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ON THE COMPUTATION OF EQUILIBRIUM CHANNELS IN COHESIONLESS ALLUVIUM

By

M. Selim YALIN
Emeritus Professor, Department of Civil Engineering, Queen's University
Kingston, Ontario, Canada

and

Ana Maria FERREIRA DA SILVA
Assistant Professor, Civil and Environmental Engineering, University of Windsor
Windsor, Ontario, Canada

SYNOPSIS

An expression for width-to-depth ratio, which covers both sand and gravel regime channels, is derived. The derivation rests on the dimensional principles and it stems from the fact that the net cross transport rate of an equilibrium channel is zero. The agreement of the derived expression with experiment is verified by the available regime data. The paper presents also a method for the regime channel computation. The results supplied by this method are compared with the corresponding characteristics of actual streams.

INTRODUCTION

In the current rational approaches (Bettess and White (1), Chang (2), (3), (4), Chang and Hill (5), Davies and Sutherland (7), Yang (18), (19), (21), Yang *et al.* (20)) the three characteristcs B_R, h_R and S_R defining a regime channel are usually determined from the following three equations (see List of Symbols)

$$Q = f_Q(B_R, h_R, S_R, c_R) \qquad \text{(resistance eq.)} \tag{1}$$

$$A_* = (A_*)_{min} \qquad \text{(minimum } A_*\text{)} \tag{2}$$

$$Q_s = f_{Q_s}(B_R, h_R, S_R, c_R) \qquad \text{(transport eq.)} \tag{3}$$

$$\text{(with } c_R = \phi_c(h_R, S_R)\text{)}.$$

Although it has not been settled yet what exactly the energy-related quantity A_* must be, or what formulae should be used for the expressions of f_Q and f_{Q_s}, no objection can be raised, in principle, with regard to the first two equations. Yet, the third equation, which has been introduced only because the first two are not sufficient to solve the three unknowns B_R, h_R and S_R requires the beforehand knowledge of the transport rate Q_s. And more often than not, the (usually unknown) Q_s is simply adjusted as to yield the "reasonable results". In the present paper, the Q_s-equation is replaced by a relation which reflects the most fundamental aspect of all regime channels – their time invariance.

Consider the uniform flow in a self-forming straight alluvial channel at an instant t ($< T_R$): the flow is tranquil ($Fr < 1$), the ratio B/h is "large" ($>\approx 10$, say), the granular material is cohesionless, the bed of the (symmetrical) channel cross-section is horizontal and, in general, it is covered by bed forms (ripples and/or dunes).

If the flow were two-dimensional ($B/h \to \infty$) then a dimensionless property Π_A of the two-phase motion "en mass" would have been a function of (at most) three independent dimensionless variables (X, Y, Z or ξ, η, Z, etc. – see Yalin (14), (15)). In the present case B/h, however large, is finite, and it is the additional variable:

$$\Pi_A = \phi_A(B/h, \xi, \eta, Z). \tag{4}$$

[Although the angle of repose φ also contributes to the determination of the cross-section geometry, it is as a rule ignored in the regime theory. First because the regime channels are usually wide, second because in the cohesionless materials the range of variation of φ is rather limited; hence φ is not present in Eq. 4.]

The sediment transported in the flow direction x tends to disperse sideways. The grains lifted from the bed diffuse (due to turbulence) toward the banks, while the grains detached from the banks tend to move in the opposite direction (because of the bank inclination as well as turbulence). Thus at any section y we have the lateral (specific) transport rates p_s' and p_s'' (see Fig. 1) and the net specific cross transport rate

$$p_s = p_s' - p_s''. \tag{5}$$

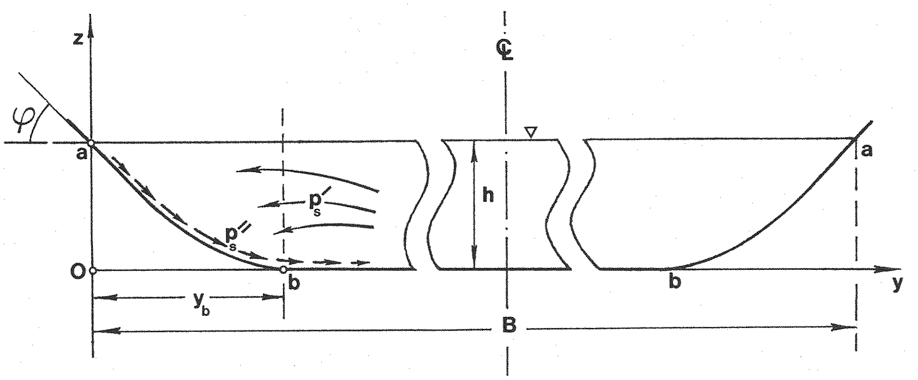


Fig. 1

Consider the dimensionless cross transport rate p_s/q_s corresponding to a specified section y (such as $y = y_b$, say). The value of p_s/q_s ($= \Pi_{p_s}$) is determined, according to Eq. 4, as

$$p_s/q_s = \phi_{p_s}(B/h, \xi, \eta, Z). \tag{6}$$

If the self-forming channel has reached the regime state, i.e. if its boundary is no longer deforming, then necessarily

$$p_s = 0 \quad \text{or} \quad p_s' = p_s'' \quad \text{for any } y (\in [0; B]). \tag{7}$$

