp(-bx)+\frac{2a(a-b)}{2a-b}\exp(-2ax)$$

$$f(y)=\frac{abc}{2a-b}\exp(-bcy)+\frac{2a(a-b)c}{2a-b}\exp(-2acy) \tag{10}$$

And when $2a-b=0$,

$$f(x)=a(1+bx)\exp(-bx) \quad , \quad f(y)=a(1+bcy)\exp(-bcy) \tag{11}$$

The following equation shows a comparison of the shape parameters of Izawa's equation and those of Freund's equation.

$$a=\frac{\beta_1}{2}\left(1+\sqrt{\frac{1-\rho}{3\rho+1}}\right) , \; b=\frac{\beta_1}{2}\left(1+\sqrt{\frac{3\rho+1}{1-\rho}}\right) \quad , \quad c=\beta_2/\beta_1 \tag{12}$$

The bivariate distribution shapes of Izawa's equation and Freund's equation are compared on the basis of the shape parameters. Figure 2 shows one example. In Fig.2 Freund's distribution (broken lines) makes a rise on the line $x/y=\beta_2/\beta_1=2.0$, showing a slight difference from Izawa's (solid lines). To locate and observe the rising point and the gap, we show the distribution on the point $\beta_2/\beta_1=2.0$ in Fig. 3. As Eqs. (1), (8) and (12) indicate, Izawa's and Freund's distributions are the same when $\rho$ is equal to zero, so the cases for $\rho=0.5$ and 0.9 are shown in Fig. 3. This figure shows that the larger the value of $\rho$, the larger the gap between Izawa' s and Freund's. distributions. It also shows that the shape at the position representing the value of the most frequent occurrence makes acute angles in Freund's formula.

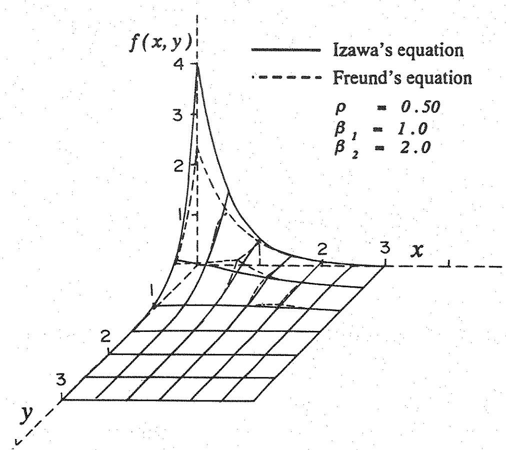
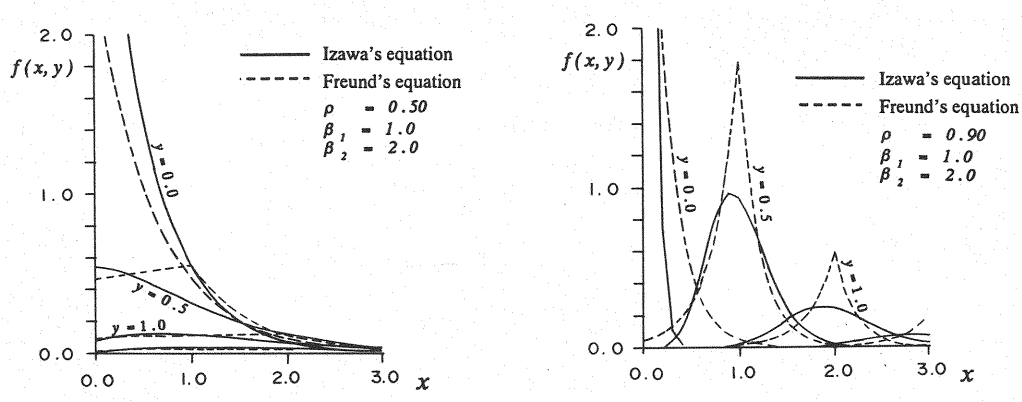



**Fig. 2 Comparison of Izawa's distribution and Freund's distribution**




$f(x, y) - x$ plane for $y$ =0, 0.5, 1.0

**Fig. 3 Comparison of Izawa's distribution and Freund's distribution**

**(3) Concurrence of Storm Surge and Rainfall**

The bivariate distribution by Izawa needs numerical analysis to calculate the excess probability because the equation contains a modified Bessel function of the first kind of degree zero. On the other hand, the bivariate distribution by Freund leads directly to the analytical solution because its equation is shown only by the exponential function. On top of that, the correlation coefficient $\rho$ of storm surge at the Osaka Port and the rainfall at the Osaka District Meteorological Observatory is 0.2, which is so small that we do not have to consider the gap between Izawa's and Freund's solutions.

We decided to adopt the bivariate extension of exponential distribution proposed by Freund because of the convenience in practical application.

