

ON THERMAL STAIRCASES FORMED IN PROCESS OF MULTIPLE-DIFFUSIVE CONVECTION IN LAKES

By

Kenji OKUBO

Associate Professor, Department of Environmental and Civil Engineering,
Okayama University, 2-1-1, Tsushima-Naka, Okayama, Japan

Shigetake NAGAI

Graduate Student, School of Civil Engineering, Okayama University,
3-1-1, Tsushima-Naka, Okayama, Japan

and

Fuminori OBAYASHI

Tokushima Prefectural Office, 1-1, Bandai-cho, Tokushima, Japan

SYNOPSIS

Thermal step structure consisting of an isothermal layer and stable/unstable interfaces is formed in lakes due to turbid or saline intrusion and the subsequent double-diffusive convection. Both the thermal and solutal/turbid Rayleigh numbers were estimated for field and laboratory staircases and little difference between silt and salt fingers was noticed when particle size was less than ten microns. A rapid settling event observed in a lake, which resulted in a staircase with finite jumps in temperature, was found to be in the finger regime but not in the equilibrium state. Thermally reversed staircase in the thermo-turbid diffusive regime moved down with a settling rate and an unstable thermal oscillation was seen, while it was almost static in the thermo-solutal case. Thermal staircase is just a fine structure in the profile, but it shows us the material fluxes which are involved in the multiple-diffusive convective process.

INTRODUCTION

There are evidences of density stratification being composed of two components or more. Such a density stratification of multiple-components in freshwater lakes is primarily thermal and secondarily turbid, and layers behave similarly to the fine-scale, double-diffusive convection known as salt fingers in deep oceans in case the settling velocity of the suspended matter does not exceed the convective velocity. Supposing the rate of vertical displacement of seasonal thermocline is typically 10^{-4} m/s, it cancels out or doubles the settling of silt particle with a diameter of 10 μm . Actual settling rates of the finer silt and nearly neutral phytoplankters would be more or less affected by the convective motion of general currents in lakes. It is the mechanism by which silt particles of a few micra can be kept in suspension for several days. The tendency is more convinced in the seasonal thermocline in deep lakes. Thus, the thermo-turbid convection similar to the oceanic thermohaline convection would take place in fresh water lakes due to stable or unstable gradient in concentrations of suspended matters. Here we will discuss the processes at the double-diffusive interfaces of thermo-turbid/solutal convection in fresh water and brackish lakes, respectively. First we collected evidences of thermo-turbid convection and examine their favorable condition, similarity to or difference from thermo-solutal convection in the oceans. Two kinds of experiments on the double-diffusive convection were separately performed, where silt (Kaolinite) and salt

were used as suspended and dissolved components, respectively. Then processes in both the finger and diffusive regimes were verified and confirmed in the lakes.

The double-diffusive convection is classified into four categories, according to the sign of gradient of each component, whether it is stable or unstable. In the domain where both the density gradients are unstable, fluids are directly mixed up and the case of stable gradients is simply overstable. Unique fingering process is due to unstable stratification of solute concentration and stronger stable thermal gradient. In contrast, the diffusive regime corresponds to a thermally unstable condition accompanied with the stronger stable concentration profile. The two regimes in the thermo-solutal/turbid systems are shown in Table 1.

Table 1 Regimes in the double-diffusive convection systems

Thermo-solutal Oceanic Finger Regime	Thermo-turbid Freshwater Finger Regime
$T > S \gg C$	$T > C \gg S$
equilibrium salt finger with downward thermal flux static staircase with subtle jumps in deep water	rapid silt finger with thermal flux fair settling growing staircase with finite jumps in shallow water
Brackish Diffusive Regime	Benthic Diffusive Regime
$S > T \gg C$	$C > T \gg S$
unstable oscillation with upward thermal flux thermally inversed layer (static and definite)	resuspension with thermal flux against settling thermally reversed staircase (movable and weak)

S: salinity; T: temperature; C: concentration. Inequalities show magnitude of contribution to determine the fluid density.

BACKGROUND

A turbid-water intrusion into thermocline of a reservoir has been treated as a density current, on which Asaeda and Tamai (1) pointed out the double-diffusive characteristics. They carried out their experiments under the diffusive regime condition using both salt and silt as the stable factors. In hypolimnion of a reservoir, Matsumoto (3) found a chemocline with locally reversed temperature profile, and Kanda et al. (2) found the reversed temperature profiles from monthly measurements in another reservoir. Yamada (10) has carried out an intensive field work on sustained thermohaline and resulting anaerobication in Lake Abashiri. Okubo et al. (6) considered a diurnal process in shallow South Basin of Lake Biwa where thermal destratification caused resuspension of silt. All these observed processes were in the diffusive regime of double-diffusive convection for various stabilizing factors of salinity, other chemicals and silt concentration.

