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STOCHASTIC RESPONSE OF DAILY RUNOFF BASED ON A FILTERED POINT PROCESS

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SYNOPSIS

A streamflow sequence is considered as a filtered point process whose input is a daily rainfall sequence. A daily rainfall sequence is assumed to be a marked point process in which the mark of the process is the magnitude of daily precipitation amount and the number of daily rainfall occurrences is a counting process represented by one of binomial, Poisson, and negative binomial probability distributions depending on the ratio of mean to variance. As a pulse-response function for a filtered point process, a tank model, called the model of 3-tanks with a parallel tank is developed. Thus, a streamflow sequence and its cumulants are able to be derived from a daily rainfall sequence based on the characteristic function of a filtered point process. Application of the proposed methodology to daily rainfall-runoff data at Sameura Dam in Kochi Prefecture, Japan is illustrated.

INTRODUCTION

In planning, management and utilization of water resources, long-term historical streamflow data is needed. However, many dam basins in the world lack such data. Even though some basins have long-term historical streamflow data, streamflow statistics are unreliable because the homogeneity of a streamflow time series is violated due to urbanization, development, and so on in such basins. Thus, some methods for deriving streamflow statistics from a rainfall time series whose homogeneity is higher than that of a streamflow time series have to be used. Up to the present, one method has usually been used. In this method, firstly, a large amount of daily rainfall data should be generated by a rainfall model; then, by using a suitable rainfall-runoff model, a daily rainfall time series can be transferred to a streamflow time series. Finally, the streamflow statistics can be estimated based on the synthetic streamflow time series.

In the present study, a filtered point process is used to derive the statistics of streamflow directly from a daily rainfall sequence. Weiss (4) and Kanda (1) studied methods used to generate streamflow data from a daily rainfall sequence based on a shot noise process—the simplest type of a filtered point process by Parzen (2) and Snyder (3) with the following assumptions: (i) the counting process of a daily rainfall sequence is a Poisson process; (ii) the distribution of daily precipitation follows an exponential distribution; and (iii) the response function for a filtered point process is one of the simplest exponential functions. These assumptions appear to be too simple to express rainfall-runoff characteristics.

This paper describes a continuous streamflow sequence as a filtered point process where: (i) a daily rainfall sequence is considered as a marked point process proposed by Snyder (3) in which the mark of the process is the magnitude of daily precipitation amount assumed to follow a gamma distribution and the number of daily rainfall occurrences is a counting process described by one of binomial, Poisson, and negative binomial probability distributions depending on the ratio of mean to variance of the daily rainfall occurrence number; and (ii) the response function for a filtered point process is expressed by the developed tank model. The methodology for deriving the cumulants of streamflow directly from characteristics of a rainfall sequence based on the filtered point process, as a generalized Poisson process, is initiated.

In addition, the proposed methodology can be used to evaluate impacts of global warming on future water resources by developing the time series model for annual changes of monthly temperature and by formulating the relationship between temperature and changes of rainfall characteristics.

STREAM FLOW AS A FILTERED POINT PROCESS AND ITS CUMULANTS

Definition

Let a daily rainfall time series $\{x_t;\ t \geqq t_0\}$ be a marked point process. Denote the n-th rainfall occurrence day and daily precipitation amount (mark) by τ_n and u_n, respectively, and let the number of rainfall occurrences $\{N_t;\ t \geqq t_0\}$ be a counting process that counts points independent of their marks. The streamflow y_t as a filtered point process can be represented by

$$y_t = \sum_{n=1}^{N_t} u_n h(t-\tau_n) \tag{1}$$

where $h(t-\tau_n)$ is the unit pulse linear response function (which is only the function of time t and has no relationship to daily precipitation) for a unit pulse (unit daily precipitation amount) at time t, and $(t-\tau_n)$ is the time lag since the pulse occurs at time τ_n. In particular, Eq. (1) is referred to as a shot noise process with parameters λ, θ, and θ_y, in which λ is the parameter for a Poisson process, θ is the mean for a random variable u_n following an exponential distribution, and θ_y is the parameter for the response function $h(s)$ being equal to $\exp(-\theta_y s)$.

In this paper, the probability distribution of the number of daily rainfall occurrences $\{N_T;\ T \geqq t_0\}$ in a period of one month follows one of binomial, Poisson and negative binomial probability distributions according to the ratio of the mean (M_N) to variance (V_N) of N_T.

If N_T follows binomial distribution, then

$$M_N = kp,\quad V_N = kpq \qquad \therefore\ M_N > V_N \tag{2a}$$

If N_T follows Poisson distribution, then

$$M_N = V_N = \Lambda \qquad \therefore\ M_N = V_N \tag{2b}$$

If N_T follows negative binomial distribution, then

$$M_N = \frac{kp}{q},\quad V_N = \frac{kp}{q^2} \qquad \therefore\ M_N < V_N \tag{2c}$$

where k, p, and q $(=1-p)$ are the parameters of binomial and negative binomial distributions, and Λ

is the occurrence rate of a Poisson distribution.

