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HYDRAULIC RESISTANCE OF FLOW WITH FLEXIBLE VEGETATION IN OPEN CHANNEL

By

Tetsuro TSUJIMOTO
Associate Professor, Department of Civil Engineering, Kanazawa University
2-40-20, Kodatsuno, Kanazawa, 920, Japan

Tadanori KITAMURA
Research Assistant, Division of Global Environmental Engineering, Kyoto University
Yoshida Honmachi, Sakyo-ku, Kyoto, 606, Japan

Yasutsugu FUJII
Osaka Prefectural Government, Osaka, Japan

and

Hiroji NAKAGAWA
Professor, Department of Environmental System, Ritsumeikan University
1916, Noji, Kusatsu, Shiga, 525, Japan

ABSTRACT

Velocity distribution of flow with deformed vegetation and deformation of plants due to shear flow are analyzed, and the hydraulic resistance of open-channel flow on a flexible vegetation-covered bed is discussed. The vegetation layer constituted by deformed plants is characterized as a layer with spatially heterogeneous permeability. On the other hand, the deformation of a plant is analyzed by assuming a constant flexural rigidity. Furthermore, the insitu values of flexural rigidity of real plants in rivers are measured to evaluate hydraulic resistance of flow there.

INTRODUCTION

Most of studies on flow with vegetation treated rigid vegetation because of simplicity, but in reality many of plants deform by flowing water, and such deformation changes the flow from that predicted under assumption of non-deformable plants. In this study, flexible plants are treated, where only the deformation of plants is taken into account. The effect of organized behavior of group of plants such as "honami" is here neglected, though the organized behavior of vegetation layer might change the turbulent structure appreciably (Tsujimoto *et al.* (1), Tsujimoto & Nagasaki (2)).

The authors proposed a method to calculate the flow with vegetation by using a k-ε turbulence model, where geometry of individual plants as boundary of the flow was neglected but it was taken into account by adding the locally averaged drag force and turbulent energy production corresponding to its work done (Shimizu & Tsujimoto (3), Tsujimoto *et al.* (4)). The effect of inclination of plants was investigated to modify the drag term (Tsujimoto *et al.* (5)). However, the deformation of plant due to the hydrodynamic force influenced by the deformation of plant is not considered, though it forms a feed-back system.

Kouwen *et al.* (6) and Kouwen & Unny (7) studied the relation between the plant deformation and hydraulic parameters by flume experiments with many model plants with different rigidity and density, and they proposed an empirical relation to predict the deformation of plants. Because of their dimensional expression, it is not reasonable from the view point of similarity. Murota & Fukuhara (8) used a cantilever model with finite deformation in calculation and tried to couple the analyses of velocity distribution and deformation of plant, but the effect of plant deformation on turbulent structure was not taken into account completely.

In this study, the plant deformation by shear flow and the velocity distribution of flow with deformed vegetation are calculated simultaneously, and the several aspects of calculation process are extracted to be verified by flume experiments. Furthermore, the calculated results are simplified to deduce a resistance law of flow with flexible vegetation layer. The representative parameter to the plant deformation is the flexural rigidity, and thus, the method to measure its insitu value is tested in a river.

MODEL FOR FLOW

In calculation of flow with vegetation, the individual plants are regarded as negative momentum sources and additional turbulent energy sources. Then, the vertical two-dimensional steady uniform flow is described by the following equations:

$$\frac{\partial}{\partial y^*}\left[\left(\nu_T^* + \frac{1}{R_e}\right)\frac{\partial U^*}{\partial y^*}\right] - \frac{\partial P^*}{\partial x^*} - F_x^* = 0 \quad (1)$$

$$\frac{\partial}{\partial y^*}\left(\frac{\nu_T^*}{\sigma_k}\frac{\partial k^*}{\partial y^*}\right) + P_k^* + C_{fk}F_x^*U^* - \varepsilon^* = 0 \quad (2)$$

$$\frac{\partial}{\partial y^*}\left(\frac{\nu_T^*}{\sigma_\varepsilon}\frac{\partial \varepsilon^*}{\partial y^*}\right) + \frac{\varepsilon^*}{k^*}\left[C_1\left(P_k^* + C_{f\varepsilon}F_x^*U^*\right) - C_2\varepsilon^*\right] = 0 \quad (3)$$

