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# A NUMERICAL MODEL FOR INTEGRAL PROPERTIES OF PERIODIC NON-LINEAR BREAKING AND NON-BREAKING WAVES

by

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## SYNOPSIS

An inter-period non-linear dispersive wave propagation numerical model is used for the evaluation of the integral properties (radiation stress and mass flux above trough level) of the non-breaking and breaking periodic waves in shallow water. A nonuniform horizontal velocity distribution is introduced to incorporate the effects of the turbulence and the surface roller in the surf zone. For non-breaking waves irrotational theory is adopted. Expressions of the time averaged horizontal kinetic energy, radiation stress and the wave mass transport are proposed in terms of the model variables i.e. the mean over the depth velocity and the surface elevation. Model results are tested against linear and non-linear wave theory and experimental data.

## INTRODUCTION

Wave induced nearshore current models (2D depth integrated, 2D vertical, quasi 3D as well as 3D) require the pre-estimation of the time- averaged wave induced forces and other integral quantities.

Several relationships for the integral properties of the waves have been proposed based both on linear and non-linear theory but only few of them are connected with a wave propagation model valid not only for purely progressive waves but also for an arbitrary two-dimensional wave field (5, 6, 14, 31, 34).

Linear theory is generally used (23, 26) but is not expected to give accurate results due to its application in a high non-linear rotational field. To overcome that, semi-empirical relations can be used (3) or non-linear theory (17, 28) both adapted for application in the surf zone, i.e. taking into account surface roller effects (26).

Instead of using one of the above methods a non-linear non-breaking and breaking wave numerical model based on the Boussinesq equations is expected to give better results not only due to its confirmed validity inside and outside the surf zone but also due to its validity to describe a confused wave field caused by the combined effects of refraction, reflection, diffraction and breaking.

The success of the Boussinesq approach in modelling nearshore wave motion has led to the development of the Boussinesq models by many researchers such as Abbot et al. (1), Brocchini et al. (2) Karambas and Koutitas (8), Karambas (10), Kabiling and Sato (11), Schaffer et al. (19), Svendsen et al. (27) for the prediction of the wave trasnformation in the shoaling region and inside surf zone. These models can also be extended to predict sediment transport and bed evolution in the nearshore zone (Karambas et al. (9), Kabiling and Sato (11), Watanabe (32), Watanabe and Dibajnia (33), Rakha et al. (18)). However the models can predict a mean over the depth mean current, including a mean over the depth below wave trough level undertow. No information is given for the distibution over the depth of the current. Thus the above sediment transport models need additional information (see 18, 32) usually provided by a wave-induced current model.

In the present work we suggest to use the non-linear wave propagation model for the evaluation of the integral properties of breaking and non- breaking waves in shallow water. Model results are compared with analytical expressions as well as experimental data.

## NUMERICAL MODEL AND VELOCITY DISTRIBUTION

Boussinesq models can be extended inside surf zone to incorporate breaking wave propagation using different methods. Brocchini et al. (2) and Schaffer et al. (19) proposed a method based on the "surface roller" concept while Karambas and Koutitas (8) on the eddy viscosity concept (see also 11). The contribution of surface roller in the flow i.e. the pressure of the mass of the roller on the surface or its propagation with the wave celerity is taken into account in the first type of models while in the second type the dissipation is introduced through a dispersion term to simulate Reynolds stresses. Karambas (10) and Svendsen et al. (27) presented two new breaking wave models by solving, simoultaneously with the Boussinesq equations, the wave energy and the vorticity transport equations respectively. In both works a nonuniform over the depth horizontal velocity profile is assumed.

In the present work a simplified version of Karambas (10) model is proposed. That model was based on the Svendsen and Madsen works (13, 25) for the extension of the classical Shallow Water Equations (SWE) to include the effect of turbulence in a bore propagation problem. In the present simplified version the wave energy equation is not solved. Turbulence and surface roller effects are simply indroduced through the assumption of a nonuniform over the depth horizontal velocity distribution.

The purpose of the present work is the derivation of the integral properties mainly inside surf zone. However, wave-induced nearshore current models also require the pre-estimation of the wave integral quantities in shallow water of the shoaling region. Thus in the shoaling region Serre type of equations (including higher order non-linear terms) are solved. The reason for this choice is outlined below. Brocchini et al. (2) concluded that in the shoaling region Serre type of equations can provide better results in comparison with standard Boussinesq equations. In addition, according to Sobey (20) standard Boussinesq equations do not satisfy the appropriate conservation laws even in shallow water. Sobey also concluded that much better results can be obtained using a new type of Boussinesq equations which in shallow water becomes similar to the Serre type of equations.

Inside surf zone the velocity distribution, used for the derivation of the Serre type of equations (Mei, (14)) is not valid. Thus in this region the momentum equation adopted in (10) and (19) is used.

