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GRANULAR FLOW MODEL OF AVALANCHE AND ITS APPLICATION

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SYNOPSIS

A theory for the mechanics of flow, erosion and deposition of avalanche is presented for delineation of the hazard area. In this model, avalanche is modeled as a granular flow including snowballs, in which the probability of snowballs' flocculation depends on the temperature of snow.
The constitutive equations are obtained taking the inelastic collision, the kinetic momentum transport and static skeleton stress into account. Using this model, the dry-type avalanche at Mt.Gongen-dake in 1986, and the wet-type avalanche at Mt.Shirouma-dake in 1992 are numerically simulated. The results of calculation agree well with the recorded phenomena.

INTRODUCTION

In Japan, many lives and a huge amount of assets have been lost since old time as a result of snow avalanche. To save from the avalanche disasters, it is important to develop the long-term strategy such as delineation of hazardous area and setting up defense hedges or check dams, as well as building up short-term warning systems based on the occurrence prediction of the avalanche itself. For these purposes, the flow model which can accurately estimate both the path and deposition area of avalanche is needed.

In the past, avalanches were mostly treated either as a non-Newtonian fluid (1) or analyzed as a big snowball (2). Recently, Nakanishi (3) presented a model in which the avalanche is treated as a set of innumerable snowballs, and Fukushima (4) proposed a powder avalanche model in which the theory of a thermal is applied. Lately, new approaches in which the avalanche is considered as a granular flow comprised of the snow particles have been proposed, wherein the internal stresses are derived from the results of flume experiments and theoretical analyses ((5),(6),(7)). Takahashi and Tsujimoto (8) carried out flume experiments using the natural snow having various characteristics to observe the flow patterns, velocity distributions and slip velocities. Based on the experiments and the constitutive law of granular flow, they obtained an avalanche flow formula which is able to explain the difference in flow patterns corresponding to snow temperature and the degree of snowballs' flocculation.

In this study, we extend our aforementioned model (8) to the plane 2-D simulation model adding the model of erosion and deposition process. To this end some flume experiments and theoretical study of the erosion and deposition mechanism are added. Thereby, all the processes of the avalanche from flow to deposition can be consistently treated. The model is applied to two actual surface snow avalanches ,one is a dry-type and another is a wet-type.

In what follows, we focused only on the motion and deposition of the main body of the avalanche which is the lower part of the flow where the volume fraction of snow particles is very high.

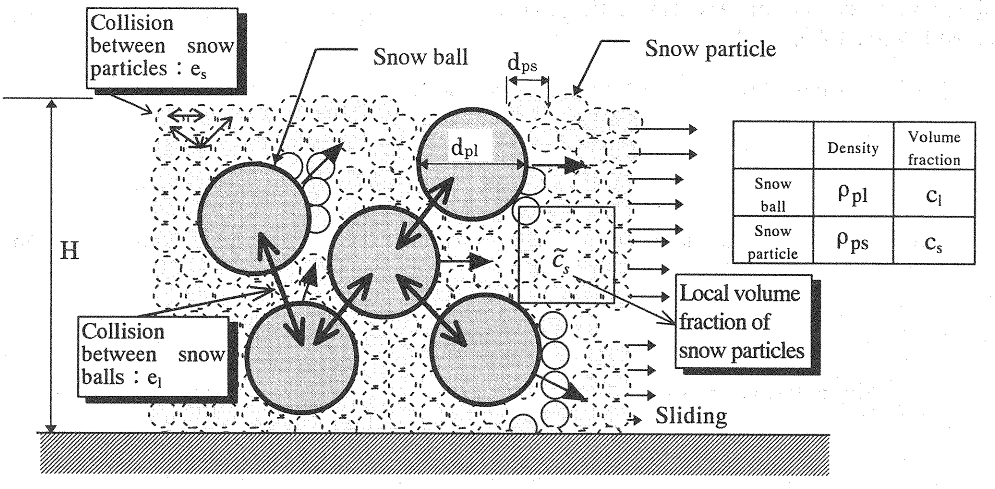

Fig. 1 Flow of the mixture of snow particles and snowballs

FLOW MECHANISM OF AVALANCHE

Internal Structure

Takahashi and Tsujimoto (8) carried out flume experiments using the natural snow of various characteristics to investigate the internal structure of the flow. As a result, the structure of the avalanche is discovered as schematically shown in Fig. 1, and the followings have been understood;

