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NUMERICAL SIMULATION MODELS ON INUNDATION FLOW IN URBAN AREA

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SYNOPSIS

In overland flood flow analysis, the Cartesian coordinate model has been commonly used for its easiness of mesh formation. However, in the application of it to urban area, blocking of inundation flow by buildings or spread of it along streets cannot be well expressed. In order to improve this, the "Generalized curvilinear coordinate model" is taken and "Street network model" in which a network comprises links of streets and nodes of intersections is developed here. The results obtained by the above two models and the Cartesian coordinate model are studied and the advantage of "Street network model" is designated.

INTRODUCTION

When inundation flow analysis is conducted by a two-dimensional unsteady flow model, the Cartesian coordinate model has been often used because of its easiness of mesh formation. But it has difficulty in considering the effects of streets and buildings which have large influence to the inundation flow in urban area. In order to improve this, such a method as evaluates the roughness coefficient by the density of buildings (3) or considers hydrodynamic force acting on the houses (2) has been adopted in the preceding studies. In these methods, however, streets are contained in each mesh and inundation flow along the streets cannot be well expressed. Therefore it must be said that there is still the limitation of improvement of inundation flow modeling in the above models. In this study, based on the idea that it is important to distinguish the street meshes from the meshes of the other part, the "Generalized curvilinear coordinate model" is adopted and a new inundation flow model, "Street network model" is developed. These three models, the "Cartesian coordinate model", the "Generalized curvilinear coordinate model" and "Street network model", are applied to storm surge flooding in Osaka City and their results are compared and discussed.

MODELS OF INUNDATION FLOW

Cartesian Coordinate Model

The analysis of inundation flow due to storm surge comprises storm surge analysis and inundation flow analysis. In this study, the Cartesian coordinate model is used for storm surge analysis according to Takeda et al. (4).

In inundation flow analysis, the shallow water equations as below are used.

$$\frac{\partial h}{\partial t} + \frac{\partial M^x}{\partial x} + \frac{\partial M^y}{\partial y} = 0 \tag{1}$$

$$\frac{\partial M^x}{\partial t} + \frac{\partial (u^x M^x)}{\partial x} + \frac{\partial (u^y M^x)}{\partial y} = -gh\frac{\partial H}{\partial x} - \frac{\tau_{bx}}{\rho_w} \tag{2}$$

$$\frac{\partial M^y}{\partial t} + \frac{\partial (u^x M^y)}{\partial x} + \frac{\partial (u^y M^y)}{\partial y} = -gh\frac{\partial H}{\partial y} - \frac{\tau_{by}}{\rho_w} \tag{3}$$

where h = water depth; u^x and u^y = x and y directional velocity, respectively; M^x and M^y = x and y directional discharge flux, respectively; H = water level; τ_{bx} and τ_{by}, which are calculated as below, = shear stress at the water bottom.

$$\tau_{bx} = \frac{\rho_w g n^2 u^x \sqrt{(u^x)^2 + (u^y)^2}}{h^{1/3}}, \quad \tau_{by} = \frac{\rho_w g n^2 u^y \sqrt{(u^x)^2 + (u^y)^2}}{h^{1/3}}$$

where n = Manning's roughness coefficient; g = gravity acceleration; ρ_w = sea water density.

Fig. 1 shows the computational meshes of the studied area and surface elevation there.

Generalized Curvilinear Coordinate Model

In considering the effects of streets whose shapes cannot be well expressed by rectangular-shaped meshes such as those in the Cartesian coordinate model, the "Generalized curvilinear coordinate model" is introduced. The basic equations in the Generalized curvilinear coordinate model can be obtained by transforming the equations in the Cartesian coordinate model on the physical plane (x, y) system into those on the computational plane (ξ, η) system. The governing equations are as follows;

$$\frac{\partial h}{\partial t} + J\frac{\partial}{\partial \xi}\left(\frac{M^\xi}{J}\right) + J\frac{\partial}{\partial \eta}\left(\frac{M^\eta}{J}\right) = 0 \tag{4}$$

