Journal of Hydroscience and Hydraulic Engineering
Vol. 18, No.1 May, 2000, 127-140

CONCEPTUAL ASPECTS OF ONE-DIMENSIONAL MATHEMATICAL MODELS FOR ALLUVIAL RIVERS

By

Zhixian Cao and Shinji Egashira

College of Science and Engineering, Ritsumeikan University,
Noji-Higashi 1-1-1, Kusatsu, Shiga 525-8577, Japan

SYNOPSIS

The complete governing equations of one-dimensional nonequilibrium mathematical models for alluvial rivers are presented, and the simplified equations and assumptions involved are briefly described. The characteristic celerities are analyzed to examine the lumped total-load transport capacity concept and decoupled solution approach widely incorporated in prior models. Existing celerity analyses for total-load transport capacity models are extended. It is revealed that suspended load transport does not adjust as quickly as flow changes. This strongly constraints the use of models in which the total-load transport rate is specified at a capacity value determined primarily by local hydraulic conditions. Further, it is shown that riverbed deformation does not occur instantaneously in response to flow change only within very limited ranges of small Froude number and low bed-load concentration (rather than the total-load concentration as of previous analyses). Otherwise, neither the "quasi-steady state" nor the "fixed bed" assumption implicit in most previous models is appropriate, which necessitates a coupled solution procedure.

INTRODUCTION

Refined modeling of flow, sediment transport, and bed evolution in alluvial rivers is essential for hydraulic and environmental engineering as well as geophysical studies. Numerous mathematical models have been developed since the 1950's and highly marketed in the last two decades. However, most suffer from serious deficiencies in the present state-of-the-art. In the context of one-dimensional models pertaining to the long-term evolution of alluvial rivers, three of the most acute features can be identified as follows.

Simplified Governing Equations

First, the governing equations for the water flow-sediment-riverbed system are over-simplified. In the aggressively marketed mobile-bed mathematical models, the governing equations are only programmatically listed, while the limitations and approximations thereof are seldom interpreted or understood. The strong coupling among the water flow, sediment transport and riverbed evolution is to a certain extent ignored without justification. Most popularly, the classical de St. Venant equations for single-phase water flow are used without considering the presence of sediment and bed mobility (e.g., Bhallamudi and Chaudhry 1991; Holly and Rahuel 1990a, b; Li 1990; Lyn 1987). Stevens (1988) claimed the importance of bed mobility, but did not consider the presence of sediment. Correia et al. (1992) introduced the term representing the change of riverbed level in the continuity equation, and found that this term is important for long-term modeling. At the same time, in the mass conservation equation for global bed material, the temporal effect associated with total- or bed-load transport (sediment storage in water column) is sometimes neglected without any justification, see for example, Li (1990), Holly and Rahuel (1990a, b), Chang (1988), Zhang and Kahawita (1987). Wei (1990) presented a set of governing equations for the phenomenon. It consists of 4 PDEs (Partial Differential Equations) based on the momentum conservation law for the sediment-water mixture flow and the mass conservation law respectively for the mixture, total bed material, and suspended sediment. Unfortunately, in this set of equations, bed-load transport is not included. Recently Cao and Egashira (1999) investigated the influences of the simplified continuity equations for both the mixture and the global bed material. This study confirms the results of Correia et al. (1992) regarding the influence of the inclusion of bed mobility, and possibly for the first time shows the considerable errors due to simplifying the bed material's continuity equation.

Lumped Total-Load Transport Capacity Concept

Second, some models are developed and analyzed on the basis of the lumped total-load concept, and the total-load transport rate is prescribed at a capacity value determined primarily by local hydraulic parameters (e.g., Hsu and Chu 1964; Cunge et al. 1980; Lyn 1987; Chang 1988; Bhallamudi and Chaudhry 1991; Correia et al. 1992; Saiedi 1994, 1997; Morris and Williams 1996; Cao and Egashira 1999). Obviously, the exchange between suspended sediment, bed-load and bed surface material cannot be explicitly represented. More deficiencies in its practical aspects have been pointed out by, for example, Holly and Rahuel (1990a).

The lumped total-load capacity concept is exclusively related to the 3-PDE models. The governing equations involve the momentum conservation and mass conservation respectively for the mixture (or water phase only) and total bed material. Criticized by many workers, the 3-PDE capacity models are limited in the applicability for refined modeling of the flow-sediment-mobile bed system.

The last three decades have seen many 4-PDE nonequilibrium models. Compared to the 3-PDE models, the new equation added is derived from the mass conservation for suspended sediment. In this kind of models, suspended and bed-load transport processes are distinctly considered (bed-load is neglected when suspension is the predominant mode of sediment transport). One may refer to among others, Dou (1963), Lin et al. (1983), Holly and Rahuel (1990a, b), Zhou and Lin (1998) etc. Nevertheless, it is still not sufficiently clear that if nonequilibrium models must always be used. Alternatively, it remains to be justified that if the total-load transport capacity approach can be used under some conditions. This appears of practical significance because of the relative simplicity of 3-PDE models in comparison with the 4-PDE nonequilibrium ones. Physically, the capacity concept for suspended load transport may be validated should the celerity of suspended load be equal to or greater than those of the flow.

Decoupled Solution Procedure

Finally, the "quasi-steady state" for the flow is often assumed when the evolution of riverbed is studied. Alternatively, the riverbed is implicitly assumed to be "fixed" within a time step and decoupled solution of the governing equations is pursued while the flow over mobile bed is of primary interest. Whether this feature is valid or not is determined by the typical time scales or relative magnitude of the characteristic celerities corresponding to the free-surface flow and riverbed evolution respectively. In the physical sense, the "quasi-steady state" or "fixed bed" assumption should be justified as the celerity related to the bed level evolution is negligible compared to those of the free surface flow.