- The volume fraction of the snow particles is so large that they keep continuous contact with each other, and the velocity distribution is almost straight.
- In the case of wet snow, snow particles stick to each other and snowballs are produced. Snowballs mutually collide while the avalanche moves on. As a result, the shape of velocity distribution becomes concave upward.
- A remarkable slip occurs at the bottom, and the lower the snow temperature, the larger the slip velocity becomes.

The appearance of the above mentioned flow pattern and velocity distribution reveals that the stresses in the snow avalanche are given by

$$\tau = \tau_s + \tau_{cs} + \tau_{cl} + \tau_{kl} \quad (1)$$

$$p = p_s + p_{cs} + p_{cl} \quad (2)$$

$$\tau_s = p_s \tan\phi_s \quad (3)$$

where τ_s = macroscopic static skeleton stress; τ_{cs} = collisional stress of snow particles; τ_{cl} = collisional stress of snowballs; τ_{kl} = kinetic stress of snowballs; p_s = inter-granular static pressure; p_{cs} = collisional pressure of snow particles; p_{cl} = collisional pressure of snowballs; ϕ_s = angle of internal friction.

For each term on the right hand sides of Eqs.1-3, various descriptions have been proposed. If Takahashi and Tsujimoto (9)'s equations for the granular flow are employed, the shear stresses in the flow, that is comprised of the snow particles and snowballs, are obtained as follows (8);

$$\tau_s = \alpha\rho_m g(H - z)\cos\theta \tan\phi_s \quad (4)$$

$$\tau_g = \tau_{cs} + \tau_{c1} + \tau_{k1} = (K_1 + K_2 + K_3)\rho_{p1} d_{p1}^2 \left(\frac{\partial u}{\partial z}\right)^2 \tag{5}$$

where $K_1 = \frac{1}{g_{01}} \frac{4\sqrt{15}}{75\sqrt{\pi}} \frac{1+e_s}{\sqrt{1-e_s}} \tilde{c}_s^2 g_{0s} \frac{\rho_{ps}}{\rho_{p1}} \frac{d_{ps}^2}{d_{p1}^2}$; $K_2 = \frac{4\sqrt{15}}{75\sqrt{\pi}} \frac{1+e_1}{\sqrt{1-e_1}} c_1^2 g_{01}$; $K_3 = \frac{1}{3g_{01}\sqrt{15\pi(1-e_1)}}$; α = ratio of the static skeleton part to the total pressure; ρ_m = bulk density; g= acceleration of gravity; H= flow depth; θ = slope gradient; e_i = coefficient of restitution; c_i = volume fraction; ρ_{pi} = density; d_{pi} = particle diameter. The suffix i represents s for snow particle or l for snowballs. Moreover, radial distribution function g_{0i} is defined by;

$$g_{0i} = \left\{1 - (c_i/c_*)^{1/3}\right\}^{-1} \tag{6}$$

where c_* = volume fraction of solids when most densely packed. Here the relation between the local volume fraction of snow particle ($\tilde{c}_s$) and the volume fraction of snowball (c_1), using the volume fraction of the snow particle($\tilde{c}_{s0}$) when snowball is not included at all, is given as follows.

$$\tilde{c}_s = \tilde{c}_{s0} - c_1 \tag{7}$$

Moreover, α is the function of bulk volume fraction of the solids ($\overline{c}_m$) ,and is expressed as follows.

