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COMPARATIVE STUDY OF CONSTITUTIVE EQUATIONS FOR DEBRIS FLOWS

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SYNOPSIS

Many constitutive equations for sediment-water mixture flows have been presented. Several sets of equations proposed for flows of coarse grain-water mixtures, and developed assuming laminar grain motion, are chosen for discussion. It is reported in the original papers that these relations successfully predict flume data obtained by the respective authors. However, the constitutive equations provide different explanations of the same phenomena. The present study examines the characteristics of these constitutive equations by solving velocity profiles, sediment concentration profiles, sediment flux and flow resistance.

INTRODUCTION

Mud and Debris flows transport a mixture of water and sediment particles. The nature of the flow changes according to the sediment concentration and characteristics of sediment size. As the shear stress structures of debris flows and mud flows are quite different from those of Newtonian fluids, it is very difficult to rationally develop constitutive equations. Many constitutive equations, therefore, have been presented; e.g., Takahashi [1], Ackermann and Shen [2], Tsubaki, Hashimoto and Suetsugi [3], Miyamoto [4], Chen [5], O' Brien and Julien [6], Egashira, Ashida, Yajima and Takahama [7], Hunt [8], Egashira, Miyamoto and Itoh [9], etc. Defining debris flows to include only laminar motion of sediment particles, constitutive equations for debris flows have been derived by several researchers in Japan [1], [3], [4], [7] and [9]. These formulas have been improved independently step by step, but debris flows structures and the mechanical interpretation for the debris flows are quite different. They give different results to the same phenomena.

It should be instructive to compare these constitutive equations from a standard viewpoint. However, the results will depend on the choice of the standard for comparison. All of the above work assumed that the flow is in an equilibrium condition, i.e. it is not eroding nor depositing sediment. It would therefore seem that an appropriate standard for comparison is the equilibrium sediment concentration and the continuation of transition from debris flow to sediment-laden flow (See figure 1.). In this study, we critically discuss several typical constitutive equations for debris flows by examining the flow resistance and the profiles of velocity and sediment concentration which are obtained by substituting these constitutive equations into momentum conservation equations.

CONSTITUTIVE EQUATIONS

Constitutive Relations

(1) Bagnold's Formulas

Bagnold [10] investigated the shear stress-strain rate relation for a mixture of neutrally buoyant particles and Newtonian fluid in a rotating double cylinder. He found the existence of an inertial region and viscous region, and proposed relationships between shear stress and strain rate in both regions. In the inertial region, the following formulas were suggested;

$$\tau_B = a_i \sin\alpha_i \left\{ \left(\frac{c_{*0}}{c} \right)^{1/3} - 1 \right\}^{-2} \sigma d^2 \left(\frac{\partial u}{\partial z} \right)^2 \tag{1}$$

$$p_B = a_i \cos\alpha_i \left\{ \left(\frac{c_{*0}}{c} \right)^{1/3} - 1 \right\}^{-2} \sigma d^2 \left(\frac{\partial u}{\partial z} \right)^2 \tag{2}$$

in which τ_B is the shear stress, p_B is the dispersive pressure, a_i is an empirical constant $(\cong 0.042)$, α_i is the interparticle collision angle, c_{*0} is the maximum possible concentration by volume, equal to $\pi/(3\sqrt{2}) \cong 0.740$ for spheres, d is the particles size, and u is the velocity. In the system which he assumed an elastic collision, energy dissipation does not occur. Therefore, it is difficult to interpret τ_B the shear stress.

(2) Takahashi's Formulas

Takahashi [1] used Bagnold's constitutive equations in the inertial region to solve the characteristics of debris flow. Afterward [12], he modified $\tan\alpha_i$ in Eqs.(1) and (2) to fit Savage's experimental data [11], as follows;

$$\tan\alpha_i = \left(\frac{c_*}{c} \right)^{1/3} \tan\phi_s \tag{3}$$

in which c_* is the sediment concentration by volume in the non-flowing layer. However, Eq.(3) should be examined more carefully because it yields a coefficient of static friction, which is larger than the dynamic coefficient.

Recently [13], Takahashi assumed the existence of a yield stress near the bed surface and modified Eqs.(1) and (2). The modified equations are;

$$\tau = p_s \tan\phi_s + \tau_B \tag{4}$$

$$p = p_s + p_B \tag{5}$$

in which p_s is the static pressure. It takes the form;

$$\frac{p_s}{p} = f(c) = \frac{c - c_3}{c_* - c_3}, \quad (c \geq c_3) \tag{6}$$

in which c_3 is the minimum sediment concentration at which p_s exists, determined from Bagnold's experimental data [14] to be $c_3 \cong 0.50$. Equation (6) indicates that the dynamic pressure due to particle collisions is much larger than the static pressure in debris flows. Besides, the formulas for τ_B and p_B are indeterminate if c equals c_*, e.g., at $z = 0$, which produces a discontinuity in τ_B and p_B at an erodible bed surface.