Characteristic Function

By using a conditional expectation, the characteristic function $\phi_{y_t}(z)=E[\exp(izy_t)]$ for streamflow y_t is defined by

$$\phi_{y_t}(z)=P_r(N_T=0)+\sum_{n=1}^{\infty}P_r(N_T=n)\cdot E\left\{\exp[iz\cdot\sum_{m=1}^{n}u_m h(t-\tau_m)]\mid N_T=n\right\} \quad (3)$$

where the summation in the expectation is unchanged by a random reordering of the occurrence times τ_m (τ_m = 1, 2, . . . , n). With this reordering, the occurrence times τ_m (τ_m = 1, 2, . . . , n) are independent and identically distributed, thus the common probability density function can be given by

$$f(\tau)=\frac{\lambda_\tau}{\int_{t_0}^{T}\lambda_\sigma d\sigma}\equiv\frac{\lambda_\tau}{E(N_T)} \quad (4)$$

where λ_τ is the daily rainfall occurrence rate at time τ. Since the mark variables u_m(m = 1, 2, . . . , n) are also independent and identically distributed, we obtain

$$E\left\{\exp[iz\cdot\sum_{m=1}^{n}u_m h(t-\tau_m)]\mid N_T=n\right\}=$$

$$\left\{E(N_T)^{-1}\cdot\int_{t_0}^{T}\lambda_\tau E\{\exp[iz\cdot u\cdot h(t-\tau)]\}d\tau\right\}^n \quad (5)$$

Substituting Eq. (5) into Eq. (3), we can obtain the characteristic function for a filtered point process y_t

$$\phi_{y_t}(z)=\sum_{n=0}^{\infty}P_r(N_T=n)\left\{E(N_T)^{-1}\cdot\int_{t_0}^{T}\lambda_\tau E\{\exp[iz\cdot u\cdot h(t-\tau)]\}d\tau\right\}^n \quad (6)$$

Let $h(t-\tau)$ be equal to zero when t is smaller than τ. Substituting Eqs. (2a), (2b), and (2c) into Eq. (6), we obtain the characteristic function for y_t based on the probability distribution of N_T as follows.

If N_T follows binomial distribution, then

$$\phi_{y_t}(z)=\left\{1+k^{-1}\int_{t_0}^{t}\lambda_\tau E\{\exp[iz\cdot u\cdot h(t-\tau)]-1\}d\tau\right\}^k \quad (7)$$

$$\left(k=\frac{\int_{t_0}^{T}\lambda_\tau d\tau}{p}\equiv\frac{E(N_T)}{p},\quad p=\frac{V(N_T)}{E(N_T)}\right)$$

If N_T follows Poisson distribution, then

$$\phi_{y_t}(z)=\exp\left\{\int_{t_0}^{t}\lambda_\tau E\{\exp[iz\cdot u\cdot h(t-\tau)]-1\}d\tau\right\} \quad (8)$$

$$(E(N_T) \equiv \int_{t_0}^{T} \lambda_\tau d\tau)$$

If N_T follows negative binomial distribution, then

$$\phi_{y_t}(z)=\left\{1-k^{-1}\int_{t_0}^{t} \lambda_\tau E\{\exp[iz \cdot u \cdot h(t-\tau)]-1\} d\tau\right\}^{-k} \tag{9}$$

$$(k=\frac{p \cdot \int_{t_0}^{T} \lambda_\tau d\tau}{q} \equiv \frac{p \cdot E(N_T)}{q}, \quad p=\frac{E(N_T)}{V(N_T)})$$

Cumulants of Streamflow

Let γ_n be the n-th cumulant for y_t at time t ($t_0 \leqq t \leqq T$). By using $i^n \gamma_n=(d^n \ln \phi_{y_t}(z)/dz^n)$, ($z=0$), the cumulants of streamflow y_t are derived from Eqs. (7), (8), and (9) as

$$\gamma_1(y_t)=E(u) \cdot \int_{t_0}^{t} \lambda_\tau h(t-\tau) d\tau \tag{10}$$

$$\gamma_2(y_t)=E(u^2) \cdot \int_{t_0}^{t} \lambda_\tau h(t-\tau)^2 d\tau \pm \frac{\gamma_1(y_t)^2}{k} \tag{11}$$

$$\gamma_3(y_t)=E(u^3) \cdot \int_{t_0}^{t} \lambda_\tau h(t-\tau)^3 d\tau \pm \frac{3\gamma_2(y_t)\gamma_1(y_t)}{k} - \frac{\gamma_1(y_t)^3}{k^2} \tag{12}$$

and the covariance function for y_t is

$$Cov(y_t, y_{t+J})=E(u^2) \cdot \int_{t_0}^{t} \lambda_\tau h(t-\tau) \cdot h(t+J-\tau) d\tau \pm \frac{\gamma_1(y_t)^2}{k} \tag{13}$$

where a plus sign (+) is adopted if N_T follows negative binomial distribution, a minus sign (-) is adopted if N_T follows binomial distribution, a term including k is absent if N_T follows a Poisson distribution, and $E(u^i)$ is the i-th moment about the origin of daily precipitation amount u.