$$\alpha = \left(\frac{\overline{c}_m - c_{s\,min}}{c_* - c_{s\,min}}\right)^m \quad ; \; c_{s\,min} \le \overline{c}_m \le c_* \tag{8}$$

where $c_{s\,min}$ = least volume fraction beyond which particles are always in touch, m = numerical constant. Assuming that the volume fraction c_i is uniform in the direction of depth in the uniform flow, the shear stress at the bottom is, from Eqs.4 and 5;

$$\tau_b = \rho_m f U^2 \tag{9}$$

$$f = \left(\frac{4}{25}K + \frac{4}{5}\sqrt{K}U_{s1} + U_{s1}^2\right)^{-1} \tag{10}$$

$$K = \frac{\rho_m}{\rho_{p1}(K_1 + K_2 + K_3)} \left(\frac{H}{d_{p1}}\right)^2 \left(1 - \frac{\alpha \tan\phi_s}{\tan\theta}\right) \tag{11}$$

where τ_b = shearing stress at the bottom; f = resistance coefficient; U= mean velocity; U_{s1} = dimensionless slip velocity.

When the condition $\tan\theta \le \alpha \tan\phi_s$ is satisfied in Eqs.9-11, K becomes zero, and the flow becomes a rigid body and the mean velocity of the flow is equal to the slip velocity. According to the experiments of Takahashi and Tsujimoto (8), the magnitude of the slip velocity depends on the property of the snow, which has an important meaning in the flow model of the avalanche. Referring to the Johnson and Jackson's (11) study in which the slip velocity is derived paying attention to the momentum transfer at the bottom, dimensionless slip velocity can be described as follows;

$$U_{sl} = U_{slc} + U_{slf} \tag{12}$$

$$U_{slc} = \frac{4\sqrt{3}c_*(1+e_s)\tilde{c}_{s0}}{\phi'\pi}\sqrt{\frac{(1-\alpha)\tan\theta}{2(1+e_s)g_{0s}\tilde{c}_{s0}}} \tag{13}$$

$$U_{slf} = \frac{2\sqrt{3}c_*(\tan\theta - \tan\delta)\alpha}{\phi'\pi g_{0s}}\sqrt{\frac{2(1+e_s)g_{0s}\tilde{c}_{s0}}{(1-\alpha)\tan\theta}} \tag{14}$$

where U_{slc} = dimensionless slip velocity corresponding to collisional stress; U_{slf} = dimensionless slip velocity corresponding to frictional stress; ϕ' = specularity coefficient whose value depends on the large-scale roughness of the surface and varies between zero for perfectly specular collisions and unity for perfectly diffuse collisions; δ = angle of friction between the surface and particle. In Eqs.13 and 14, the lower the temperature and the smoother the bed, the smaller ϕ' and δ become.

Flocculation of snow particles

In the constitution law of the avalanche proposed in this paper, the difference in the flow characteristics between powder avalanche and dense one can be expressed by the volume fraction of snowballs included in the flow and slip velocity in the bottom. The slip velocity has been already shown (Eqs.12-14). More interesting peculiarity of the avalanche is the great difference in the ratio of flocculation of snow particles between dry snow and wet one. It is necessary to model the flocculation process to use the proposed constitution law more effectively. Here, we assume that snowballs are produced in the process of collision of snow particles (8).

If the particle k, whose diameter is k, is produced by the collision between particles i and j, whose diameters are i and j, respectively, the production speed is written as;

$$\frac{dn_k}{dt} = \frac{1}{2}\sum_{i+j=k} N_{ij} - \sum_{i=1}^{\infty} N_{ik} \tag{15}$$

where n_k = number density of the particle k, N_{ij} = number of binary collisions per unit time per unit volume.