The concept that the cross-transport rate of a regime channel must vanish was first introduced by Parker (12). The relations 7 are equally valid for both, sand and gravel bed channels: if sand, then the (equal to each other) p_s' and p_s'' are finite; if gravel, then they are zero. Substituting Eq. 7 in Eq. 6, one obtains

$$0 = \phi_{p_s}(B_R/h_R, \xi, \eta_R, Z_R) \tag{8}$$

and thus

$$B_R/h_R = \phi_{B/h}(\xi, \eta_R, Z_R). \tag{9}$$

It follows that a regime channel is distinguished (from the rest of the analogous alluvial channels) by the fact that its properties Π_{A_R} are determined by three (rather than four) dimensionless variables. Indeed eliminating, in principle, $B/h = B_R/h_R$ from Eqs. 9 and 4, one obtains for any $\Pi_A = \Pi_{A_R}$

$$\Pi_{A_R} = \phi_{A_R}(\xi, \eta_R, Z_R). \tag{10}$$

[This also explains why is a regime channel determined by (only) six characteristic parameters (as the channel R in Yalin (14)).]

The development of the regime aspect ratio B_R/h_R must be expected to be strongly affected by the granular skin roughness of the flow boundaries as well as by the bed forms present at the time $t = T_R$. Consequently the introduction of the friction factors $\bar{c}_R$ and c_R into the expression of B_R/h_R would be appropriate.

The consideration of the forms

$$\bar{c}_R = \phi_{\bar{c}}(\xi, \eta_R, Z_R) \qquad \text{and} \qquad c_R = \phi_c(\xi, \eta_R, Z_R) \tag{11}$$

is equivalent to that of

$$\frac{c_R}{\bar{c}_R} = \frac{\phi_c}{\phi_{\bar{c}}} = \phi_\lambda(\xi, \eta_R, Z_R) \quad (= \lambda_R) \tag{12}$$

and

$$c_R\sqrt{\eta_R} = \sqrt{\eta_R}\,\phi_c = \phi_v(\xi, \eta_R, Z_R) \quad (\approx v/v_{*cr}). \tag{13}$$

Eliminating η_R and Z_R from Eqs. 9, 12 and 13, one determines

$$\frac{B_R}{h_R} = \phi_{B/h}(\xi, \lambda_R, c_R\sqrt{\eta_R}\,). \tag{14}$$

THE EXPRESSIONS OF THE REGIME FLOW WIDTH AND DEPTH

Since the derivation of the function 14 rests entirely on $p_s = 0$ (which is valid for the regime channels formed by any granular material), its form $\phi_{B/h}$ cannot be expected to vary depending on the nature of material: the difference between materials (between gravel and sand, say) being reflected solely by the (different) numerical values of the variables ξ, λ_R and $c_R\sqrt{\eta_R}$. The consideration of this fact makes it possible to reveal how the ratio B_R/h_R depends on the (most significant) variable $c_R\sqrt{\eta_R}$. Indeed, consider a gravel bed regime channel: the influence of viscosity (and thus of ξ) is negligible, the bed forms (if they exist at all) are insignificant ($\lambda_R \approx 1$) and the gravel boundary of the channel is at its critical stage ($\eta_R \approx 1$). Thus for a gravel bed regime channel, Eq. 14 reduces into

$$\frac{B_R}{h_R} \approx \phi_{B/h}(1, c_R\sqrt{1}\,), \tag{15}$$

where the friction factor c_R is given by

$$c_R \approx \bar{c}_R \approx 6.82 \left(\frac{h_R}{D}\right)^{1/6}. \tag{16}$$