The momentum equation of the present model is written in the form (8, 19) inside surf zone:

$$\frac{\partial U}{\partial t}+\frac{1}{h}\frac{\partial\left(\int_{-d}^{\zeta}u^2dz\right)}{\partial x}-\frac{1}{h}U\frac{\partial(Uh)}{\partial x}+g\frac{\partial\zeta}{\partial x}=\frac{d^2}{3}\frac{\partial^3 U}{\partial x^2\partial t}+d_x d\frac{\partial^2 U}{\partial x\partial t}+\frac{1}{h}\frac{\partial}{\partial x}\left(\nu_t h\frac{\partial U}{\partial x}\right) \tag{1a}$$

and in the shoaling region the Serre type of equations by Mei (24) and Brocchini et al. (2):

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+g\frac{\partial\zeta}{\partial x}=\frac{h^2}{3}\frac{\partial^3 U}{\partial x^2\partial t}+\frac{h^2}{3}\left(U\frac{\partial^3 U}{\partial x^3}-\frac{\partial U}{\partial x}\frac{\partial^2 U}{\partial x^2}\right)+h\frac{\partial\zeta}{\partial x}\frac{\partial^2 U}{\partial x\partial t}+d_x h\frac{\partial^2 U}{\partial x\partial t} \tag{1b}$$

where u=u(z)=horizontal velocity; $\zeta$=surface elevation; $\nu_t$= eddy viscosity coefficient for the simulation of Reynolds stresses ($\overline{u'^2} - \overline{w'^2} \approx \nu_t \, \partial U/\partial x$); d= still water depth; h=d+$\zeta$; and the quantity U(x, t) is the mean over the depth velocity:

$$U = \frac{1}{h} \int_{-d}^{\zeta} u \, dz \qquad (2)$$

Assuming a similarity in the flow between an hydraulic jump and a breaking wave in the inner region, a non-uniform velocity distribution is adopted (13) to describe the flow in the turbulent region $\delta$=h-a, (Fig. 1):

$$u(z)=u_o+u_d \, f(\sigma) \quad ; \qquad u_d=u_s-u_o \qquad (3)$$

$$f(\sigma)=-A\sigma^3 +(1+A)\sigma^2 \quad ; \qquad \sigma=(d+z-a) / (h-a) \text{ and } A=1.4$$

where $u_s$ =surface velocity; and $u_o$=velocity at the lower part of the wave z<-(d-a).

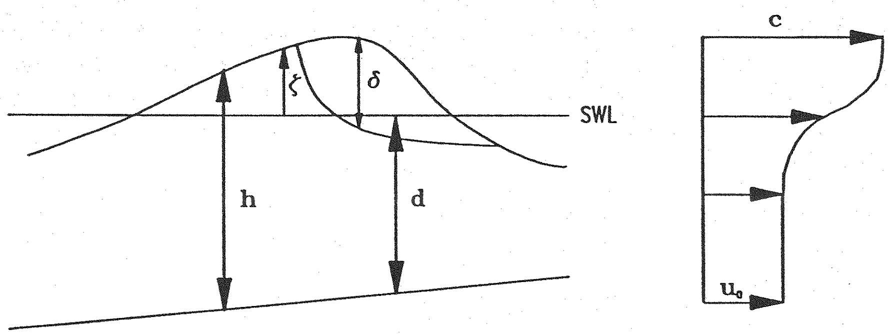


Fig. 1. Definition sketch and velocity distribution.

The velocity simulation in the lower region which is adopted based on the results obtained by the experiments of Nadaoka et al. (15). The authors separated the velocity u into an rotational part $u_r$ and an irrotational part $u_p$. In the non-turbulent region (Fig. 1 and Fig. 20 of their paper) $u_r$ is not significant and we can assume that u=$u_p$, where $u_p$ the non-breaking wave horizontal velocity which is given by (16):

$$u_p(z) = U - \left( \frac{(d+z)^2}{2} - \frac{h^2}{6} \right) \frac{\partial^2 U}{\partial x^2} \qquad (4)$$

Since the velocity is rather constant in the lower non turbulent region we can assume that for the lower non turbulent region (of a breaking wave where $\delta$>0):

$$u_p(-d) = u_o = U + \frac{h^2}{6} \frac{\partial^2 U}{\partial x^2} \quad \text{for} \quad -d<z<-(d-a) \qquad (5)$$

Experiments by Stive (21) confirm the above assumption. Nadaoka et al. (15) also used non-linear irrotational theory to estimate the velocity field from surface elevation measurements of a breaking wave. Using the definition of the mean velocity U Eqs (2) and (3) the turbulent region depth δ (i.e. the thickness of the surface roller) can be estimated from (10):

$$h - a = \delta = \frac{U - u_o}{u_d} \frac{h}{0.45} \tag{6}$$

applied in a length detected geometrically as in Schaffer et al. (19).

In the irrotational region of a breaking wave (where δ=0) a uniform distribution u(z)=U is adopted.

In the shoaling region Eq. (4) is assumed to be valid.

The connection of the two different Eqs (1a) and (1b), outside and inside surf zone, does not requires any specific treatment.