In the highly concentrated flow, as shown in Fig. 1, particles move more or less parallel to each other like a laminar flow. Therefore, the number of collision is given by;

$$N_{ij} = \frac{(d_{pi} + d_{pj})^3}{6}\frac{du}{dz}n_i n_j \tag{16}$$

Hence, for $d_{pi} \approx d_{pj} \approx d_m$, substituting Eq.16 into Eq.15 and summing up over all k, the change of total number of particles is given by the following equation;

$$\frac{dn_\infty}{dt} = -\frac{4}{\pi}VE_T\frac{du}{dz}n_\infty \tag{17}$$

where $V = \pi d_m^3 n_\infty/6$, E_T = coefficient representing stickiness that depends on the snow temperature. Herein, we assume;

$$E_T = \exp\left(-\frac{|T_s|}{a}\right) \tag{18}$$

where T_s = the snow temperature, a = constant.

Using Eq.17, the change in number density of the solids by the flocculation of the snow particle can be calculated. Moreover, if the flow is assumed to be comprised only of two diameters of the snow particle and the snowball;

$$n_\infty = \frac{6c_1}{\pi d_{p1}^3} + \frac{6\tilde{c}_s}{\pi d_{ps}^3} \tag{19}$$

Using Eqs.7 and 19, the temporal change of the volume fraction of the snowball which depends on the snow temperature is obtained.

EROSION AND DEPOSITION MECHANISM OF AVALANCHE

To simulate the actual avalanche or to delineate hazardous area by using the proposed avalanche flow model, the model of the erosion and deposition should be added. In case of a large-scale avalanche, existence of the cut bank along the path and the densely compressed snow at the surface of the path have been reported (12). However, there is few experimental and theoretical research about the process of erosion and deposition. Accordingly, an experimental flume 5m long and 10cm wide was set at the Hodaka Sedimentation Observatory, D.P.R.I, Kyoto Univ., and flume experiments to make the fundamental mechanics of erosion and deposition clearer were carried out.

As shown in Fig. 2, a volume of model avalanche comprised of new snow was supplied into the flume on the smooth bed upstream, the flow downstream was recorded from the side through the glass wall with high-speed video. Variable combinations of the channel slope and the snow temperature were used in the experiments. Fig. 3 shows the velocity distribution in the process of deposition. As is evident from the figure, the velocity near the bed decreases at first, the slip velocity, however, is still remarkable, and the velocity decreases gradually showing the characteristic movement of a rigid body. Such characteristics show much difference to the debris flow or mud flow in which the process of deposition is explained by the conservation law of volume fraction. In this paper, we assume that the avalanche in flat area, where K=0 in Eq.10 is satisfied; decelerates gradually downstream and when the velocity becomes smaller than a critical speed(U_{min}),the entire layer(H), stops in a short duration(ΔT). Using this idea, the deposition speed becomes;

$$i_d = -H/\Delta T \tag{20}$$

In Eq.10, K becomes zero when the slope gradient is lower than about 38° although it slightly changes depending on α and ϕ_s.