The equations are made dimensionless by using the flow depth h as the length scale and the depth-averaged velocity U_m as the velocity scale, in which ρ=mass density of water; x and y=longitudinal and vertical directions; U=flow velocity; $R_e \equiv U_m h/\nu$; U_m=depth-averaged velocity; h=depth; ν=kinematic viscosity; P=pressure; F_x=longitudinal component of locally averaged drag force acting on plants per unit mass of water; ν_T=kinematic eddy viscosity; k=turbulence energy; ε=dissipation rate of turbulence energy; P_k=turbulence energy production due to locally averaged velocity gradient; σ_k, σ_ε, C_{fk}, $C_{f\varepsilon}$, C_1, C_2=model parameters; and * indicates the non-dimensional quantities normalized by velocity scale U_m and length scale h.

The turbulent energy production due to the locally averaged velocity gradient is written as,

$$P_k^* \equiv \nu_T^*\left(\frac{\partial U^*}{\partial y^*}\right)^2 \quad (4)$$

The kinematic eddy viscosity is related to k and ε as follows:

$$\nu_T = C_\mu \frac{k^2}{\varepsilon} \quad (5)$$

in which C_μ=a numerical parameter.

The longitudinal component of drag acting on plants per unit mass of water is written as

$$F_x^* = \frac{1}{2}\frac{C^*}{l_0^*}\left(U^*\cos\theta\right)^2 \quad (6)$$

in which l_0=length of plant; $C^* \equiv C_D\lambda l_0$; C_D=drag coefficient of a plant; λ=projected area of plants to the vertical plane perpendicular to the x axis per unit mass of water; and θ=inclination angle of plant (θ=0 means the vertical). The standard values for the numerical constants are employed such that, C_μ=0.09; C_1=1.44; C_2=1.92; σ_k=1.0; and σ_ε=1.3. The parameters C_{fk} and $C_{f\varepsilon}$ are determined such that the numerical calculation agrees with the experiments for flow with rigid vertical vegetation (Tsujimoto *et al.* (4)), as follows: C_{fk}=1.0; and $C_{f\varepsilon}$=1.3.

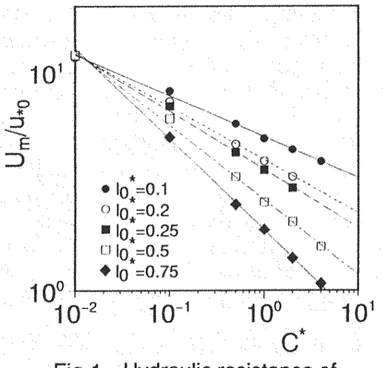

Fig.1 Hydraulic resistance of vertical rigid vegetation

The calculated results for flow with vertical rigid vegetation (θ=0) are depicred in Fig.1 to demonstrate the relation between U_m/u_{*0} and C^* with l_0^* as a parameter, in which $u_{*0} \equiv \sqrt{ghI_e}$; g=gravitatinal accelaration; and I_e=energy gradient. The solid lines to connect the calculated results are approximated by the following equations, and they represent the hydraulic resistance of flow with rigid vertical vegetation.

$$\frac{U_{\mathrm{m}}}{u_{*0}} = \alpha C^{*\mathrm{m}} \qquad (0.01 \le C^* < 4.0) \qquad (7)$$

$$\alpha = 1.46\text{-}1.55\ln l_0^* \; ; \quad m = -0.17\text{-}0.34 l_0^* \qquad (0.1 \le l_0^* \le 0.75) \qquad (8)$$

MODEL FOR PLANT DEFORMATION

Each plant might be regarded as a cantilever with finite deformation. The following assumptions are employed to calculate the plant deformation: (i) the cross-section of plant is perpendicular to the neutral axis even after deformation; and (ii) the only hydrodynamic force causes the deformation of the plant.