As is well known, $B_R \sim Q^{1/2}$ can be regarded as valid for all (sand and gravel) regime channels (see e.g. Table 4.1 in Yalin (14)); and for gravel channels we have $h_R \sim Q^{3/7}$ (see the same Table; see also p. 144 in (14)). Using these proportionalities in Eqs. 15 and 16, one determines respectively (for a given experiment, i.e. for $D = const$)

$$\frac{B_R}{h_R} \sim \frac{Q^{1/2}}{Q^{3/7}} \sim Q^{1/14} \tag{17}$$

and

$$c_R \sim h_R^{1/6} \sim (Q^{3/7})^{1/6} \sim Q^{1/14}. \tag{18}$$

Hence both B_R/h_R and c_R are proportional to (the same) $Q^{1/14}$, and they must thus be proportional to each other:

$$\frac{B_R}{h_R} \sim c_R\,. \tag{19}$$

In the function 14, c_R appears only in conjunction with η_R as $c_R\sqrt{\eta_R}$, and therefore Eq. 19 is, in fact, $B_R/h_R \sim c_R\sqrt{\eta_R}$, where $\eta_R = 1$. Since, however, the form of the function 14 does not vary depending on material, the proportionality $B_R/h_R \sim c_R\sqrt{\eta_R}$ (though found from the consideration of gravel channels only) must be valid also for sand channels as well. But this means that Eq. 14 can be expressed (for all materials) in the following manner

$$\frac{B_R}{h_R} = c_R\sqrt{\eta_R}\,[\phi(\xi,\lambda_R)]^2 \quad \left(= \frac{\Pi_{B_R}}{\Pi_{h_R}}\right). \tag{20}$$

The resistance equation of wide regime channels, viz

$$Q = (B_R h_R)\, c_R \sqrt{g S_R h_R}\,, \tag{21}$$

can be expressed identically as

$$\left(\frac{Q/v_{*cr}}{B_R^2}\right)\left(\frac{Q/v_{*cr}}{h_R^2}\right) = c_R^2 \frac{g S_R h_R}{v_{*cr}^2} \quad \text{i.e.} \quad \Pi_{B_R}^{-1}\Pi_{h_R}^{-1} = c_R\sqrt{\eta_R}\,. \tag{22}$$

Solving Eqs. 20 and 22, one determines

$$\Pi_{B_R} = B_R\sqrt{\frac{v_{*cr}}{Q}} = \phi(\xi,\lambda_R) \tag{23}$$

and

$$\Pi_{h_R} = h_R\sqrt{\frac{v_{*cr}}{Q}} = [c_R\sqrt{\eta_R}\,\phi(\xi,\lambda_R)]^{-1}. \tag{24}$$

From Eqs. 20, 23 and 24, it is clear that these relations can be rendered practically useful if the form of only one function, viz $\phi(\xi,\lambda_R)$ is revealed; and at the present state of knowledge this can be done only with the aid of experiment.

On the basis of data-plots it has been established (Yalin (14), Lei (10)) that ξ affects Π_{B_R} by means of "its own" function, $\phi_\xi(\xi)$ say, which, with an accuracy sufficient for all practical purposes, can be formulated as follows:

$$\text{If } \xi \leq 15 \text{ then } \phi_\xi(\xi) = 0.45\xi^{0.3} \quad \text{and} \quad \text{if } \xi > 15 \text{ then } \phi_\xi(\xi) = 1\,. \tag{25}$$

Consider now the role of λ_R. If ξ has "its own" function, then so must λ_R [i.e. $\phi(\xi, \lambda_R) = \phi_\xi(\xi)\phi_\lambda(\lambda_R)$]. The gravel regime data, which can be taken to correspond to $\phi_\xi(\xi) = 1$ and $\lambda_R = 1$, indicates (see Fig. 4.8 in Ref. (14)) that

$$\Pi_{B_R} = 1\,\phi_\lambda(1) \approx 1.42. \tag{26}$$

Furthermore, in the course of their recent work, the authors realized that the sand-data pattern in Fig. 4.7 in Ref. (14) can be made congruent with the gravel-data pattern in Fig. 4.8 in Ref. (14) if the ordinates of the sand-data points are multiplied by their respective values of 1.42λ. (The trials with $1.42\lambda^n$, for various n did not produce any improvement). Hence, one arrives at

$$\phi(\xi, \lambda_R) = 1.42\,\lambda_R\,\phi_\xi(\xi) \tag{27}$$

which has been adopted for the evaluation of Eqs. 20, 23 and 24 (with $\phi_\xi(\xi)$ given by Eq. 25).

The validity of the expressions 23 and 24 derived above was tested by using the alluvial stream data of *all* the available sources (Refs. (1a) to (18a)). The (unified) plots in Figs. 2 and 3 contain the points ranging from fine-sand to gravel (the larger is the number of the point symbol, the larger is the grain size $D \sim \xi$). In spite of this, no "grouping" of the points (as to form "their own" lines) is detectable: the scatter is purely experimental.

[Not all the data included in Refs. (1a) to (18a) correspond to the regime state. As should be clear from Ref. (14), the regime value of η for gravel channels is unity, while for sand-bed channels it is comparable with 10. The data points possessing η-values that differ too much (more than twice, say) from the η_R-values mentioned were excluded.