(3) Formulas of Tsubaki, Hashimoto and Suetsugi

Tsubaki et al.[3] and Hashimoto et al.[15] conducted flume tests to investigate the mechanism of debris flows over erodible beds by using sediment particles and artificial material. Based on their experimental results, as well as a theoretical formulation of the momentum exchange from particle to particle, they derived the following equations;

$$\tau = K_M \sigma d^2 (c/c_*)^2 / (1 - c/c_*) (\partial u / \partial z)^2 \tag{7}$$

$$p = \frac{1}{\alpha}\tau + K_p \frac{c}{c_*}\frac{c - c_s}{c_s} \quad (8)$$

$$K_M = \frac{\pi}{6}(0.0762 + 0.102\mu)\beta^2 k_M,\ K_p = \chi c_s \rho(\sigma/\rho - 1) g h_t \cos\theta$$

$$\alpha = \frac{\gamma}{1 + \rho/(2\sigma)},\ \beta = c_* \Big/ \frac{\pi}{6},\ \gamma = \frac{0.0762 + 0102\mu}{0.0898 - 0.067\mu}$$

in which c_s is the sediment concentration at the free surface, μ is the dynamic friction coefficient for interparticle contacts $(= 0.05 - 0.1)$, k_M is an empirical constant $(= 5.0 - 7.5)$ and χ is an empirical constant $(= 1/5 - 1/3)$. In equation (7), the effect of interparticle contact on shear stress is neglected. The first term on the right of Eq.(8) expresses the pressure due to particle collisions, and the second term is the pressure due to sustained contact between particles. It is interesting that they regarded collisions between particles as inelastic collisions of many particles. Equations (7) and (8) yield a discontinuity at $c = c_*$, as it found with Takahashi's fomulas. This discontinuity is eliminated by substituting Eq.(22) into Eqs.(7) and (8). Thus the continuity condition of τ and p is satisfied at the bed surface. In the derivation of Eq.(7), the virtual mass of sediment particle is introduced and estimated at about 10 to 100 times larger than the real mass. It seems that one of the reasons why virtual mass is very large is the lack of shear stress due to interparticle contact.

(4) Formulas of Egashira, Miyamoto and Itoh

Egashira, Ashida, Yajima and Takahama [7] suggested that energy dissipation in debris flows was mainly dominated by static interparticle contacts, turbulence of interstitial water and inelastic particle collisions, and assumed that they were able to be summed linearly.

$$\tau = \tau_y + \tau_f + \tau_d \quad (9), \quad p = p_s + p_w + p_d \quad (10)$$

Moreover, they applied Miyamoto's study [4] on the inelastic particle to particle collisions. Their constitutive equations are as follows;

$$\tau_y = p_s \tan\phi_s \quad (11)$$

$$\tau_f = \rho \nu_f (1 - c)(\partial u/\partial z) + \rho k_f d^2 (1 - c)^{5/3} c^{-2/3} (\partial u/\partial z)^2 \quad (12)$$

$$\tau_d = k_d (1 - e^2) \sigma d^2 c^{1/3} (\partial u/\partial z)^2 \quad (13), \quad p_d = k_d \sigma e^2 d^2 c^{1/3} (\partial u/\partial z)^2 \quad (14)$$

$$p_s/(p_s + p_d) = 1/(1 + \alpha),\ (\alpha \cong 0.25) \quad (15)$$

in which τ_y is the yield stress, τ_f is the shear stress supported by interstitial water, ν_f is the kinematic viscosity of the liquid phase, τ_d is the shear stress due to inelastic particle to particle collisions, p_s is the pressure of static interparticle contacts, p_d is the dynamic pressure due to inelastic particle collisions, and $k_d = 0.0828$ and $k_f = 0.16 - 0.25$ are empirical constants. The first term on the right of Eq.(12) expresses the shear stress due to viscosity, and the second term is the shear stress due to turbulence in interstitial water. In equation (12), the shear stress due to viscosity is usually neglected. Equation (15) cannot simultaneously describe flows on the both of erodible beds and rigid beds. In stead of Eq.(15), Egashira, Miyamoto and Itoh [9] and [17] suggested a form for p_s so that total pressure, p, could satisfy the condition $p = p_s$ at the erodible bed surface:

$$p_s/(p_s + p_d) \equiv f(c) = (c/c_*)^{1/n} \quad (16)$$

in which n was set equal to 5.0 based on their experimental data.

Characteristics of Uniform Debris Flows Predicted by Each Formula

Momentum conservation equations for a steady, longitudinally uniform, one-dimensional flow of a sediment-water mixture are described, with z the bed-normal coordinate as shown in Fig. 1 by

$$0=\int_z^{h_t}\rho_m g\sin\theta dz-\tau(z) \tag{17}$$

$$0=\int_z^{h_t}\rho_m g\cos\theta dz-p(z) \tag{18}$$

in which h_t is the flow depth, g is the acceleration due to gravity, θ is the inclination above horizontal, τ is the shear stress, p is the isotropic component of stress, or 'pressure'. ρ_m is the mass density of sediment mixture; $\rho_m=(\sigma-\rho)c+\rho$, in which σ is the mass density of sediment particles, ρ is the mass density of water and c is the sediment concentration by volume in the mixture. If we approximate p in Eq.(18) by the hydrostatic pressure, we obtain ;

$$0=\int_z^{h_t}\rho(\sigma/\rho-1)cg\cos\theta dz-p(z) \tag{19}$$

(1) Results Inferred from Takahashi's Formulas

Takahashi [1] derived a formula for depth averaged sediment concentration by using Eqs.(1), (2), (17) and (19);

$$\bar{c}=\frac{\tan\theta}{(\sigma/\rho-1)(\tan\alpha_i-\tan\theta)} \tag{20}$$

However, the above equation (20) over-estimates sediment concentration, and provides a constant sediment concentration vertically. He modified Eq.(20) into the following to fit experimental data.

