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STABLE LONGITUDINAL PROFILE OF RIVER CHANNELS

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SYNOPSIS

The longitudinal profiles of bed elevation, mean diameter of bed materials, width, and depth of flow in alluvial rivers are investigated theoretically. In order to close the system of equations, the concept that the non-dimensional bed shear stress is constant in the longitudinal direction is introduced. This concept has been derived from analysis of the stable channel cross-section. The theoretical analyses are tested by field data from several rivers in Hokkaido.

INTRODUCTION

Usually rivers originate from mountains and flow down into the sea. The width and the depth of river channels increase and the diameter of bed material decrease as they flow downstream. Is it possible to explain this morphology by the present knowledge of the river hydraulics? Are there a stable longitudinal profiles of bed elevation of river channels?

Some attempts have been made to obtain the stable longitudinal bed profiles (1). There are more unknown variables than equations. It is necessary to make one or more assumptions for the longitudinal variation of width, depth, or size of the bed material to close the system and to solve this problem.

Recently some analyses have been developed to solve the problems of bank erosion and the stable cross-section of river channels(2). A new attempt is made in this paper to evaluate the stable longitudinal profiles of river channels by introducing the results of the analysis of the stable channel cross-section.

BASIC EQUATIONS

The flow rate of rivers varies time to time. Although it is a problem how to define the effective flow rate which controls the geometry of river channels, the effective flow rate, Q, is assumed to be known in the following analysis.

The equation of motion of river flow is

$$\frac{1}{2g}\frac{d}{dx}\left(\frac{Q}{Bh}\right)^2+\frac{dh}{dx}=I_b-I_e \qquad (1)$$

where B= width of channel, h= depth of flow, x= distance from the upstream and in the downstream direction, I_b= bed slop (=-dz/dx), z= bed elevation at x and I_e= energy slope evaluated by a law of resistance.

The resistance to flow is described by Eq.2 assuming that the channel bed is flat.

$$\frac{Q}{Bh\sqrt{ghI_e}}=6.9\left(\frac{h}{d}\right)^{1/6} \qquad (2)$$

where d= mean diameter of bed material and g= acceleration due to gravity.

As for the motion of bed material a dynamic equilibrium state is considered. The equation of continuity of the bed material, and the equation of motion of bed material corresponding to the equation of the bed load transport rate are described as

$$\frac{d}{dx}(q_B B)=0 \quad \text{or} \quad q_B B = C_1 \qquad (3)$$

$$\frac{q_B}{\sqrt{sgd^3}}=8(\tau_*-\tau_{*c})^{3/2} \qquad (4)$$

where q_B= bed load transport rate per unit width, s=(1.65) specific weight of bed material in water, τ_*= non-dimensional bed shear stress, τ_{*c}=(0.05) non-dimensional critical bed shear stress and C_1= an unknown constant.

These four equations are commonly employed in analysis of the stable longitudinal bed profiles. In the following analysis one more equation, Eq.5, is introduced. This was derived from the analysis of the stable cross-section of erodible channels.

$$\tau_*\left(=\frac{hI_e}{sd}\right)=C_2 \qquad (5)$$

in which C_2= 1.23τ_{*c}. Eq.5 was derived by Ikeda et al.(2) for gravel bed channels. Eq.5 implies that the non-dimensional bed shear stress is constant at the central part of the channel where the depth of water is uniform. This is a simple and important conclusion.

DERIVATION OF GOVERNING EQUATIONS

Eq.6 is derived from Eqs.3, 4 and 5 and Eq.7 is derived from Eqs.2 and 5, respectively,

$$Bd^{3/2}=\frac{C_1}{C_3} \qquad (6)$$

$$\frac{Q}{\sqrt{sI_e}Bd^{3/2}}=6.9\left(\frac{C_2 s}{I_e}\right)^{5/3} \qquad (7)$$

in which $C_3=8\sqrt{sg}\,(C_2-0.05)^{3/2}$.

The energy slope, I_e, can be derived from Eqs.6 and 7 and is expressed only by the flow rate, Q, as Eq.8.

$$I_e=\left[\frac{Q}{\sqrt{g}}\frac{C_3}{C_1}\frac{1}{6.9(sC_2)^{5/3}}\right]^{-6/7} = C_4Q^{-0.86} \tag{8}$$

The wellknown regime theory states that there exists a relationship between the energy slope I_e and a power of the flow rate Q; Eq.8 is such a relationship. Takayama reviewed, in his book (1), Leopold's works which introduced the concept of entropy. Leopold's equation takes the form of

$$I_b \propto Q^{-0.74} \tag{9}$$

The power of Q is -0.48 ~ -1.07 according to the observations of Leopold. Eq.8 is very similar to Eq.9. It should be noted that Eq.8 is by contrast derived theoretically from hydraulic relationships.