On the other hand, the finger regime has not been easily found in fresh water lakes by the presence of general currents. In the North Basin of Lake Biwa, however, turbid intrusions around the seasonal thermocline are frequently seen after river flooding, as is the case in reservoirs, and after sediment resuspension event due to boundary mixing.

THE FINGER REGIME

Silt Finger Regime below Turbid Intrusion into the Upper Thermocline

A water column containing intrusive turbid layer was isolated in an enclosure with a diameter of 10 m at a fixed station in the North Basin during the Lake Biwa Transport Experiment (BITE'93). A sign of fingering was noticed (7) inside the enclosure, as it was free from advection and further sediment supply from the river. Considering the observed condition of the trapped thermocline containing turbid water, we have conducted laboratory experiments to find that a silt finger regime was potentially possible below turbid intrusion into the upper thermocline in the lake.

Temperature and turbidity not only compose a density stratification of double components but also regulate the fluxes mutually through the interaction in which

the vertical silt flux (settling) controls the thermal flux downward by keeping the buoyancy flux ratio constant. The dependence of the flux ratio, $F = B_T/B_S$ on the density ratio, $R_\rho = \alpha\Delta T/\beta\Delta S$ was actually the same as that of the salt finger(7). The buoyancy flux, B_S due to weak settling of silt with a convective enhancement was evaluated from an actual settling velocity of $w_s = 5 \times 10^{-5}$ m/s (7 μ m silt equivalent) and the thermal buoyancy flux, B_T was estimated including the surface cooling rate of the tank, in which α , β are the coefficients of thermal expansion and solutal contraction; ΔT , ΔS are differences in temperature and concentration between the layers, respectively. And $\beta = \sigma$ (submerged specific weight of sediment) when ΔS is replaced by ΔC , the difference in volume concentration of silt.

Figure 1 shows the diagram of the finger regime (quadrant III) specified with the Rayleigh numbers, $Ra = \alpha g \Delta T d^3 / \kappa \nu$ and $Rs = \beta g \Delta S d^3 / \kappa \nu$ in which both the gradients of temperature and concentration are negative downward (positive z direction) in the convective layer with the thickness, d . Normally Rs is defined with respect to concentration for solute, usually for salinity, but here it is extended for weight concentration (turbidity) to deal with the case of suspended silt conditions as well. Laboratory experiments (Runs FA, FB and FC) were conducted using a cylindrical tank with the inner diameter of 9 cm, for which the values of Ra and Rs were estimated from measurements at every 20 minutes (7). In the attached table to Fig. 1 the initial and the final values of the experimental valuables are shown in the first and the second rows, respectively. The experimental points were asymptotically shifting from left to right closer toward the limit of $Ra = \tau^{-1} Rs$, with elapsed time, which suggests that the diffusivity ratio is close to the thermohaline case, $\tau = \kappa_s / \kappa = 10^{-2}$, where κ_s and κ are the molecular diffusivities of salinity and heat, respectively.

Finger Regime						
RUN	U-L	U-L	U/L	T/S	d	diffu-
time	DT	DS	depth	Rp	(cm)	sive
hr	(°C)	(mg/l)	cm/cm			mater
FA1	4.88	793	20/80	1.48	-	silt
	1.66	2.17	133	-	3.95	2
FA2	4.77	786	20/80	1.46	-	silt
	1.66	2.27	132	-	4.15	2
FA3	4.28	786	20/80	1.31	-	sugar
	1.66	1.91	130	-	3.56	2
FB1	5.66	1	20/80	1737	-	fluor.
	1.66	3.35	0	-	6236	2
FB2	5.61	393	20/80	3.44	-	silt
	1.66	3.09	66	-	11.3	2
FB3	5.40	94	20/80	13.7	-	silt
	1.66	3.32	16	-	50.7	2
FC1	6.09	189	20/80	7.78	-	silt
	1.66	3.48	28	-	30	2
FC2	7.67	189	20/80	9.8	-	salt
	1.66	3.96	33	-	29.3	2
FC3	5.42	472	20/80	2.76	-	silt
	1.66	3.17	78	-	9.84	2
FD1	8.75	317	12/100	11.4	-	creap
	19	0.13	(0.5)	-	62.7	2.6
FD2	5.22	317	12/100	11.4	-	silt
	18	0.10	(0.5)	-	48.2	4.4