THREE-TANK MODEL

Pulse Response Function for a Filtered Point Process

Since a streamflow consists of surface, rapid and delayed subsurface, and groundwater runoffs, a tank model consisting of three tanks, called the three-tank model (as shown in Fig. 1), is developed. Based on the linear system of a filtered point process (see Eq. (1)), only one horizontal hole from which runoff occurs is set up on the right side at the bottom of each tank. The sizes of the holes are denoted by a_1, a_2, and a_3. In order to indicate the infiltrations from Tank1 to Tank2 and from Tank2 to Tank3, the vertical holes on the bottoms of Tank1 and Tank2 are opened and their sizes are denoted by b_1 and b_2. In Fig. 1, q_1, q_2 and q_3 are interpreted as surface, subsurface, and groundwater runoffs occurring from Tank1, Tank2, and Tank3, respectively; and f_1 and f_2 are referred to as the infiltrations from Tank1 to Tank2 and from Tank2 to Tank3, respectively.

Assuming that $q_i = a_i s_i$ ($i=1, 2, 3$) and $f_i = b_i s_i$ ($i=1, 2$) are the function of storage height s_i

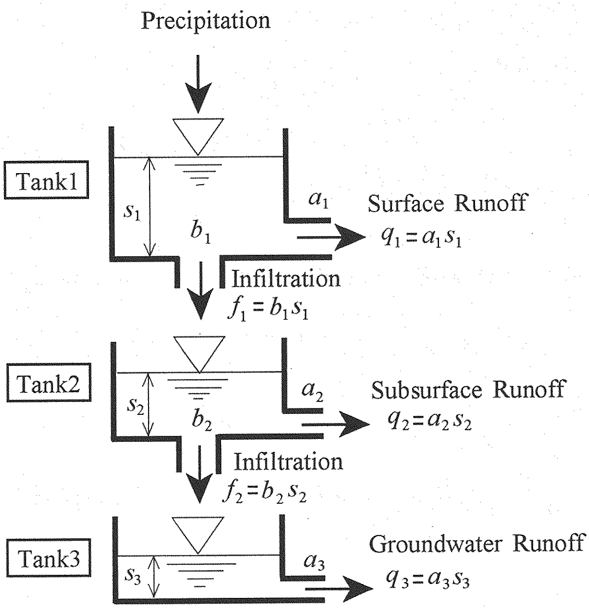

Fig. 1 Schematic illustration of the three-tank model

in tank i, we have the linear response function $h(s)$

$$h(s) = 1 + D_{11}\exp(-C_1 \cdot s) + D_{21}\exp(-C_2 \cdot s) + D_{31}\exp(-a_3 \cdot s) \quad (0 \leqq s < 1) \tag{14a}$$

$$h(s) = D_{12}\exp(-C_1 \cdot s) + D_{22}\exp(-C_2 \cdot s) + D_{32}\exp(-a_3 \cdot s) \quad (1 \leqq s) \tag{14b}$$

where

$$C_1 = a_1 + b_1 \ ; \ C_2 = a_2 + b_2 \ ; \ D_{11} = \left\{ \frac{a_1}{b_1} - \frac{C_1 \cdot a_2 - C_2 \cdot a_3}{(C_1 - C_2)(C_1 - a_3)} \right\} \cdot \frac{b_1}{C_1}$$

$$D_{21} = \frac{b_1(a_2 - a_3)}{(C_1 - C_2)(C_2 - a_3)} \ ; \ D_{31} = \frac{b_1 b_2}{(C_1 - a_3)(C_2 - a_3)} \ ; \ D_{12} = D_{11}[\exp(C_1) - 1]$$

$$D_{22} = D_{21}[\exp(C_2) - 1] \ ; \ \text{and } D_{32} = D_{31}[\exp(a_3) - 1]$$

From Eqs. (14a) and (14b), it can be seen clearly that $h(s)$ is given by the summation of some exponential functions with different decreasing constants.