$$\bar{c}=\frac{\tan\theta}{(\sigma/\rho-1)(\tan\phi_s-\tan\theta)} \tag{21}$$

in which ϕ_s is the interparticle friction angle. Equation (21) indicates that the averaged sediment concentration $\bar{c}$ depends monotonically on the bed inclination. However, Eq.(21) is different from the resulting form obtained by Eqs. (3), (4), (5) and (6) if c equals c_* at $z=0$.

Substitution of Eqs.(3), (4), (5) and (6) into Eqs.(17) and (19) yields the estimated equations on velocity and sediment concentration profiles. They are as follows;

$$\frac{\partial u'}{\partial z'}=\left(\frac{h_t}{d}\right)\left[\frac{\left\{(c_*/c)^{1/3}-1\right\}^2}{a_i\sin\alpha_i\,\sigma/\rho}\right]^{1/2}\left\{\sin\theta(1-z')\right.$$
$$\left.-f(c)(\sigma/\rho-1)\cos\theta(\tan\phi_s-\tan\theta)\int_{z'}^{1}c\,dz'\right\}^{1/2} \tag{22}$$

$$\int_{z'}^{1}c\,dz'=\frac{1}{(\tan\alpha_i-\tan\theta)-(\tan\alpha_i-\tan\phi_s)f(c)}\cdot\frac{\tan\theta(1-z')}{\sigma/\rho} \tag{23}$$

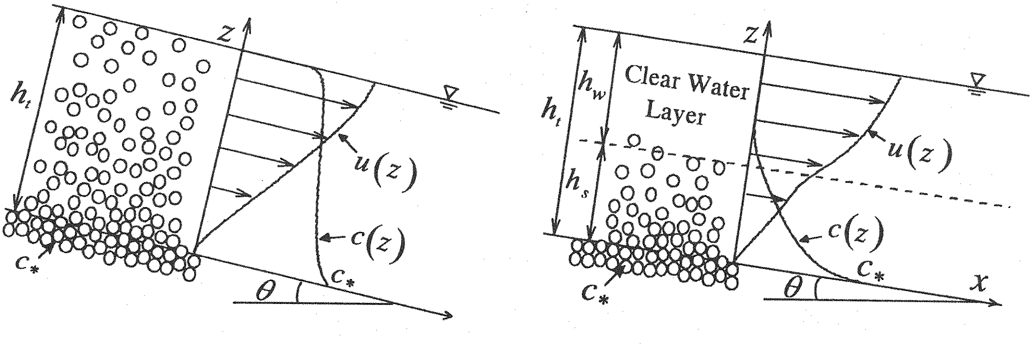

Fig. 1 (a) Sketch of a uniform debris flow

Fig. 1 (b) Sketch of a uniform sediment-laden flow

in which $u' = u/\sqrt{gh_t}$, $z' = z/h_t$.

Moreover, Takahashi [13] suggested an equation for sediment-laden flows. The velocity profile of water-layer is derived as follows. He assumed that the boundary surface (hereafter called 'interface') between the sediment mixture layer and the clear water layer was the surface at which the sediment concentration c equals 0.2, and, defining a virtual origin ς'_B, derived the following equation by imposing the continuity of the velocity gradient near the interface.

$$\partial u'/\partial z' = \{\sin\theta(1-z')\}^{1/2}/l', \quad l' = \kappa(z' - h'_s + \varsigma'_B) \tag{24}$$

$$\varsigma'_B = \{\sin\theta(1-h'_s)\}^{1/2} \Big/ \left\{\kappa(\partial u'/\partial z')\big|_{z'=h'_s}\right\}$$

in which $l' = l/h_t$, κ is the Kármán constant, h_s is the distance from the bed surface to the interface, $h'_s = h_s/h_t$.

(2) Results Inferred from the Formulas of Tsubaki et al.