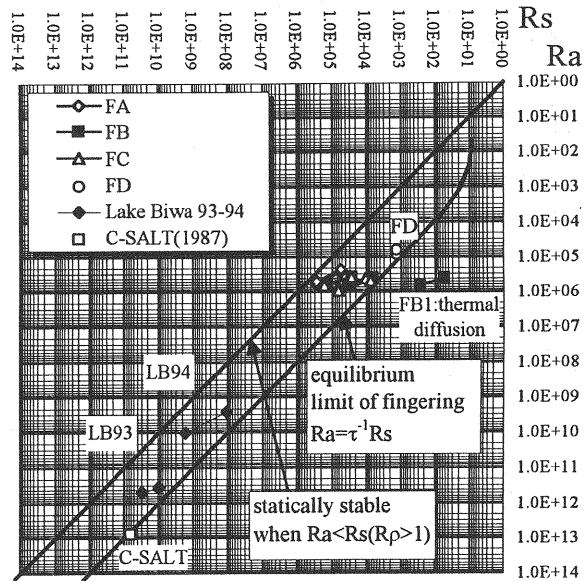


Fig. 1 Rayleigh numbers diagram of the finger regime

Some of the experimental runs were conducted using warmer brine of the same weight concentration as that in the run of silt finger. An exceptional run (FB1 out of the regime) was a pure thermal diffusion in which only 1 mg/l of fluorescent dye in the upper warmer and turbid fluid at the initial setup. The same amount of dye was added to the upper fluid in FB and FC, and its temporal dilution was traced by using a continuous pumping to a fluorometer. The fraction of the third component in the upper layer decreased gradually, keeping the pace with settling of silt which

drove the fingers. In this manner, silt finger would involve dissolved matter as well as heat to produce a multiple-diffusive convection. The density ratio, R_ρ , an index of stability, was lowest in FA with highest concentration among the runs. Profiles at later stage of runs FC, in which the most stable fingers were observed, are shown in Fig. 2(a). The profiles were taken at 100 minutes after the removal of the horizontal plate which separated the warm and turbid water from the cool and clear water, in which signs of stable staircases are noticed. Fig. 2(b) shows the multiple staircases established at 16 to 19 hours after fingering in a larger rectangular tank with the cross section of 21 cm by 25 cm in Run FD1. Powder milk was used in FD1, but the same structure was formed in the silt finger experiment, FD2. The structure looks like thermal steps with infinitesimal jumps as seen in deep oceans. Assuming the ΔS equals to a resolution of turbidity, it should be found close to the fingering limit as shown in Fig. 1.

During the enclosure experiment in Lake Biwa, temperature fluctuation with period of several ten seconds was observed only at the depth of 20 m below the turbid water which came from the Yasu River. The fluctuation lasted for six hours inside the enclosure after the curtain was closed, when the turbidity difference across the layer, ΔS , was 60 mg/l, and it decreased to half the initial value and shifted toward the equilibrium of the finger regime, $R_\rho = Ra/Rs = \tau^{-1}$. Mormorino et al.(5) showed similar thermal staircase with an 8 m thick isothermal layer at 250 m depth in the Caribbean Sea (C-SALT experiment). It was in the equilibrium state of the finger regime.

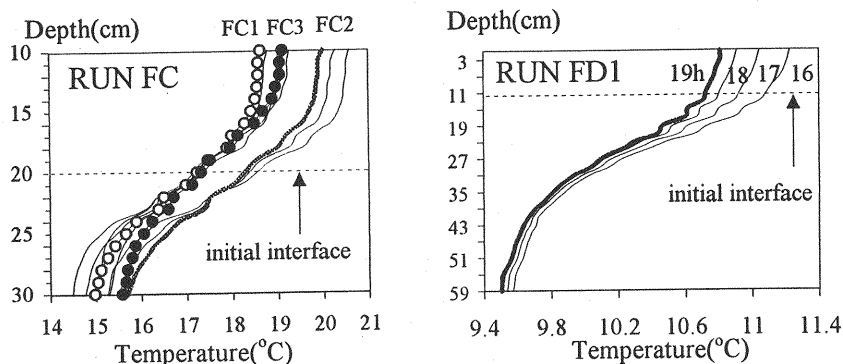


Fig. 2 Thermal staircases in the finger regime:
(a) rapid fingering (FC); (b) equilibrium after fingering (FD)

Stable Thermal Staircase with the Sheets and Layer

In layers directly above the sloping bottom close to the perimeter of a lake, instantaneous and local vertical motion with velocity around 0.01 m/s arises from breakings of surface and internal waves, which mix the water column thoroughly or partially. During the boundary mixing event, bottom sediment of 100 μ m would be suspended, but coarser sediments sink rapidly forming an isothermal layer in the thermocline, in which silt remains and forms turbid intrusion with a concentration of several mg/l at the bottom of staircase. In 1994 September, Okubo et al. (7) measured a formation of such an isothermal layer with a thickness of 0.7 m as shown in Fig. 3(a). It took only an hour or less to construct the staircase. The suspended particles in the layer should come from the bottom, since the chemical composition was the same as sediments collected in the same area of the day taken by Sugiyama and Hori (9). The staircase is shown in Fig. 1 as the points labeled LB94, which also shows a tendency toward the equilibrium. Water temperature rose at the depths of 16 m of the staircase and 17.5 m close to the bottom. It was therefore suggested that thermal fluxes from the upper layer to the isothermal layer, and from the staircase to the bottom layer existed, and that those fluxes were driven by settling of the suspended sediments.