The longitudinal and the vertical components of hydrodynamic force acting on a plant per unit length in the vertical direction are written as,

$$D_x^* = \frac{1}{2} C_D d^* (U^* \cos\theta)^2 = C^* \frac{l_0^*}{\gamma} (U^* \cos\theta)^2 \qquad (9)$$

$$D_y^* = -\frac{1}{2} C_D d^* (U^* \cos\theta)^2 \tan\theta = -C^* \frac{l_0^*}{\gamma} (U^* \cos\theta)^2 \tan\theta \qquad (10)$$

in which $\gamma \equiv l_0^2 N_0$; N_0=number of plants per unit area; d=diameter of plant; and these forces are made dimensionless by U_{m}, h and ρ. The moment to bend M is given by

$$M^*(y^*) = \int_{y^*}^{l^*} \left\{ (\eta^* - y^*) D_x^*(\eta^*) + [\delta^*(\eta^*) - \delta^*(y^*)] D_y^*(\eta^*) \right\} d\eta^* \qquad (11)$$

in which l=height of inclined plant; δ=longitudinal displacement of the plant (see Fig.2); and δ is written as follows:

$$\delta^*(y^*) \equiv \int_0^{\eta^*} \left(\frac{d\delta^*}{dy^*}\right) dy^* \qquad (12)$$

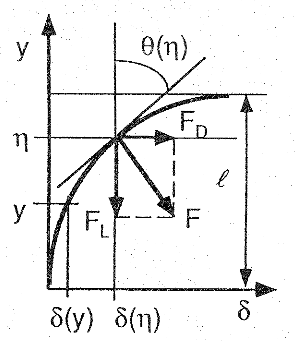

Fig.2 Definition sketch

The following relation exists between M and flexural rigidity EI:

$$\frac{M^*}{(EI)^*} = \frac{\dfrac{d^2\delta^*}{dy^{*2}}}{\left[1 + \left(\dfrac{d\delta^*}{dy^*}\right)^2\right]^{3/2}} \qquad (13)$$

in which $(EI)^* \equiv EI/(\rho U_{\mathrm{m}}^2 h^4)$. The following geometrical relation should hold:

$$l_0^* = \int_0^{l^*} \sqrt{1 + \left(\frac{d\delta^*}{dy^*}\right)^2} \, dy^* \qquad (14)$$

The boundary condition is as follows:

$$\frac{d\delta^*}{dy^*} = \delta^* = 0 \quad \text{at} \quad y^* = 0 \qquad (15)$$

When the velocity distribution is given, the deformation of the plant is calculated by solving Eqs.9~13 numerically with Runge-Kutta method. The value of l_0* which satisfies Eq.14 is obtained by trial and error: the procedure is iterated until Eqs.9~15 are satisfied simultaneously.

When only one plant is set into the flow, it hardly disturbs the velocity distribution. If there are no other vegetation in the flow over a smooth bed, the log-law profile of velocity distribution is still valid. Such a situation was tested in a flume, where a filament made by a transparent sheet for an over-head projector (EI=867dyne•cm^2, l_0=4.6cm, d=0.2cm, C_D=2.0) was used as a model plant. Figure 3 shows the experimental results on the velocity profile and the deformed geometry of model plant. They agree with the calculated results reasonably.

Even if a vegetation layer is prepared, the velocity distribution is known to be almost uniform when the flow with vegetation has a water depth smaller than the vegetation height. The deformation of the model plant in such a flow was measured and compared with the calculated results in Fig.4.

These experiments demonstrate the applicability of the method proposed in this paper to calculate the deformation of plant of which flexural rigidity is known in advance.