De Vries (1965, 1973) considered the 3-PDE, total-load transport capacity model and analyzed the relative celerities when the volumetric sediment concentration is negligible. Morris and Williams (1996) confirmed the results of De Vries and extended the analysis to cases with finite sediment concentrations. It has been found that water flow, sediment transport, and riverbed evolution can be considered to be mathematically independent of each other only within very limited ranges of total-load concentration and Froude number. Beyond these ranges, the "quasi-steady state" or "fixed bed" assumption is no longer reasonable. Lyn (1987) identified the multiple time scales of the 3-PDE, total-load transport capacity model. He showed that previous models, which reduce the number of conservation equations solved simultaneously from three to two under the "quasi-steady state" or "fixed bed" assumption, are unable to satisfy exactly either a general boundary condition or an arbitrary initial condition. And in situations with highly variable discharge and sediment inputs, the aforementioned assumption is not justified. Cao and Egashira (1999) have studied the behavior of decoupled models with complete or simplified governing equations. It has been found that an asynchronous solution procedure may either render the physical process mathematically ill posed or cause appreciable errors for long-term simulations. More analyses on this feature can be found in Hsu and Chu (1964). Despite the studies stated above, it is recognized that previous analyses are mostly for 3-PDE, total-load transport capacity models. To these writers' knowledge, there has been no analysis, in the configuration presented in this paper, of the relative celerities or time scales associated with the 4-PDE, nonequilibrium models with distinct separation of suspended and bed-load. In sharp contrast to this fact, the "quasi-steady state" or "fixed bed" assumption is frequently introduced in such type of models and decoupled solution is mostly pursued. Holly and Rahuel (1990a, b) presented a framework of nonequilibrium models and coupled solutions to many problems, but without analysis of its simplifying or decoupling conditions. The need for clarifying the assumptions and decoupling conditions is evident.

Present Work

This paper first seeks to present the complete set of governing equations for the flow-sediment-riverbed system, which are rigorously based on mass and momentum conservation laws. Discussions are given in relation to the simplified forms. Then, the characteristic celerities are analyzed to examine the aforesaid acute features common to most existing mathematical models. The results discourage the use of models which represent the suspended load transport rate as a capacity value specified by local hydraulic conditions, especially in very small Froude number cases. Also, the finding severely constraints the use of the "quasi-steady state" or "fixed-bed" assumption, and accordingly challenges the decoupled solution of the governing equations. The need for fully coupled, nonequilibrium modeling is conceptually demonstrated.

FORMULATIONS — GOVERNING EQUATIONS AND CLOSURE

Complete Conservation-Based Equations

Consider the unsteady, sediment-laden flow over erodible bed in an alluvial channel. No lateral inflows are included. A definition sketch is shown in Fig. 1. At this time, sediment is assumed to be uniform. Extension to graded sediments will be straightforward. The governing equations are formulated by utilizing the Reynolds Transport Theorem (Roberson and Crowe 1990), based on the momentum conservation law for the water-sediment mixture flow and the mass conservation law respectively for the mixture, global bed material, and suspended sediment.

Apparently, to properly account for the mass exchange across the mobile bottom boundary is essential for describing the water-sediment mixture's mass conservation. Traditionally, the control volume is defined to include only the flow area A. In this case, one has to formulate separately the mass conservation in an extended alluvial area A_0 above some datum, Fig. 1(b), in which the time-averaged streamwise velocity vanishes. It follows from the mass conservation law respectively for the flow and alluvial areas that

$$\frac{\partial}{\partial t}(\alpha_1 A\rho_m)+\frac{\partial}{\partial x}(AU\rho_m)=F_t \tag{1}$$

$$\rho_0\frac{\partial A_0}{\partial t}=-F_t \tag{2}$$

where t = time; x = streamwise coordinate; U = cross section-averaged streamwise velocity; F_t = net mass flux of total water-sediment mixture exchange with bed surface; $\rho_0=\rho_w p+\rho_s(1-p)$, bulk density of water-saturated bed; p = bed sediment porosity (constant); $\rho_m=\rho_w(1-C_t)+\rho_s C_t$, bulk density of water-sediment mixture; C_t = flux-averaged total-load concentration in volume; ρ_w = water density (constant); ρ_s = sediment density (constant); and α_1 = modification coefficient denoting the difference of the mixture's geometrically and flux-averaged densities.

Adding Eqs. (1) to (2) leads to the continuity equation for the mixture flow over a movable bed

$$\frac{\partial}{\partial t}(\alpha_1 A\rho_m)+\frac{\partial}{\partial x}(AU\rho_m)+\rho_0\frac{\partial A_0}{\partial t}=0 \tag{3}$$

The water-sediment mixture's momentum conservation equation reads

$$\frac{\partial}{\partial t}(AU\rho_m)+\frac{\partial}{\partial x}(\beta_1 AU^2\rho_m)+gA\rho_m\frac{\partial h}{\partial x}+gAh_G(\rho_s-\rho_w)\frac{\partial C_t}{\partial x}=g\rho_m A(I_b-I_e) \tag{4}$$

where g = gravitational acceleration; h = flow depth; h_G = distance from water surface of the shape center of cross section; $I_b=-\partial Y/\partial x$, bed slope; Y = bed elevation; I_e = friction slope; and β_1 = momentum modification coefficient.

The mass conservation equations respectively for the global bed material and suspended sediment read

$$(1\text{-}n_p)\frac{\partial A_0}{\partial t}+\frac{\partial}{\partial t}(\alpha_2 A C_t)+\frac{\partial}{\partial x}(AUC_t)=0 \tag{5}$$

$$\frac{\partial}{\partial t}(\alpha_3 AC)+\frac{\partial}{\partial x}(AUC)=BF_n \tag{6}$$

where B = channel width; C = flux-averaged suspended load concentration in volume; F_n = net flux of suspended sediment exchange with bed surface per unit width; and α_2 and α_3 = modification coefficients expressing the difference of geometrically and flux-averaged concentrations respectively of total- and suspended loads.