Svendsen and Madsen (25) proposed a system of four equations consisting from the continuity, total momentum and energy equation as well as from an additional momentum equation for the non turbulent region. The variables $u_d$ and h-a were determined in terms of energy E, U, ζ and $u_o$. Karambas (10) extended the above work incorporating the wave energy equation into a non-linear breaking wave propagation system based on Boussinesq equations. Here a simplified version of the model is used. As in Karambas (10) it is assumed that in the front side of the wave in the region between the toe and the top of the surface roller, $u_s$=c, with c the wave celerity (19). Another reason to adopte $u_s$=c is that we not consider a single bore propagation, as Svendsen and Madsen (25), but succeed breakers where the turbulence kinetic energy which remain from the previous wave has to be added in the next one. Thus essentially there is not a non-turbulent region but a region with nearly constant horizontal velocity u (21) extending in a height a to be determined rather empirically than analytically.

The integral in the momentum equation is estimated applying Eq. (3):

$$\int_{-d}^{\zeta} u^2 dz = h u_o^2 + \delta\left(0.312 u_d^2 + 0.9 u_o u_d\right) \tag{7}$$

Eq. (7) is the only difference between the present model and the Karambas (10) one. In Karambas work (10) the above integral is estimated from the simoultaneously solution of the wave energy equation.

The eddy viscosity coefficient $\nu_t$ is estimated from the numerical solution of a turbulent kinetic energy model as in Karambas and Koutitas (8) and Karambas (10). The transport equation for the mean over the depth turbulent kinetic energy $k_t$ is written (4):

$$\frac{\partial(k_t h)}{\partial t} + \frac{\partial(k_t h U)}{\partial x} = \frac{\partial}{\partial x}\left(\nu_t \frac{\partial(k_t h)}{\partial x}\right) + D - \varepsilon h \tag{8}$$

with

$$\nu_t = k_t^{1/2}\, l$$

where D=production term (see Karambas (10)); ε=depth mean dissipation rate of $k_t$ which are related through:

$$\varepsilon = C_d\, k_t^{3/2} / l$$

where $C_d$ is a constant $C_d$ =0.08; and $l$=characteristic length of Energy Containing Eddies which is usually related to the local depth d. Here, as in Karambas (10) we adopte $l$=0.3d (see also (24)).

In the shoaling region vertical velocity w(z) is assumed to vary linearly from the bottom up to surface:

$$w(z) = -(d+z)\frac{\partial U}{\partial x} \tag{9}$$

this distribution satisfies the free surface boundary condition after the assumption that $\partial U/\partial x \approx (\partial\zeta/\partial t + u_s\partial\zeta/x)/h$ which has been used for the derivation of the Boussinesq equations.

From continuity equation and Eq. (3) (in the turbulent region, where δ>0) the following expression or w(z) is derived:

$$w(z) = -\frac{\partial u_o}{\partial x}(d+z-a) + \frac{\partial u_d}{\partial x}\left(-0.35\frac{(d+z-a)^4}{\delta^3} + 0.8\frac{(d+z-a)^3}{\delta^2}\right) \tag{10}$$

Breaking point is determined using Goda's breaking criterion as in Karambas and Koutitas (8).

The numerical scheme is presented in the Appendix. The model has been succesfully tested against experimental data and non linear theories concerning the wave energy, height, elevation and horizontal velocity prediction inside as well as outside surf zone (8, 9, 10). The results obtained by the present version of the model are very close (almost identical) to the Karambas (10) model. In the present work only the numerical results concerning the integral properties will be presented.

## RADIATION STRESS AND MASS TRANSPORT

In a set-up 1D model or in a 2D horizontal wave induced current model the driving forces are the well known radiation stress $S_{xx}$ defined as (3):

$$S_{xx} = \rho\overline{\int_{-d}^{\zeta}(u^2 - w^2)dz} + \frac{1}{2}\rho g\overline{(\zeta - \bar{\zeta})^2} \tag{11}$$

with $\bar{\zeta}$ = mean water level and an overbar denotes time averaging.

Undertow models require also the estimation of the wave mass flux M in the region between the wave crests and troughs, including the contribution of the surface roller, in order to balance the flow below trough level $\zeta_{tr}$. Mass flux M is defined in the Eulerian description as:

$$M = \overline{\int_{-\zeta_{tr}}^{\zeta} u\,dz} \tag{12}$$

The above defined integral quantities i.e. radiation stress $S_{xx}$ and mass flux M are evaluated in the next section in terms of the wave model dependent variables U and ζ.

Outside the surf zone, in the shoaling region, the u(z) distribution of Eq. (4) is adopted, given by Peregrine (16) after the assumption of irrotational waves. For the vertical velocity w(z) the linear distribution given by Eq. (9) is used. Inside surf zone u(z) and w(z) have already been given in the previous section. After that we have all the information required for the evaluation of the integral quantities.

Yasuda et al. (34) derived the integral quantities as a function of the mass flux and the potential energy to be similar to the expressions from non linear theory (12). Here another derivation should be adopted relating the integral quantities directly with the results (mean velocity U and surface elevation $\zeta$) of the wave propagation model.

Time and depth averaged horizontal and vertical kinetic energy are defined as:

$$E_u = \rho \overline{\int_{-d}^{\zeta} u^2 dz} \quad \text{and} \quad E_w = \rho \overline{\int_{-d}^{\zeta} w^2 dz} \tag{13}$$

where $E_u$ , $E_w$ =time and depth averaged horizontal and vertical kinetic energies respectively.