Furthermore, from the point of view of application to irrigation planning and so forth, a streamflow time series $\{ y_t \}$ is usually averaged over a given period of time (J), for example, five days in Japan. Thus, the averaged $\{ Y_t \}$ is defined as

$$Y_t = J^{-1} \int_{t-J}^{t} y_s \, ds \tag{15}$$

Substituting Eq. (1) into Eq. (15) gives

$$Y_t = \sum_{n=1}^{N_t} u_n \cdot h_J(t-\tau_n) \tag{16}$$

where $h_J(s)$ is expressed as follows:

1) For $0 < s < 1$

$$h_J(s) \equiv h_{J1}(s) = J^{-1}\int_0^s h_1(\sigma)d\sigma$$

$$= E_{001} + E_{01}\cdot s + E_{11}\exp(-C_1\cdot s) + E_{21}\exp(-C_2\cdot s) + E_{31}\exp(-a_3\cdot s) \tag{17a}$$

2) For $1 \leqq s < J$

$$h_J(s) \equiv h_{J2}(s) = J^{-1}\left[\int_0^1 h_1(\sigma)d\sigma + \int_1^s h_2(\sigma)d\sigma\right]$$

$$= E_{002} + E_{02}\cdot s + E_{12}\exp(-C_1\cdot s) + E_{22}\exp(-C_2\cdot s) + E_{32}\exp(-a_3\cdot s) \tag{17b}$$

3) For $J \leqq s < (J+1)$

$$h_J(s) \equiv h_{J3}(s) = J^{-1}\left[\int_{s-J}^1 h_1(\sigma)d\sigma + \int_1^s h_2(\sigma)d\sigma\right]$$

$$= E_{003} + E_{03}\cdot s + E_{13}\exp(-C_1\cdot s) + E_{23}\exp(-C_2\cdot s) + E_{33}\exp(-a_3\cdot s) \tag{17c}$$

4) For $(J+1) \leqq s$

$$h_J(s) \equiv h_{J4}(s) = J^{-1}\int_{s-J}^s h_2(\sigma)d\sigma$$

$$= E_{004} + E_{04}\cdot s + E_{14}\exp(-C_1\cdot s) + E_{24}\exp(-C_2\cdot s) + E_{34}\exp(-a_3\cdot s) \tag{17d}$$

where E_{00i}, E_{0i}, E_{1i}, E_{2i} and E_{3i}, ($i = 1, 2, 3, 4$) are the functions of D_{0j}, D_{1j}, D_{2j}, and D_{3j}, ($j = 1, 2$), and are also the functions of a_i and b_i.

Therefore, the cumulants and covariance of Y_t can be obtained by substituting Y_t for y_t and $h_J(s)$ for $h(s)$ in Eqs. (10), (11), (12), and (13).

In order to obtain the cumulants and covariance of Y_t, the definite integral of $\int h_J^n(s)$ has to be calculated. Because this definite integral becomes too complicated to be analytically solved without help of a computer as the order n and the number of terms increase, the system REDUCE for computer algebra is used to solve this problem. The covariance needs 150 lines and the 3-rd order cumulant needs 260 lines in the form of FORTRAN output. Due to space limitations, the analysis results are omitted from this paper.

Identification of Parameters for the Three-tank Model

In order to verify the applicability and validity of the proposed method, we make use of the daily rainfall-streamflow data observed at an actual reservoir, Sameura Dam, which has a catchment area of 472 km^2, and is located in the upper reaches of the Yoshino River Basin in Shikoku, Japan.

In identification of the parameters a_i and b_i of the three-tank model, two other correction parameters also need to be identified. One is the parameter f_E for modifying the evapotranspiration

Table 1 Identified parameters for the three tank model

a_1(1/day)	b_1(1/day)	a_2(1/day)	b_2(1/day)	a_3(1/day)
0.421	1.305	0.140	0.192	0.049

Table 2 Identified values of f_E and f_P

Season	May - June	July - September	October - November	December - April
f_E	0.37	0.91	0.52	0.54
f_P	0.73	0.89	0.76	0.76