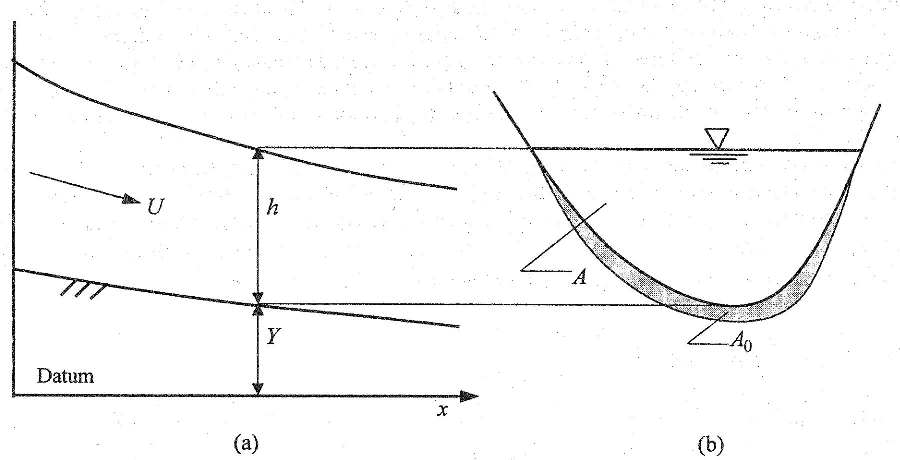

FIG. 1. Definition Sketch of Problem. (a) Streamwise Profile; and (b) Cross-Section

Simplified Equations

In applications, the complete governing Eqs. (3) through (6) are often simplified by assuming
(i) sediment concentration is very low as is in most natural rivers; and
(ii) the modification coefficients α_1, α_2, α_3, and β_1 are approximately equal to unity.
Using Eq. (5), the mixture's continuum Eq. (3) can be rewritten as

$$\frac{\partial A}{\partial t}+U\frac{\partial A}{\partial x}+A\frac{\partial U}{\partial x}+\frac{\partial A_0}{\partial t}=0 \tag{7}$$

From Eq. (5) it can be inferred that the bed evolution rate (i.e., the last term in the left-hand-side of Eq. (7)) is of second order-of-magnitude compared to other terms due to the low sediment concentration assumption. In most existing models, it is neglected. However, Correia et al. (1992) suggested that this term in Eq. (7) is important when long-term river modeling is pursued, whereas Rahuel (1993) challenges the reliability of this argument. The recent study of Cao and Egashira (1999) demonstrates that neglecting the bed mobility in Eq. (7) leads to appreciable inaccuracy, depending on the changes of flow discharge and sediment inputs. Especially, this inaccuracy becomes more pronounced progressively with increasing computational time. In view of this status respect to this term, it is retained in the present paper as strictly is.

When Eq. (3) is substituted into Eq. (4) and the terms of second order-of-magnitude related to the low sediment concentration are neglected, one yields

$$\frac{\partial U}{\partial t}+g\frac{\partial h}{\partial x}+U\frac{\partial U}{\partial x}+g\frac{\partial Y}{\partial x}=-gI_{\text{e}} \tag{8}$$

Evidently, Eqs. (7) and (8) reduce to the classical St. Venant equations that are used widely in prior models (e.g., Holly and Rahuel 1990a, b) if the bed evolution rate in Eq. (7) is not included. In view of the unavoidable uncertainty due to the flow resistance relationship that must be invoked to close the momentum equation, the simplified Eq. (8) is acceptable for most cases and has been widely used.

Generally speaking, the phenomenon to be modeled is the water-sediment mixture flow over erodible bed, rather than the fluid-phase (water) flow only. The continuity and momentum equations naturally should be formulated for the mixture, despite that under the low sediment concentration assumption they may reduce to the same forms (St. Venant equations) as for equivalent single-phase flows. One may be interested in two-phase formulation-based models. Yet, the interactions between the two phases are still so poorly understood as not to be accurately expressed. In some prior studies (Morris and Williams 1996), the continuity equation for the fluid-phase (water) only is used instead of the mixture's continuity Eqs. (3) or (7), while the momentum equation thereof is for the mixture flow. In essence, this inconsistency is reconcilable as the fluid-phase continuity equation can be simply derived by subtracting the global bed material's conservation Eq. (5) from Eq. (3) for the mixture. In other words, the three continuity equations respectively for water, total bed material, and their mixture are not independent of each other, and any two of them may be used.

Subtracting Eq. (6) from Eq. (5) leads to

$$(1\text{-}n_{\text{p}})\frac{\partial A_0}{\partial t}+\frac{\partial}{\partial t}(AC_{\text{b}})+\frac{\partial}{\partial x}(AUC_{\text{b}})=-BF_{\text{n}} \tag{9a}$$

$$(1\text{-}n_{\text{p}})\frac{\partial A_0}{\partial t}+\frac{\partial}{\partial t}(q_{\text{b}}B/U)+\frac{\partial}{\partial x}(q_{\text{b}}B)=-BF_{\text{n}} \tag{9b}$$

where $C_{\text{b}}=C_{\text{t}}-C$ the bed-load concentration in volume; $q_{\text{b}}=AUC_{\text{b}}/B$, the bed-load transport rate in volume per unit width.

It is interesting to mention that when bed-load is negligible ($q_{\text{b}}=0$), Eq. (9) reduces to the so-called riverbed deformation equation (Wei 1990) distinctly derived from the kinematics of the mobile bottom boundary, which is subject to prior prescription of the net flux of suspended sediment exchange. In this case, the bed evolution is caused by suspended sediment exchange with the mobile bed only, which apparently assumes local characteristics. In contrast, as bed-load transport dominates (or suspended load is negligible, $F_{\text{n}}=0$), Eq. (9) is similar to the total-load mass conservation equation as used by *inter alia*, Lyn (1987), Bhallamudi and Chaudhry (1991), Correia et al. (1992) as well as Morris and Williams (1996). It is noted that the second terms in the left-hand-side of Eqs. (9a, b) and (5) represent the temporal effects in relation to bed- and total-load transport. However, in some models (e.g., Holly and Rahuel 1990a, b; Li 1990; Chang 1988; Zhang and Kahawita 1987) it is erroneously neglected without any explanation. Evidently it remains to be justified. Part of the efforts on this line can be found in Cao and Egashira (1999), in which the effect of this term is clearly shown to be substantial for long-term simulations.