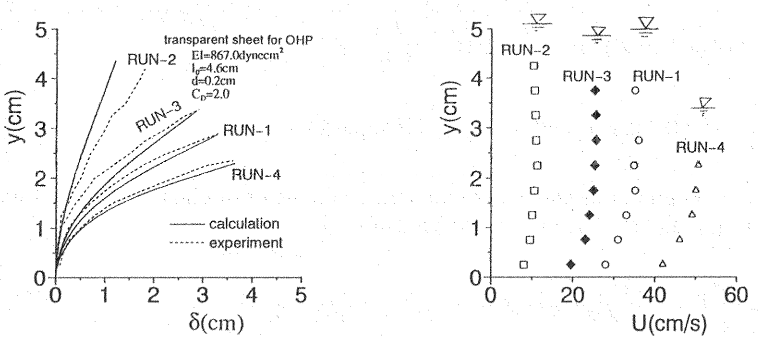

Fig.3 Comparison between the calculated result of defomation of plant and the experimantal result in the flow with log-low profile

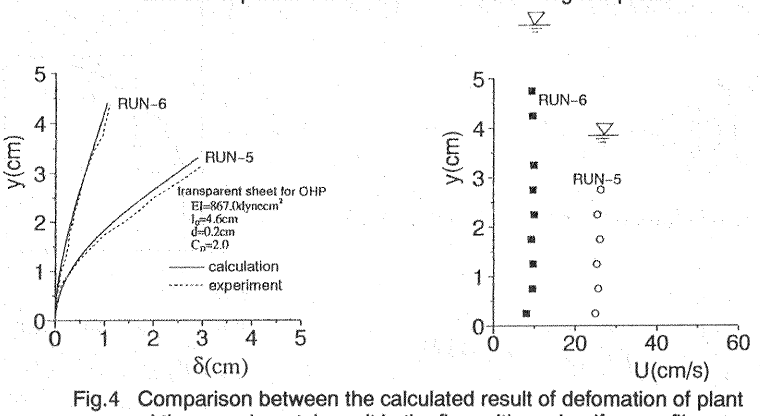

Fig.4 Comparison between the calculated result of defomation of plant and the experimantal result in the flow with qusi-unifom profile

COUPLING OF MODEL FOR FLOW WITH PLANT-DEFORMATION MODEL

The model for flow and the model for plant deformation are coupled with each other to predict the velocity distribution of flow and the deformation of plants simultaneously. The procedure for calculation is as follows: [i] Assume the deformation of vegetation, [ii] solve the flow field, [iii] and calculate the deformation of vegetation under the condition of the solved flow field. [iv] If the new solution is well approximated by the previous assumption (the error of the vegetation height was here employed to check the convergence of the iterate calculation), the calculation is stopped. Otherwise, return to [i].

For calculation, the following 4 parameters should be known in advance: $(EI)^*$; γ; C^*; and l_0^* (combinations of flexural rigidity, diameter, initial height (length), drag coefficient of the plant, and vegetation density).

Figure 5 shows an example of the calculations on the deformed geometry of the plant, the velocity distribution and the Reynolds-stress distribution. Comparing them with the results for vertical rigid vegetation, one can recognize the following features: When the vegetation is so flexible to be inclined by the flow, the inflection of the velocity profile near the vegetation boundary becomes unclear and the peak of the Reynolds stress at the vegetation boundary is no longer sharp.

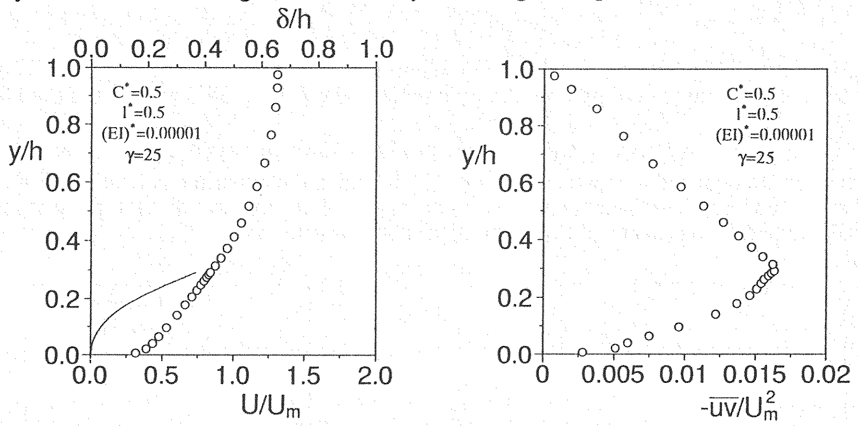

Fig.5 Calculated result of flow with flexible vegetation

The decrease of the vegetation-layer thickness due to inclination of plants was focused, and the relation between l/l_0 and the special parameter $\gamma EI/(\rho u_{*0}^2 l_0^4)$ was investigated. The calculated results in Fig.6 suggest a unique relation between them, and it is consistent to the experimental data (Murota & Fukuhara (8), Tsujimoto *et al.* (5)). The solid line fit to the calculated result is approximated by the folowing equation.