Tsubaki, Hashimoto and Suetsugi [3] derived equations for the velocity and sediment concentration by substituting Eqs. (7) and (8) into Eqs.(17) and (19):

$$\frac{\partial u'}{\partial z'} = \left(\frac{h_t}{d}\right)\left\{\frac{\sin\theta}{(\sigma/\rho)K_M\Psi}\frac{1-c/c_*}{(c/c_*)^2}\right\}^{1/2}\left\{(1-z') + \chi(\sigma/\rho-1)(c-c_s)\frac{c}{c_*}\right\}^{1/2} \tag{25}$$

$$2\frac{c_*-c}{c_*} - \frac{2c_\alpha - c_s}{c_*}\ln\left(\frac{c-c_\alpha}{c_*-c_\alpha}\right) = \frac{\Psi}{\chi}z' \tag{26}$$

$$c_\alpha = \frac{\tan\theta}{(\sigma/\rho-1)(\alpha-\tan\theta)}, \quad \Psi = (\alpha - \tan\theta)/\alpha$$

In the sediment-laden flow, Hashimoto et al. [15] assumed the velocity profile in the clear water layer to be uniform. On the other hand, they applied Eq.(7) to which Reynold's stress was added and Eq. (8) to Eqs.(17) and (19). The estimated equations on the velocity and sediment concentration profile are as follows;

$$\frac{\partial u'}{\partial z'} = \left(\frac{h_t}{d}\right)\left\{\frac{\sin\theta}{(\sigma/\rho)K_M\Psi}\frac{1-c/c_*}{(c/c_*)^2}\right\}^{1/2}\left\{(h'_s - z') + \chi_d(\sigma/\rho-1)h'_s(c-c_s)\frac{c}{c_*}\right\}^{1/2} \tag{27}$$

$$z' = \frac{\chi_d h'_s}{\Psi c_*}\left\{2(c_0 - c) - (2c_\alpha - c_s)\ln\left(\frac{c-c_\alpha}{c_0-c_\alpha}\right)\right\} \tag{28}$$

in which χ is the empirical constant$(= 1/5 - 1/3)$, h'_s is the thickness of sediment mixture layer, which is determined experimentally. c_0 is the sediment concentration at $z = 0$, given by $c_0 \cong 1.978c_s$, in which c_s is the sediment concentration at the interface. This equation indicates that the sediment concentration c_0 at the bed of sediment-laden flows is smaller than that of debris flows. Moreover, according to Eq.(27), the velocity gradient at the bed of sediment laden flows is non-zero, unlike that at the bed of debris flows. Recently, Hashimoto et al.[16] suggested modified constitutive equations for the Reynold's stress in the clear water layer.

(3) Results Inferred from the Formulas of Egashira et al.

Egashira, Miyamoto and Itoh [9] emphasized that their equations can predict the position of the theoretical bed surface in debris flows over erodible beds. Substitution of Eqs.(11), (12), (13), (14) and (16) into Eqs.(17) and (19) yields $\partial u/\partial z = 0$ at the theoretical bed surface in debris flows over erodible beds, and results in the following depth averaged sediment concentration [9, 17];

$$\int_0^{h_t} c\,dz \Big/ h_t \equiv \bar{c} = \frac{\tan\theta}{(\sigma/\rho-1)(\tan\phi_s-\tan\theta)} \tag{29}$$

Equation (29) is applicable to sediment-laden flow, because no restrictions are employed between debris flows and sediment-laden flows.

The estimated equations for velocity and sediment concentration are obtained by substituting (11)-(14) and (16) into Eqs.(17) and (19);

$$\partial u'/\partial z' = (h_t/d)\left[(G-Y)/(f_d+f_f)\right]^{1/2} \tag{30}$$

$$(1-z')\frac{\partial F}{\partial c}\frac{\partial c}{\partial z'} = F - c \tag{31}$$

in which $G = \sin\theta\int_{z'}^{1}\{(\sigma/\rho-1)c+1\}dz'$, $Y = (c/c_*)^{1/n}\cos\theta\tan\phi_s\int_{z'}^{1}(\sigma/\rho-1)c\,dz'$

$F = f_{pd}\tan\theta/\{(\sigma/\rho-1)(F_1-F_2)\}, F_1 = f_f + f_d - f_{pd}\tan\theta$,

$F_2 = (c/c_*)^{1/n}(f_f + f_d - f_{pd}\tan\phi_s)$, $f_d = k_d(1-e^2)(\sigma/\rho)c^{1/3}$, $f_f = k_f(1-c)^{5/3}/c^{2/3}$,

$f_{pd} = k_d e^2(\sigma/\rho)c^{1/3}$

With respect to sediment-laden flows [9], the mathematical formulation in the clear water layer is as follows. If the position of the interface is selected as the point where sediment concentration is 0.05, a logarithmic velocity profile may be derived with virtual origin η_0.

$$u(z)/u_{\tau w} = u_i/u_{\tau w} + (1/\kappa)\ln\{(z-h_s+\eta_0)/\eta_0\} \tag{32}$$

in which $u_{\tau w} = \sqrt{gh_w \sin\theta}$, h_w is the thickness of the clear water layer, u_i is the velocity at the interface, and η_0 is the length scale defined by Egashira et al. [7];

$$\eta_0 = al_0 = a\sqrt{k_f}\{(1-c)/c\}^{1/3}d, \ (a \cong 1.0) \tag{33}$$

APPLICATION OF CONSTITUTIVE EQUATIONS

Profiles of Velocity and Sediment Concentration

Several equations for velocity and sediment concentration were cited above. It is instructive to examine simultaneous solutions for the velocity and sediment concentration profiles in order to understand the structures of constitutive equations. We call these solutions 'Exact solutions'.