$$\begin{aligned} \frac{\partial M^\xi}{\partial t} = & -J^2 y_\eta \left\{ \frac{\partial}{\partial \xi}\left(\frac{u^\xi M^x}{J}\right) + \frac{\partial}{\partial \eta}\left(\frac{u^\eta M^x}{J}\right) \right\} \\ & + J^2 x_\eta \left\{ \frac{\partial}{\partial \xi}\left(\frac{u^\xi M^y}{J}\right) + \frac{\partial}{\partial \eta}\left(\frac{u^\eta M^y}{J}\right) \right\} \\ & - J^2 gh \left\{ (x_\eta^2 + y_\eta^2)\frac{\partial H}{\partial \xi} - (x_\xi x_\eta + y_\xi y_\eta)\frac{\partial H}{\partial \eta} \right\} \\ & - \frac{g n^2 M^\xi \sqrt{(M^x)^2 + (M^y)^2}}{h^{7/3}} \end{aligned} \tag{5}$$

$$\begin{aligned} \frac{\partial M^\eta}{\partial t} = & -J^2 y_\xi \left\{ \frac{\partial}{\partial \xi}\left(\frac{u^\xi M^x}{J}\right) + \frac{\partial}{\partial \eta}\left(\frac{u^\eta M^x}{J}\right) \right\} \\ & + J^2 x_\xi \left\{ \frac{\partial}{\partial \xi}\left(\frac{u^\xi M^y}{J}\right) + \frac{\partial}{\partial \eta}\left(\frac{u^\eta M^y}{J}\right) \right\} \\ & - J^2 gh \left\{ -(x_\xi x_\eta + y_\xi y_\eta)\frac{\partial H}{\partial \xi} + (x_\xi^2 + y_\xi^2)\frac{\partial H}{\partial \eta} \right\} \\ & - \frac{g n^2 M^\eta \sqrt{(M^x)^2 + (M^y)^2}}{h^{7/3}} \end{aligned} \tag{6}$$

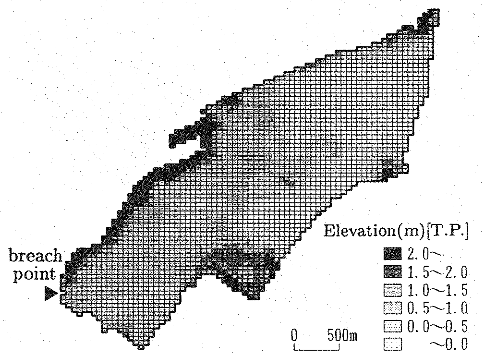


Fig. 1 Computational meshes in the Cartesian coordinate model

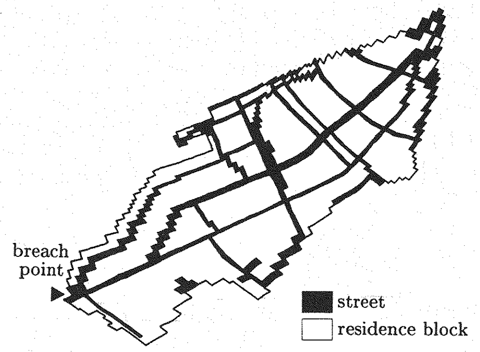


Fig. 2 Street meshes in the generalized curvilinear coordinate model

Where J = Jacobian defined by

$$J = \frac{1}{x_\xi y_\eta - x_\eta y_\xi} \tag{7}$$

A subscript denotes partial differentiation in terms of its variable and a superscript denotes the direction of the velocity or discharge flux.

The Generalized curvilinear coordinate model makes it possible to evaluate the effects of streets and buildings by distinguishing street meshes from the meshes of the other part (residence block). Fig. 2 shows the street meshes and the residence blocks used in this study. About 25% of the studied area can be regarded as the street meshes which has no buildings (the net streets area read from the map is about 35%).