Considering the empiricism inherent in existing closure relations for the net flux of suspended sediment exchange with the mobile bed, it is reasonable to simplify Eq. (6) based on the low sediment concentration assumption. Substituting Eqs. (7) into (6) yields

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial x}=\frac{BF_{\text{n}}}{A} \tag{10}$$

In some existing models (especially developed by Chinese workers, see for example Xie 1990; Zhou and Lin 1998), the right-hand-side of Eqs. (9) and (10) are *priori* replaced with specific empirical relations for the net flux of sediment exchange. Recognizing the hypotheses is definitely important to interpret the results and limitations of these models.

In prior models (e.g., Holly and Rahuel 1990a), there may be more PDEs in addition to the mass and momentum conservation-based governing equations as stated above. Strictly speaking, the governing equations should be distinguished from these auxiliary PDEs complemented empirically to represent the temporal and spatial delay effects associated with bed-load transport and the sorting of heterogeneous sediments.

Closure of Equations

It is necessary to recognize that Eqs. (3) through (5) constitute the governing equations of 3-PDE models based on the capacity concept for the lumped total-load. By nature the total-load transport rate (alternatively, total-load concentration) is whereby specified empirically as a function of local hydraulic conditions. To close the 3-PDE models, one must also specify a flow resistance relation, in addition to a total-load transport capacity.

For closure of the nonequilibrium models (i.e., Eqs. (3) through (6)), one must

(i) determine the flux of exchange between suspended sediment and bed surface material;
(ii) specify a flow resistance relation; and
(iii) represent the bed-load transport rate, where the spatial and temporal delay effects (Phillips and Sutherland 1989, 1990) may need to be considered.

Because the present analysis is conceptual in nature and concerned primarily with the structure of the governing equations, more details for the friction resistance and net flux of sediment exchange are not given here as they can be found not to influence the results of this study. However, a relation for the unit bed-load transport rate has to be specified. It is not difficult to incorporate the spatial and temporal bed-load delay effects in Eqs. (9a, b) as they are well formulated based on enhanced understanding of the mechanism. For simplicity, the spatial and temporal delay effects associated with bed-load transport are not considered herein as in most existing models. Thus the bed-load transport rate q_b is equal to the equilibrium value, which can be determined approximately by the simplest power form

$$q_b = \delta U^{\alpha} h^{\beta} d^{\sigma} \tag{11}$$

where d = sediment particle diameter; α, β and σ are the exponents; and δ is the coefficient.

CHARACTERISTIC CELERITIES AND IMPLICATIONS

Without losing generality, the cross-section of the channel is in this study assumed to be rectangular with constant width. Thus $A = Bh$, $\partial A_0 / \partial t = B \partial Y / \partial t$, and $C_{\mathrm{b}} = q_{\mathrm{b}} / Uh$. From Eq. (11), one obtains

$$\frac{\partial q_{\mathrm{b}}}{\partial x} = \alpha C_{\mathrm{b}} h \frac{\partial U}{\partial x} + \beta C_{\mathrm{b}} U \frac{\partial h}{\partial x} \tag{12}$$

$$\frac{\partial}{\partial t}\left(\frac{q_{\mathrm{b}}}{U}\right) = (\alpha - 1) C_{\mathrm{b}} h U^{-1} \frac{\partial U}{\partial t} + \beta C_{\mathrm{b}} \frac{\partial h}{\partial t} \tag{13}$$

Substituting Eqs. (7) and (8) into Eq. (13), and then the resulting formulation with Eq. (12) into Eq. (9b), neglecting the term of second order-of-magnitude and introducing the Froude number $Fr = U(gh)^{-0.5}$, one has

$$\frac{\partial Y}{\partial t} + \psi Fr^{-2} U \frac{\partial h}{\partial x} + \varphi h \frac{\partial U}{\partial x} + \psi Fr^{-2} U \frac{\partial Y}{\partial x} = -\frac{F_{\mathrm{n}}}{1 - n_{\mathrm{p}}} - \psi Fr^{-2} U I_{\mathrm{f}} \tag{14}$$

where

$$\psi = \frac{-(\alpha - 1) C_{\mathrm{b}}}{(1 - n_{\mathrm{p}})}; \qquad \varphi = \frac{-(\beta - 1) C_{\mathrm{b}}}{(1 - n_{\mathrm{p}})} \tag{15a, b}$$

After rearrangement, Eqs. (7), (8), (10), and (14) can be expressed as

$$\frac{\partial \mathbf{F}}{\partial t} + \mathbf{K}\frac{\partial \mathbf{F}}{\partial x} = \mathbf{R}_{\mathrm{m}} \tag{16}$$

where

$$\mathbf{K} = \begin{bmatrix} (1-\psi Fr^{-2})U & (1-\varphi)h & 0 & -\psi Fr^{-2}U \\ g & U & 0 & g \\ 0 & 0 & U & 0 \\ \psi Fr^{-2}U & \varphi h & 0 & \psi Fr^{-2}U \end{bmatrix} \tag{17}$$

$$\mathbf{F} = (h, U, C, Y)^{\mathrm{T}} \tag{18}$$

$$\mathbf{R}_{\mathrm{m}} = (R_1, R_2, R_3, R_4)^{\mathrm{T}} \tag{19}$$

$$R_1 = -R_4\,; \quad R_2 = -gI_{\mathrm{f}}\,; \quad R_3 = F_{\mathrm{n}}/h\,; \quad R_4 = -F_{\mathrm{n}}/(1-n_{\mathrm{p}}) - \psi Fr^{-2}UI_{\mathrm{f}} \tag{20a,b,c,d}$$