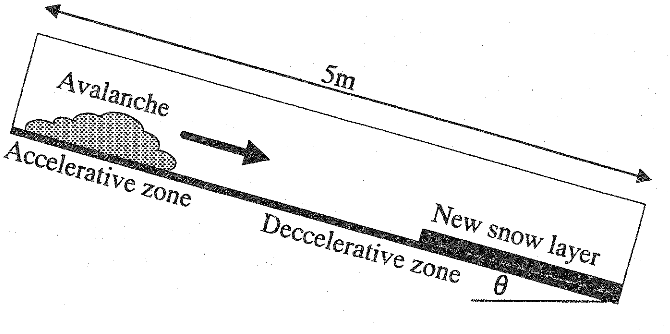

Fig. 2 Experimental flume

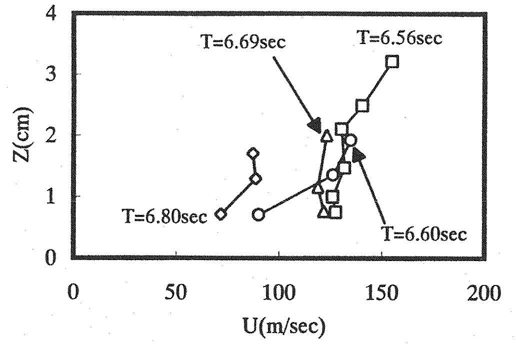

Fig. 3 Velocity distribution during the process of deposition

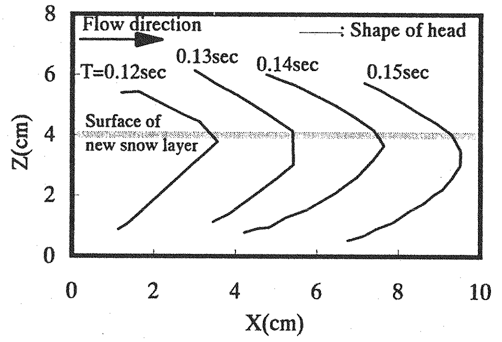

Fig. 4 Variation of front shape during the process of erosion

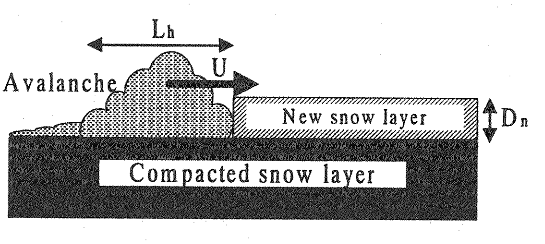

Fig. 5 Erosion model of the avalanche head

Fig. 4 shows the hem line of the front of the experimental avalanche at every 1/100 seconds, which flows on the new snow layer of 4cm deep accumulated naturally and softly on the flume bed. In this figure it is clear that the avalanche flows down compacting and eroding the deposited new snow layer at the head. In the actual avalanche (2), it is reported that the front part continued to flow down pushing out or pushing forward the surface snow like a 'bulldozer'. The result of the experiments was very similar to the actual flow. In consideration of above-mentioned results, we modeled the process of erosion that the new snow of thickness D_n is eroded while the head of length L_h passes over (see Fig. 5).

$$i_e = D_n U / L_h \tag{21}$$

SIMULATION of TWO-DIMENSIONAL FLOW AND DEPOSITION

Basic equations

The flowing avalanche is considered incompressible, then depth-averaged plane two-dimensional continuity and momentum equations are shown as follows (13);

$$\frac{\partial H}{\partial t}+\frac{\partial M}{\partial x}+\frac{\partial N}{\partial y}=i \tag{22}$$

$$\frac{\partial M}{\partial t}+\frac{\partial (UM)}{\partial x}+\frac{\partial (VM)}{\partial y}=gH\sin\theta_{bx0}-gH\cos\theta_{bx0}\frac{\partial (H+z_b)}{\partial x}-\frac{\tau_{bx}}{\rho_m} \tag{23}$$

$$\frac{\partial N}{\partial t}+\frac{\partial (UN)}{\partial x}+\frac{\partial (VN)}{\partial y}=gH\sin\theta_{by0}-gH\cos\theta_{by0}\frac{\partial (H+z_b)}{\partial y}-\frac{\tau_{by}}{\rho_m} \tag{24}$$

where, U, V = mean velocity in x,y direction, respectively; $M=UH$; $N=VH$; z_b = elevation of the bed; $\theta_{bx0}, \theta_{by0}$ = slope gradients in x,y direction, respectively; i = deposit, $i=i_d$ (Eq.20 when $\sqrt{U^2+V^2}\le U_{min}$) or erosion speed, $i=i_e$ (Eq.21); τ_{bx}, τ_{by} = shear stresses acting at the bottom.

Moreover, the conservation law of solids number density is as follows;

$$\frac{\partial (n_\infty H)}{\partial t}+\frac{\partial (n_\infty M)}{\partial x}+\frac{\partial (n_\infty N)}{\partial y}=in_b-I_\upsilon H \tag{25}$$

where n_b = particle number density of the new snow layer when erosion takes place, or = particle number density in the flow when deposition takes place; I_υ = decreasing speed of particle number density by the collision and the cohesion that is given by Eq.17.