Street Network Model

In urban area, the inundation water spreads along streets since buildings are constructed closely on both sides of streets. Then, in this model, the studied area is divided into the street part and the other part (named "residence block"). The street part consists of links (streets) and nodes (intersections). The concept of this modeling is shown in Fig. 3 and the street meshes (link meshes and node meshes) used in this study are shown in Fig. 4. About 33% of the studied area can be regarded as street meshes.

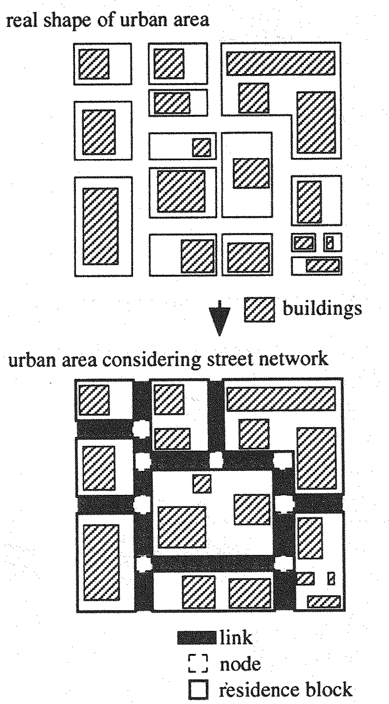


Fig. 3 Modeling of street network

In calculation, in link mesh, which is divided into finer meshes, one-dimensional shallow water equations are used as below. In this case, each link is regarded as a uniform rectangular channel and assumed to have an x-positive direction from a start node to an end node.

$$\frac{\partial h}{\partial t} + \frac{\partial M}{\partial x} = \frac{q_{in}}{B} \tag{8}$$

$$\frac{\partial M}{\partial t} + \frac{\partial (uM)}{\partial x} = -gh\frac{\partial H}{\partial x} - \frac{gn^2|M|M}{h^{7/3}} \tag{9}$$

Where u and M = x-directional velocity and discharge flux, respectively; q_{in} = lateral inflow; B = channel width. The viscous term is neglected since its effect is supposed to be expressed by the roughness coefficient n.

Node mesh or residence block is regarded as one mesh for itself. The water depth there is calculated from the following continuity equation.

$$\frac{\partial h}{\partial t} = \frac{1}{A}\sum_{k=1}^{m} Q_k \qquad (10)$$

Where h = water depth of node mesh or residence block; A = its area; Q_k = the discharge from the mesh side k; m = the total number of the mesh sides.

To calculate the discharge flux between link, node and residence block, the following equation is used.

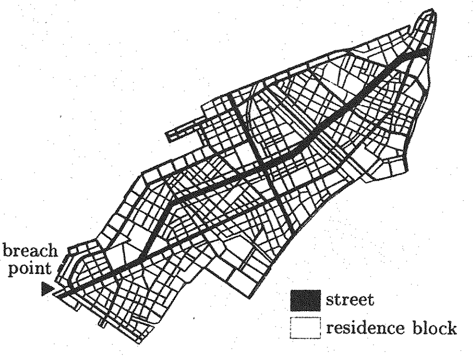


Fig. 4 Street meshes in street network model

$$\frac{\partial M}{\partial t} = -gh\frac{\partial H}{\partial x} - \frac{gn^2|M|M}{h^{7/3}} \qquad (11)$$

In treating the water surface slope, Δx, the distance between the adjacent two meshes is defined in each case as below. In the case between link and node, as shown in Fig. 5, Δx is defined by the distance between the centroids of the node mesh and the link mesh at the end. In the case between link and residence block, as shown in Fig. 6, a perpendicular line is drawn from the centroid of the link mesh to the boundary of the link mesh and the residence block, and the centroid of the residence block is defined as the middle point of the part of this perpendicular line separated by the residence block, then Δx is defined by the distance between these centroids. In the case between node mesh and residence block, Δx is defined in the same way as shown in Fig. 6.

In making street network, a long straight-shaped street often has to be divided into a few links. If the angle made by two links is less than 10°, then, these two links are assumed to be straight connected and Eq. 9 is used for calculation of the discharge flux between link and node, instead of Eq. 11.