Eq. (16) forms a fourth-order hyperbolic system and is thus associated with four characteristic celerities λ_1, λ_2, λ_3 and λ_4. They are related to the propagation of small disturbances of free-surface flow (λ_1 and λ_2), suspended sediment transport (λ_3) and mobile bed evolution (λ_4) respectively. These celerities can be found to satisfy the following characteristic equation in λ,

$$(U-\lambda)[\lambda^3 - 2U\lambda^2 + (U^2 - gh + gh\psi)\lambda + Ugh(\varphi - \psi)] = 0 \tag{21}$$

Defining the relative celerity $f = \lambda U^{-1}$, one yields from Eq. (21) that

$$(f-1)[f^3 - 2f^2 + (1 - Fr^{-2} + \psi Fr^{-2})f + Fr^{-2}(\varphi - \psi)] = 0 \tag{22}$$

The first solution of Eq. (22) can be found easily, i.e.,

$$f_3 \equiv 1 \tag{23}$$

which represents the propagation of small disturbance in relation to suspended load transport. The other three relative celerities can be found from the following cubic equation in f

$$f^3 - 2f^2 + (1 - Fr^{-2} + \psi Fr^{-2})f + Fr^{-2}(\varphi - \psi) = 0 \tag{24}$$

It is noted that Eq. (24), in its structure, is similar to a previous characteristic equation (De Vries 1973; Cunge et al. 1980) when φ is neglected. However, it is necessary to recognize the distinct physical background involved. In De Vries' study, a total-load transport capacity approach (i.e., 3-PDE model) was adopted, and therefore ψ is associated with the total-load concentration rather than the bed-load concentration as in the present analysis.

Special Solution

For the special situation in which sediment transport is predominantly in the suspension mode, i.e., $C_b \approx 0$, Eq. (24) reduces to

$$f^3 - 2f^2 + (1 - Fr^{-2})f = 0 \tag{25}$$

which gives the well-known results for the free surface flow,

$$f_1 = 1 + Fr^{-1}; \qquad f_2 = 1 - Fr^{-1} \tag{26a, b}$$

and for riverbed evolution,

$$f_4 = 0 \tag{27}$$

Conventionally, Eqs. (26) and (27) are correct only for single-phase flows over fixed bed. However, it is presently shown that the results can be extended to sediment-laden flows over mobile bed when bed-load is negligible compared to suspended sediment. Under this condition, the free surface flow and bed evolution can be considered to be mathematically independent of each other and decoupled solution is applicable. Specifically, in a given time step, the flow equations are first solved, and then the riverbed evolution equation is solved. The need to simultaneously solve the whole set of equations is thus obviated. This result justifies the flow-bed decoupling in numerous existing nonequilibrium models for suspended load-dominated alluvial rivers. The present clarification is important, recalling the prior results that the decoupling should be valid only under the conditions of low total-load concentration and small Froude number (De Vries 1965, 1973; Morris and Williams 1996).

Moreover, as can be derived from Eqs. (23) and (26),

$$f_3 < Max(f_1, |f_2|) = f_1 \tag{28}$$

It is revealed by Eq. (28) that the transport of suspended load will never adjust itself to local hydraulic conditions as quickly as the flow changes. Consequently, it is inappropriate to represent the suspended load and accordingly, the total-load transport rates at capacity values specified primarily by local hydraulic parameters. The use of 3-PDE total-load capacity models is thus open to question. More discussions on this aspect follow in the next subsection.

General Solution

For a general solution of Eq. (24), ψ and φ need to be addressed. Roughly, the typical values of the exponents in the relation Eq. (11) for equilibrium bed-load transport rate are specified as $\alpha = 4.0$, $\beta = -0.2$ (Xie 1981), and the porosity $n_p = 0.4$. Thus, from Eq. (15),

$$\psi = -5.0C_b; \qquad \varphi = 2.0C_b \tag{29a, b}$$

Fig. 2 shows the relative celerities determined by Eqs. (24) and (23) versus Froude number Fr and bed-load concentration C_b. It is not surprising that the three relative celerities f_1, f_2 and f_4 vary with Fr and C_b in the manner similar to those with Fr and a total-load transport parameter (De Vries 1973) because of the similar structure of Eq. (24) to De Vries' characteristic equation, as described before. The first celerity f_1 is invariably positive and almost unaffected by C_b. The second celerity f_2 is always negative, independent of

Fr and finite bed-load concentration C_b, although it is correct only for subcritical flows ($Fr<1$) when $C_b=0$ as is apparent from (26b). Algebraically, this results from the fact that the last term in the left-hand-side of Eq. (24) is consistently positive. Physically, it is characterized that any disturbance in the downstream will definitely influence the upstream flow and bed evolution, wherever $C_b \neq 0$. The third celerity $f_3 \equiv 1$, indicating that the propagation of small disturbance pertaining to suspended load transport always follows the flow velocity. The fourth celerity f_4, in relation to riverbed evolution, is seen to increase with Fr slowly or rapidly for finite values of C_b.

Discussions

Define the following ratios of the relative celerities of suspended load transport (f_3) and bed evolution (f_4) to those of the free surface flow (f_1 and f_2),

$$R_{31}=f_3/f_1; \quad R_{32}=|f_3/f_2|; \quad R_{41}=f_4/f_1; \quad R_{42}=|f_4/f_2| \tag{30}$$

Figs. 3 and 4 show respectively the variations of R_{31}, R_{32}, R_{41}, and R_{42} with Fr and C_b. It is obvious from Fig. 3 that for very small Fr, both R_{31} and R_{32} are much smaller than unity, characterizing that suspended load transport does not adjust immediately to any flow change. As Fr increases, both R_{31} and R_{32} become larger. Approximately when $Fr \geq 0.5$, R_{32} becomes larger than unity independent on C_b. However, R_{31} will never be as large as up to unity except when $Fr \to \infty$, noting the lower bound of f_1 given by Eq. (26a). For finite values of Fr practically possible, there invariably will be

$$R_{31}<1 \tag{31}$$

Consequently, suspended load transport will not adapt itself to local hydraulic conditions as quickly as flow varies as seen in the special case with negligible bed-load stated above. This upsets and highly constraints the use of the transport capacity concept for suspended load and subsequently total-load in which the transport rate is specified primarily by local hydraulic parameters. Naturally, this finding could not be derived from the lumped total-load transport capacity concept as in previous studies (e.g., De Vries 1965, 1973; Lyn 1987; Morris and Williams 1996).