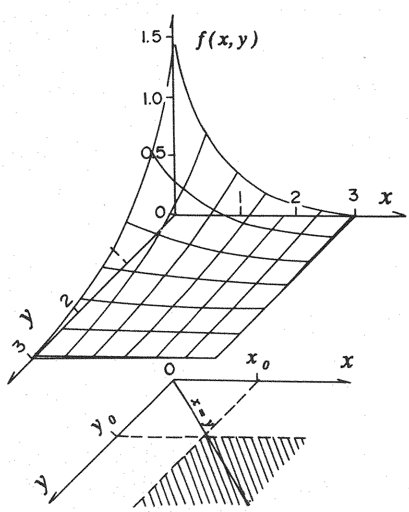



**Fig. 4 Probability Density Function and Integration Area**

When the storm surge is set to $x$ and the rainfall is set to $y$, the probability that both values exceed any value $(x_0, y_0)$— that is, the joint excess probability $P_F(x_0, y_0)$— lies in the area of $x_0 < x \leqq \infty$, $y_0 < y \leqq \infty$ in Fig. 4 and is given by integrating the probability density function shown in Eq. (4). However, there is a difference between the areas of $x \leqq y$ and $x > y$, so integration for each area is required.

Consequently, the formula showing the joint excess probability $P_F$ is given by the following analytical solution with a very simple form, which is needed from a practical viewpoint.

$$P_F = \begin{cases} \dfrac{b_1 \exp(-a_2 x_0)}{a_1+b_1-a_2}[\exp\{-(a_1+b_1-a_2)y_0\} - \exp\{-(a_1+b_1-a_2)x_0\}] \\ \qquad + \exp\{-(a_1+b_1)x_0\} & \text{for } x_0 > y_0 \\ \exp\{-(a_1+b_1)x_0\} = \exp\{-(a_1+b_1)y_0\} & \text{for } x_0 = y_0 \\ \dfrac{a_1 \exp(-b_2 y_0)}{a_1+b_1-b_2}[\exp\{-(a_1+b_1-b_2)x_0\} - \exp\{-(a_1+b_1-b_2)y_0\}] \\ \qquad + \exp\{-(a_1+b_1)y_0\} & \text{for } x_0 < y_0 \end{cases} \tag{13}$$

Moreover, Eq. (13) gives $y_0$ and $x_0$ for given $P_F$ and $x_0$ or given $P_F$ and $y_0$, respectively. This is very convenient for drawing the equi return period curves in practice. The equations are expressed as follows:

When $y_0 > x_0$, we have

$$x_0 = -\frac{1}{a_1+b_1-b_2}\ln\Big[\frac{a_1+b_1-b_2}{a_1\exp(-b_2 y_0)} \times \langle P_F - \exp\{-(a_1+b_1)y_0\}\rangle + \exp\{-(a_1+b_1-b_2)y_0\}\Big] \tag{14a}$$

When $x_0 > y_0$

$$y_0 = -\frac{1}{a_1+b_1-a_2}\ln\Big[\frac{a_1+b_1-a_2}{b_1\exp(-a_2 x_0)} \times \langle P_F - \exp\{-(a_1+b_1)x_0\}\rangle + \exp\{-(a_1+b_1-a_2)x_0\}\Big] \tag{14b}$$

However, in transforming the excess probability into the return period, the annual joint excess probability $P_{Fa}$ is required in place of the joint excess probability $P_F$. The relation between $P_{Fa}$ and $P_F$ is shown approximately by the following equation (see APPENDIX).

$$T = 1/P_{Fa} = 1/(\lambda P_F) \tag{15}$$

where $\lambda$ : annual average number of typhoons

Figure 5 shows an example of equi return period curves for storm surge and rainfall. The figure attached to each curve refers to the recurrence interval in years. The circles show the relationship between the maximum storm surge and the maximum rainfall per 4 hours during the period that the storm surge level is positive. The correlation coefficient between the maximum storm surges and the maximum rainfalls per 4 hours is about 0.2. On each equi return period curve, there is a tendency that a typhoon that causes high maximum storm surge brings a small amount of rainfall. Figure 5 also shows that the equi return period of Typhoon Jane whose rainfall was adopted as the design rainfall of the current storm drainage project is 180 years. Considering the recorded 41.7 mm/4hrs during Typhoon Jane and the 3.0 m design maximum storm surge, the equi return period can be calculated to be a longer period than 200 years, which assures the high safety margin of the storm drainage project.