The structure containing turbid intrusion initiated from resuspension of the bottom sediment (Kaolinite) by internal wave breaking on the sloping boundary and

the resultant staircase accompanied with fingering, were observed in laboratory experiments by Morikawa et al. (4). For field experiments, we have been using a turbidity-temperature and depth profiler in Lake Biwa since 1996, by which turbid intrusions and thermal staircases were simultaneously measured more in detail. Staircases like in Fig. 3(b) were frequently formed in a short duration after the onset of internal wave breaking. The movement featuring the structure seemed to be a "rapid" fingering with a smaller density ratio, R_ρ and larger Rayleigh numbers compared with the deep oceanic thermal steps in the equilibrium state.

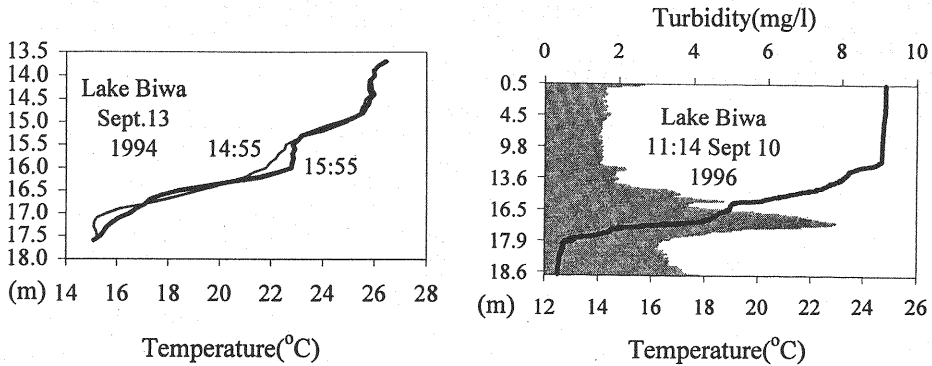


Fig. 3 Thermal staircases in the North Basin of Lake Biwa measured by (a) thermistor chain; (b) profiler

THE DIFFUSIVE REGIME

Thermally Reversed Layers in Laboratory

Laboratory experiments on the diffusive regime were conducted with initial well mixed profile of silt or two-layered stratification of salt in the rectangular tank (used in FD). Thermal boundary condition was open; exposing the water surface to the air temperature varying in the laboratory overnight with an average buoyancy flux of about $10^{-8} \text{ m}^2/\text{s}^3$. Under nocturnal natural cooling, thermally reversed successive steps were observed as shown in Fig. 4(a). In thermo-turbid system, the thermally reversed layers were slowly moving down at a rate of $7.4 \times 10^{-6} \text{ m/s}$ ($2.5 \mu\text{m}$ silt equivalent), and thermal oscillation with a period of several minutes was confined within the vicinity of the moving layers, as shown in Fig. 4(b) with the elapsed time.

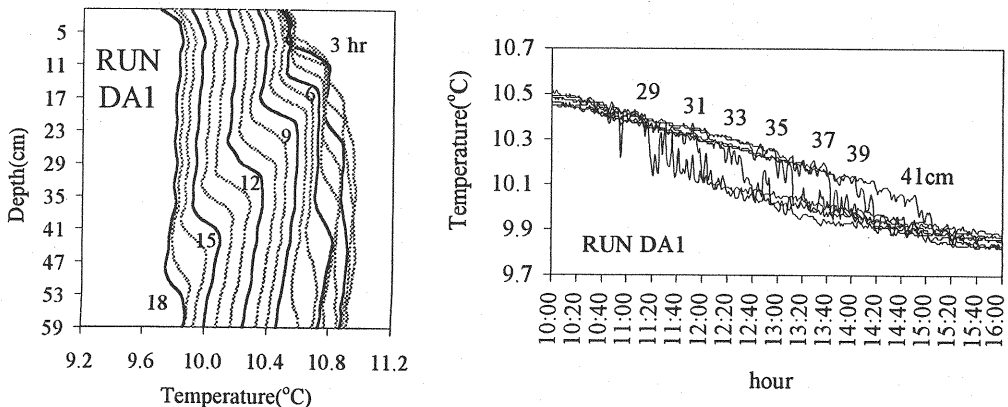


Fig. 4 The diffusive regime experiment (DA1): (a) temperature profiles; (b) unstable thermal oscillation