In the shoaling region, using (5) and (9) $E_u$ and $E_w$ are given by:

$$E_u = \rho \overline{\left( U^2 h + \frac{h^5}{45}\left(\frac{\partial^2 U}{\partial x^2}\right)^2 \right)} \quad \text{and} \quad E_w = \rho \overline{\frac{h^3}{3}\left(\frac{\partial U}{\partial x}\right)^2} \tag{14}$$

Inside surf zone Eqs (3) and (10) give:

$$E_u = \overline{\rho h u_o^2 + \rho\delta\left(0.312 u_d^2 + 0.9 u_o u_d\right)} \tag{15}$$

and

$$E_w = \overline{\rho a^3\left(\frac{1}{3}\frac{\partial u_o^2}{\partial x}\right) + \rho\delta^3\left(\frac{1}{3}\frac{\partial u_o^2}{\partial x} + 0.035\frac{\partial u_d^2}{\partial x} + 0.203\frac{\partial u_o}{\partial x}\frac{\partial u_d}{\partial x}\right)} \tag{16}$$

Radiation stress $S_{xx}$ can now be defined as:

$$S_{xx} = E_u - E_w + \frac{1}{2}\rho g \overline{\left(\zeta - \bar{\zeta}\right)^2} \tag{17}$$

Application of the above u(z) distribution in Eq. (12) can give the mass flux M above trough level:

$$M = \overline{\left(Uz - \frac{\partial^2 U}{\partial x^2} F(z)\right)\Big|_{\zeta_{tr}}^{\zeta}}$$

with $F(z)=(d+z)^3/6 - h^2 z/6$

$$M = \overline{\left(u_o z - u_d F_{br}(z)\right)\Big|_{\zeta_{tr}}^{\zeta}}$$

with $F_{br}(z)=-0.35(d+z-a)^4/(h-a)^3 + 0.8(d+z-a)^3/(h-a)^2$

or

$$M = \overline{U(\zeta+\zeta_{tr}) - \left(F(\zeta) - F(-\zeta_{tr})\frac{\partial^2 U}{\partial x^2}\right)}$$

$$M = \overline{u_o(\zeta + \zeta_{tr}) + \left(F_{br}(\zeta) - F_{br}(-\zeta_{tr})u_d\right)} \tag{18}$$

The first relation is used outside surf zone while the second inside surf zone in the turbulent region.

The period mean quantities in the above expressions are estimated through numerical integration.

A different way to indroduce breaking in a Boussinesq model is described by Brocchini et al. (2) and Schaffer et al. (19) assuming that a surface roller of velocity c and thickness $\delta$ is present in the front side of a breaker, which can be detected geometrically. In this type of models the horizontal kinetic energy is written:

$$E_u = \rho \overline{\int_{-d}^{\zeta} u^2 dz} = \overline{\rho(h-\delta)u_o^2 + \rho\delta c^2} \tag{19}$$

in which $\delta$ is determined geometrically while $u_o$ is calculated from the continuity Eq. (2) which is written: $Uh=u_o(h-\delta) + c\,\delta$.

For E we may suppose the same expression as in Eq. (9). Mass flux M above trough level is calculated from:

$$M = \overline{\int_{-\zeta_{tr}}^{\zeta} udz} = \overline{\int_{-\zeta_{tr}}^{\zeta-\delta} u_o dz} + \overline{\int_{\zeta-\delta}^{\zeta} cdz} = \overline{u_o z\Big|_{\zeta_{tr}}^{\zeta-\delta} + cz\Big|_{\zeta-\delta}^{\zeta}} = \overline{u_o(\zeta + \zeta_{tr}) + \delta(c - u_o)} \tag{20}$$

## COMPARISON WITH LINEAR AND NON LINEAR WAVE THEORY

Most of the studies use small amplitude linear wave theory for the evaluation of the radiation stress S and the mass flux M. The application of this theory gives the well-known expressions for $S_{xx}$ and M:

$$S_{xx} = \left(\frac{1}{2} + \frac{2kd}{\sinh(2kd)}\right)\frac{1}{8}\rho g H^2 \tag{21}$$

$$M = \frac{\frac{1}{8}\rho g H^2}{c} \tag{22}$$

where k = wave number; H=wave height.

More accurate results can be expected from the use of high order non linear theories which account the effects of finite amplitude waves. Cokelet's relationships for radiation stress (12) as well as Stream Function theory (7) are used for comparison with non breaking wave results of the present model.

Fig. (2) shows a comparison between the present model results ($T/(g/d)^{1/2}=10$) for dimensionlessradiation stress $P_{xx} = S_{xx}/(\rho g d^2)$, as a function of dimensionless wave height H/d, and Cokelet's relationships using Stokes's first definition of phase velocity (i.e. $\overline{u} = 0$ below trough level (12)). In the same figure results from linear theory are also presented. The differences become significant only in very shallow water where the linear theory overestimate the radiation stress $P_{xx}$ predicted from non-linear theory and present model.