Figures 2(a), (b), (c) and (d) compare exact solutions for velocity and sediment concentration for $\theta = 5.0$ and $\theta = 15.0$ degrees, which yield sediment laden flow and debris flow respectively. Constants which are common to all or some equations are set to the same value, for example $\phi_s = 34.0$ degrees, $c_* = 0.52$, and $h_t/d = 10.0$, other constants used in the calculations were those suggested by each researcher. In the calculation using Takahashi's equations, the concentration at the bed surface is set equal to $0.983c_*$ because the results for τ_B and p_B have indeterminate forms if the bed concentration takes the value of c_*.

First, let us discuss the exact solutions for the debris flows ($\theta = 15.0$ degrees). The forms of the velocity profiles are concave near the bed surface and convex near the free surface. The velocity profile provided by Takahashi's formula has a smaller magnitude than the other results. The results for sediment concentration are fairly similar in magnitude. However, the forms of profiles are quite different; the form provided by Egashira et al. has a 'Reverse S' form, while the form of results by Takahashi and Tsubaki et al. are concave. Those differences depend on whether the pressure of static interparticle contacts or the yield stress is regarded as significant, or not.

Secondly, we examine the exact solutions in the sediment laden flows ($\theta = 5.0$ degrees) in Figs. 2(b) and (d). The forms of velocity profile in the sediment mixture layer by Takahashi and Egashira et al. are similar. However, in the solution of Hashimoto et al.'s equations, the velocity gradient at the bed surface is

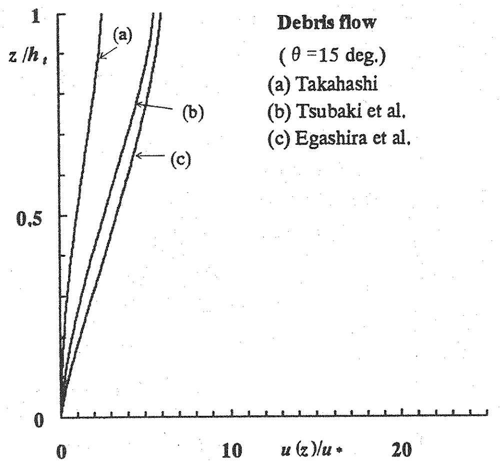

Fig. 2 (a) Comparison of the velocity profiles in the debris flow ($\phi_s = 34.0$ degrees)

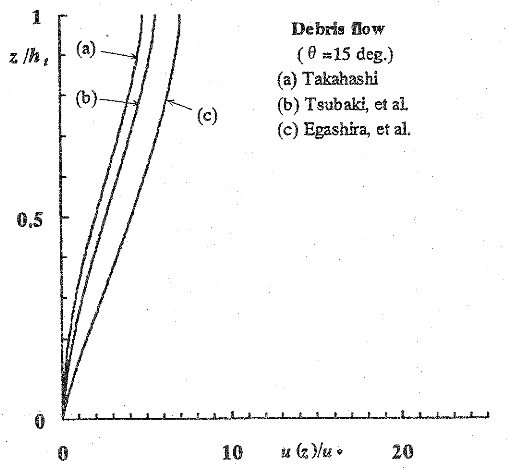

Fig. 3 (a) Comparison of the velocity profiles in the debris flow ($\phi_s = 38.5$ degrees)

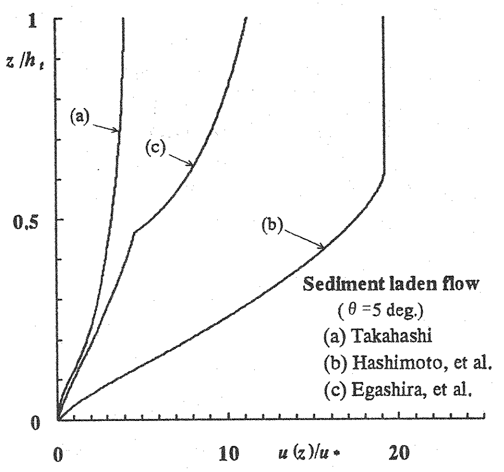

Fig. 2 (b) Comparison of the velocity profiles in the sediment-laden flow ($\phi_s = 34.0$ degrees)

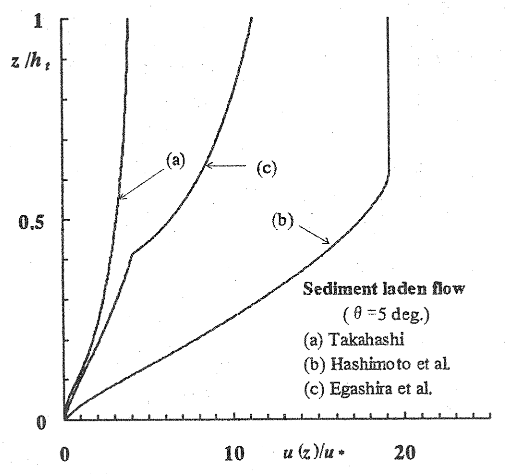

Fig. 3 (b) Comparison of the velocity profiles in the sediment-laden flow ($\phi_s = 38.5$ degrees)

