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PARAMETERS OF RAIN DROP SIZE DISTRIBUTION AND THEIR VERTICAL DEPENDENCE ON RAINFALL TYPE

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SYNOPSIS

The rainfall intensity on the ground was usually not equivalent to the rainfall intensity estimated from the conventional radar. To improve the accuracy of the radar-estimated rainfall, we observed and analyzed the vertical profile of the rain drop size distribution (DSD), then developed a new formulation of the vertical profile of DSD. Based on the obtained information, first, a new formulation of DSD which considers rainfall type, rainfall intensity, and observed DSD from the Disdrometer was developed. Then, the analysis of the vertical profile of DSD in the selected rainfall type was carried out to investigate the possibility of determing relationships among parameters for the DSD.

INTRODUCTION

There are many uncertainties involved in the estimation of rainfall intensity or amount of rainfall near the ground using a radar. A partial list includes an anomalous propagation echo, bright bands, beam filling errors by rain drops, random errors in reflectivity measurement, natural variability of rain drop size distribution (DSD), the vertical profile of reflectivity, and conversion of radar reflectivity to rain rate (Doviak and Zrnic, 1984; Krajewski, 1993). The last three uncertainties of this list are focused on in this paper.

The change of DSD with distance fallen is computed by relating the micro-physical processes of precipitation which are coalescence, accretion, and evaporation (Hardy, 1963 ; Srivastava, 1971). Hodson (1986) divided exponential rain drop size distributions into three categories by rainfall intensity. Kozu (1991) suggested the statistical properties of DSD parameters through observations and modeling. Nakakita et. al (1995) have formulated height dependent relationships of rainfall intensity, which are estimated from the vertical profiles of DSD observed by a vertical pointing VHF Doppler radar. The purposes of this paper are to analyze both the DSD observed on the ground and the observed vertical profile of DSD by radar, subsequently formulating a vertical profile of DSD.

This paper is divided into three parts. First, the outline of both the observation and the estimation of the vertical profile of the DSD from the Doppler spectra are explained. Second, the observed DSD on the ground is used to formulate the relationships between rainfall type and rainfall intensity. In this paper, the rainfall type is limited to rainfall categorized as large scale atmospheric conditions such as the Baiu front, typhoon etc. The DSD on the ground is observed using a Disdrometer RD69. Finally, the observed vertical profile of DSD is used to analyze the differences in rainfall type and subsequently, to formulate height dependent relationships of DSD.

OUTLINE OF THE OBSERVATIONS

Observations of DSD are currently carried out by a Disdrometer RD69 on the ground surface, and by a vertical pointing VHF Doppler radar in Japan named the Middle and Upper atmosphere radar (MU radar). This radar is managed by the Radio Atmospheric Science Center at Kyoto University . The ground based measurements are carried out at the radar site. The temporal resolution of the accumulated information of DSD from the Disdrometer is 1 minute. Time series of 1 minute rainfall is also observed at the radar site by an optical rain gage.

The MU radar is capable of detecting vertical profiles of the Doppler spectrum which is composed of the velocities of both the rain drop itself and the air movements (Fukao et. al, 1985). Temporal and vertical resolutions of the Doppler spectrum from the MU radar are 3 minutes and 150m, respectively. Because the detailed discussion on the computer processing required to derive the DSD from the Doppler spectra is documented in Sato et. al (1990), only the principles of the derivation are outlined here.

The Doppler velocity spectrum $S_p(v)$ forms an echo from precipitation by a vertical radar beam in conditions where atmospheric turbulence and wind are non existent. The spectrum is expressed in terms of the diameter of rain drop D as ;

$$S_p(v) = CN(D)D^6\left|\frac{d(v(D))}{dD}\right|^{-1}, \qquad (1)$$

where $N(D)\mathrm{d}D$ is the number of drops with a diameter between D and D + dD, $v(D)$ is the drop fall velocity for drop-size D (positive being the upwards direction), and C is a constant. The relationship proposed by Gunn and Kinzer (1949) can be used as a function for $v(D)$.

$N(D)$ is approximated by the following Gamma distribution;

$$N(D) = \begin{cases} N_0 D^{\mu} \exp(-\Lambda D) & \text{for } (v_{\max} \le v(D) \le 0), \\ 0 & \text{for } (v(D) < v_{\max},\ 0 < v(D)) \end{cases} \qquad (2)$$

where N_0, Λ and μ are parameters which depend on the type of rainfall. The parameter v_{max} is introduced so that the existence of a maximum sized rain drop D_{max}, say 6mm or so in diameter, could be taken into consideration in the calculations.