As shown in Fig. 4, both R_{41} and R_{42} increase with C_b and Fr when $Fr<1$. While $Fr>1$, R_{42} becomes greater than unity and R_{41} increases with Fr slightly dependent on C_b. This is similar but not identical to the results of De Vries (1973) and Morris and Williams (1996). The bed-load concentration C_b in Fig. 4 must be distinguished from the total-load transport parameter in prior analyses. It follows from Fig. 4 that under certain conditions of bed-load concentration and Froude number, the celerity of bed evolution is comparable to those of the flow. The rate of bed level is not small enough as to be negligible in comparison with flow change. This upsets the aforesaid "quasi-steady state" and "fixed bed" assumptions and subsequently the flow-bed decoupling. As a result, the whole set of governing equations must be solved simultaneously. Further, because $R_{42} \geq R_{41}$ and f_2 is invariably negative wherever $C_b \neq 0$, the bed evolution is subject to the disturbance from the downstream more sensitively than from the upstream. Therefore, when the process of flow-sediment-bed evolution in response to downstream factors is to be modeled, coupled solution is necessitated to a greater extent. Practically there exist some processes, in which the changes of water flow and bed evolution stem purely from the downstream disturbances. A couple of examples can be provided here. The first is reservoir flushing-induced bed degradation in the reservoir (while the upstream flow and sediment inputs remain unchanged). The second is the bed evolution resulting from the downstream base level lowering. For these processes, coupled solution approach has the promise to give more refined results.

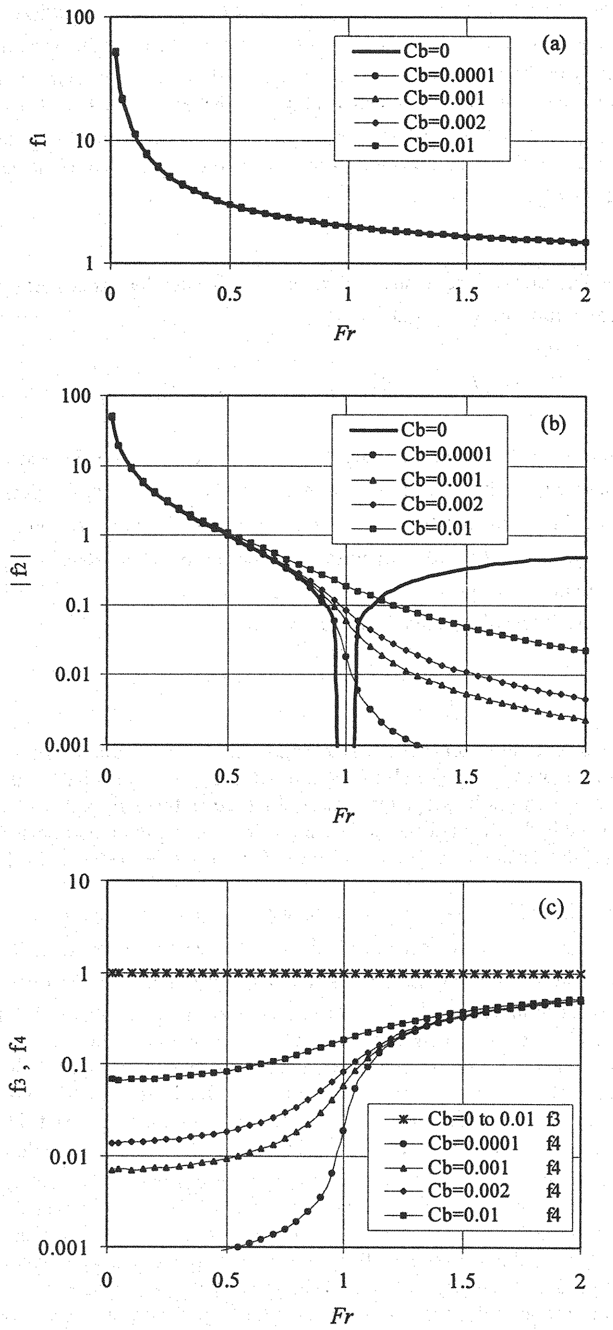

FIG. 2. Relative Celerities versus *Fr* and C_b. (a) and (b), f_1 and f_2 Associated with Free-Surface Flow; (c) f_3 and f_4, Related to Suspended Load Transport and Riverbed Evolution Respectively. f_2 is always negative when $C_b > 0$.

Qualitatively,

$$R_{41} << 1; \qquad R_{42} << 1 \tag{32a, b}$$

must be satisfied for validity of the aforementioned "quasi-steady state" and "fixed-bed" assumptions and therefore mathematical decoupling of the routing of free surface flow with riverbed evolution. A quantitative criterion is, however, unavailable at this time. For the purpose of interpretation of the implications, the following relations

$$R_{41} \leq 0.01; \qquad R_{42} \leq 0.01 \tag{33a, b}$$

are tentatively suggested. Then, the maximum Fr corresponding to a specific C_b can be found as given in Table 1. It is obvious that the maximum Fr decreases as C_b becomes greater. As C_b decreases, the constraint on Fr is relaxed. And especially, when $C_b = 0$, the constraint on Fr vanishes as is evident from Eq. (27). In spite of that the quantitative decoupling conditions are dependent on the exponents α and β in Eq. (11) as well as the bed porosity, it is demonstrated that the "quasi-steady state" or "fixed-bed" assumption and therefore the flow-bed decoupling are reasonable only within very limited ranges of small values of Fr and C_b. In the special situations with negligible bed-load transport, i.e., when suspended sediment transport dominates, the flow-bed decoupling is valid. Previous studies (e.g., De Vries 1965, 1973; Morris and Williams 1996) suggest that low total-load concentration, instead of bed-load concentration as in this study, should validate the decoupling between flow and bed evolution at small Froude numbers. The clarification by the present analysis is obviously important for applications.