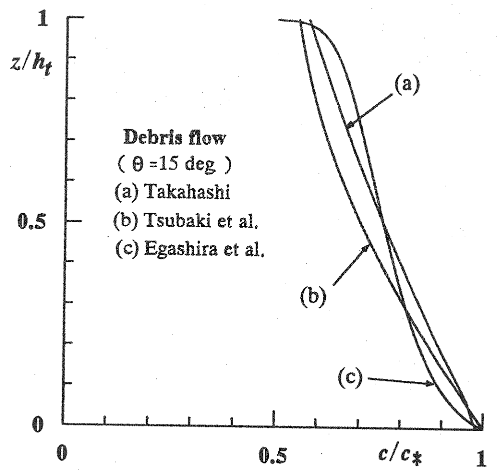

Fig. 2 (c) Comparison of the sediment concentration profiles in the debris flow ($\phi_s = 34.0$ degrees)

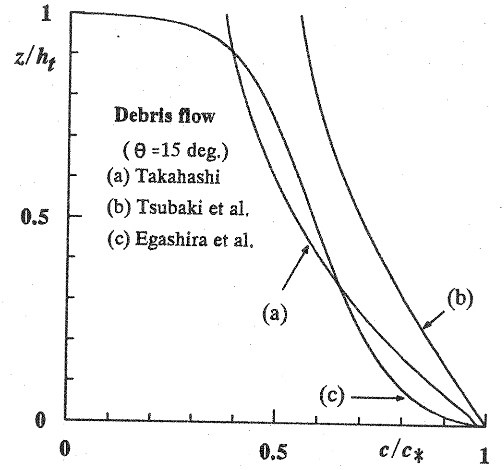

Fig. 3 (c) Comparison of the sediment concentration profiles in the debris flow ($\phi_s = 38.5$ degrees)

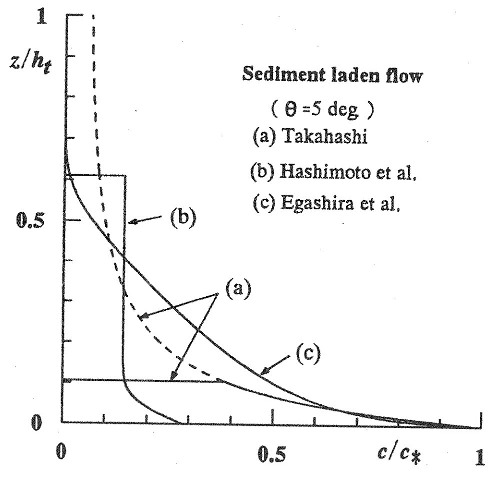

Fig. 2 (d) Comparison of the sediment concentration profiles in the sediment-laden flow ($\phi_s = 34.0$ degrees)

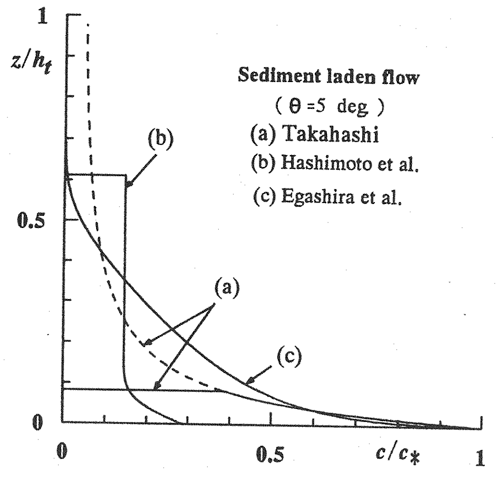

Fig. 3 (b) Comparison of the sediment concentration profiles in the sediment-laden flow ($\phi_s = 38.5$ degrees)

non-zero, and the absolute value of the velocity is larger than the other authors' results. In the clear water layer, the velocity profile by Hashimoto et al. is constant. On the other hand, those by Takahashi and Egashira et al. are based on the logarithmic law. At the interface, Takahashi impose continuity of the velocity gradient, while Egashira et al. require only continuity of velocity.

Consider now the sediment concentration profile in sediment laden flow, focusing on the definition of the interface. The application of the each author's interface definition (Takahashi : $c = 0.2$, Hashimoto et al. : empirical formula, Egashira et al. : $c = 0.05$) yields the results in Fig. 2(d). If the existence of an interface is ignored, the results on the profile by Takahashi and Hashimoto et al. have a concave shape and approach a constant value near the free surface. By contrast, the result of Egashira et al. vanishes at a finite height from the bed surface, indicating the existence of a clear water layer. In other words, the formulas of Egashira et al. predict the thickness of sediment layer automatically.

In order to test the sensitivity of exact solutions to ϕ_s, Figs. 3(a) to (d) show results on the profiles of velocity and sediment concentration for $\phi_s = 38.5$ degrees. As shown in these figures, the result by Takahashi changes sensitively with ϕ_s. That by Egashira et al. depends somewhat less on the change in ϕ_s, showing an increase in the magnitude of velocity due to increasing of the sediment concentration when ϕ_s increases. On the other hand, the results by Hashimoto et al. do not depend on the value of ϕ_s.