As the cooling was rather moderate inside the laboratory, little motion was observed in the thermohaline experiments DB4, DB5 and DB6, where the upper and lower layer depths were set 72 cm and 40 cm, respectively. In order to reproduce a realistic field condition in winter, we floated a metal bowl on the upper surface of the tank, filling it up with ice and water. Under the forced cooling, density structure for both Runs DB7 and DB8 became marginally stable, in which the initial salinity differences were 3 and 10‰, and upper and lower depths were 17 cm and 95 cm, respectively. Thermal oscillation observed in DB7 is shown in Fig. 5. Temperature fluctuation with an amplitude of one degree Celsius was seen in the convective layer of the depth range, 13 cm to 19 cm, and the apparent period was a few minutes, while the temperature measurement was done every minute with the thermistors array.

In Run DB7, the forced cooling flux was imposed once a day, the record is shown in Fig. 6(a) as a time series of temperature and salinity differences between the layers of 6.5 cm and 28 cm. Magnitude of thermal oscillation was found larger in DB7 than the runs above, where the motion which fluctuated the density field was visible in a shadow graph within the convective layer. However, when the cooling was absent, the reversed temperature difference, DT was mostly released in the diffusive way. Salinity difference, DS was reduced during the cooling period, but remained constant during non-cooling period. While in DB8 with the higher initial difference of salinity, the convective layer during the cooling was thinner, and the salinity difference was diffusive corresponding to the higher value of temperature inversion even without surface cooling as shown in Fig. 6(b).

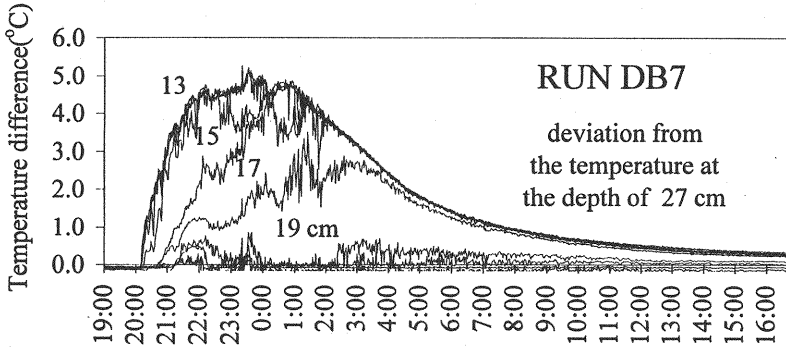


Fig. 5 Thermal oscillation observed in Run DB7
The record started at 19:00, Oct.23, 1996

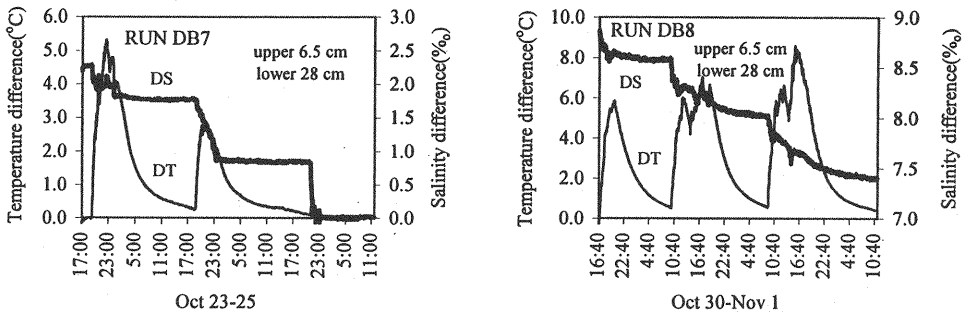


Fig. 6 Temperature and Salinity differences in (a) DB7 and (b) DB8

Unstable Oscillation in the Brackish Lakes in Winter

As thermohaline in brackish lakes is generally much stronger than in the oceans, largely reversed temperature difference and the corresponding high Rayleigh numbers are seen in winter. Instability is possible only when the cooling and resulting

convection takes place. While in the thermo-turbid system, larger difference of density is normally found near the bottom region because of settling, so the stratification in the middle or surface layers is marginally stable and there often develops an unstable oscillation. In 1996 November, field surveys were conducted in Lake Shinji and Lake Nakaumi, in order to see the actual stability range of thermally reversed layers. It was found 2°C in Lake Shinji and 3°C in Lake Nakaumi, and typical profiles of water temperature and turbidity are shown in Fig. 7. Surface inversion in Lake Nakaumi was due to river inflows and turbid intrusions were seen at the lower bottom of the thermally reversed layer, which suggested that salt water was lying beneath the intrusion.