The velocity of flow is derived from Eqs.5 and 8.

$$\frac{Q}{Bh}=\frac{C_3}{C_1}\left(\frac{C_4}{sC_2}\right)^{3/2}h^{1/2}Q^{-2/7} = C_5h^{1/2}Q^{-2/7} \tag{10}$$

Eq.11 is obtained from Eqs.1 and 10.

$$\left(1+\frac{2g}{C_5^2}Q^{4/7}\right)\frac{dh}{dx}-\frac{4}{7}\frac{1}{Q}\frac{dQ}{dx}h=\frac{2g}{C_5^2}Q^{4/7}(I_b-I_e) \tag{11}$$

LONGITUDINAL DISTRIBUTION OF FLOW RATE AND STABLE BED PROFILE

It is assumed that the flow is locally uniform and $I_b=I_e$. If the longitudinal distribution of flow rate is given, the stable longitudinal bed profile can be obtained by integration of Eq.8.

The longitudinal distribution of flow rate varies according to the characteristics of each river basin. An exponential distribution is adopted in the following analysis, which agrees with most rivers.

$$Q=Q_0\exp(qx) \tag{12}$$

in which q= constant, Q_0=flow rate at x= 0.

The boundary conditions are,

$$\begin{aligned} z&=z_* \quad \text{at} \quad x=0 \\ z&=0 \quad \text{at} \quad x=L \end{aligned} \tag{13}$$

Eq.14 is obtained by integrating Eq.8.

$$\frac{z}{z_*}=\frac{\exp(-a\xi)-\exp(-a)}{1-\exp(-a)} \tag{14}$$

where z_*= relative bed elevation in the reach under consideration, L= length of river channel, a=(6/7)qL and ξ = x/L.

Eq.14 is depicted in Fig.1 for several values of 'a'. For the value a=1, bed slop is relatively uniform in longitudinal direction. Incresing the value of 'a', bed slop becomes steeper at upper reach and milder at lower reach.

Eq.15 is obtained by integration of Eq.11 assuming that $I_e=I_b$, in which h_0= depth of flow at x= 0.

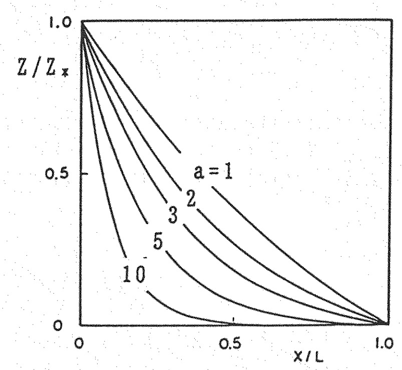

Fig.1 Theoritical change of longitudinal profile of river bed with the value of 'a'.

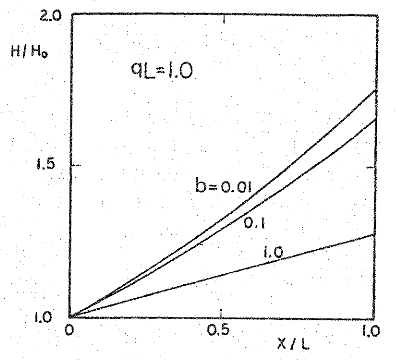

Fig.2 Theoritical change of longitudinal distribition of depth of flow with the value of 'b'.

$$\frac{h}{h_0}=\frac{(1+b)\exp\left(\frac{2}{3}a\xi\right)}{1+b\cdot\exp\left(\frac{2}{3}a\xi\right)} \tag{15}$$

where $b=(2g/C_5^2)Q_0^{4/7}$ is a constant and is proportional to $(Q_o/q_B B)$.

The relative bed load transport rate decreases as the value of 'b' increases. Fig.2 shows some examples of the longitudinal variations of the depth of flow along the river channel in the equilibrium state. It appeares from Fig.2 that the longitudinal variation of the depth of flow highly depends on the value of 'b' and is small when the bed load transport rate is small.