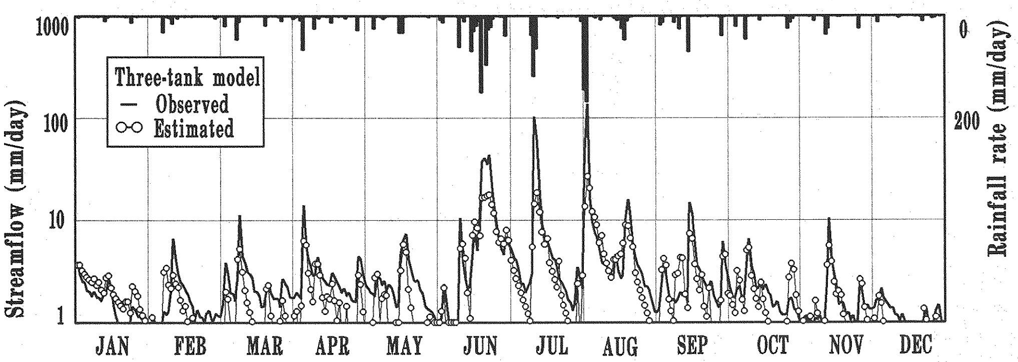

Fig. 2 Daily streamflow hydrograph by the three-tank model

calculated by the Hamon method based on average monthly temperature, and the other is the parameter f_P for modifying the daily precipitation error caused by substituting the point precipitation at Motoyama located down stream from Sameura Dam for the areal precipitation in the Sameura Dam basin. Because there is little snow-fall in this basin, the structure of runoff process is assumed to be invariant throughout the year, so that the model's parameters are constant over a year. But the parameters f_E and f_P are assumed to be variant in different seasons (May—June, July—September, October—November, and December—April).

The Simplex method is used to search the optimal values of the parameters that give minimum values of the objective function F expressed by

$$F(f_E, f_P;\ a_1, a_2, a_3, b_1, ,b_2)=\frac{1}{n}\sum_{t=1}^{n}\left\{\frac{[Q(t)-y(t)]^2}{Q(t)}\right\} \tag{18}$$

where $y(t)$ and $Q(t)$ are the calculated and observed streamflow on day t, respectively; and n is the number of days in a calculation period.

We calculate $y(t)$ by the method of finite differences and let the time interval Δt be equal to 1/10 day because the parameters of the model may be larger than 1 in day. Based on daily rainfall-streamflow data from 1977 to 1989, the parameters are identified and the averaged values of the parameters for the three-tank model are shown in Table 1, and the averaged values of f_E and f_P are shown in Table 2. The calculated daily streamflow time series is shown in Fig. 2.

By comparing the computed daily streamflows with the observed ones, it is found that the com-

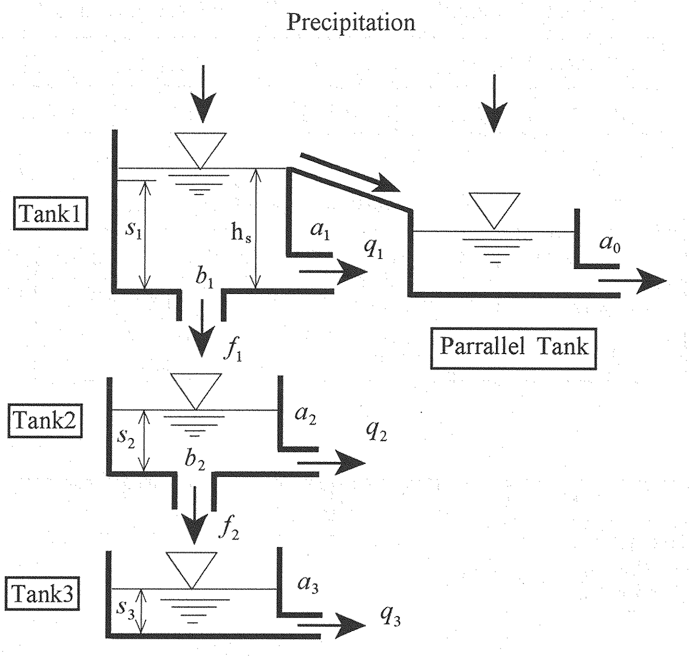

Fig. 3 Model of 3-tanks with a parallel tank

puted values fit the observed data well for low flow periods (May—June, October—November, and December—April), while in the flood period (July—September) the computed peak flows are much smaller than the observed peaks. The reason is likely to be that Tank1 in the three-tank model shown in Fig. 1 can not represent surface runoff adequately.

MODEL OF 3-TANKS WITH A PARALLEL TANK

In order to express surface runoff accurately, a tank is added parallel to the three-tank model to receive the surface runoff overflowing from Tank1 where a storage height s_1 is higher than h_s, as shown in Fig. 3. We call this model as the model of 3-tanks with a parallel tank.

The response function of this model is given by the summation of two kinds of linear response functions. One is the response function for the three-tank model, and the other is the response function for the parallel tank like that of Tank3 in the three-tank model.

In addition, generally, in order to express surface runoff well, another horizontal hole is usually set up on the upper tank (Tank1) and good results can be also obtained. But this tank model does not belong to a linear system because the response function for the model is the function of not only time t, but also daily precipitation amount.