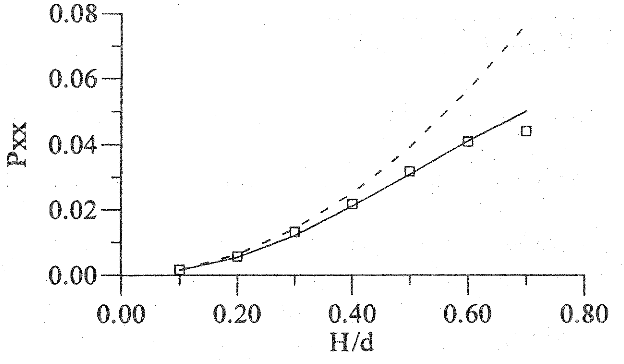


Fig. 2. Dimensionless radiation stress $P_{xx}=S_{xx}/(\rho gd)$ for a dimensionless period $T/(g/d)^{1/2}=10$. Solid line=Model results, Symbols=Non-linear theory (12), Dashed line=Linear theory.

In Table (1) results are presented of the mass flux M from the present model, Stream Function and linear theory for waves on shallow water (d=3.05 m and T=10 secs). The application of the present model and the Stream Function theory gave very similar results while the use of Airy linear theory (Eq. 22) significally overestimates the mass flux M.

Table 1. Mass transport computed by Linear theory (Eq. 22), Stream Function theory (7) and present numerical model. Water depth d=3.05 m, period T=10 secs.

| H (m) | $M/\rho$ ($m^2$/sec) | | |
|---|---|---|---|
| | Linear Theory | Stream Function | Present model |
| 0.60 | 0.082 | 0.071 | 0.070 |
| 0.91 | 0.189 | 0.143 | 0.145 |
| 1.10 | 0.277 | 0.222 | 0.228 |
| 1.52 | 0.529 | 0.315 | 0.319 |
| 1.77 | 0.717 | 0.380 | 0.375 |
| 1.92 | 0.844 | 0.412 | 0.413 |

## COMPARISON WITH MEASUREMENTS

Experimental data are needed to confirm the model validity inside surf zone and in the shoaling region where the surface roller and turbulence effects as well as the vertical assymetry of the profile characterize the waves which non linear theory is not able to predict. Stive's measurements (22, 23) will be used for comparison having in mind that above trough level the given values are extrapolated from those below trough level. Wave conditions are shown in Table 2 representing the two types of initial breaking i.e. spilling breaker in test 1 and plunging breaker in test 2.

Table 2. Wave conditions of Stive's test 1 and test 2 (Stive (22) and Stive and Wind (23)). Slope 1/40, water depth in the horizontal section d=0.7 m ($H_o$, $L_o$ =wave height and wave lenght in deep waters respectively).

| Test | $H_o$ (m) | T (secs) | $H_o/L_o$ |
|---|---|---|---|
| 1 | 0.159 | 1.79 | 0.032 |
| 2 | 0.142 | 3.00 | 0.010 |

Mean velocity $\bar{u}$ below wave trough level is not zero in the wave model due to the presence of the two-dimensional impermeable beach where the net mass flux below trough level (undertow) balances that above trough level (24). Thus the Eulerian mass balance requires for the mean wave momentum (24):

$$\overline{I = \int_{-d}^{\zeta} u dz = 0}$$

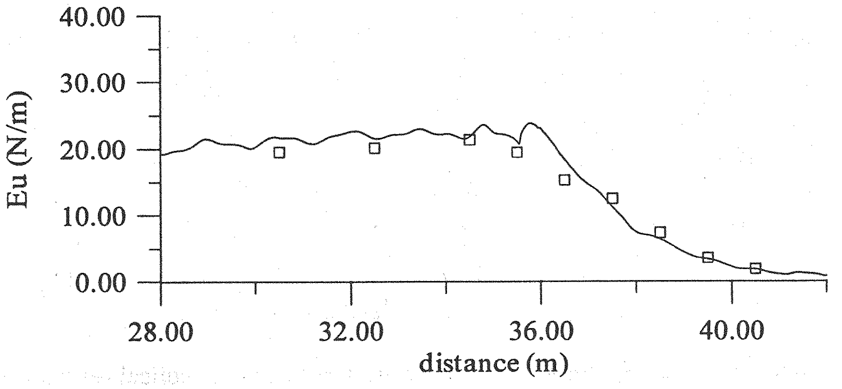


Fig. 3. Horizontal kinetic energy $E_u$ for test 1. Comparison between numerical results and experimental data from Stive (22) and Stive and Wind (23). Solid line=Model results, Symbols=Data.