The distribution of mean diameter of bed material in the longitudinal direction is described by Eq.16 obtained by substituting Eq.15 into Eq.5. The longitudinal variation of the width of river channel is deduced from Eqs.5, 6, 15 and 16 as Eq.17.

$$\frac{d}{d_0}=\frac{(1+b)\exp\left(-\frac{1}{3}a\xi\right)}{1+b\exp\left(\frac{2}{3}a\xi\right)} \tag{16}$$

$$\frac{B}{B_0}=\left[\frac{(1+b)\exp\left(-\frac{1}{3}a\xi\right)}{1+b\exp\left(\frac{2}{3}a\xi\right)}\right]^{-3/2} \tag{17}$$

where d_o= mean diameter of bed material and B_o= width of river channel at x=0, respectively.

The sediment diameter d and the width of channel B as fuctions of ξ and 'b' are shown in Figs.3 and 4. The longitudinal distributions of mean diameter of bed material and width of the river channel also highly depend on the value of 'b'. Their longitudinal variations are emphasized when the bed load transport rate is small.

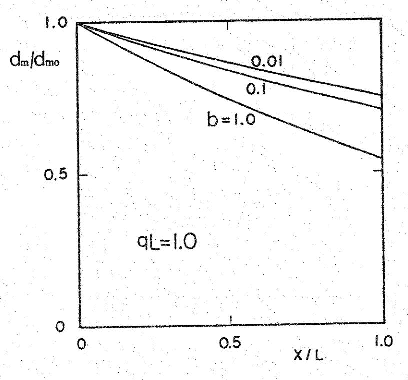

Fig.3 Theoritical change of longitudinal distribution of mean diameter of bed material with the value of 'b'.

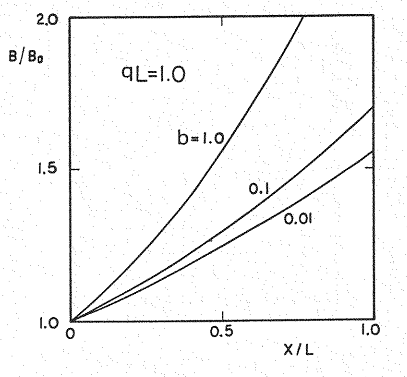

Fig.4 Theoritical change of longitudinal distribution of width of river channel with the value of 'b'.

APPLICATIONS TO RIVERS

The preceding analysis is tested by rivers in Hokkaido. They are the Biei R., the Yubetsu R., the Tokoro R., the Tokachi R. and the Ishikari R., all of which have at least three gaging stations along their trunk channels. General descriptions of these rivers are shown in Table 1.

Although rivers in Hokkaiso are not artifitially modified relative to ones in the other districts in Japan, they do not keep a natural state that the present theory is based on.

Table 1. General descriptions of rivers

Name of River	Drainage Area(km^2)	Length(km)
Biei R.	717	68
Yubetsu R.	1,480	87
Tokoro R.	1,930	120
Tokachi R.	9,010	156
Ishikari R.	14,330	268

The average of the yearly maximum flow rate is adopted in the analysis as the effective flow rate which controls the geometrical change of the river channels. It has been verified by one of the authors that the regime criteria of the formation of sand bars in rivers in Japan can be well described by the average of yearly maximum flow rates (3). The formation of sand bars has an important role on the development of the morphology of rivers in Japan. It is reasonable to assume that the flow rate which develops the river morphology is the effective flow rate.

Figs.5a and 5b show two typical examples of the longitudinal distributions of the effective flow rates, Q, in the Yubetsu R. and the Ishikari R., respectively. The Biei R., the Yubetsu R. and the Tokoro R. have comparatively small river basins. The flow rates of these rivers can be described well by Eq.12 as an example of Fig.5a. On the other hand, the Tokachi R. and the Ishikari R. have comparatively large river basins. In these rivers, Eq.(12) is difficult to be adapted for their whole reach. Therefor the reaches are divided into two parts

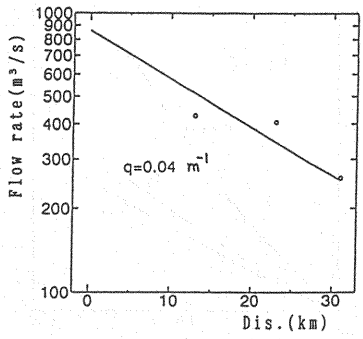

Fig.5a The Yubetsu R.

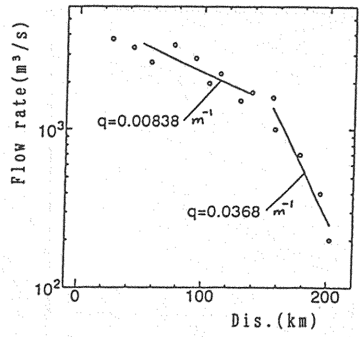

Fig.5b The Ishikari R.