Cumulants for the Model

Adding the correction parameters for precipitation and evaporation, similar to that of the three-tank model, we have the cumulants for *J*-day averaged streamflow as

$$\gamma_1(Y_t)=f_P\left[E(u)\,\lambda\int_0^t h_J(s)\,ds+P_{rs}\cdot E(u_s)\int_0^t h_{Js}(s)\,ds\right]-\frac{f_E\,E_P(D_t-M_N)}{D_t} \qquad (19)$$

$$\gamma_2(Y_t)=f_P^2\left[E(u^2)\lambda\int_0^t h_J(s)^2ds+P_{rs}\cdot E(u_s^2)\int_0^t h_{Js}(s)^2ds\right]\pm\frac{\gamma_1(Y_{1t})}{k^2} \tag{20}$$

$$\gamma_3(Y_t)=f_P^3\left[E(u^3)\lambda\int_0^t h_J(s)^3ds+P_r\cdot E(u_s^3)\int_0^t h_{Js}(s)^3ds\right]$$

$$\pm\frac{3\gamma_2(Y_{1t})\gamma_1(Y_{1t})}{k}-\frac{\gamma_1(Y_{1t})^3}{k^2} \tag{21}$$

$$Cov(Y_t,\ Y_{t+J})=f_P^2\left[E(u)^2\lambda\int_0^t h_J(s)\cdot h_J(s+J)ds+P_{rs}\cdot E(u_s)^2\int_0^t h_{Js}(s)\cdot h_{Js}(s+J)ds\right] \tag{22}$$

where P_{rs} is the occurrence probability of surface runoff from the parallel tank; $\gamma_1(Y_{1t})$ and $\gamma_2(Y_{1t})$ are the mean and variance of J-day averaged streamflow from the three-tank model for $P_{rs}=0$; D_t is the number of days in an object month; M_N is the mean of the number of rainy days in an object month; $h_J(s)$ is the response function for the three-tank model; $h_{Js}(s)$ is the response function for the parallel tank; $E(u^i)$ $(i = 1,\ 2,\ 3)$ is the i-th order moment of daily precipitation amount on rainy days when surface runoff does not take place; and $E(u_s^i)$ $(i = 1,\ 2,\ 3)$ is the i-th order moment of daily precipitation amount on rainy days when surface runoff takes place ($y_t \geqq (a_1+b_1)\ h_s$ = 40.3mm/day) from the parallel tank.

Identification of Parameters for the Model

The identification method of the parameters is the same as that mentioned in the three-tank model. The parameters identified are shown in Table 3. The values of f_E and f_P are the same as those in the three-tank model. The calculated daily streamflows are shown in Fig. 4. From this, it is clear that the calculated flows fit the observed ones over a year.

Table 3 Identified parameters for the model of 3-tanks with a parallel tank

a_0(1/day)	h_s(mm)	a_1(1/day)	b_1(1/day)	a_2(1/day)	b_2(1/day)	a_3(1/day)
4.213	22.22	0.269	1.545	0.094	0.038	0.022

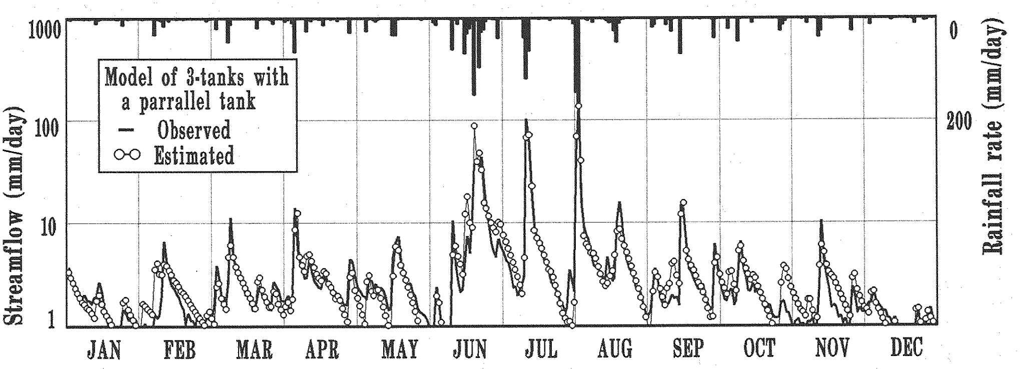

Fig. 4 Daily streamflow hydrograph by the model of 3-tanks with a parallel tank

Estimation of Cumulants for 5-day Averaged Streamflow

In Japan, 5-day averaged streamflow data is usually used for water resources planning. Based on Eqs. (19), (20), (21), and (22), the mean, variance, coefficient of skewness, and auto-correlation for 5-day averaged streamflow are able to be estimated. The calculation procedures and results are summarized as follows:

1. Monthly characteristics of a daily rainfall sequence

(a) Daily rainfall occurrence number. The monthly mean and variance of the number of daily rainfall occurrences are estimated from the observed data. Then, the distribution and its parameters (p, q (=1-p), k, and Λ) are estimated by Eqs. (2a), (2b), and (2c), as shown in Table 4.