APPLICATION TO URBAN AREA

Effects of Streets and Buildings

To consider the effects of streets and buildings, the concepts of the occupying ratio λ (defined as the ratio of buildings area to the

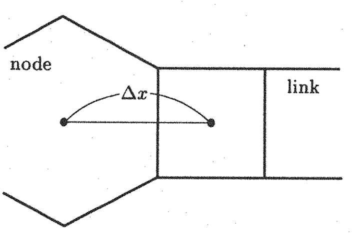


Fig. 5 Distance of link mesh and node mesh

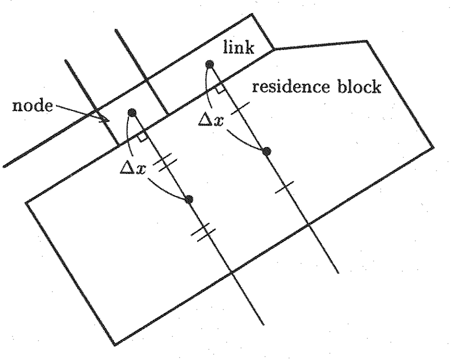


Fig. 6 Distance of link mesh and residence block or node mesh and residence block

mesh area in each mesh) and the invasion ratio β, suggested by Nakagawa(3), are introduced.

a) Cartesian coordinate model and Generalized curvilinear coordinate model

For the mesh (i, j), the invasion ratio $\beta_{i,j}$ is defined from the occupying ratio $\lambda_{i,j}$ as

$$\beta_{i,j} = \sqrt{1-\lambda_{i,j}} \tag{12}$$

Then the $x(\xi)$-directional discharge flux between the mesh (i, j) and $(i+1, j)$ is corrected by

$$\tilde{M}^x_{i+1/2,j} = \beta_{a,j} M^x_{i+1/2,j} \tag{13}$$

where a is $i+1$ or i according as $M^x_{i+1/2,j}$ is positive or negative, respectively. The $y(\eta)$-directional discharge flux $M^y_{i,j+1/2}$ is corrected in the same way.

In the case of the Cartesian coordinate model, for example, the water depth is calculated by substituting this corrected discharge for the following equation,

$$(1-\lambda_{i,j})\frac{\partial h}{\partial t} + \frac{\partial \tilde{M}^x}{\partial x} + \frac{\partial \tilde{M}^y}{\partial y} = 0 \tag{14}$$

b) Street network model

Following the same way as a), the invasion ratio β_k of the residence block k is defined from the occupying ratio λ_k as $\beta_k = \sqrt{1-\lambda_k}$. The discharge $Q_{k,l}$ at the mesh side l of the residence block k is corrected as

$$\tilde{Q}_{k,l} = \alpha Q_{k,l} \tag{15}$$

where $\alpha = \beta_k$ or α=1 according as the inundation water comes into or out the residence block, respectively. The water depth is calculated by substituting this for the following equation,

$$\frac{\partial h}{\partial t} = \frac{1}{(1-\lambda_k)A_k}\sum_{k=1}^{m} \tilde{Q}_{k,l} \quad . \tag{16}$$

On the other hand, in considering the effect of blocking of inundation flow by buildings in the residence block and inundation water flow along streets, the roughness coefficient of street meshes (link meshes and node meshes) is set smaller than that of residence blocks.

Computational Condition

The following two cases are assumed: one is the non-urbanized case (λ=0.0, n (residence block) = n (street) =0.067), and the other is the urbanized case (λ (residence block) $\neq$ 0.0, λ (street) = 0.0, n (residence block) = 0.067, n (street) = 0.043). These are summarized in Table. 1. The reason why the occupying ratio value is different among the three models in the urbanized case is that the ratio of the area occupied by residence blocks is different among them. But total buildings area in the whole studied area is constant.

The "designed typhoon" of Osaka City, which has the Muroto typhoon course and the Isewan typhoon scale, is adopted as the external force. The discharge hydrograph (Fig. 7) at the assumed breach point obtained from the storm surge analysis is used. Except the breach point, the studied area is assumed to be surrounded by an extremely high vertical wall.