COMPUTATION OF REGIME CHANNELS

i) Pertinent Equations

Two equations 23 and 24 involve all three unknowns (B_R, h_R and S_R), because η_R, and thus c_R and λ_R which vary with η_R, are determined among others by S_R. Following Jia (9) and Yalin (14) it will be assumed that

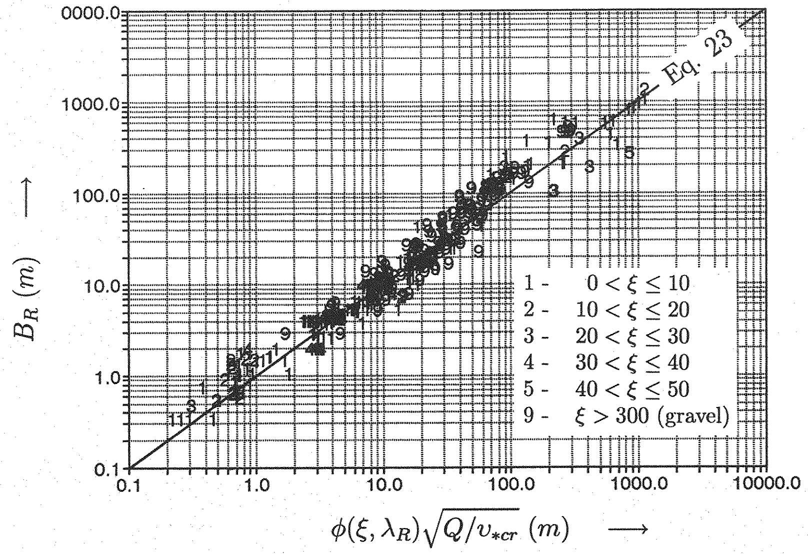
$$A_* = Fr = v^2/gh = c^2 S = c^2 \eta v_{*cr}^2/(gh), \tag{28}$$

and consequently that B_R and h_R satisfy Eqs. 23 and 24 for the smallest value of Fr.

In the present method the regime channel is specified by the numerical values of the following six characteristic parameters

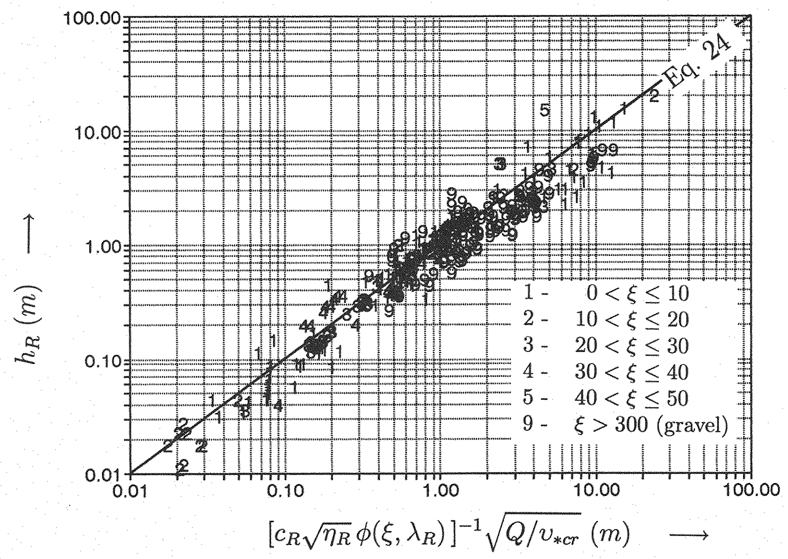
$$Q, \rho, \nu, v_{*cr}, D, g, \tag{29}$$

where v_{*cr} stands *in lieu* of γ_s (see List of Symbols).



$B_R\ (m)$ →
$\phi(\xi, \lambda_R)\sqrt{Q/\upsilon_{*cr}}\ (m)$ ⟶
Eq. 23
1 - $0 < \xi \le 10$
2 - $10 < \xi \le 20$
3 - $20 < \xi \le 30$
4 - $30 < \xi \le 40$
5 - $40 < \xi \le 50$
9 - $\xi > 300$ (gravel)

Fig. 2



$h_R\ (m)$ →
$[c_R\sqrt{\eta_R}\,\phi(\xi, \lambda_R)]^{-1}\sqrt{Q/\upsilon_{*cr}}\ (m)$ ⟶
Eq. 24
1 - $0 < \xi \le 10$
2 - $10 < \xi \le 20$
3 - $20 < \xi \le 30$
4 - $30 < \xi \le 40$
5 - $40 < \xi \le 50$
9 - $\xi > 300$ (gravel)