(b) Daily rainfall amount. According to the condition of surface runoff occurrence (surface runoff occurs when daily streamflow y_t exceeds 40.3mm/day), the observed daily precipitation data is divided into two groups—one without surface runoff occurrence and the other with surface runoff occurrence. Their monthly mean and variance are estimated, respectively, as shown in Table 4.

2. The mean, variance, and skew coefficient of daily streamflow for each month are calculated from the observed data. Daily streamflow probability distributions are approximated by gamma distributions, and the surface runoff occurrence probabilities P_{rs} ($y_t \geqq 40.3$mm/day) are computed using these distributions.

3. The correction parameter f_P is calculated from Eq. (19).

4. Substituting P_{rs}, f_P, and values for a daily precipitation sequence into Eqs. (20), (21) and (22), the variance, coefficients of skewness, and auto-correlation of 5-day averaged streamflow for each month are calculated as shown in Fig. 5.

From Fig. 5, it can be seen that the computed variances are almost the same as the observed ones during the low flow periods, while during the flood period, the computed values are smaller than observed ones, especially in August. The main reason is likely to be that a daily rainfall process is assumed as a random process. It is thought that if the characteristics of a daily rainfall sequence is modeled in a daily unit, even though heavy rain continues to fall for 2 or 3 days, the storm is separated into independent daily rainfalls in detached days. Thus, the theoretical variances are smaller than the observed ones.

In order to solve this problem, the authors (5) formulated the statistics of streamflow based on a daily rainfall sequence modeled as a storm unit, where the cluster nature of continuous rainfall over

Table 4 Characteristics of a daily rainfall sequence

Month	Daily rainfall occurrence number					Daily precipitation			
						Model of 3-tanks		Parallel tank	
	M_N	V_N	Distribution	k	p	$E(u)$	$V(u)^{1/2}$	$E(u_s)$	$V(u_s)^{1/2}$
1	8.3	15.5	negative binomial	9.57	0.536	8.9	12.2	0	0
2	9.3	7.4	binomial	45.52	0.204	11.0	15.3	0	0
3	12.0	20.7	negative binomial	16.55	0.580	15.4	19.3	0	0
4	12.5	12.5	Poisson	Λ= 12.5		19.1	21.9	0	0
5	12.2	22.3	negative binomial	14.74	0.547	19.6	25.1	125.0	95.6
6	15.3	18.1	negative binomial	83.60	0.845	22.7	31.1	89.1	93.4
7	12.9	28.5	negative binomial	10.67	0.453	19.3	30.7	84.3	90.7
8	12.3	25.3	negative binomial	11.64	0.486	21.0	35.7	104.8	106.0
9	10.9	12.5	negative binomial	74.26	0.872	20.4	37.8	102.9	98.1
10	8.7	12.7	negative binomial	18.92	0.685	15.4	23.1	63.3	54.4
11	7.3	9.4	negative binomial	25.38	0.777	13.7	25.4	0	0
12	6.3	6.5	negative binomial	198.25	0.969	8.3	11.0	0	0