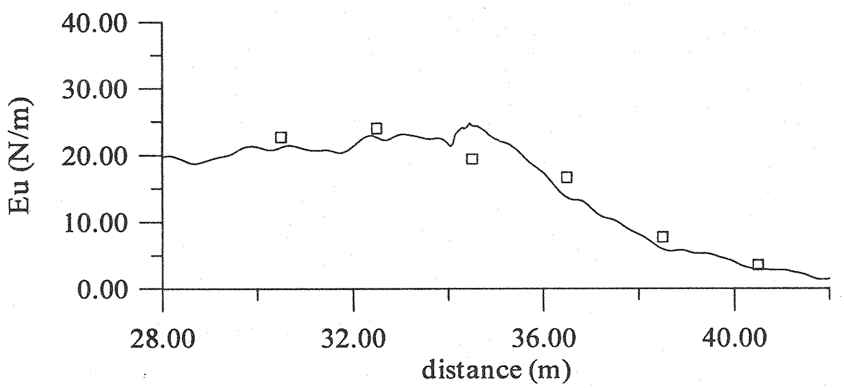


Fig. 4. Horizontal kinetic energy $E_u$ for test 2. Comparison between numerical results and experimental data from Stive (22) and Stive and Wind (23). Solid line=Model results, Symbols=Data.

Frequently the reference frame in which we define the radiation stess is not stationary with respect to the beach and the integral I has a non zero value. The radiation stress in this reference frame can be easily transformed to the present frame (23).

The above experiments give also the opportunity to apply Svendsen's (26) expressions for radiation stress including the effects of surface roller and to compare with model results:

$$S_{xx} = \rho g H^2 \left( 1.5B_o + 0.9 \frac{(d + \bar{\zeta})}{L} \right) \tag{23}$$

where L=wave length; and $B_o$ defined as:

$$B_o = \overline{\left(\frac{\left(\zeta-\bar{\zeta}\right)^2}{H}\right)}$$

where surface roller effects are introduced through the last term.

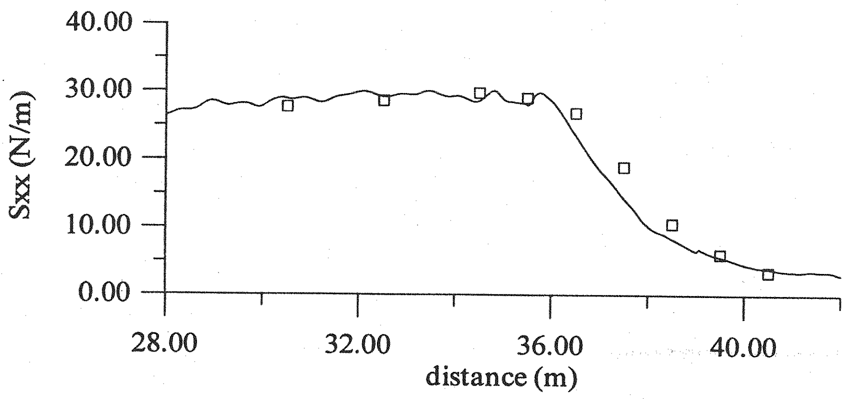


Fig. 5. Radiation stress $S_{xx}$ for test 1. Comparison between numerical results and experimental data from Stive (22) and Stive and Wind (23). Solid line=Model results, Symbols=Data.

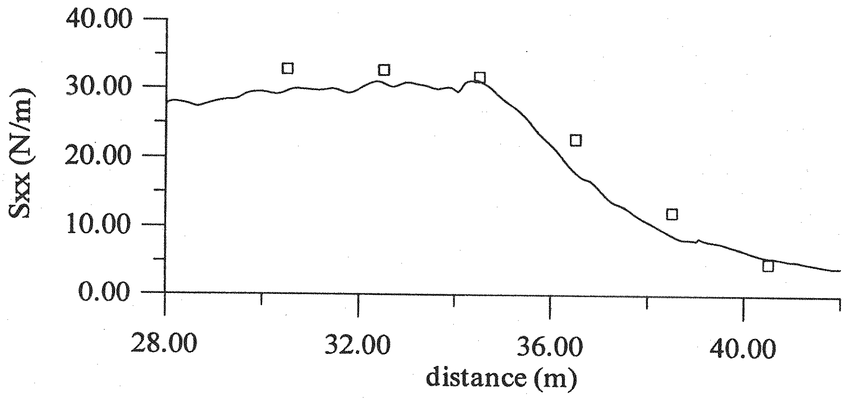


Fig. 6. Radiation stress $S_{xx}$ for test 2. Comparison between numerical results and experimental data from Stive (22) and Stive and Wind (23). Solid line=Model results, Symbols=Data.

Eq. (23) is based on the linear theory relation between the orbital velocity and the surface elevation. Since linear theory overestimates the magnitude of the particle velocity Buhr Hansen (3) used an empirical reduction coefficient $\beta^2$ which is defined as:

$$\beta^2 = \frac{\left(d+\bar{\zeta}\right)}{c^2}\frac{\overline{u^2}}{\overline{\zeta^2}}$$

Incorporation of $\beta^2$ into Eq. (23) gives:

$$S_{xx} = \rho g H^2\left(\left(\beta^2 + 0.5\right)B_o + 0.9\frac{\left(d+\bar{\zeta}\right)}{L}\right) \qquad (24)$$

Buhr Hansen (3) proposed the following relation for $\beta^2$ :

$$\beta^2=0.8-0.5\tanh\left(2.5\left(d'/d_b'\right)^2\right)$$

where $d'$ and $d_b'$= total depth at any point inside surf zone and at the breaking point respectively.