Fig. 3

For the friction factors $\bar{c}$ and c, the following relations are adopted (see e.g. Schlichting (13), Engelund (8), Yalin (14), (17))

$$\bar{c} = \frac{1}{\kappa} \ln\left(\psi(X)\frac{Z}{2}\right) \qquad (\kappa \approx 0.4) \tag{30}$$

$$\frac{1}{c^2} = \frac{1}{\bar{c}^2} + \sum_{j=d,r} a_j \left(\frac{\Delta}{\Lambda}\right)_j^{b_j} \frac{\Lambda_j}{h} \qquad (d = \text{dunes};\ r = \text{ripples}) \tag{31}$$

where $\psi(X) = e^{\kappa B_s - 1}$ with B_s given by

$$B_s = 8.5 + [2.5\, ln(2X) - 3]e^{-0.217[ln(2X)]^2} \tag{32}$$

(see e.g. Yalin (14), p.10).

The bed form properties $(\Delta/\Lambda)_j$ and Λ_j/h (in Eq. 31) are evaluated with the aid of the expressions (see e.g. Yalin (14), (16))

$$\left(\frac{\Delta}{\Lambda}\right)_d = 0.013\psi_{X_d}(X)(\eta - 1)e^{-\frac{(\eta-1)}{13(1-e^{-0.008Z})}} \tag{33}$$

$$\left(\frac{\Delta}{\Lambda}\right)_r = 0.035\psi_{X_r}(X)(\eta - 1)e^{-0.1(\eta-1)}, \tag{34}$$

and

$$\frac{\Lambda_d}{h} = 6\left[1 + 0.01\frac{(Z-40)(Z-400)}{Z}e^{(-0.055\sqrt{Z}+0.04X)}\right]; \quad \frac{\Lambda_r}{h} = \frac{1000}{Z} \tag{35}$$

where

$$\psi_{X_d}(X) = 1 - e^{-(X/10)^2}; \tag{36}$$

$$\psi_{X_r}(X) = e^{-[(X-2.5)/14]^2} \text{ when } X > 2.5 \quad \text{and} \quad \psi_{X_r}(X) = 1 \text{ when } X < 2.5. \tag{37}$$

For the case of ripples, $a_r = 1/2$ and $b_r = 2$; for the case of dunes $a_d = 1/2$ and $b_d = 2$, except if $D \in [0.1mm\,; 1.1mm\,]$, in which case b_d is given by

$$b_d = 2.0 - 2.4(D - 0.1)(D - 1.1)[0.25\, log_{10}N - 0.75] \tag{38}$$

(with D in mm).

The relation 38 has been established recently on the basis of available data by the authors and it supersedes Table 3.1 in Ref. (14).

Note that Eqs. 30 to 37 give the friction factor c as a function of X, η and Z. However, since a regime channel is specified by the nature of granular material and Q, it would be more appropriate (and convenient) if a related dimensionless characteristic which is given originally as $\Pi_A = \phi_A(X, \eta, Z)$ is converted into the form $\overline{\phi}_A(\xi, \eta, N)$. This can be achieved with the aid of the relations

$$Z = N/c\sqrt{\eta} \qquad \text{and} \qquad X = [\eta\, \Psi(\xi)\, \xi^3]^{1/2}. \tag{39}$$

The first of these relations follows from the division of the resistance equation by v_{*cr}; the second, from the definition of ξ, viz $\xi^3 = X^2/Y = X^2/\eta\Psi(\xi)$.

It can be shown (see Yalin (14)) that Fr $(= v^2/gh = c^2S)$ and c are interrelated as

$$Fr = (\alpha/N)(c^2\eta)^{3/2} \tag{40}$$

where $\alpha = (\gamma_s/\gamma)Y_{cr} = v_{*cr}^2/gD$. Figs. 4 and 5 show as example the Fr-curve families computed for $\xi = 4.3$ and $\xi = 1265$, respectively: η is the abcissa; each individual curve corresponds to a particular value of N. Observe from these Figures that the Fr-curves tend to be monotonous in the case of gravel channels (where no bed forms are present); and that they tend to exhibit "dips" in the case of sand channels (where bed forms are prominent). Note also that in both cases the regime data points tend to converge towards the location of the smallest Fr.