The Doppler spectrum $S_t(v)$ due to atmospheric turbulence by a vertical radar beam is approximated by the following Gaussian function ;

$$S_t(v) = P_0 \exp[\frac{-(v-w)^2}{2\sigma^2}], \qquad (3)$$

where w is the mean wind velocity in the direction of the radar beam, σ is the spectral broadening and P_0 is the peak value of the power spectral.

If the rain drops completely coincide with the motion of atmospheric turbulence, the theoretical Doppler spectrum $S(v)$ due to the combination of rain drop and atmospheric turbulence is expressed by;

$$S(v) = S_t(v) + S_p(v) * S_0(v) + P_n, \qquad (4)$$

where $S_0(v)$ is normalized from $S_t(v)$, $P_n(v)$ is the noise level of the spectra and the asterisk denotes the convolution operation.

The theoretical spectrum in Eq.(4) is defined entirely by the parameters P_n, P_0, w, and σ for the background atmosphere, and N_0, Λ, μ, and v_{max} for rain drops. These eight parameters can be identified by fitting $\log(S_{obs}(v))$ to $\log(S(v))$ using a nonlinear fitting algorithm. These values for the parameters can be obtained for heights between 2km to 4.5km in intervals of 150m.

Before turning to a closer examination, the accuracy of the estimation of parameters (N_0, Λ, and μ) are estimated should be mentioned. Teraoka and Sato et al.(1993) showed the accuracy of the estimation. In the estimation of the DSD parameters by the Gamma distribution function (such as Eq.(2)), the estimated DSD parameters are different from the true DSD parameters, because the relationship between Λ and μ is correlative. Then, Teraoka and Sato et al. investigated the estimation errors by a simulation. In the simulation, as the observed spectrum has statistical error, a model spectrum is expressed by the spectrum that has already known true parameters(Λ_0 and μ_0). Then, they analyzed the accuracy of the estimation to the model spectrum. As a result, the relationship among the parameters can be expressed, to the 100 samples, as follows;

$$\Lambda - \Lambda_0 = 8(\mu - \mu_0). \qquad (5)$$

Where, Λ and μ are the estimated parameters. However, a new parameter Λ' is redefined as follows;

$$\Lambda' = \Lambda - 8\mu = \Lambda_0 - 8\mu_0. \qquad (6)$$

The accuracy of the estimation of Λ is about ± 20 [mm-1], and that of Λ' is about ± 3. Namely, the accuracy of estimation errors can be improved. The physical meaning of Λ' is the slope of the exponential distribution. As their conclusion, for time series analysis on the estimated parameters, we should use the DSD parameters formulated by the exponential distribution.

GROUND BASED MESUREMENTS OF DSD

In this section, we attempt to formulate the DSD based on rainfall type from the surface observations. The observed DSD is formulated by the Gamma distribution in Eq.(2).

First of all, we discuss the accuracy of the estimation of DSD parameters. As the observed DSD on

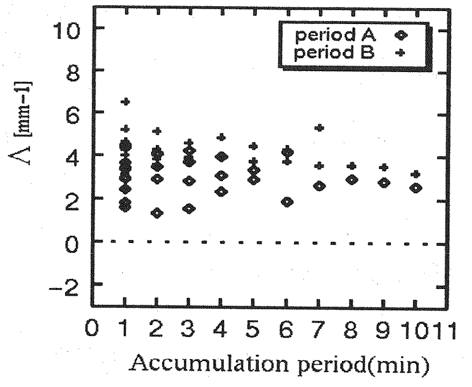


Fig. 1: Relationship between the scattering of parameter Λ and the accumulation period.

the ground can be obtained in every 1 minute, the relationship between the scattering of parameter Λ and the accumulation period. Fig. 1 shows the relationship between the scattering of parameter Λ and the accumulation period is analyzed. This figure shows, when the accumulation period is short, the scattering of parameter Λ is large and the estimation errors are also large. On the other hand, when the accumulation period is getting large, about 5 minutes, the scattering of parameter Λ is getting small. Namely, when the accumulation period is 5 minutes, we can analyze the DSD parameters based on rainfall type without influencing the statistical errors.