The boundary conditions which must be considered are;

a) At the release meshes, the hydrograph of inflow is given.

b) At the forefront of the flow, if the computation yields a flow depth less than a certain small threshold value in a mesh, no flux is generated from this mesh.

Reproduction of the actual phenomena by calculation

Reproduction of the actual phenomena by calculation has been done respectively for dry and wet snow avalanches. Numerical values used in the calculation are determined as $\tilde{c}_{s0}$ = 0.56, $d_{ps}=0.002$m, $d_{pl}=0.1$m, $e_s=0.4$, $e_l=0.4$, m = 0.2, a = 0.8 based on the authors' experiments (8) and the result of theoretical consideration as well as the report of field investigation. U_{min}=0.5m/s and L_h=100m are adopted in calculating deposition and erosion speeds irrespective of the type of the avalanche. Only the parameters ϕ' and δ that relate to the slip velocity differ depending on the wet or dry snow.

a) Dry-type avalanche

For the reproduction of the dry snow avalanche, the avalanche which occurred in Nou-machi, Niigata Pref, Japan on January 26, 1986 by which 13 people were killed is taken as an example. The temperature of the snow was estimated as -6℃ from the local meteorological data. The values of ϕ' and δ have not known although our experiments (8) suggest that they are smaller in case of dry snow avalanche than in case of the wet snow one. Accordingly, some trial calculations were done, and $\phi'=0.04$ and $\delta=2$ were determined. The thickness of erosive new snow layer is set to $D_n=2.5$m for the altitude higher than 600m $D_n=0.5$m for the area lower than 600m based on the field investigation. As for the ground height, some smoothing was added to 25m lattice data supplied from the Geographical Survey Institute.

From the presumed shape of the occurrence region, the upstream boundary condition was set as 100m in inflow width, 2.5m in depth and 24m^2/s in the unit width discharge. The discharge was continued for 10 seconds. At the inflow point $\tilde{c}_s = 0.56$ and $c_l = 0.0$.

The calculation shows that the avalanche accelerated on the steep slope of the average 45° down to about an altitude of 600m as illustrated in Fig. 6, and flow velocity became about 60m/s immediately before it came to the flatter slope. Snowballs were produced by this time, and the maximum volume fraction c_l was about 10%. Afterwards, the volume fraction of snowballs decreases due to entrainment of snow particles by erosion and due to suppression of collisions because of the low shear rate inside the flow on the flatter slope region. Small content of snowballs in this particular avalanche was evident in the actual deposit and therefore we can understand that the calculation well reproduced the natural phenomena. Fig. 7 shows the actual affected area and the results of calculation as they are at 200 seconds after initiation of avalanche. By this time the avalanche volume has developed eight times as big as that at the upstream boundary and the front has reached to the entrance of Maseguchi village. One will understand that the calculated path of the avalanche excellently reproduces the actual events. The amount of the avalanche volume at 200 seconds after initiation was calculated as $2.2 \times 10^5 m^3$, while the estimated total volume by the field survey was $(1 - 3) \times 10^5 m^3$. These results also verify that the calculation is satisfactory.

This avalanche was recognized as a type that accompanied by snow cloud layer beyond the main body. So far, the process of formation of the snow cloud layer has been uncertain. However, Hopfinger (15) pointed out that the snow cloud is formed when the flow speed reached to 10m/s. In the calculation, the speed reaches 60m/s and the snowball content is little. These results suggest that the avalanche accompany the snow cloud layer due to effects of turbulence at the upper boundary of the main body.