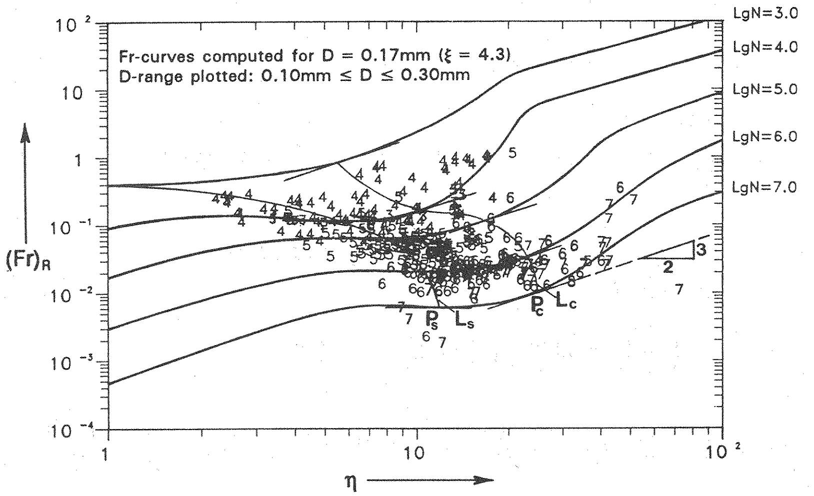


Fig. 4 (from Yalin (14))

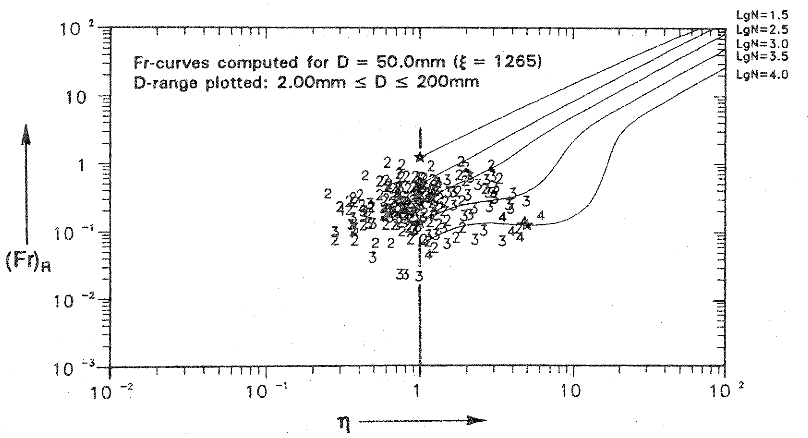


Fig. 5 (from Yalin (14))

ii) Computational procedure

Given Q and D, and thus Q, ξ and v_{*cr}, the following steps are adopted:

Step 1: Adopt $(B_R)_i$.

Step 2: Compute $(N_R)_i$ $(= Q/((B_R)_i D v_{*cr}))$.

Step 3: Knowing $(N_R)_i$, compute with the aid of Eq. 40, viz

$$Fr_i = (\alpha/(N_R)_i)((c_R)_i^2\eta)^{3/2}, \quad \text{where} \quad (c_R)_i = \phi_c(\xi, \eta, (N_R)_i)$$

such an $\eta = (\eta_R)_i > 1$ which yields $Fr = (Fr_i)_{min}$; or the value of Fr which corresponds to $\eta = 1$, whichever applicable.
Observe that $(\bar{c}_R)_i$, and thus $(\lambda_R)_i$, must be computed in the process of solving Eq. 40.

Step 4: Using $(c_R)_i$ and $(\lambda_R)_i$ determined in step 3, use Eq. 23 to compute $(B_R)_{i+1}$.
If $(B_R)_{i+1} = (B_R)_i$, then the problem is solved: $B_R = (B_R)_i$.
If $(B_R)_{i+1} \neq (B_R)_i$, then repeat steps 2 to 4 until such an i is reached which yields $(B_R)_{i+1} = (B_R)_i$.

Step 5: Knowing B_R, compute S_R from $(Fr)_R = c_R^2 S_R$.

Step 6: Compute h_R from $\eta_R = gS_R h_R/v_{*cr}^2$.

iii) Examples

Some examples of regime characteristics computed by the above described method are given below ($\gamma_s/\gamma = 1.65$, $\nu = 10^{-6}m^2/s$).

Example 1: $Q = 1669.7m^3/s$, $D = 0.18mm$ (Bhagirathi River, Chitale (6))
Computed: $B_R = 192.6m$ $h_R = 9.69m$ $S_R = 0.000028$
$[N_R \approx 10^{6.5}$; $(Fr)_R = 0.0084$; $\eta_R = 14.60$; $c_R = 17.35]$
Reported: $B = 218.1m$ $h = 5.95m$ $S = 0.000058$
$[N \approx 10^{6.5}$; $Fr = 0.028$; $\eta = 18.60]$

Example 2: $Q = 848.9m^3/s$, $D = 0.80mm$ (Savannah River, Chitale (6))
Computed: $B_R = 151.8m$ $h_R = 6.39m$ $S_R = 0.000057$
$[N_R \approx 10^{5.5}$; $(Fr)_R = 0.0122$; $\eta_R = 7.44$; $c_R = 14.59]$
Reported: $B = 106.7m$ $h = 5.18m$ $S = 0.00011$
$[N \approx 10^{5.6}$; $Fr = 0.05$; $\eta = 11.55]$