Figs.5 The example of longitudinal distributions of effective flow rates. Solid lines show the theoritically assumed distributions (Eq.12).

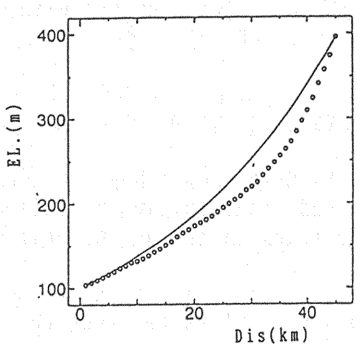

Fig.6a The Biei R.

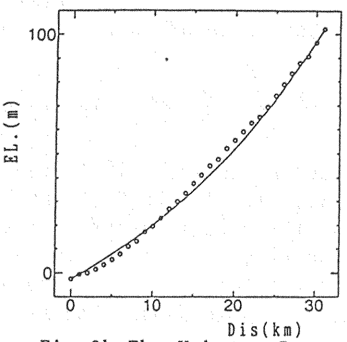

Fig.6b The Yubetsu R.

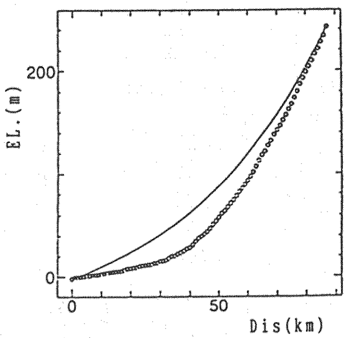

Fig.6c The Tokoro R.

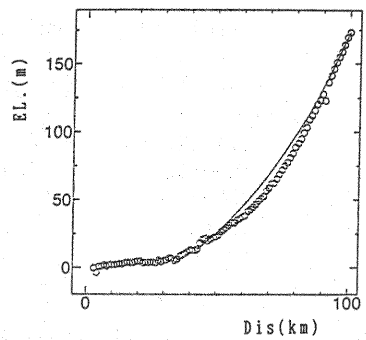

Fig.6b The Tokachi R.

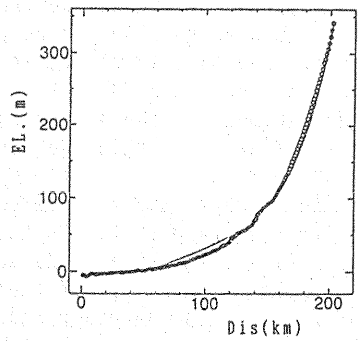

Fig.6d The Ishikari R.

Figs.6 Comparison between the theoritical longitudinal plofiles of bed elevation with field data. Solid lines show the theoritical one (Eq.14).

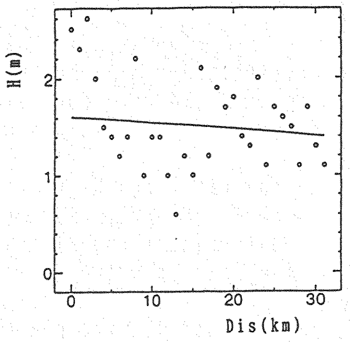

Fig.7a Depth of flow.

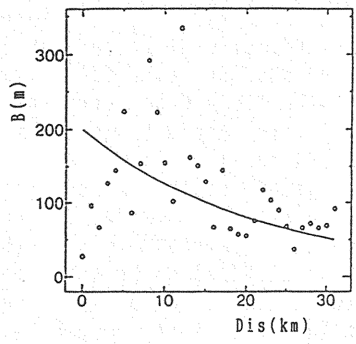

Fig.7b Width of channel.

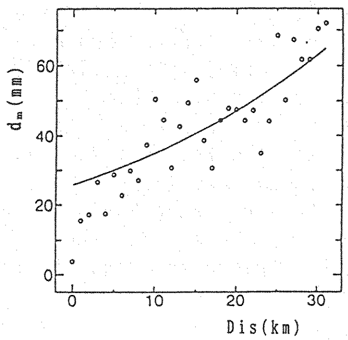

Fig.7c Mean diameter of bed material

Figs.7 Comparison between the theoritical distributions of the depth of flow, the width of channel and the mean diameter of bed material with field data of the Yubetsu R.