$$\frac{l}{l_0} = 1 - 0.89\left(-4.66\frac{\gamma EI}{\rho u_{*0}^2 l_0^4}\right) \qquad \left(0.03 \leq \frac{\gamma EI}{\rho u_{*0}^2 l_0^4} \leq 2.0\right) \qquad (16)$$

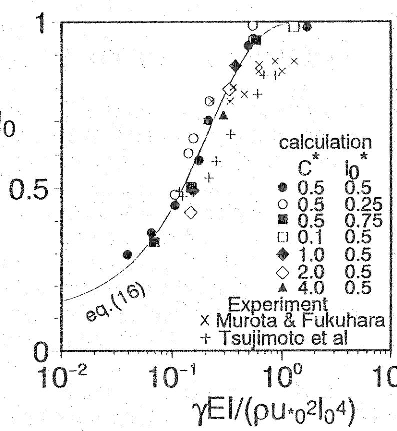

Fig.6 Decrease of the vegetation-layer thickness

The calculation suggests a relation of the hydraulic resistance by deformation of vegetation, because the inclination of plants decreases the vegetation-layer thickness. As for the relation between U_m/u_{*0} and $\gamma EI/(\rho u_{*0}^2 l_0^4)$, the predicted relation where only the decrease of vegetation-layer thickness is taken into account (Eqs.7 and 8 have been applied) is depicted by a dotted line in Fig.7. The calculated results obtained by coupling the flow model with the model for plant deformation are plotted in Fig.7, and one can recognize that the dotted line underestimates U_m/u_{*0}. The reason of such an underestimation implies the fact that the reduction of drag due to plant inclination is more appreciable than the decrease of the vegetation layer-thickness. To take such an effect into account, the following convenient method to predict the hydraulic resistance is proposed: Replace θ and C^* to θ_e and C^*_e respectively in Eqs.7 and 8, which are defined as follows:

$$\theta_e = \cos^{-1}\left(\frac{l}{l_0}\right) \quad ; \quad C^*_e = C^*\cos^2\theta_e = C^*\left(\frac{l}{l_0}\right)^2 \tag{17}$$

The solid curve in Fig.7 to express the above-mentioned approximated method is well consistent to the plots to represent the calculated results by coupling the flow model with the model for plant inclination.

In Fig.8, the relations between U_m/u_{*0} and $\gamma EI/(\rho u_{*0}^2 l_0^4)$ obtained by the above-mentioned convenient method are depicted for several values of l_0^*, in which the experimental data by Tsujimoto *et al.* (5) are also plotted. The agreement between the curves and plots suggests that the present method is useful to predict hydraulic resistance of flow with deformable vegetation.

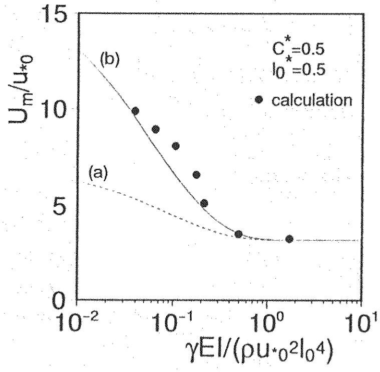

Fig.7 Effect of the deformation of plant to hydraulic resistance

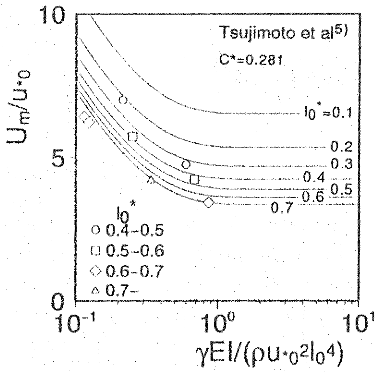

Fig.8 Comparison between the predicted hydraulic resistance of flow with flexible vegetation and the experimental result