TABLE 1. Maximum Fr Corresponding to Specific C_b to Satisfy Criteria Eq. (33a, b)

C_b	0.000005	0.00001	0.0001	0.001	0.002	0.01	0.02	0.05
Max. *Fr*	0.959	0.943	0.824	0.519	0.387	0.135	0.082	0.043

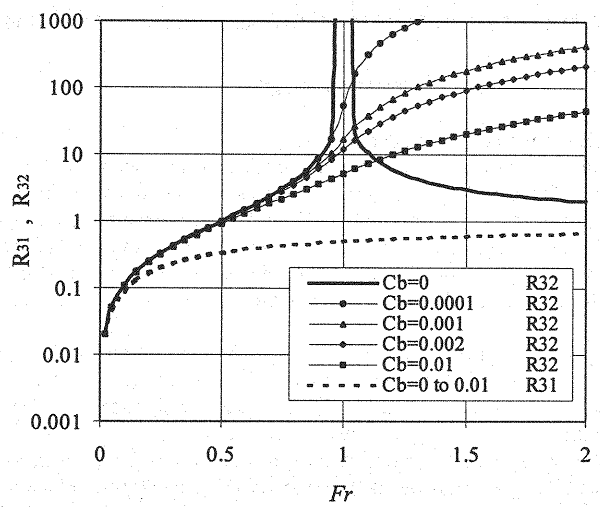

FIG. 3. Ratio of Relative Celerity of Suspended Load Transport to Those of Flow

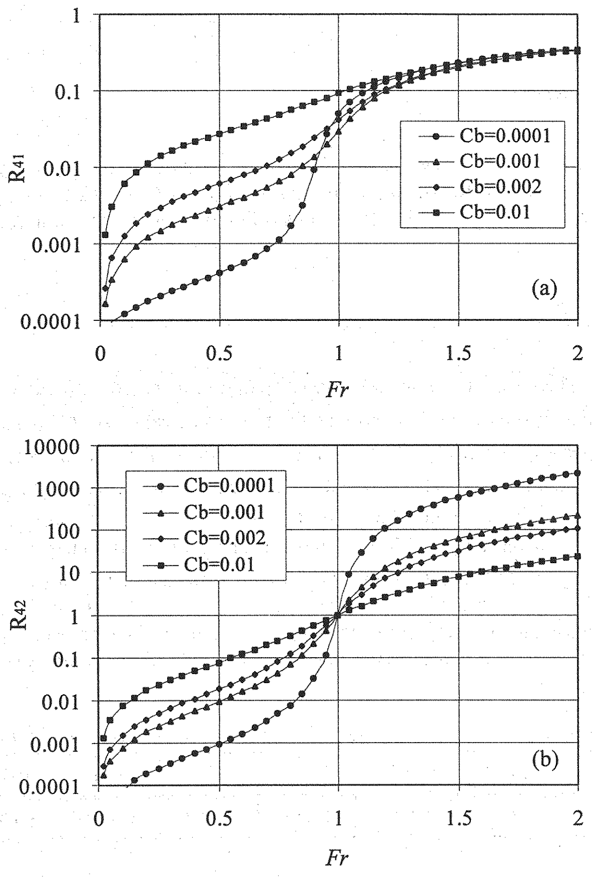

FIG. 4. Ratio of Relative Celerity of Riverbed Evolution to Those of Flow

CONCLUSIONS

The present study leads to the following conclusions:

1. The governing equations in existing one-dimensional mathematical models for alluvial rivers are usually oversimplified. Some of the simplifications and hypotheses involved remain to be justified.
2. Suspended sediment transport does not adapt itself to local hydraulic conditions as quickly as flow changes, particularly at small Froude numbers. This result severely constraints the use of the 3-PDE models, in which the total-load transport rate is specified as a capacity value determined primarily by local hydraulic parameters. Thus, 4-PDE nonequilibrium models need to be used with separate accounts for suspended sediment and bed-load.
3. Riverbed does not adjust instantaneously in response to any flow change only within very limited ranges of small Froude number and low bed-load concentration (instead of the total-load concentration as of previous studies). Beyond these limited ranges, both the "quasi-steady state" and "fixed bed" assumptions and accordingly the artificial decoupling of flow and bed evolution involved in most existing mobile-bed flow models are not valid. This finding necessitates synchronous solution of the governing equations.
4. Particularly as bed-load is negligible, decoupled solution is justified independent upon the Froude number. This fact validates the flow-bed evolution decoupling used in most existing nonequilibrium models for suspended sediment-dominated rivers.

This study is conceptual and qualitative. Further efforts are necessary for quantitative results.

ACKNOWLEDGEMENTS

The financial support from the Gakujutsu Frontier Program of the Promotion and Mutual Aid Corporation for Private Schools of Japan and the National Natural Science Foundation of China (under Grants No. 59609006 and 59890200-3) is gratefully acknowledged. The constructive comments of the anonymous reviewers are helpful to the revised paper.