In each brackish lake, salinity difference was measured at a fixed point, and a correlation was found between temperature and salinity. Salinity difference and the Rayleigh number at other points were estimated through the temperature-salinity correlation within each lake. As the lake waters were in the diffusive regime, the density ratio is defined as $R_\rho = \beta \Delta S / \alpha \Delta T$. The ratio was around 3 in Lake Shinji and more than 10 in Lake Nakaumi. Thermal oscillations with the periods of several minutes at around the reversed thermocline in the lakes are shown in Fig. 8(a) and (b).

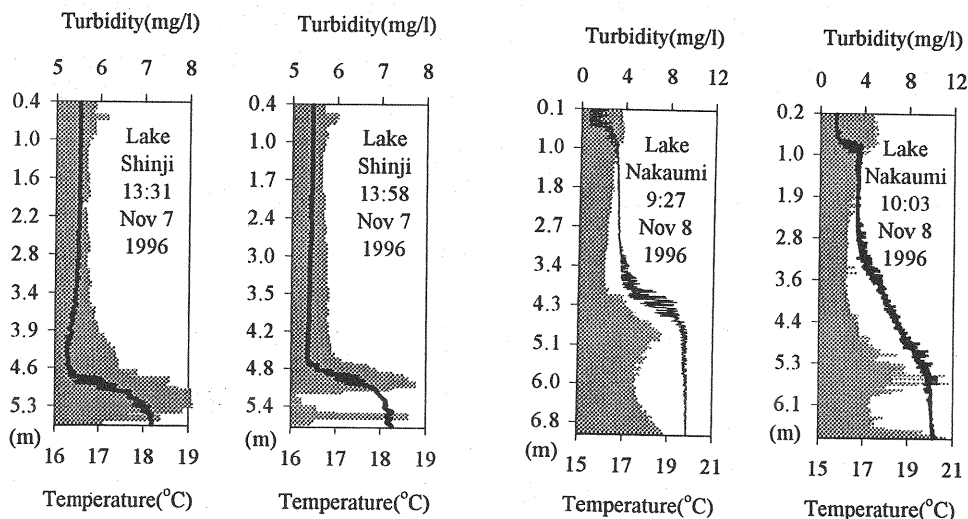


Fig. 7 Reversed temperature profiles in the brackish lakes

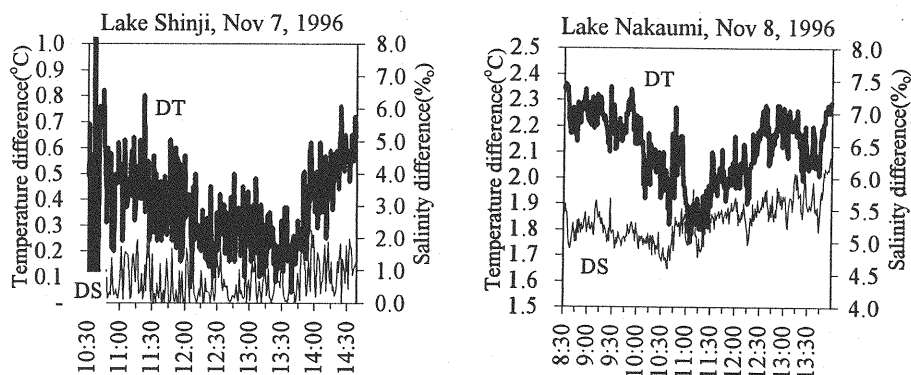


Fig. 8 Unstable thermal oscillation in the brackish Lakes

The minimum differences were seen around noon because of the diurnal thermal stratification, and variation with the higher frequency was larger in Lake Shinji.

Figure 9 shows the diffusive regime of the Rayleigh numbers (quadrant I) with the experimental and field data and attached table shows the experimental condition. The thermo-turbid experiments DA1 and DA2 are close to the Asaeda and Tamai's data (1), which have the higher Ra than that of Shirtcliffe (8), and are close to the critical stability curve, beyond which unstable condition is observed being accompanied by the forming, coupling, or settling of convective layers. Unstable thermal oscillation is found when Ra is close to the critical curve and its frequency is about one-fifth of the buoyancy frequency for the stable salinity stratification. Those instabilities were seen only in the forced cooling the top surface or heating