INSITU SURVEY OF FLEXURAL RIGIDITY AND DENSITY OF PLANTS AND PREDICTION OF HYDRAULIC RESISTANCE OF VEGETATION IN RIVERS

In order to predict the hydraulic resistance of vegetation in rivers, it is necessary to know the height, diameter of representative stems, flexural rigidity and vegetation density of the real plants. Such a survey was performed in the flood plain of the Kizu River and the Katsura River (Kyoto Prefecture) in August 1994. Yoshi (Reed, *P. communis Trin.*) and Seitaka Awadachiso (Tall goldenrod, *S. altissima L.*) were chosen, because they are representative plants in flood plain of sand-bed rivers and they make obvious colonies. The observations were conducted for the respective colonies in the areas which are submerged about once a year. The vegetation density was measured by counting the stems in the prescribed areas of a few square-meters. The height, the diameter and the flexural rigidity of the stem were obtained as respective means of 10 representative sample plants in the prescribed area. The diameter of the stem was measured at the level 1m high above the ground.

The flexural rigidity was estimated through a bending test of stems, where the horizontal and vertical displacements, δ_B and l, were measured by changing the horizontal tractive force W (see Fig.9 and Photo 1). The horizontal force W was measured with a spring balance. Assuming that the diameter and the rigidity are constant from the foot to the top of the stem, one can use the theory of finite deformation of a cantilever (9): the following relation between l/l_0 and a is expected:

$$\frac{l}{l_0} = \frac{2b}{a}\cos\varphi_0 \tag{18}$$

in which

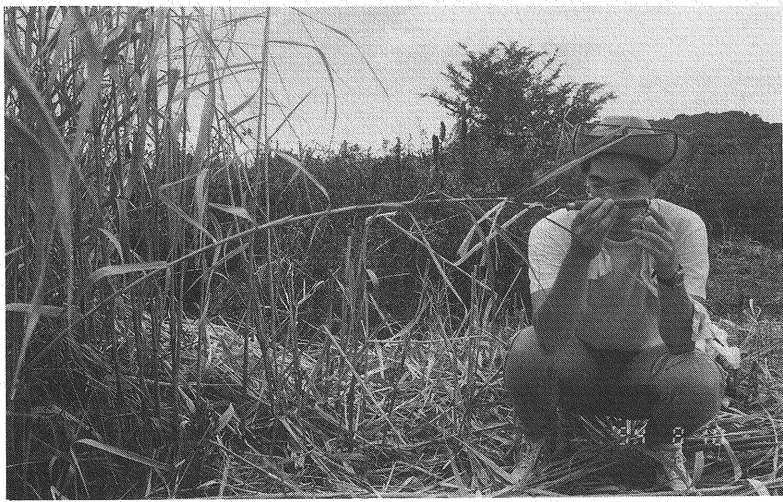

Photo 1 Test of flextual rigidity of plant

$$a \equiv \sqrt{\frac{Wl_0^3}{EI}} = F\left(b, \frac{\pi}{2}\right) - F(b, \varphi_0) \qquad (19)$$

$$b \equiv \sqrt{\frac{1}{2\sin^2\varphi_0}} \qquad (20)$$

$$F(b, \varphi_0) \equiv \int_0^{\varphi_0} \frac{d\varphi}{\sqrt{1-b^2\sin^2\varphi}} \qquad (21)$$

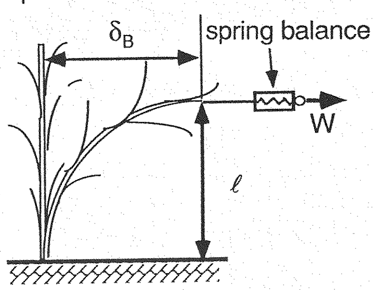

Fig.9 Illustration of the test of flexural rigidity of plant

Figure 10 shows the relation between l/l_0 and a given by Eqs.18~21 with field data. The flexural rigidity EI is estimated by the comparison between the field data and the theoretical curve. The estimated values of EI are summarized in Table 1 along with other data.