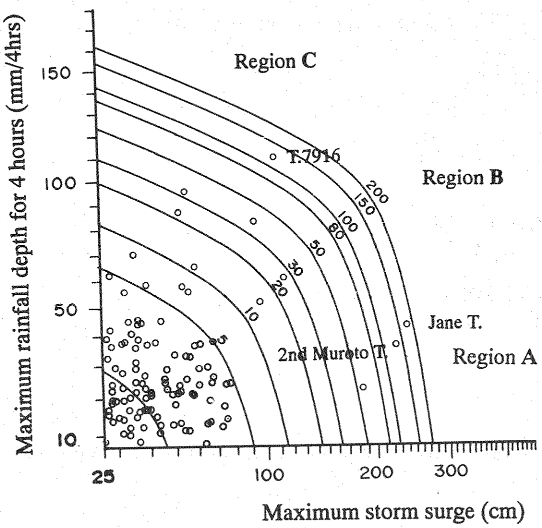



**Fig. 5 Typical Diagram of Equi Return Period Curves for Storm Surge and Rainfall**

## APPLICATION TO PLANNING FOR STORM DRAINAGE

The main target of the storm drainage project examined in the current study is to control the large maximum storm surges in region A in Fig. 5. For this purpose, the tide gates are to be closed before the tide level reaches the design high tide level or even before the maximum storm surge occurs. Usually this is done several hours before the maximum storm surge occurs when the tide level is still several tens of centimeters below the design high tide level. Sometimes the tide does not reach the forecasted level. This offers no problem as long as the purpose of the gates is to protect the river against a storm surge. However, if rainfall is larger than the expected volume, a shortage of pump capacity will result.

For the above stated reason, in a system using gates against storm surges, the design rainfall for the storm drainage project should cover the rainfall brought about by the typhoons shown in region B of Fig. 5 and not region A.

We propose that the design rainfall for the specified safety level should be set by the following procedure. The target maximum storm surge should be set to the lowest level of the expected maximum storm surge which occurs after closing the gate. During Typhoon 7916 the maximum storm surge at the gates of the Neya River system reached 1.07 m after the gates had been closed. This suggests that the appropriate tide level for the target maximum storm surge is approximately 1 m.

The design rainfall with respect to the 1 m maximum storm surge is determined as the point where the perpendicular line from the point of the 1 m maximum storm surge meets the equi return period curve of the given safety level. By preparing diagrams similar to Fig. 5 for $N$ ($N$ = 1, 2,$\cdots$,12) hours of rainfall and repeating the above procedure, the design hyetograph can be obtained.

## CONCLUDING REMARKS

The authors presented their argument that the concurrent excess probability can be expressed by a considerably simplified expression by the use of the bivariate extension of exponential distribution by Freund. In a storm drainage project, by drawing several equi return period curves for the maximum storm surge and $N$ hours of rainfall, a design hyetograph can be created easily for a river with tide gates.

## ACKNOWLEDGEMENTS

The authors would like to thank the staff of the Urban Rivers Division, Public Works Department, Osaka Prefectural Government for giving them the opportunity to publish this paper.

## APPENDIX

When the partial duration series (non-annual exceedance series) is followed by the exponential distribution, their excess probability is set to $P_{FE}$. Then the annual maximum series is followed by Gumbel distribution, whose excess probability is set to $P_{FG}$. The excess probability $P_{FE}$ and $P_{FG}$ are expressed as follows:

$$P_{FE} = \exp(-\beta x) \tag{16}$$

$$P_{FG} = 1 - \exp\{-\lambda \exp(-\beta x)\} \tag{17}$$

λ: annual average occurrence frequency

When $\lambda \exp(-\beta x) << 1$,

$$P_{FG} = 1 - [(1 - \lambda \exp(-\beta x) + \frac{1}{2}\{\lambda \exp(-\beta x)\}^2 + \cdots\cdots$$

$$= \lambda \exp(-\beta x) - \frac{1}{2}\{\lambda \exp(-\beta x)\}^2 + \cdots\cdots$$

$$= \lambda P_{FE} - \frac{1}{2}\{\lambda P_{FE})^2 + \cdots\cdots \tag{18}$$

$$\fallingdotseq \lambda P_{FE} \tag{19}$$

As the return period exceeds 5 years, Eq. (18) can be approximately expressed by Eq. (19), that is, $P_{FG} \approx \lambda P_{FE} < 1/5$. The first term is under 0.2 and the second term is under 0.02 in Eq. (18) so the error is under 10 % if the second term is ignored. An error of under 10 % can be ignored for the excess probability.

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## APPEDIX-NOTATION

The following symbols are used in this paper:

$a_1, a_2, b_1, b_2$ = shape parameter in Freund's distribution;

$f(x, y)$ = joint probability density function;

$f(x)$, $f(y)$ = marginal probability density functions;

$I_0$ = modified Bessel function of the first kind of degree zero;

$N_1$, $N_2$ = number of each data that fit condition $x \leqq y$ and $x > y$;

$P_F$ = joint excess probability;

$P_{Fa}$ = annual joint excess probability;

$P_{FE}$ = excess probability by exponential distribution;

$P_{FG}$ = excess probability by Gumbel distribution;

$(x_j^1, y_j^1), (x_j^2, y_j^2)$ = $(x, y)$ data that fit conditions $x \leqq y$ and $x > y$;

$\beta_1$, $\beta_2$ = shape parameter in Izawa's distribution;

$\lambda$ = annual average occurrence frequency; and

$\rho$ = correlation coefficient.

(Received August 22, 2000 ; revised December 22, 2000)