Example 3: $Q = 4386.0m^3/s$, $D = 3.10cm$ (North Saskatchewan River, Neill (11))
Computed: $B_R = 236.3m$ $h_R = 6.63m$ $S_R = 0.00038$
$[N_R \approx 10^{3.6}$; $(Fr)_R = 0.1206$; $\eta_R = 1.00$; $c_R = 17.68]$
Reported: $B = 244.0m$ $h = 7.62m$ $S = 0.00035$
$[N \approx 10^{3.6}$; $Fr = 0.074$; $\eta = 1.12]$

The ever-varying (in space and time) characteristics of a natural river cannot be represented by exact numerical values, and therefore the "reported" values are to be viewed only as some (vague) typical values. The computer results appear to compare with them satisfactorily.

REFERENCES

1. Bettess, R., White, W.R.: "Extremal hypothesis applied to river regime". in "Sediment Transport in Gravel-Bed Rivers", C.R. Thorne, J.C. Bathurst and R.D. Hey eds., John Wiley and Sons, 1987.
2. Chang, H.H.: "Stable alluvial canal design". J. Hydr. Engrg., ASCE, Vol. 106, No. HY5, 1980.
3. Chang, H.H.: "Geometry of gravel streams". J. Hydr. Div., ASCE, Vol. 106, No. HY9, 1980.
4. Chang, H.H.: "Minimum stream power and river channel patterns". J. Hydrology, Vol. 41, 1979.
5. Chang, H.H., Hill, J.C.: "Minimum stream power for rivers and deltas". J. Hydr. Div., ASCE, Vol. 103, No. HY12, 1977.
6. Chitale, S.V.: "River channel patterns". J. Hydr. Div., ASCE, Vol. 96, No. HY1, 1970.
7. Davies, T.H.R., Sutherland, A.J.: "Extremal hypothesis for river behaviour". Water Resour. Res., Vol. 19, No. 1, 1983.
8. Engelund, F.: "Hydraulic resistence of alluvial streams". J. Hydr. Div., ASCE, Vol. 92, No. HY2, March, 1966.
9. Jia, Y.: "Minimum Froude number and the equilibrium of alluvial sand rivers". Earth Surf. Processes and Landforms, Vol. 15, 1990.
10. Lei, S.: "A regime theory based on the minimization of the Froude number". Ph.D. Thesis, Dept. of Civil Engrg., Queen's Univ., Kingston, Canada, 1992.
11. Neill, C.R.: "Hydraulic geometry of sand rivers in Alberta". Proc. Hydrology Symp., Alberta, May, 1973.
12. Parker, G.: "Self-formed straight rivers with equilibrium banks and mobile bed. Part 1. The sand-silt river". J. Fluid Mech., Vol. 89, 1978.
13. Schlichting, H.: "Boundary layer theory". McGraw-Hill Book Co. Inc., Verlag G. Braun, 1968 (6th edition).
14. Yalin, M.S.: "River mechanics". Pergamon Press, Oxford, 1992.
15. Yalin, M.S.: "Mechanics of sediment transport". Pergamon Press, Oxford, 1977.
16. Yalin, M.S.: "On the determination of ripple length". J. Hydr. Div., ASCE, Vol. 103, No. HY4, 1977.
17. Yalin, M.S.: "On the average velocity over a mobile bed". La Houille Blanche, No. 1, Jan/Feb., 1964.
18. Yang, C.T.: "Energy dissipation rate approach in river mechanics". in Sediment Transport in Gravel-Bed Rivers, C.R. Thorne, J.C. Bathurst and R.D. Hey eds., John Wiley and Sons, 1987.
19. Yang, C.T.: "Unit stream equation for gravel". J. Hydr. Engrg., ASCE, Vol. 110, No. 12, 1984.
20. Yang, C.T., Song, C.C.S., Woldenberg, M.J.: "Hydraulic geometry and minimum rate of energy dissipation". Water Resour. Res., 17, 1981.
21. Yang, C.T.: "Minimum unit stream power and fluvial hydraulics". J. Hydr. Div., ASCE, Vol. 102, No. HY7, 1976.

REFERENCES A: Sources of data for regime plots

1a. Ackers, P. et al.: "The geometry of small meandering streams". Proc. Instn. Civ. Engrs. Paper 7328S, London, 1970.

2a. Ackers, P.: "Experiments on small streams in alluvium". J. Hydr. Div., ASCE, Vol. 90, No. HY4, 1964.

3a. Bray, D.: "Estimating average velocity in gravel-bed rivers". J. Hydr. Div., ASCE, Vol. 105, No. HY9, 1979.

4a. Brownlie, W.R.: "Compilation of alluvial channel data: laboratory and field representation". Report No. KH-R-43B, W. M. Keck Lab. for Hydr. and Water Res., Calif. Inst. of Tech., Pasadena, California, 1981.