APPENDIX I. REFERENCES

1. Bhallamudi, S.M. and Chaudhry, M.H.: Numerical modeling of aggradation and degradation in alluvial channels, *J. Hydr. Engrg.*, ASCE, vol.117(9), pp.1145-1164, 1991.
2. Cao, Zhixian and Egashira, Shinji: Coupled mathematical modeling of alluvial rivers, *J. Hydroscience and Hydr. Engrg.*, JSCE, vol.17(2), pp.71-85, 1999.
3. Chang, H.H.: *Fluvial Processes in River Engineering,* Wiley Interscience, New York, N.Y., 1988.
4. Correia, L.R.P., Krishnappan, B.G. and Graf, W.H.: Fully coupled unsteady mobile boundary flow model, *J. Hydr. Engrg.*, ASCE, vol.118(3), pp.476-494, 1992.
5. Cunge, J.A., Holly, F.M.Jr. and Verwey, A.: *Practical aspects of Computational River Hydraulics.* 1st Ed., Pitman, London, pp.273-311, 1980.
6. De Vries, M.: Considerations about non-steady bed-load transport in open channels, *Proc. 11th Int. Congr.*, IAHR, Delft, the Netherlands, pp.3.8.1-3.8.11, 1965.
7. De Vries, M.: River bed variations-aggradation and degradation, *Proc. Int. Seminars on Hydr. of Alluvial Streams*, IAHR, Delft, the Netherlands, pp.1-10, 1973.
8. Dou, Guoren: Transport of suspended sediment in tidal flow and the calculation of sedimentation and erosion, *Chinese J. Hydr. Engrg./Shuili Xuebao*, vol.4, 1963.
9. Holly, F.M. and Rahuel, J.L.: New numerical/physical framework for mobile-bed modeling, part 1: Numerical and physical principles, *J. Hydr. Res.*, IAHR, vol.28(4), pp.401-416, 1990(a).
10. Holly, F. M. and Rahuel, J.L.: New numerical/physical framework for mobile-bed modeling, part 2: test applications, *J. Hydr. Res.*, IAHR, vol.28(5), pp.545-564, 1990(b).
11. Hsu, H.C. and Chu, P.C.: Study of river bed degradation and aggradation by the method of characteristics, *Chinese J. Hydr. Engrg./Shuili Xuebao*, vol.5, pp.1-19, 1964.
12. Li, Yitian: One-dimensional mathematical model, *River Modeling*, Xie, Jianheng eds., Chapt. 3, China Water and Power Press, Beijing (in Chinese), 1990.
13. Lin, P.N., Huan, J. and Li, X.: Unsteady transport of suspended load at small concentrations, *J. Hydr. Engrg.*, ASCE, vol.109(1), pp.86-98, 1983.
14. Lyn, D.A.: Unsteady sediment transport modeling, *J. Hydr. Engrg.*, ASCE, vol.113(1), pp.1-15, 1987.
15. Morris, P.H. and Williams, D.J.: Relative celerities of mobile bed flows with finite solids concentrations, *J. Hydr. Engrg.*, ASCE, vol.122(6), pp.311-315, 1996.
16. Phillips, B. and Sutherland, A.J.: Spatial lag effects in bed-load sediment transport, *J. Hydr. Res.*, IAHR, vol.27(1), pp.115-133, 1990(a).
17. Phillips, B. and Sutherland, A.J.: Temporal lag effects in bed-load sediment transport, *J. Hydr. Res.*, IAHR, vol.28(1), pp.5-23, 1990(b).
18. Rahuel, J.L.: Discussion on "Fully coupled unsteady mobile boundary flow model," *J. Hydr. Engrg.*, ASCE, vol.119(4), pp.528-530, 1993.
19. Roberson, J.A. and Crowe, C.T.: *Engineering Fluid Mechanics,* Houghton Mifflin, Boston, MA, 1990.
20. Saiedi, S.: A non-dimensional coupled numerical model of alluvial flow, *Int. J. Sediment Res.*, IRTCES, vol.9(2), pp.59-79, 1994.
21. Saiedi, S.: Coupled modeling of alluvial flows, *J. Hydr. Engrg.*, ASCE, vol.123(5), pp.440-446, 1997.
22. Stevens, M.A.: Discussion on "Unsteady sediment transport modeling," *J. Hydr. Engrg.*, ASCE, vol.114(8), pp.954-956, 1988.
23. Wei, Liangyan: Basic equations of mathematical models." *River Modeling*, Xie, Jianheng eds., Chapt. 1, China Water and Power Press, Beijing (in Chinese), 1990.
24. Xie, Jianheng (Eds): *River Sedimentation Engineering*, China Water Resources Press, Beijing (in Chinese), 1981.

25. Xie, Jianheng (Eds): *River Modeling*, China Water and Power Press, Beijing (in Chinese), 1990.
26. Zhang, H. and Kahawita, R.: Nonlinear model for aggradation in alluvial channels, *J. Hydr. Engrg.*, ASCE, vol.113(3), pp.353-369, 1987.
27. Zhou, Jianjun and Lin, Bingnan: One-dimensional mathematical model for suspended sediment by lateral integration, *J. Hydr. Engrg.*, ASCE, vol.124(7), pp.712-717, 1998.

APPENDIX II. NOTATION

The following symbols are used in this paper:

A = area of flow cross-section;
A_0 = extended alluvial area above some datum;
B = channel width;
C = flux-averaged suspended sediment concentration in volume;
C_b = flux-averaged bed-load concentration in volume;
C_t = flux-averaged total-load concentration in volume;
d = sediment particle diameter;
F_n = net flux of suspended sediment exchange with bed per unit width;
F_t = net mass flux of total water-sediment mixture exchange with bed surface;
Fr = Froude number;
f = relative celerity;
g = gravitational acceleration;
h = flow depth;
h_G = distance from free surface of shape center of cross-section;
I_b = bed slope;
I_e = friction slope;
p = bed sediment porosity;
q_b = bed-load transport rate per unit width;
q_t = total-load transport rate per unit width;
t = time;
U = cross section-averaged streamwise velocity;
x = streamwise coordinate;
Y = bed elevation;
$\alpha, \beta, \sigma, \delta$ = parameters in bed-load discharge formula;
ψ, φ = parameters related to bed-load concentration;
λ = characteristic celerity;
ρ_w = water density;
ρ_s = sediment density;
$\rho_m = \rho_w(1 - C_t) + \rho_s C_t$, bulk density of water-sediment mixture; and
$\rho_0 = \rho_w p + \rho_s \cdot (1 - p)$, bulk density of water-saturated bed.

(Received February 9, 1999 ; revised March 2, 2000)