Figure 11 shows the relation between the value of EI and the diameter of sample plants. One can expect a unique relation between the rigidity and the diameter of the stem for each species of plants.

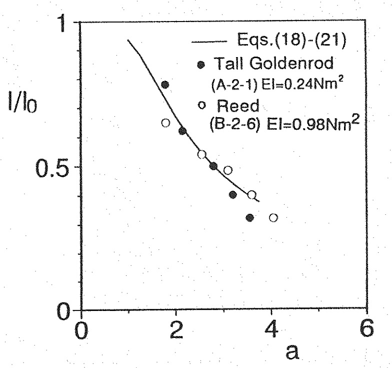

Fig.10 Estimation of flexural rigidity of plant

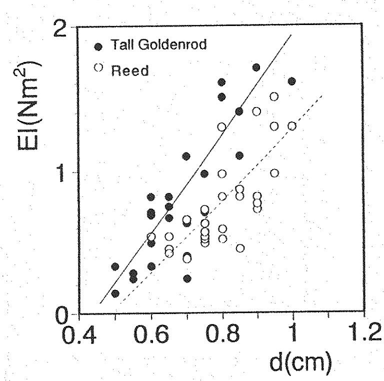

Fig.11 Relation between measured values of EI and diameter of the plant

Table 1 Result of the field test of the physical properties of plant

	N_0 (m^{-2})	l_{0mean} (m)	l_{0rms} (m)	d_{mean} (m)	d_{rms} (m)	EI_{mean} (Nm^2)	EI_{rms} (Nm^2)
A-1	38.3	1.782	0.123	0.0067	0.0009	0.970	0.330
A-2	40.0	1.482	0.114	0.0062	0.0007	0.675	0.290
A-3	27.7	2.060	0.134	0.0085	0.0012	0.720	0.310
B-1	21.3	2.112	0.141	0.0088	0.0009	0.830	0.330
B-2	25.7	1.657	0.116	0.0071	0.0009	0.430	0.250
B-3	22.3	1.700	0.140	0.0081	0.0007	1.280	0.330

1994.8.9-8.11 A : Tall Goldenrod B : Reed

According to the above-mentioned survey, the representative values for respective species of plants have been chosen as shown in Table 2 in order to discuss the hydraulic resistance. The predicted relation between U_m/u_{*0} and h/l_0 (initial relative depth) is shown in Fig.12. Furthermore, it is converted to the relation between Manning coefficient n and h/l_0 as shown in Fig.13. Both of them change with the energy gradient I_e. One should be aware of the fact that the Manning coefficient would change appreciably with the energy gradient.

Table 2 Representative values by the field survey

	N_0 (m^{-2})	l_0 (m)	d (m)	EI (Nm^2)	C*	γ
Tall Goldenrod	35.3	1.775	0.00713	0.788	0.447	111.2
Reed	23.1	1.823	0.00800	0.847	0.337	76.8