Next, in the formulation, the focus is placed on the relationships among the parameters, N_0, Λ and μ, in Eq.(2). The relationship between Λ and μ can be expressed as follows;
Namely, the relationship between Λ and μ is expressed by;

$$\bar{D} = \frac{\int_0^\infty DN(D)dD}{\int_0^\infty N(D)dD} = \frac{\mu+1}{\Lambda}, \quad (7)$$

$$\mu = \bar{D}\Lambda - 1. \quad (8)$$

Here, $\overline{D}$ [mm] is the average diameter of the DSD. This relationship is satisfied if the DSD can be approximated by Gamma distribution. Fig. 2(a) shows the relationship between Λ and μ. The rainfall type in Fig. 2(a) corresponds to the Baiu front rainfall. This figure shows that parameter $\overline{D}$ in Eq.(8) can be defined, because the relationship between Λ and μ is linear. Fig. 2(b) shows the observed relationship between N_0 and Λ. This figure shows that the relationship between N_0 and Λ can be expressed as follows;

$$\ln N_0 = a\Lambda + b, \quad (9)$$

because the relationship between N_0 and Λ is also linear. These relationships in Eq.(8) and Eq.(9) can be expressed also to another rainfall type.

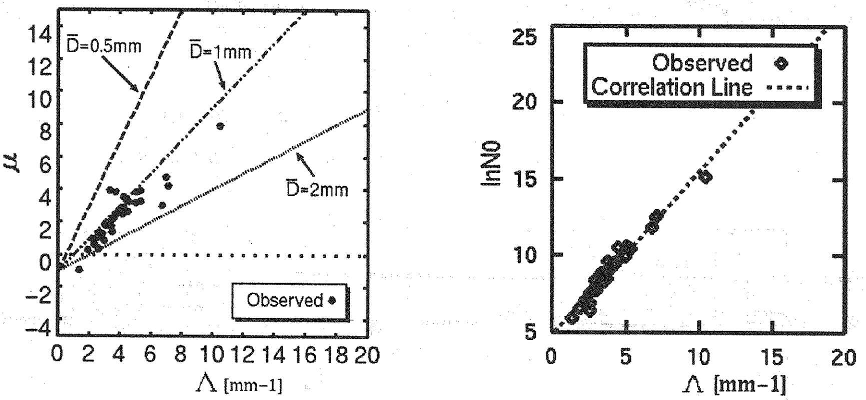
Substituting Eq.(8) and Eq.(9) into Eq.(2), results in the formulated DSD, $N(D)$;

$$\ln N(D) = (a\Lambda + b) + (\bar{D}\Lambda - 1)\ln D - \Lambda D, \quad (10)$$

where the value(s) of the parameters a, b, $\overline{D}$ and Λ are shown in Table 1. Note that Eq.(10) can be applied to the limited conditions where the rainfall type is either a Baiu front, typhoon caused rainfall event (typhoon) or Akisame front caused rainfall event (Akisame front), and when the rainfall intensity is greater than 15 mm/h. Eq.(10) is expressed as a function of Λ because it is influenced greatly by both the rainfall type and rainfall intensity. This table shows that the value(s) of parameters is almost constant in Baiu front

Table 1: Value(s) of the parameters a, b, $\overline{D}$ and Λ based on rainfall type

Rainfall type	Number of observed hours	$\overline{D}$	a	B	Λ[mm-1]
Baiu front	14	1.3～1.5	1.0	4.7	3～4
Typhoon	9	0.95～1.4	0.8～1.0	6.0～7.6	4～7
Akisame front	19	0.7～1.7	1.0	5.3～6.5	4～6



(a)Relationship between Λ and μ (b)Relationship between $\ln N_0$ and Λ

Fig. 2 : Relationships among parameters for Baiu front-caused rainfall which occurred on July 7, 1994.

rainfall. On the other hand, in other rainfall types, values of parameters vary very much. Therefore, rainfall type should be categorized in smaller scale such as convective and stratiform rainfall than large scale atmospheric conditions such as Baiu front, typhoon, and so on.

VERTICEL PROFILE OF DSD

In the analysis on the vertical profile of DSD, we use the observed DSD data by MU radar. Fig. 3 shows temporal rainfall intensity, occurred on Sep. 29, 1994, observed by an optical rain gage. This rainfall type is typhoon. From this figure, we decide that the period of analysis is 9 hours (from pm.2 to pm.11).