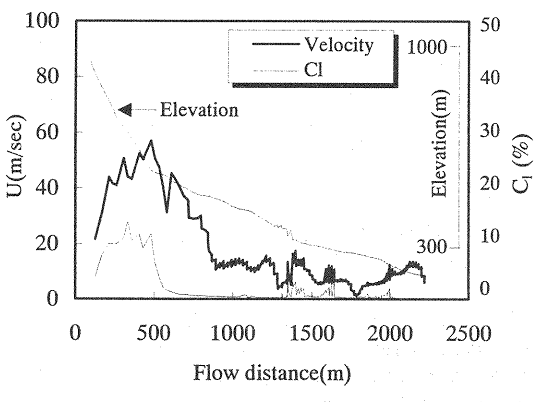

Fig. 6 Variations of the speed, the volume fraction of the snowball and the position of the front by the simulation (dry-type avalanche)

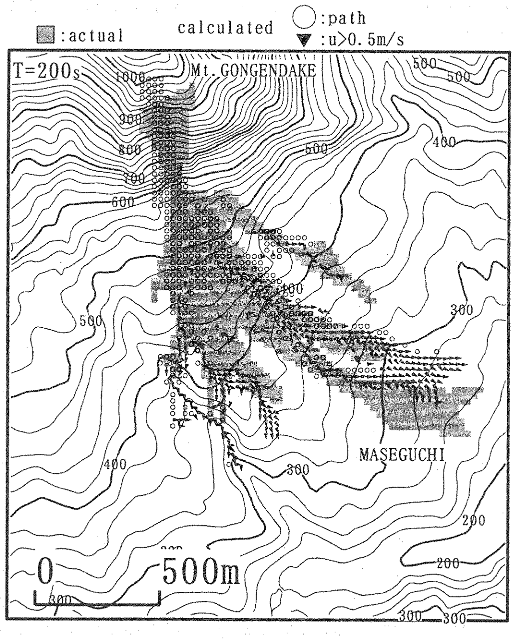

Fig. 7 Comparison between the calculated path and deposition area and the actually affected area (dry-type avalanche)

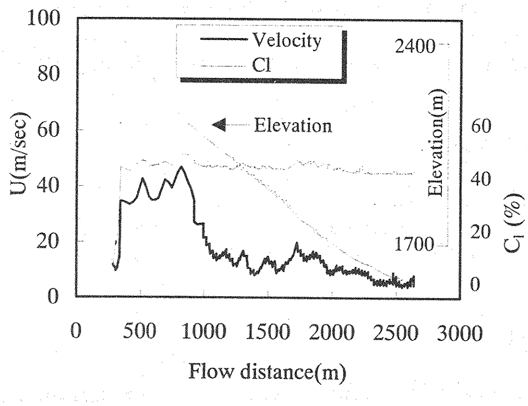

Fig. 8 Variations of the speed, the volume fraction of the snowball and the position of the front by the simulation (wet-type avalanche)

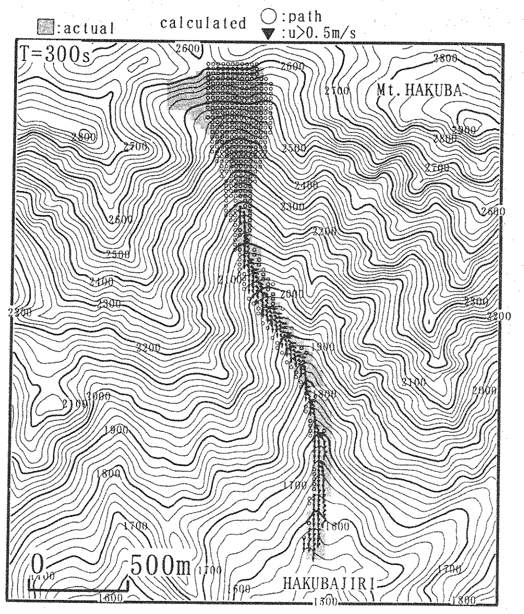

Fig. 9 Comparison between the calculated path and deposition area and the actually affected area (wet-type avalanche)