5a. Center Board of Irrigation and Power: "Library of canal data (Punjab and Sind)". Technical Report No. 15, June, 1976.

6a. Center Board of Irrigation and Power: "Library of canal data (Upper Ganga)". Technical Report No. 15, June, 1976.

7a. Center Board of Irrigation and Power: "Library of canal data (U.S. canals)". Technical Report No. 15, June, 1976.

8a. Chitale, S.V.: "River channel patterns". J. Hydr. Div., ASCE, Vol. 96, No. HY1, 1970.

9a. Church, M., Rood, R.: "Catalogue of alluvial river channel regime data". Dept. of Geography, Univ. of British Columbia, Vancouver, Canada, 1983.

10a. Colosimo, C., Coppertino, V.A., Veltri, M.: "Friction factor evaluation in gravel-bed rivers". J. Hydr. Engrg., ASCE, Vol. 114, No. 8, 861-876, 1988.

11a. Hey, R.R., Thorne, C.R.: "Stable channels with mobile beds". J. Hydr. Engrg., ASCE, Vol. 112, No. 8, 1986.

12a. Higginson, N.N.J., Johnston, H.T.: "Estimation of friction factor in natural streams". in "River Regime", W. R. White ed., J. Wiley and Sons, 1988.

13a. Kellerhals, R., Neill, C.R., Bray, D.I.: "Hydraulic and geomorphic characteristics of rivers in Alberta". Alberta Cooperative Research Program in Highway and River Engineering, 1972.

14a. Khan, H.R.: "Laboratory study of alluvial river morphology". Ph.D. Dissertation, Colorado State University, Fort Collins, Colorado, 1971.

15a. Lan, Y.Q.: "Dynamic modeling of meandering alluvial channels". Ph.D. Dissertation, Colorado State University, Fort Collins, Colorado, 1990.

16a. Odgaard, A.J.: "Stream bank erosion along two rivers in Iowa". Water Resour. Res., Vol. 25, No. 7, 1987.

17a. Schumm, S.A.: "River adjustments to altered hydrologic regime – Murrumbidgee River and Paleochannels, Australia". U.S. Geol. Survey Prof. Paper 598, 1968.

18a. Simons, D.B., Albertson, L.: "Uniform water conveyance channels in alluvial material". J. Hydr. Div., ASCE, Vol. 86, No. HY5, 1960.

APPENDIX - NOTATION

The following symbols are used in this paper:

A_*	energy related property of flow (subjected to minimization during the regime channel formation)
B	flow width
c	dimensionless Chézy friction factor over a bed covered by bed forms
$\overline{c}$	flat bed value of c
D	typical grain size (usually D_{50})
Fr	Froude number ($= v^2/gh$)
g	acceleration due to gravity
h	flow depth
Q	flow rate
Q_s	volumetric transport rate (through the whole cross-section of flow)
q_s	longitudinal specific volumetric total transport rate (averaged over the bed $\overline{bb'}$)
p'_s	specific cross transport rate directed *toward the bank* (volumetric and total (= bed + susp.))
p''_s	specific cross transport rate directed *toward the centreline* (volumetric and total (= bed + susp.))
p_s	specific *net* cross transport rate (volumetric and total (= bed + susp.))
S	slope of the uniform flow
t	time
T_R	duration of regime channel formation
v	average flow velocity
v_*	shear velocity
x	direction of flow
y, z	cross-sectional abcissa and ordinate (see Fig. 1)
γ	specific weight of fliud
γ_s	specific weight of grains in fluid
Δ	developed bed form height
κ	Von Karman constant
$\lambda = c/\overline{c}$	friction factor ratio
Λ	developed bed form length
ν	kinematic viscosity
ρ	fluid density
f_A	dimensional function determining a quantity A
ϕ_A, ψ_A	dimensionless function determining a quantity A
Π_A	dimensionless counterpart of A

Subscript R: signifies the regime value of a quantity
Subscript cr: signifies the value corresponding to the initiation of sediment transport (to the "critical stage")

Dimensionless combinations:

$N = \frac{Q}{BDv_{*cr}}$	dimensionless flow rate
$X = \frac{v_* D}{\nu}$	grain size Reynolds number
$Y = \frac{\rho v_*^2}{\gamma_s D}$	mobility number
$Y_{cr} = \Psi(\xi)$	modified transport initiation function (Yalin (14), (15))
$Z = \frac{h}{D}$	dimensionless flow depth
$\xi^3 = \frac{\gamma_s D^3}{\rho \nu^2}$	material number $(= X^2/Y)$
$\eta = \frac{Y}{Y_{cr}} = \frac{gSh}{v_{*cr}^2}$	relative flow intensity

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