Fig. 4 shows the relationship between Λ and μ, formulated by the Gamma distribution such as Eq.(2), during the analyzed period observed by MU radar. This figure shows, however, the relationship between Λ and μ is linear, the values of Λ', in Eq.(8), vary during the period. Therefore, the focus is placed on how long the relationship between Λ and μ is changing. Fig. 5(a) shows the relationship between Λ and μ for 15 minutes period. This figure shows that the relationship between Λ and μ during 10 minutes(19:00-19:10) is different from the relationship during 5 minutes(19:10-19:15). Namely, the value of Λ' changes every 10 minutes. It is also clear for two different 10 minutes period in Fig. 5(b). Therefore, temporal changes of Λ' can be followed up. As the physical meaning of Λ' is the slope of the exponential distribution, we analyze the vertical profile of DSD and the DSD parameter formulated by the exponential distribution as following;

$$N(D) = N_0' \exp(-\Lambda' D). \quad (11)$$

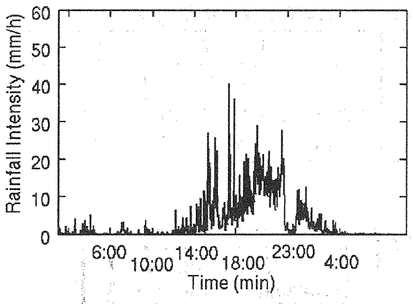


Fig. 3: Hyetograph (Sep. 29, 1994)

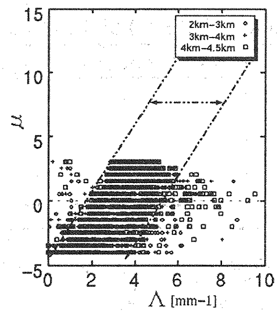
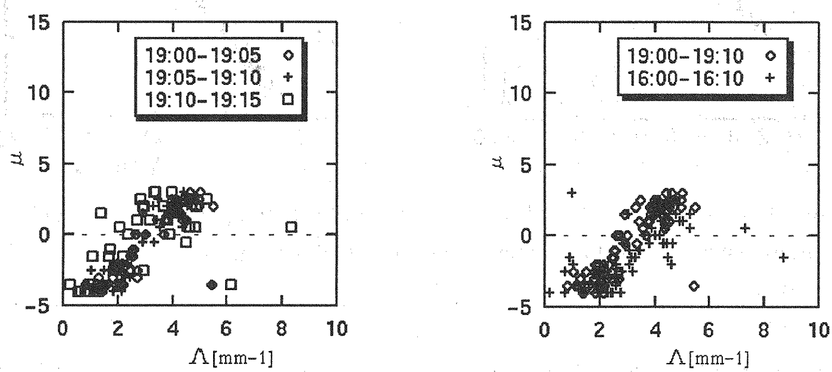


Fig. 4: Relationship between Λ and μ along the analyzed period



(a)Relationship between Λ and μ for 15 minutes period

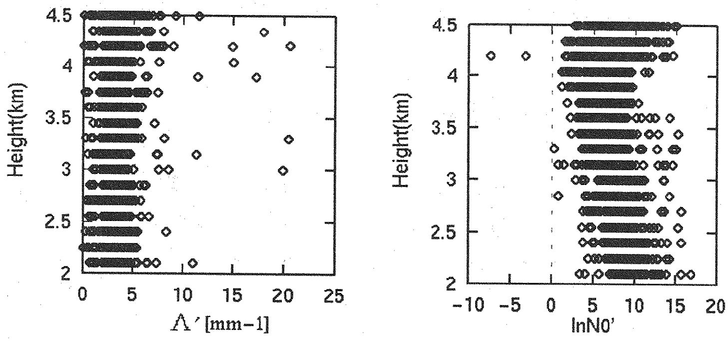
(b)Relationship between Λ and μ for two different 10 minutes period

Fig. 5: Relationship between Λ and μ.

Next, for investigating the height dependency of the vertical profile of DSD depending on the rainfall type, we focus on the vertical profile of the DSD parameters, N_0' and Λ'. Fig. 6(a) shows the vertical profile of Λ', and Fig. 6(b) shows the vertical profile of N_0' during the analyzed period. From these figures, it is clear that to define the height dependency of the DSD parameter depending on the rainfall type of typhoon is difficult, because each vertical profile of DSD parameter changes during the period.

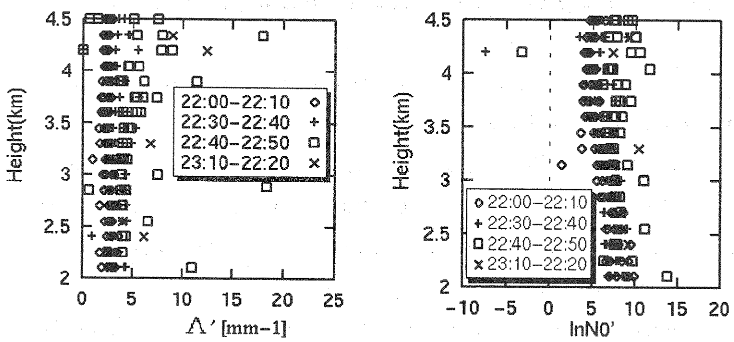
Therefore, we attempt to analyze the temporal variation of the vertical profile of each DSD parameters. Fig. 7 shows the vertical profile of N_0' and Λ' before and after typhoon is going through the MU Radar site. The time that typhoon goes through the MU Radar site was at 22:40. These figures show that the vertical profile of each parameter varies before and after typhoon is going through the MU Radar site. Then, the variation of vertical profile of DSD parameters can be categorized in smaller scale such as convective and stratiform rainfall than large scale atmospheric condition such as the Baiu front, typhoon, and so on.