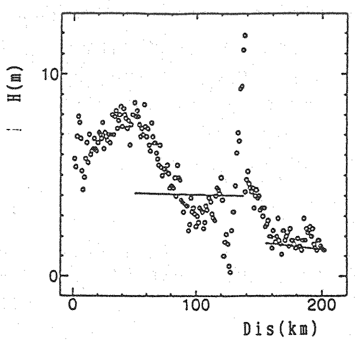

Fig.8a Depth of flow.

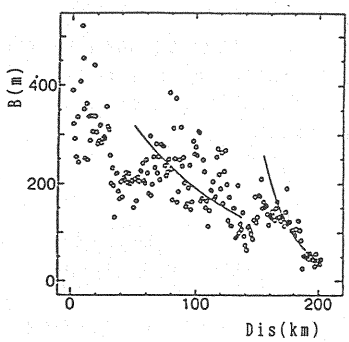

Fig.8b Width of channel.

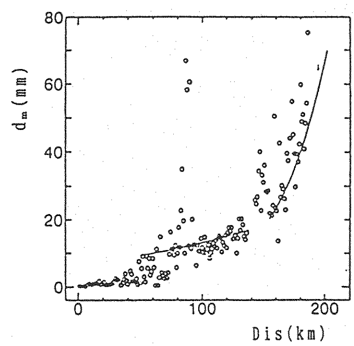

Fig.8c Mean diameter of bed material

Figs.8 Comparison between the theoritical distributions of the depth of flow, the width of channel and the mean diameter of bed material with field data of the Ishikari R.

and Eq.(12) are adapted to them respectively as shown in Fig.5b.

The longitudinal profiles of bed elevation for five rivers are shown in Figs. 6a, 6b, 6c, 6d and 6e. In these figures, theoretical curves given by Eq.14 are compared with the field data represented by circles. Field data for relative bed elevation 'z_*' are used to calculate the curves because these values can not be evaluated theoritically.

The adaptation of theory were restricted to the river reach where the mean diameter of bed material d>2 mm, because Eq.5 is verified only for gravel bed rivers. The theory agrees well with the field data except the Biei R. and the Tokoro R. The difference between the preceding analysis and the field data for these two rivers has not been explained yet. Although the effects of artificial modification in these rivers should be checked, difference in the plane shape of drainage basin is interesting.

The longitudinal distributions of depth of flow, width of the river channel and the mean diameter of the bed material for the Yubetsu R. and the Ishikari R. are shown, as example, in Figs.7a, 7b and 7c, and Figs.8a, 8b and 8c, respectively.

For the Yubetsu R., the theoritical results agree well with the field data. For the Ishikari R. however, there is a great discrepancy in the lower reach. This is because the Ishikari R. has a narrow gorge 120km upstream from the river mouth and also has some large tributaries. Furthermore the fact must be considered that the Ishikari R. was shortend about 100 km before 1960's. The bed elevation of the Ishikari R. does not show a remarkable change in the recent 20 years, however, it may beyond the scope of the theory.

CONCLUSION

It is derived theoretically that the relative longitudinal distributions of bed elevation, depth of flow, width of the river channel and mean size of bed material can be evaluated only by the longitudinal distribution of the effective flow rate. The theoretical analyses are tested by field data of rivers in Hokkaido. The theory agrees well with data of some rivers tested, however, there exist the considerable number of disagreement. Tests by data of rivers in natural state are necessary.

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APPENDIX-NOTATION

The following symbols are used in this paper:

a	$=(6/7)qL$;
B	=width of river channel;
B_o	=B at x=0;
b	$=(2g/C_5{}^2)Q_o{}^{4/7}$;
C_1	=constant in Eq(3);
C_2	$=1.23\tau_{*c}$;
C_3	$=8\sqrt{sg}\,(C_2-0.05)^{3/2}$;
C_4	=constant in Eq(8);
C_5	=constant in Eq(10);
d	=mean diameter of bed material;
d_o	=d at x=0;
g	=acceleration due to gravity;
h	=depth of flow;
h_o	=depth of flow at x=0;
I_b	=-dz/dx bed slope;
I_e	=energy slope;
L	=length of river channel;
Q	=effective flow rate;
Q_o	=Q at x=0;
q	=longitudinal increasing rate of Q(Eq.12);
q_B	=bed load transport rate per unit width;
s	=specific weight of bed material in water;
x	=distance from upstream end;
z	=bed elevation at x;
z_*	=relative bed elevation of the reach;
ξ	=x/L;
τ_*	=non-dimensional bed shear stress; and
τ_{*c}	=non-dimensional critical bed shear stress.

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