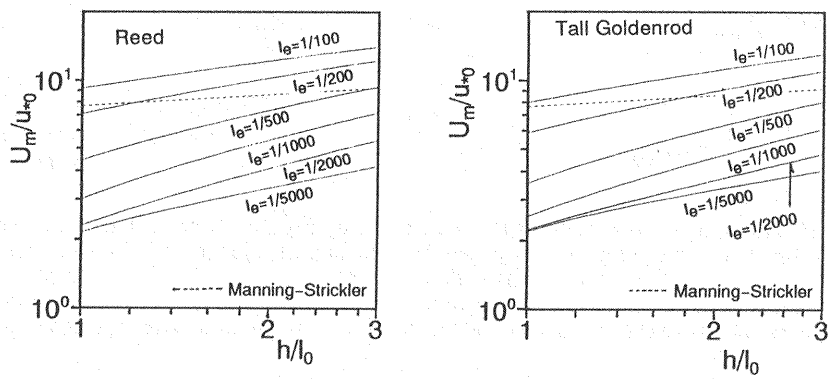

Fig.12 Predicted hydraulic resistance

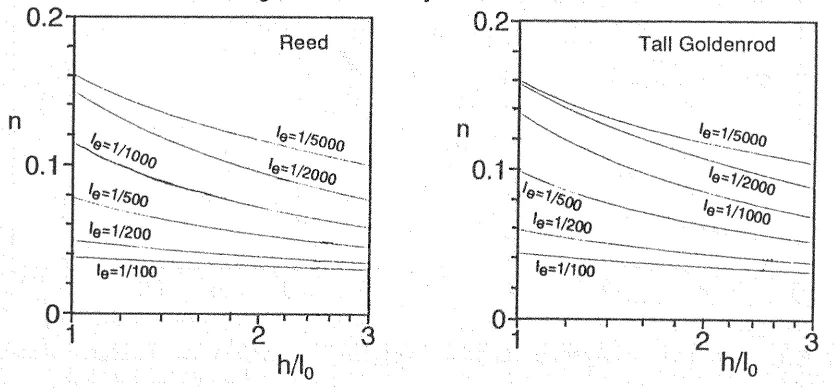

Fig.13 Predicted Manning coefficient

CONCLUSIONS

In this study, the velocity distribution and the deformation of vegetation have been analyzed in order to estimate the hydraulic resistance of flexible vegetation. The flow with vegetation has been analyzed by using k-ε turbulence model where the vegetation has been treated as a rather porous layer. On the other hand, the deformation of individual plants has been analyzed by using the finite deformation theory of a cantilever. These two are coupled with each other, and the velocity profile and deformation of vegetation are simultaneously solved. The result has been summarized as a resistance law. A convenient method modify the parameters in the method for flow with rigid vegetation has been proposed.

Furthermore, insitu measurement of the characteristic parameters of vegetation was conducted in a flood plain of rivers. By using the data obtained by the field study, the hydraulic resistance of flow with vegetation has been discussed, and it has been found that the resistance coefficient changes appreciably with the energy gradient as well as the relative depth (the ratio of the depth to the vegetation height).

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APPENDIX - NOTATION

The following symbols are used in this paper:

C_D = drag coefficient of vegetation;

C_1, C_2, C_μ = model parameters for turbulence model;

$C_{fk}, C_{k\varepsilon}$ = model parameters for turbulence model with drag term;

C^* = dimensionless vegetation density defined by $C_D \lambda l_0$;

C_e^* = effective vegetation density;

D_x, D_y = longitudinal and vertical components of hydrodynamic force acting on a plant per unit length with vertical axis;

d = diameter of plant;

EI = flexural rigidity of plant;

F_x	= longitudinal component of locally averaged drag force acting on plants per unit mass of water;
g	= gravity acceleration;
h	= water depth;
I_e	= energy gradient;
k	= turbulence energy;
M	= bending moment;
m	= constant;
n	= Manning coefficient;
l	= height of inclined plant;
l_0	= plant length;
l_0^*	= relative plant length;
N_0	= number of plants per unit area;
P	= pressure;
P_k	= turbulence energy production;
R_e	= Reynolds number defined by $U_m h/\nu$;
U	= mean primary velocity;
U_m	= depth averaged value mean primary velocity;
u_{*0}	= shear velocity defined by $\sqrt{ghI_e}$;
W	= horizontal tractive force;
x, y	= longitudinal, vertical coordinates;
α	= constant;
δ	= longitudinal displacement of plant;
δ_B	= longitudinal displacement of plant at the top of plant;
ε	= dissipation rate of turbulence energy;
γ	= non-dimensional parameter defined by $l_0^2 N_0$;
η	= distance along y axis;
λ	= projected area of plants to vertical plane per unit mass of water;
ν	= kinematic viscosity;
ν_T	= kinematic eddy viscosity;
θ	= inclination angle of plant;
θ_e	= effective inclination angle of plant;
ρ	= mass density of water;
$\sigma_k, \sigma_\varepsilon$	= model parameters for turbulence model; and

Superscripts

*	= non-dimensional quantities notmalized by velocity scale U_m and length scale h.

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