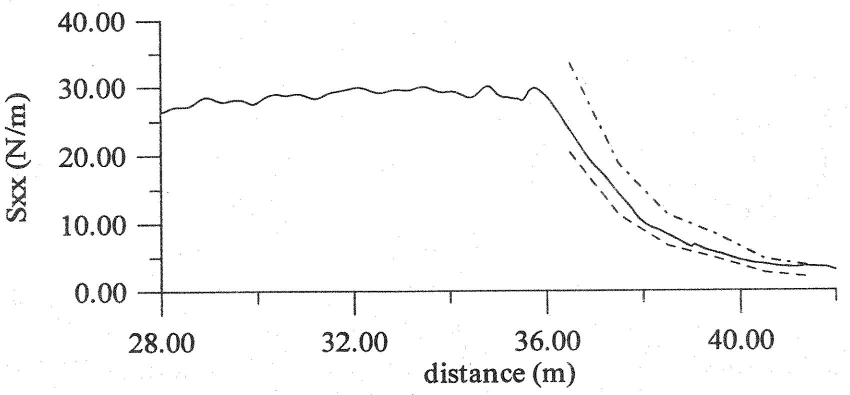


Fig. 7. Radiation stress $S_{xx}$ as predicted from present model, linear theory and Hansen's expression. Test 1. Solid line=Model results, Dashed line=Eq. (24) (3), Irregular dashed line=Linear theory with surface roller effects (Eq. 23).

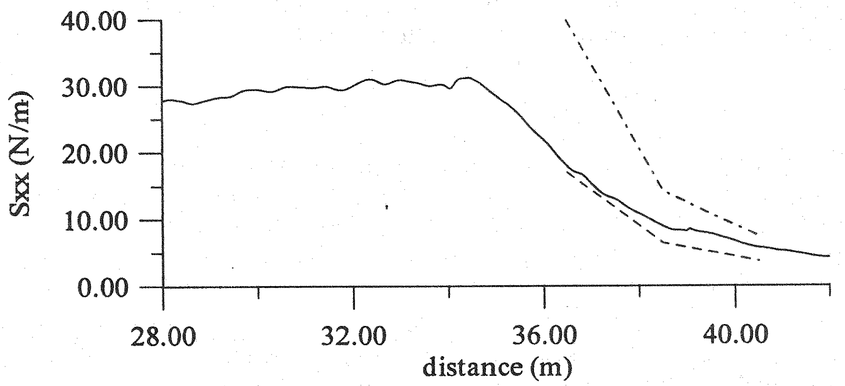


Fig. 8. Radiation stress $S_{xx}$ as predicted from present model, linear theory and Hansen's expression. Test 2. Solid line=Model results, Dashed line=Eq. (24) (3), Irregular dashed line=Linear theory with surface roller effects (Eq. 23).

Both in the shoaling region and in the surf zone a semi empirical expression for $B_o$ is given by Buhr Hansen (3) in terms of the Ursell parameter, wave steepness and depth.

In the outer region which is extended several times the breaking depth and where rapid tranformation of wave shape occurs the present model for breaking waves as well as Eq. (23) are not valid since the surface roller is not yet been formed. A linear variation of the radiation stress in this region could be assumed to avoid the fluctuations shown in Figs 5 and 6 near breaking point. The description of the large vortices which are formed in that region and play an important role both in the momentum and energy balanced, is not easily indroduced in a Boussinesq model. Some new efforts (4) seem to give encouraging results.

In Figs (3) to (6) horizontal kinetic energy $E_u$ and radiation stresses $S_{xx}$ predicted by the present model and experimental data (with $S_{xx}$ calculated from mean water level data) are compared. The model predicts well the two above integral properties. In Figs (7) and (8) the application of the Eq. (24) is also presented. The values of the wave height H and the mean depth d′, for the estimation of the $S_{xx}$, are taken directly from the Stive's data. In the same figures results obtained from Eq. (21) including surface roller effects (as in Eq. 23) are also presented. Inside the surf zone and especially in the outer region linear theory overestimates radiation stresses $S_{xx}$, while Buhr Hansen's semi empirical relations give better results.

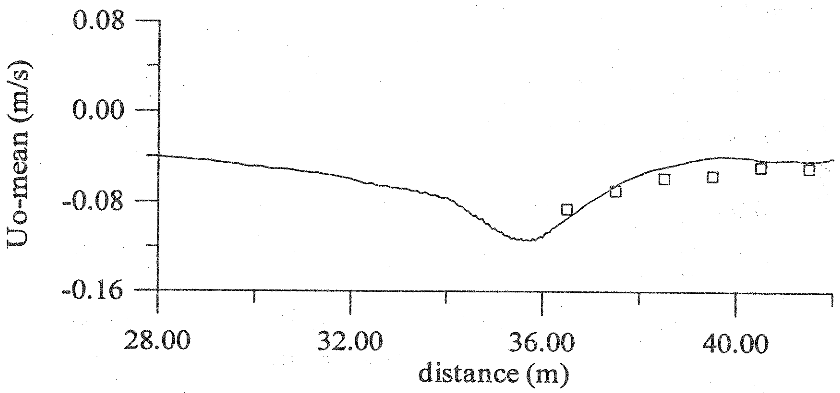


Fig. 9. Mean undertow velocity. Comparison with experimental data by Stive and Wind (23). Solid line=Model results, Symbols=Data.