Figure. 4 shows the equilibrium depth-averaged sediment concentration; $\bar{c} = \int_0^{h_t} c\,dz \Big/ h_t$, for the exact solutions. The result by Egashira et al. indicates that $\bar{c}$ has a one to one dependence on θ, and shows a continuous change between debris flows and sediment laden flows. It furthermore corresponds perfectly to the result of Eq.(21) or (29). The result by Takahashi corresponds to Eq.(21) or (29) in the region of more than 13.0 degrees, but in the region of sediment laden flows, $\bar{c}$ becomes much smaller than the value predicted by Eq.(21) or (29). The result by Tsubaki et al. is smaller than that of Eq.(21) or (29).

Flux sediment Concentration and Flow Resistance

It is quite difficult to obtain accurate experimental data on the profiles of velocity and sediment concentration for debris flows and sediment laden flows over erodible beds. However, the data on the flux sediment concentration and mean flow velocity are reliable. So, based on the exact solutions for velocity and sediment concentration, the calculated results for flux sediment concentration c_t $\left(\equiv \int_z^{h_t} cu\,dz \Big/ \int_z^{h_t} u\,dz\right)$ and the flow resistance are examined.

Figure 5 shows the relationships between c_t and θ for the exact solutions. Eq.(21) or (29) and the

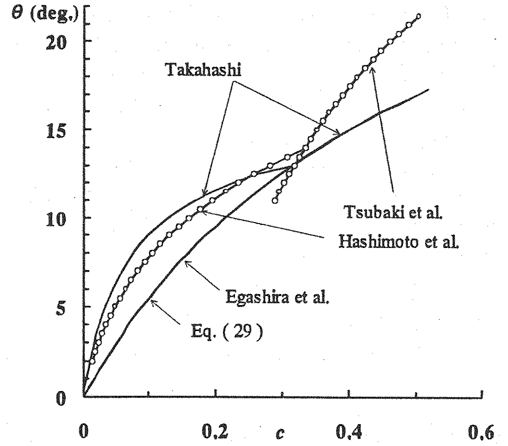

Fig. 4 Calculated curves for mean sediment concentration ($\phi_s = 34.0$ degrees)

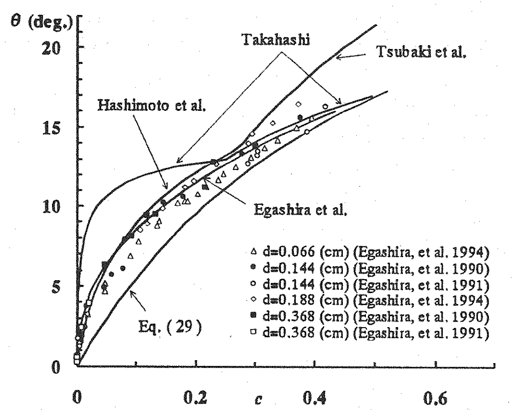

Fig. 5 Calculated curves for flux sediment concentration ($\phi_s = 34.0$ degrees)

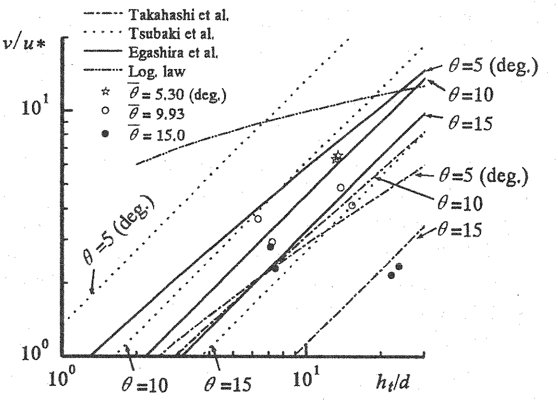

Fig. 6 Calculated curves of flow resistance
($\phi_s = 34.0$ degrees)

experimental data by Egashira et al. [18]-[20] are shown for comparison. According to the experimental data, c_t changes smoothly with θ. The result by Egashira et al. varies smoothly and the absolute value of c_t is quite similar to the experimental data. This emphasizes that the constitutive equations of debris flows by Egashira et al. are very useful for sediment laden flows, and in addition the Eq.(29) is the formula for the theoretical bed whether the flows are debris flows, or not. On the other hand, the results by Takahashi and Tsubaki et al. have a discontinuous shape near the boundary between debris flows and sediment laden flows.

Figure 6 is a graph of v/u_τ versus relative depth h_t/d, where v is the average flow velocity and u_τ is friction velocity defined by $u_\tau = \sqrt{gh_t \sin\theta}$. The experimental data by Egashira et al. [18, 19] are shown for comparison. The logarithmic law (the equivalent sand roughness; $k_s = 2d$) is shown for reference in Fig. 6. These results indicate clearly the tendency of the velocity profiles shown in Fig. 2 and Fig. 3.; in the region of debris flows, the flow resistance from Takahashi's equations is the largest of all the results, and in the region of sediment laden flows, the resistance predicted by Tsubaki et al.'s equations is lowest. This is because their constitutive equations have been considered separately for debris flows and sediment laden flows..