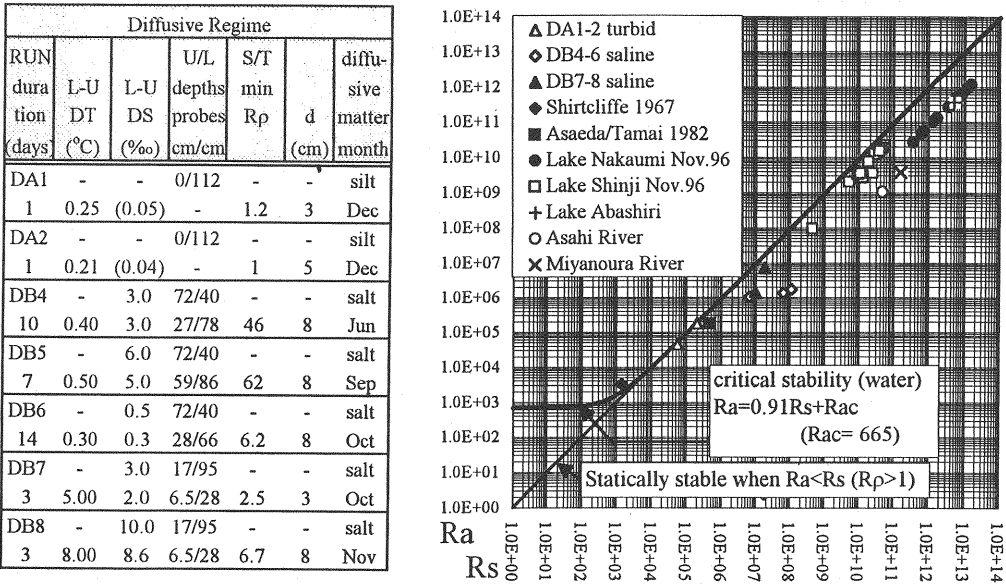


Fig. 9 The diffusive regime of the positive Rayleigh numbers

the lower bottom. Under natural cooling conditions, in DB4, DB5 to DB6, the salinity jumps were too strong and even an overstable condition, in which both the thermal and salinity gradients were stable, prevailed during the daytime when the mixing events were generally unexpected. The Rayleigh numbers in lakes, especially the Rs in Lake Nakaumi, were larger and are close to the values in Lake Abashiri (in February) which is ice-covered in winter time. For comparison, the Rayleigh numbers obtained in two rivers are shown. The data from the Asahi River (Okayama, in January) and the Miyanoura River (Yaku-Is., Kagoshima, at night in August) are also categorized into the diffusive regime. Density ratios of the estuaries are generally high, which means that those are classified into a weakly-mixed type (saline wedge).

It was found that the thermohaline in Lake Nakaumi is extraordinary stable with little oscillation even in winter time, under condition of normal cooling and usual wind velocity. On the other hand the thermohaline in Lake Shinji would have been broken and oscillation like autumn of 1994 would exhibit the diffusive regime, if the cooling was a few times larger than that in the day of the measurement, which corresponds to the larger thermal Rayleigh number, Ra. In the lake, shortage of river discharge due to 1994 summer drought caused large amount of salt intrusion from the Japan Sea through Lake Nakaumi. A measurement using a thermistor chain in the October revealed the fluctuations with abrupt temperature rise with an apparent period of several minutes, which was not identified but thought as the unstable oscillation in the diffusive regime. In the brackish lakes, summer stratification is overstable and the oxygen depletion of the bottom layer is known as

a problem of concern. In October when the buoyancy flux changes its direction, thermally stable and unstable profiles are seen in turn and the thermally reversed layer of the diffusive regime is established in early winter. The mixing effect of the convection is weak but not negligible, especially for Lake Shinji.

CONCLUSION

The processes accompanied by the diffusive convection at the interfaces in both the finger and diffusive regimes were investigated in field and laboratory. Turbid intrusion into the seasonal thermocline resulting in thermal staircases in fresh water lakes and thermally reversed (unstable) layers in brackish lakes are often observed and halt there for a long duration affecting on the biochemical processes. Double-diffusive convective processes allowing those thermal staircases, are also important to consider water quality problems in such lakes. The processes would be generalized as multiple-diffusive convections in which the direction of vertical material transport is determined by the double-diffusive, thermo-turbid /solutal convection, and other constituents, for example, plankters, chemicals and dissolved oxygen and so on, should be involved in the process.

In the finger regime, the thermo-turbid/solutal convections were identified with respects to the density and flux ratios of thermal buoyancy, whether unstable component is dissolved or suspended. It is known that a flux path of material transport forms a loop consisting of resuspension, intrusion and settling. Thermal staircases are formed especially relating to the intrusion and settling in the form of fingering. The step structures at mid depths of lakes are neither in equilibrium state nor static, and regarded as rapid fingers. These are different from the deep oceanic steps in fully equilibrium state, but the resulting transport is the same.