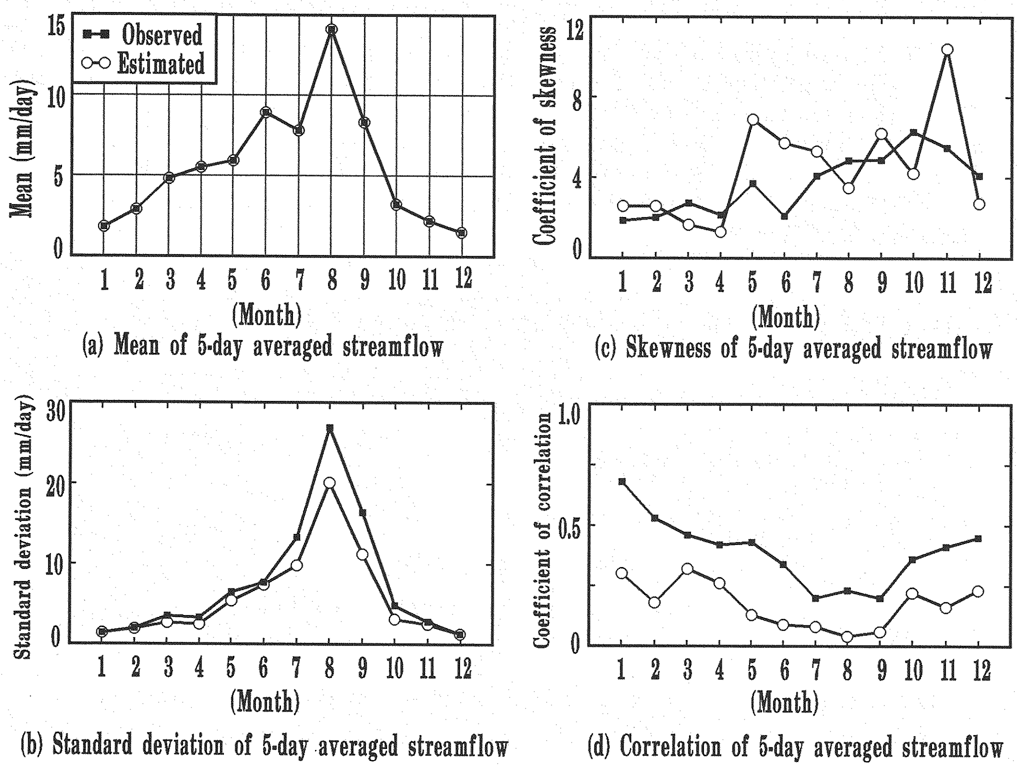

Fig. 5 Estimated cumulants based on the model of 3-tanks with a parallel tank

a few days and auto-correlation of daily rainfall of the storm had been taken into account, and obtained good fitness between estimated and observed values (mean, variance, skewness, and auto-correlation).

CONCLUSION

For the purpose of water resource planning and so forth, the basic statistics of daily streamflow (mean, variance, skewness, and auto-correlation) are formulated based on the characteristic function and the suitable response function of a filtered point process. The proposed method was applied to an actual data and we reached the following conclusions:

1. As a linear response function for a filtered point process, the three-tank model and the model of 3-tanks with a parallel tank, are developed and compared. It is found that the model of 3-tanks with a parallel tank is a suitable model for a basin where surface runoffs occur frequently.
2. If a daily rainfall sequence is modeled as a daily unit, then the cluster nature of heavy rain continuing over a few days and the auto-correlation of daily rainfall in a storm are ignored. Consequently, the theoretical cumulants are much smaller than the observed ones in months when heavy rain occurs frequently.

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APPENDIX — NOTATION

The following symbols are used in this paper:

a_1, a_2, a_3	= parameters of tank models;
b_1, b_2	= parameters of tank models;
C_1, C_2	= parameters for the linear response functions;
$Cov(y_t, y_{t+J})$	= covariance function for daily streamflow y_t;
$Cov(Y_t, Y_{t+J})$	= covariance function for J-day averaged streamflow Y_t;
D_{ij} ($i=1, 2, 3$; $j=1, 2$)	= parameters for the linear response functions;
D_t	= the number of days in an object month;
E_{00i}, E_{0i}, E_{1i}, E_{2i}, E_{3i}, ($i=1, 2, 3, 4$)	= functions of D_{ij} ($i=1, 2, 3$; $j=1, 2$)
$E(N_T)$	= mean of the number of daily rainfall occurrences;
$E(u^i)$ ($i=1, 2, 3$)	= the i-th order moment of daily precipitation amount on the rainy days when surface runoff does not occur;
$E(u_s^i)$ ($i=1, 2, 3$)	= the i-th order moment of daily precipitation amount on the rainy days when surface runoff occurs from the parallel tank;
f_E, f_P	= correction parameters for evaporation and precipitation respectively;
$f(\tau)$	= common probability density function defined by Eq. (4);
$h(s)$	= unit pulse response function for the unit daily precipitation;
$h_J(s)$	= J-day averaged response function for the three-tank model;
$h_{Js}(s)$	= J-day averaged response function for the parallel tank;
k	= parameters of binomial and negative binomial distributions, defined by Eq. (7) and Eq. (9);
M_N	= mean of the number of daily rainfall occurrences in a month;
N_T	= the number of daily rainfall occurrences in a month;

p	= parameter of binomial, negative binomial distribution;
P_r	= occurrence probability of daily rainfall;
P_{rs}	= occurrence probability of surface runoff from the parallel tank;
q	= parameter of binomial, negative binomial distribution ($=1-p$);
s_i	= storage height of tank i ;
u_n	= the n-th daily precipitation amount;
V_N	= variance of the number of daily rainfall occurrences in a month;
y_t	= daily streamflow as a filtered point process;
Y_t	= J-day averaged streamflow as a filtered point process;
$\gamma_i(y_t)$ ($i=1, 2, 3$)	= the i-th cumulant of y_t at time t ;
$\gamma_i(Y_t)$ ($i=1, 2, 3$)	= the i-th cumulant of Y_t at time t ;
$\gamma_i(Y_{1t})$ ($i=1, 2$)	= mean and variance for the three-tank model when $P_{rs}=0$;
θ, θ_y	= means of exponential distributions;
λ	= occurrence rate of N_T for a Poisson process;
λ_τ	= occurrence rate;
τ_n	= the daily rainfall occurrence day; and
Λ	= parameter for a Poisson distribution.

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