Depth mean undertow velocity variation inside surf zone is shown in Fig. 9 in comparison with experimental data (24). Due to the inclusion of the roller effects Eq. (18) simulates the depth-mean over undertow velocity and consequently the mass flux M well.

## SUMMARY AND CONCLUDING REMARKS

A non-linear dispersive wave model extended to the surf zone is used for the evaluation of the wave integral properties. Turbulence and surface roller effects on the vertical distribution of u(z) are indroduced in the model. Radiation stress $S_{xx}$ and mass flux M above trough level are expressed in terms of the model dependent variables, i.e depth mean velocity U and surface elevation $\zeta$. The results are compared with experimental data and analytical results from linear and non linear wave theory.

The basic conclusion is that a non-linear wave numerical model in the surf zone can provide a nearshore current model with the required integral wave properties. The model, valid both in the shoaling region and inside surf zone, advances over the linear and non-linear theory which has been developed for irrotational non-breaking waves.

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## APPENDIX - THE NUMERICAL SCHEME

A third order accuracy Finite Differences numerical scheme is used for the numerical solution of the Boussinesq equation system using central finite-differences both in time and space.

The partial derivatives of a variable F, centered in point idx, at time ndt are approximated:

$$\left.\frac{\partial F}{\partial x}\right|_i^n \approx \frac{F_{i+1}^n - F_{i-1}^n}{2\,dx}, \qquad \left.\frac{\partial F}{\partial t}\right|_i^n \approx \frac{F_i^{n+1} - F_i^{n-1}}{2\,dt} \tag{A.1}$$

where F=(U, ζ).

Abbott et al. (1) inrceased significally the accuracy of their numerical Boussinesq model introducing new terms in order to correct the significant first order terms. A similar procedure is adopted here.

The truncation errors introduced from the above approximation can be easily estimated using Taylor series expansion:

$$\left.\frac{\partial F}{\partial x}\right|_i^n \approx \frac{F_{i+1}^n - F_{i-1}^n}{2\,dx} - \frac{dx^2}{6}\frac{\partial^3 F}{\partial x^3}$$

$$\left.\frac{\partial F}{\partial t}\right|_i^n \approx \frac{F_i^{n+1} - F_i^{n-1}}{2\,dt} - \frac{dt^2}{6}\frac{\partial^3 F}{\partial t^3} \tag{A.2}$$

The higher order derivatives are replaced using the linearized Shallow Water Equations as proposed by Abbott et al. (1):

$$\frac{\partial^3 \zeta}{\partial x^3} \approx -\frac{1}{g}\frac{\partial^3 U}{\partial x^2 \partial t}$$

$$\frac{\partial^3 \zeta}{\partial t^3} \approx -gd\,\frac{\partial^3 \zeta}{\partial x^2 \partial t} \qquad \text{(A.3)}$$

similar expression can be also derived for U.

After the substitution of A.3 in A.2 the correction terms which are introduced in the right hand side of the momentum and the continuity equations are written:

$$\left(-\frac{\Delta x^2}{6} + gd\,\frac{\Delta t^2}{6}\right)\frac{\partial^3 U}{\partial x^2 \partial t}$$

$$\left(-\frac{\Delta x^2}{6} + gd\,\frac{\Delta t^2}{6}\right)\frac{\partial^3 \zeta}{\partial x^2 \partial t} \qquad \text{(A.4)}$$

The third order derivative dispersion terms are expressed:

$$\frac{\partial^3 F}{\partial x^2 \partial t} \approx \frac{\left(F_{i+1}^{n+1} - 2F_i^{n+1} + F_{i-1}^{n+1}\right) - \left(F_{i+1}^{n-1} - 2F_i^{n-1} + F_{i-1}^{n-1}\right)}{dx^2\, 2dt} \qquad \text{(A.5)}$$

Finally, the non linear terms are indroduced explicitly i.e.:

$$U\frac{\partial U}{\partial x} \approx U_i^n\,\frac{U_{i+1}^n - U_{i-1}^n}{2dx} \qquad \text{(A.6)}$$

Instability problems appear for very small values of dx and dt and/or when the wave period is large. To overcome these problems a weighting factor $\alpha$ is introduced, in the approximation of the linear terms only, as follows:

$$\left.\frac{\partial F}{\partial x}\right|_i^n \approx \alpha\,\frac{F_{i+1}^{n+1} - F_{i-1}^{n+1}}{2\,dx} + (1-2\alpha)\frac{F_{i+1}^{n} - F_{i-1}^{n}}{2\,dx} + \alpha\,\frac{F_{i+1}^{n-1} - F_{i-1}^{n-1}}{2\,dx} \qquad \text{(A.7)}$$

where $\alpha$ takes values near and less than 0.25.

The resulting system of linear equations is solved using the bi- tridiagonal method.

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