For the diffusive regime, some differences were seen between the thermo-turbid and thermo-solutal convections due to settling. It is essential if larger time span and depth range are concerned. Typical diffusive regime is seen in brackish lakes in winter time. The stability of the thermohaline in the regime depends on the salinity Rayleigh number and magnitude and type of atmospheric cooling. Generally the thermohaline is stronger in brackish lakes than in the oceans, so that the thermal effects have been neglected, so far. However, thermohaline convective process is important in monitoring the brackish lakes.

To combine the hydraulic process with water quality problems, multiple-diffusive process of thermal, dissolved and suspended matters should be considered. In case that the thermo-turbid or thermo-solutal, double-diffusive convective process is dominant, and contribution of concentrations of other constituents to density gradient are lower enough, the process can be treated as the double-diffusive process that controls transport rates of such minor materials. In this reason, it must be important to describe the double-diffusive process considering the flux of the second factor and the buoyancy flux ratio of the first to the second factor.

The authors wish to thank to Mr. Hiroshi Morikawa, DC student of Kyoto University, for his help in the field measurements in the lakes.

REFERENCES

1. Asaeda, T. and N. Tamai : Convection induced by input of heat to continuously stratified layers, *Jour. Hydraulic, Coastal and Environmental Engineering, JSCE*, Vol.323, pp.109-119, 1982 (in Japanese).
2. Kanda, T., K. Michioku, T. Nishikawa, M. Higashino, T. Itoh and K. Ishikawa : Field survey on temperature and quality of reservoir water with unstable thermal stratification in bottom layer, *Annual Jour. of Hydraulic Engineering, JSCE*, Vol.40, pp.601-606, 1996 (in Japanese).
3. Matsumoto, H. : Behavior of water and the index substances of water quality in a Reservoir with the dicho-thermal stratification, *Lecture Notes of the 32nd Summer Seminar on Hydraulic Engineering, Vol.32A, A-8, pp.1-14, 1996 (in Japanese).*

4. Morikawa, H., K. Okubo and Y. Muramoto : Intrusion of turbid water in stratified lakes driven by resuspension of sediments on sloping boundaries, Annual Jour. of Hydraulic Engineering, JSCE, Vol.40, pp.607-612, 1996 (in Japanese).
5. Mormorino, G.O., W.K. Brown, W.D. Morris : Two-dimensional temperature structure in the C-SALT thermohaline staircase, Deep-Sea Res., Vol.34, pp.1676-1697, 1987.
6. Okubo, K., Y. Muramoto and H. Morikawa : Transport of bottom sediment in Lake Biwa, Annuals Disaster Prevention Research Institute, Kyoto Univ., Vol.36B-2, pp. 499-518, 1993 (in Japanese).
7. Okubo, K., Y. Muramoto, H. Morikawa and S. Ezoe : Simultaneous variation process of water temperature and turbidity in Lake Biwa, Annuals Disaster Prevention Research Institute, Kyoto Univ., Vol.38B-2, pp.407-422, 1995 (in Japanese).
8. Shirtcliffe: Thermosolutal convection; Observation of an overstable mode, Nature, Vol.213, pp.489-490, 1967.
9. Sugiyama, M. and T. Hori : Distribution and dynamics of particulate chemical constituents; - results from the Thermocline Dynamics'94-, Proc. Jap. Limnol. Soc., Vol.60, p.A4, 1995 (in Japanese).
10. Yamada, T. : Study on salt intrusion into the lakes in Hokkaido, Scientific Report, The Ministry Grant No.05555147, 1996 (in Japanese).

APPENDIX - NOTATION

The following symbols are used in this paper:

B_T, B_S	= buoyancy fluxes in temperature and concentration differences
d	= thickness of the convective layer;
DT, DS	= $\Delta T, \Delta S$ measured;
F	= buoyancy flux ratio (= B_T/B_S);
g	= gravity acceleration;
Ra	= Rayleigh number (= $\alpha g \Delta T d^3 / \kappa \nu$);
Rs	= salinity Rayleigh number (= $\beta g \Delta S d^3 / \kappa \nu$);
$R\rho$	= density ratio (= $\alpha \Delta T / \beta \Delta S$) in the finger regime; (= $\beta \Delta S / \alpha \Delta T$) in the diffusive regime;
w_a	= actual settling velocity of silt particle;
z	= vertical depth coordinate positive downward;
α, β	= coefficients of thermal expansion and saline contraction;
$\Delta T, \Delta S$	= temperature and concentration (in weight) differences;
ΔC	= concentration (in volume) difference;
σ	= submerged specific weight of sediment;
κ, κ_s	= thermal and salinity diffusivity (molecular);
ν	= kinematic viscosity; and
τ	= diffusivity ratio (= κ_s / κ) of about 10^{-2} in the oceans.