b) wet snow avalanche

The target of the reproduction by calculation is a large-scale surface avalanche that occurred in the snow valley on the eastern slope of Hakuba-dake in Northern Japan Alps, Nagano Pref., on May 4, 1992. According to the photographs and the local witness, this avalanche was possible to be a wet snow type, although it could not be clearly distinguished from the temperature at the time of occurrence. Based on the local meteorological data, the temperature of the snow was presumed as -2℃, and $\phi' = 0.125$ and $\delta = 5$ were determined after some trial calculations. The thickness of erodible snow layer is set to 2.0m if the altitude is higher than 2,200m and 0.2m if the altitude is lower than 2,200m. At the upstream boundary, the width of the inflow is assumed 275m from the shape of the presumed avalanche occurrence region, and the other conditions necessary in the calculation are the same as in the case of the previous dry snow avalanche. According to the calculation, the avalanche accelerated up to about 40m/s, then it decelerated on the flatter slope to about 10m/s as shown in Fig. 8. Within 300 seconds the volume of the avalanche became about six times as big as the amount of the inflow in the upstream boundary and the front part reached to an altitude of 1500m as shown in Fig. 9. The calculated velocity before stoppage is about 5-6m/s, which is a value close to the witness that is expressed as it was just at a run. The calculated path also excellently reproduces the actual situation. Snowballs are produced in early stage up to the volume fraction becomes about 45%. Photograph taken in the flowing stage shows concentrated snowballs at the head (12), so that the calculation is also valid in this point. The calculated volume of the avalanche at 300 seconds is $4.2\times10^5 m^3$, and it is almost corresponding to the amount $3.9\times10^5 m^3$ of the avalanche debris obtained from the field survey.

CONCLUSION

In this research, two actually occurred avalanches were reproduced by the calculation using our new avalanche model based on the constitutive law of the granular flow. The reproduced paths, deposition areas and content of snowballs both for the dry avalanche and the wet avalanche well coincided with the actual situations. Thus, the applicability of our model to two cases of avalanches was proved. Verification of the parameter included in the model, however, needs further data. Further investigations are also necessary for development of the model to be able to simulate the snow cloud layer formed in the upper part of the dry snow avalanche.

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APPENDIX - NOTATION

The following symbols are used in this paper:

c_* = volume fraction of most densely packed material;

$\tilde{c}_s, \tilde{c}_l$ = local volume fraction for snow particle and snowball, respectively;

$\tilde{c}_{s0}$ = volume fraction of the snow particle when snow ball is not included at all;

c_s, c_l = volume fraction for snow particle and snowball, respectively;

$c_{s\,min}$ = least volume fraction beyond which particles are always in touch;

$\bar{c}_m$ = bulk volume fraction of the solids;

d_{ps}, d_{pl} = particle diameter for snow particle and snowball, respectively;

e_s, e_l = coefficient of restitution for snow particle and snowball, respectively;

E_T = coefficient defined by Eg.18;

f = resistance coefficient defined by Eq.10;

g = acceleration of gravity;

g_{0s}, g_{0l} = radial distribution function defined by Eq.6 for snow particle and snowball, respectively;

H = flow depth;

i_d = deposition speed;

i_e = erosion speed;

m = numerical constant;

M, N = flux(M=UH,N=VH) along x and y, respectively;

n_k = number density of the particle k;

N_{ij} = number of binary collisions between particles i and j per unit time per unit volume;

p_s = inter-granular static pressure;

p_{cs} = collisional pressure of snow particles;

U, V = mean velocity along x and y, respectively;

U_{sl} = dimensionless slip velocity;

U_{slc}, U_{slf} = dimensionless slip velocity corresponding to collisional stress and frictional stress, respectively;

z_b = elevation of the bed;

α = the function of bulk volume fraction defined by Eq.8;

δ = angle of friction between the surface and particle;

ϕ_s = angle of internal friction;

ϕ' = specularity coefficient;

θ = channel slope;

$\theta_{bx0}, \theta_{by0}$ = slope gradients in x,y direction, respectively;

ρ_m = bulk density;

ρ_{ps}, ρ_{pl} = density for snow particle and snowball, respectively;

τ_{cs} = collisional stress of snow particles;

τ_{cl} = collisional stress of snowballs;

τ_{kl} = kinetic stress of snowballs; and

τ_{bx}, τ_{by} = shear stress at the bottom in x,y direction, respectively.

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