Table. 1 Cases for analysis

	non–urbanized case	urbanized case
Cartesian coordinate	C_0	C_1 (street $\cdots$ 0%)
Generalized curvilinear coordinate	G_0	G_1 (street $\cdots$ 25%)
Street network model	N_0	N_1 (street $\cdots$ 33%)

non-urbanized case

$$\begin{cases} \lambda(\text{residence block}) = \lambda(\text{street}) = 0.0 \\ n(\text{residence block}) = n(\text{street}) = 0.067 \end{cases}$$

urbanized case

$$\begin{cases} \lambda(\text{residence block}) = \\ \qquad 0.48 \cdots \text{Cartesian coordinate} \\ \qquad 0.64 \cdots \text{Generalized curvilinear coordinate} \\ \qquad 0.72 \cdots \text{Street network model} \\ \lambda(\text{street}) = 0.0 \\ n(\text{residence block}) = 0.067 \\ n(\text{street}) = 0.043 \end{cases}$$

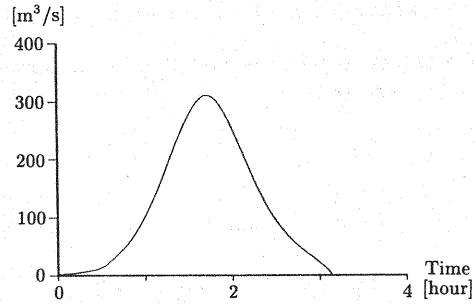


Fig. 7 Inundation discharge from the coastal dike

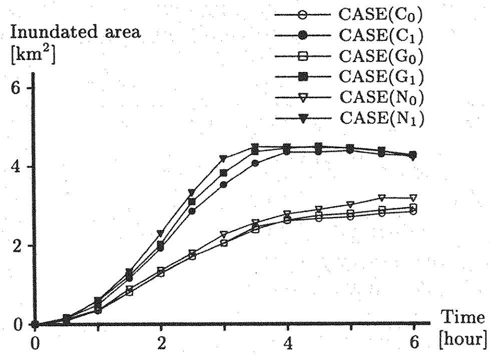


Fig. 8 Temporal change of inundated area (1)

NUMERICAL RESULTS

Comparison of the Obtained Results

Fig. 8 shows the temporal change of the inundated area (including the buildings area within the inundated area) of all cases. The difference between the non-urbanized case and the urbanized case is due to the spread of the inundation water by waterproof buildings.

The comparison of inundated area spread 2 hours after inundation start in the non-urbanized case is shown in Fig. 9(a). From Fig. 8 (CASE(C_0), CASE(G_0), CASE(N_0)) and Fig. 9(a), the results of the three models have little difference. Therefore under the simple condition without considering the effects of streets and buildings, the Generalized curvilinear coordinate model and Street network model bring almost the same results as the Cartesian coordinate model which has been used most commonly.

Fig. 9(b) shows the comparison of inundated area spread 2 hours after inundation start in the urbanized case. From Fig. 8 (CASE(C_1), CASE(G_1), CASE(N_1)) and Fig. 9(b), the inundation water spreads fastest in the case of Street network model, followed by the Generalized curvilinear coordinate model and the Cartesian coordinate model. The larger the street area regarded as street meshes is, the faster the inundation water spreads out.