CONCLUSIONS

Several Japanese studies on constitutive equations for debris flows and sediment laden flows have been selected and their characteristics examined. The essential differences between the constitutive equations by Takahashi, Tsubaki et al. and Hashimoto et al. and those by Egashira et al. are the existence of a yield stress and a shear stress due to turbulence at the interstitial scale. The results clarified in this study are as follows;
(1) The constitutive equations modified by Takahashi can explain the flux sediment concentration c_t in the region of debris flows. However, they underestimate it in the region of sediment-laden flows. From the viewpoint of applicability, it seems that the previous studies are superior to the modified ones [21].
(2) The constitutive equations by Tsubaki et al. and Hashimoto et al. can explain the flux sediment concentration in the region of sediment-laden flows, but underestimate it in the debris flow region. The flow resistance in the region of debris flows can sufficiently estimate the tendency of experimental data. However, it is difficult to compare their studies directly with other authors' studies because several empirical coefficients such as k_M, χ, etc., and empirical relations for the thickness and sediment concentration at the boundary in sediment laden flows, which cannot be resolved by using other authors' relations, are introduced in their method.
(3) On the characteristics of the constitutive equations derived by Egashira et al., the transition from debris flows to sediment laden flows is predicted reasonably as well as the phenomena transit continuously from the debris flows to sediment laden flows. Moreover, if the sediment concentration c is equal to c_* at the theoretical bed surface, equation (29) derived by Egashira et al. reasonably applies to not only the regime of

debris flows but also a general bed load regime.

It may seem that our interpretations of the calculation results are partial because we examine other studies from our point of view. However, it seems useful to study in this way in order to stimulate discussions.

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APPENDIX-NOTATION

The following symbols are used in this paper;

a_i	=	empirical constant by Bagnold $(\cong 0.042)$;
c	=	sediment concentration by volume in mixture;
$\bar{c}$	=	depth averaged sediment concentration (the equilibrium concentration), $\bar{c} = \int_0^{h_t} c\,dz \big/ h_t$;
c_*	=	sediment concentration by volume in the non-flowing layer;
c_{*0}	=	maximum possible concentration by volume by Bagnold;
c_s	=	sediment concentration on the free surface by Hashimoto et al.;
c_3	=	minimum sediment concentration in which p_s drives by Takahashi, $(= 0.50)$;
c_t	=	depth averaged flux concentration of sediment;
c_0	=	sediment concentration at the $z = 0$ in the sediment laden flow by Hashimoto et al., $c_0 \cong 1.978 c_s$;
d	=	sediment particles size;
g	=	acceleration due to gravity;
h_t	=	flow depth;
h_t/d	=	relative depth
h_s	=	distance from the bed surface to the interface
h_s'	=	non dimensional distance from the bed surface to the interface; $h_s' = h_s/h_t$;
h_w	=	thickness of clear water layer;
k_d, k_f	=	empirical constants by Egashira et al., $k_d = 0.0828$ and $k_f = 0.16 - 0.25$;
k_M	=	empirical constant $(= 5.0 - 7.5)$;
k_s	=	equivalent sand roughness, $k_s = 2d$
l'	=	non dimensional scale by Takahashi, $l' = l/h_t$;
n	=	empirical constants by Egashira et al. $(n = 5.0)$;
p	=	isotropic pressure;
p_B	=	dispersive pressure by Bagnold;
p_d	=	dynamic pressure due to inelastic particle collisions by Miyamoto;
p_s	=	pressure of static interparticle contacts by Egashira et al.;
u	=	velocity in the flow direction;
u_i	=	velocity at the interface;
u'	=	non dimensional velocity in the flow direction, $u' = u\big/\sqrt{gh_t}$;
u_τ	=	shear velocity on the bed surface, $u_\tau = \sqrt{gh_t \sin\theta}$
$u_{\tau w}$	=	shear velocity on the inteface, $u_{\tau w} = \sqrt{gh_w \sin\theta}$;
v/u_τ	=	depth averaged velocity;
z'	=	non dimensional height, $z' = z/h_t$;
α_i	=	collision angle to particle-particle by Bagnold;

ζ_B	=	virtual point by Takahshi;
η_0	=	scale length defined by Egashira et al.;
θ	=	inclination from horizontal line;
κ	=	Karmán's constant;
μ	=	kinematic friction coefficient to particle to particle $(= 0.05 - 0.1)$;
ν_f	=	kinematic viscosity in liquid phase;
ρ	=	mass density of water;
ρ_m	=	mass density of sediment mixture, $\rho_m = (\sigma - \rho)c + \rho$;
σ	=	mass density of sediment particles;
τ	=	sheer stress at any distance from the bed surface;
τ_B	=	shear stress by Bagnold;
τ_d	=	shear stress due to inelastic collisions of particle to particle by Egashira et al.;
τ_f	=	shear stress supported by interstitial water by Egashira et al.;
τ_v	=	shear stress in liquid phase by Egashira et al.;
τ_y	=	yield stress by Egashira et al.;
ϕ_s	=	interparticle friction angle; and
χ	=	empirical constant $(= 1/5 - 1/3)$ by Tsubaki et al..

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