Fig. 8(a) shows the relationship between Λ' and $\ln N_0'$ during before and after period that typhoon is going through the MU Radar site and Fig. 8(b) shows the relationship between Λ' and $\ln N_0'$, categorized by height, from 22:00 to 22:10. These figures show, when the height is high, the value of Λ' is big, the slope of the DSD becomes heavy and the total number of the DSD decreases. On the other hand, when the



(a) Vertical profile of Λ'　　(b) Vertical profile of N_0'

Fig. 6: Vertical profile of N_0' and Λ' during the analyzed period.



(a) Vertical profile of Λ'　　(b) Vertical profile of N_0'

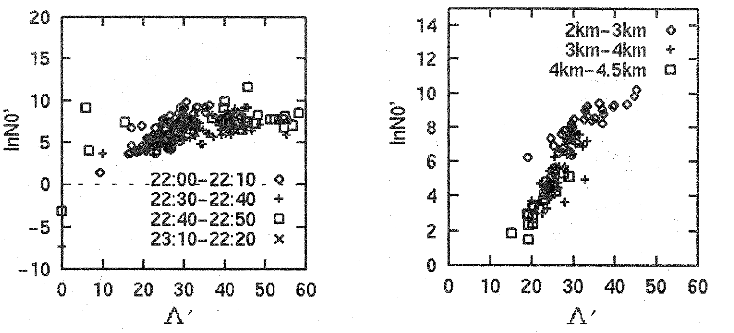
Fig. 7: Vertical profile of N_0' and Λ' before and after the typhoon is going through the MU Radar site. The time that typhoon goes though the MU Radar site is at 22:00.

higher the height is, the smaller the value of lnN_0' is, the total number of the DSD also decreases. These results correspond to the real phenomenon that the higher the height is, the smaller the rainfall intensity is. Then, the height dependency of the DSD should be categorized and analyzed in smaller scale such as convective and stratiform rainfall than large scale atmospheric condition such as the Baiu front, typhoon, and so on.

In this chapter, it is clear that the height dependency of the DSD can be categorized in smaller scale such as convective and stratiform rainfall, therefore, the smaller scale changes every 10 minutes. This phenomenon corresponds to the life cycle of a rain cell.

CONCLUSTIONS

Analysis on the parameters of rain drop size distribution and their vertical dependence on rainfall were shown in three parts as the following. First, we formulated the DSD using observations by the Disdrometer RD69, which was dependent on both rainfall type and rainfall intensity. The observed DSD was formulated by the Gamma distribution. We focused on the relationships among the parameters. The



(a)Relationship between Λ' and $\ln N_0'$ during before and after period that typhoon is going through the MU Radar site

(b)Relationship between Λ' and $\ln N_0'$ categorized by height

Fig. 8: Relationship among the DSD parameters.

relationships between Λ and μ was expressed such as Eq.(8). The relationship between $\ln N_0$ and Λ was linear. A new formulation of the DSD was expressed such as Eq.(10). Eq.(10) applied into the limited conditions where rainfall type was either a Baiu front rainfall, typhoon and Akisama front rainfall, and the rainfall intensity was greater than 15mm/h, and from this analysis, it was clear that rainfall type should be categorized smaller scale such as convective and stratiform rainfall than large scale atmospheric condition such as Baiu front rainfall, typhoon, and so on.

Secondly, we analyzed the vertical profile of DSD that was observed by the MU radar based on rainfall type and was formulated by the Gamma distribution. The rainfall type was typhoon. However the DSD was formulated by the Gamma distribution, the relationship between Λ and μ was linear, and this relationship changed every 10 minutes. Namely, it was not able to formulate the vertical profile of DSD. And we analyzed the vertical profile of DSD and DSD parameters formulated by exponential distribution such as Eq.(11).

Finally, we analyzed the vertical profile of DSD parameters based on rainfall type. From this analysis, it was clear that the height dependency of the DSD should be categorized in smaller scale such as convective and stratiform rainfall. The smaller scale also changes every 10 minutes. This phenomenon correspond to the life cycle of a rain cell.

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