Consideration of Invasion Ratio

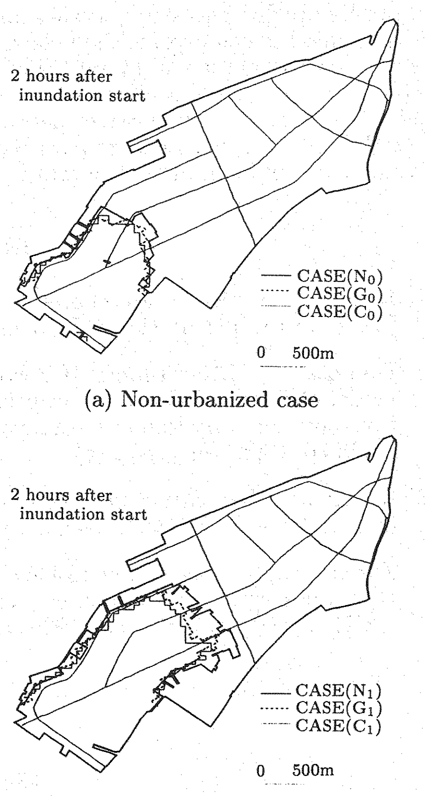
The inundation water is supposed to invade into residence block through opening between buildings, as shown in Fig. 10. In the previous section, though the invasion ratio β is defined as a function of the occupying ratio λ, β should be essentially independent of λ. Then in Street network model, the invasion ratio β is redefined as the ratio of the opening length to the side length of the residence block independently of the occupying ratio λ. In either case when the inundation water come into or out of the residence block, α in Eq. 15 is replaced by β.

The value of β used in the previous section was 0.53, and the case with β=0.6 is defined "CASE(N_α)". But in fact, the value of β is supposed to be a little smaller than 0.6, so the cases with β=0.4, 0.2 are adopted as "CASE(N_β)", "CASE(N_γ)", respectively. The temporal change of the inundated area of these four cases, including CASE(N_1), is shown in Fig. 11. From this figure, it is found that the smaller the invasion ratio is, the slower the inundation water spreads. This may be due to the reason that the inundation water which comes out from the residence block is restricted and it is stored in the residence block for a longer period. On the other hand, as β gets smaller, the inundation water is generally expected to spread faster along the streets. This contradiction should be further investigated in detail.

In this way, by means of Street network model, the invasion ratio can be defined independently of the occupying ratio and more practical results can be obtained.

CONCLUSION

In urban area, inundation water spreads along main streets and then it invades into detailed streets. In this study, the Generalized curvilinear coordinate model and Street network model which can evaluate the street as meshes are developed and their computational results, including the Cartesian coordinate model, are compared. In



(a) Non-urbanized case

(b) Urbanized case

Fig. 9 Comparison of inundated area spread

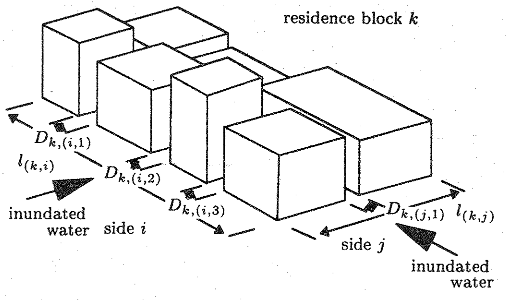


Fig. 10 Redefinition of invasion ratio

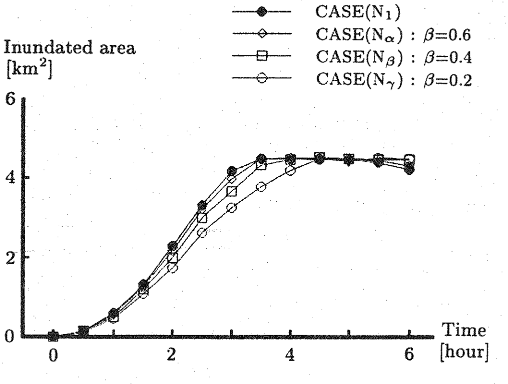


Fig. 11 Temporal change of inundated area (2)

the cases of the Generalized curvilinear coordinate model and Street network model, the studied area can be divided into street meshes and the meshes of the other part. It has been found that these models can express the behavior of the inundation water more practically than the Cartesian coordinate model by means of the simple way that the parameters (the occupying ratio and the roughness coefficient) can be given to street meshes and the residence blocks, respectively. Especially in Street network model, the invasion ratio of the inundation water can be defined as an independent parameter directly and the behavior of the inundation water in urban area can be evaluated suitably.

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