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A FORMULA FOR BED LOAD CAPACITY OF FLOW ON SMOOTH AND FIXED BEDS

By

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SYNOPSIS

This paper describes a new formula for evaluating the capacity of flows on smooth and fixed beds to transport, with regard to bed load only, the supplied sediment under the condition of non settling particles. The capacity, termed "the bed load capacity" in this paper, is expressed as a product of the volume of moving particles per unit area of bed surface and their mean velocity. Both these factors are formulated by considering the influence of moving particles on the representative velocity of fluid drag and difference in the friction coefficient of particles between static and dynamic states. A velocity distribution function for sediment laden flows is proposed, on the basis of Prandtl's Mixing Length Theory, to be incorporated with the present formula. The characteristics of the friction coefficient of moving particles are clarified empirically. The proposed formula shows a significant correlation with existing flume data.

INTRODUCTION

In certain reaches of mountain streams, the bed configuration often changes from granular to rocky and vice versa, depending on flood conditions and the degree of sediment supply from upstream reaches. Such changes in bed configuration appears to considerably affect morphological processes of the streams, since, as explained later, there is a remarkable difference in characteristics of sediment transport between granular beds and rocky beds. Similar features of sediment transport and morphological processes can also be observed in man-made channels and tunnels used for water intake or sediment detour whose beds are lined with concrete or steel. In this paper, these inerodible beds are termed "fixed beds", and the characteristic of bed load transport on this type of stream bed is discussed.

On a granular bed, with exception of a short reach downstream from its upstream end, the bed load rate at a cross-section is related uniquely to the local conditions of flow and sediment mixture in the exchange layer. In the case of a fixed bed, however, the bed load rate is not closely correlated with the flow condition. The flow on a fixed bed merely conveys all of the sediment supplied from the upstream reaches, as long as the rate of supply does not exceed a certain critical rate. In flume experiments on this type of sediment transport, it is observed that no sediment particles settle on the bed surface. All of them being transported as if they were the materials of wash load. When the above condition of sediment supply is not satisfied, the bed surface is covered by a settled sediment layer in a short time and, consequently, the bed configuration changes to the granular bed.

The above mentioned critical rate of sediment supply can be interpreted as the maximum rate of sediment that can be transported without settling on the surface of a fixed bed. From this point of view, hereafter, it is termed "bed load capacity" of the flow on fixed beds. Establishment of a method for its accurate evaluation is essential to enable reliable prediction of the morphological process in mountain streams in addition to that of the sediment transport efficiency of man-made channels and tunnels.

The problem of bed load capacity has been investigated experimentally and theoretically with respect to uniform sediment on smooth fixed beds. Ishihara et al. (7), based on experimental observations, showed a significant influence of moving sediment on the velocity distribution of water flow. Pedroli (13) performed comprehensive experiments using uniform test sand of various sizes and presented empirical formulae for bed load capacity and flow resistance. Furthermore, Shima & Hayakawa (14), Tubaki et al. (15) and Michiue (11) attempted to theoretically formulate the bed load capacity and resistance law of sediment laden flows. However, because of the complexity of this problem, sufficiently reliable formulation has not yet been completed. On the other hand, recently, some models have been proposed for numerical simulation of solid/liquid two phase flows in bed load layers and sheet flow layers (2)(5)(6). These models are couplings of the Euler or the Lagrange models for solid phase and first or higher order turbulent flow models for liquid phase. They have been proved to be powerful tools for exact clarification of the two phase flow phenomena. At present, however, it is difficult to incorporate them with a long term prediction model of the morphological processes in actual mountain streams due to the high cost of calculation.

The purpose of this paper is to improve formulation of bed load capacity. In the following, firstly, its general expression is proposed by modifying Ashida & Michiue's (3) theory of bed load on loose granular beds. The resultant equation, however, includes an unknown velocity distribution function of flow affected by moving sediment as well as several unknown parameters to be determined empirically. Therefore, secondly, the velocity distribution function is derived theoretically on the basis of Prandtl's Mixing Length Theory. Its applicability is tested with flume data collected by the authors and Ishihara et al. (7). Finally, the unknown parameters are identified by referring to related research achievements and flume data presented in published papers.

GENERAL EXPRESSION OF BED LOAD CAPACITY

The bed load capacity, q_T, can be expressed as

$$q_T = V \cdot v_s \tag{1}$$

where V = volume of moving particles per unit area of bed surface; and v_s = mean velocity of particles. In the present study, the two factors were formulated on the basis of Ashida & Michiue's (3) theory of equilibrium bed load transport on granular beds.

Based on Bagnold's (4) experimental study on air flows in blown-sand layers, Ashida & Michiue (3) employed a simple assumption that bed shear stress in the liquid phase is kept equal to critical shear stress, τ_c, for incipient motion of bed materials. This assumption is applicable to the present case. Thus, the first factor, V, can be formulated as follows by considering the balance between tractive and resisting forces acting on moving particles of volume V:

$$\mu_m(\sigma - \rho)gV = \tau_G = \tau_o - \tau_c \tag{2}$$

where τ_o and τ_G = apparent and tractive bed shear stresses of flow, respectively; μ_m = coefficient of friction force between moving particles and fixed bed; σ = density of sediment; ρ = density of water; and g = gravitational acceleration.

For formulation of the second factor, v_s, let us assume that all particles are transported in isolation with uniform and constant velocity. Then, the balance of fluid and friction forces acting on a representative particle may be expressed as

$$\frac{1}{2}\rho C_{Dm}(u_b - v_s)^2 K_2 d^2 = \mu_m(\sigma - \rho)gK_3 d^3 \tag{3}$$

where d = normal diameter of sediment; C_{Dm} = drag coefficient of particles in motion; u_b = representative velocity of fluid drag; and K_2 and K_3 = coefficients of the projected area and particle volume, respectively. At the critical condition of particle movement, where $v_s = 0$, Eq. 3 is reduced to

$$\frac{1}{2}\rho C_{Dc} u_{bc}^2 K_2 d^2 = \mu_c(\sigma - \rho)gK_3 d^3 \tag{4}$$

where C_{Dc} = drag coefficient of particle at rest; u_{bc} = representative velocity of fluid drag at critical condition; μ_c = coefficient of friction force between resting particles and fixed bed. From Eqs. 3 and 4, the following expression of v_s is obtained.

$$\frac{v_s}{u_*} = \frac{u_b}{u_*} - \sqrt{\frac{\mu_m}{\mu_c}}\sqrt{\frac{C_{Dc}}{C_{Dm}}}\frac{u_{bc}}{u_{*c}}\frac{u_{*c}}{u_*} \tag{5}$$

where u_* = apparent shear velocity ($= \sqrt{\tau_o/\rho}$); and u_{*c} = critical shear velocity ($=\sqrt{\tau_c/\rho}$). If the local flow velocity at a point where $y = d$ is selected for u_b and u_{bc}, and if the surface of the fixed bed is regarded as being hydraulically smooth, then u_b/u_* and u_{bc}/u_{*c} will be functions of grain Reynolds numbers, u_*d/ν and $u_{*c}d/\nu$, respectively. However, because the distribution of flow velocity just above the bed surface is significantly affected by moving particles, the functions u_b/u_* and u_{bc}/u_{*c} will take different forms,

$$\frac{u_b}{u_*} = f\left(\frac{u_* d}{\nu}\right) \tag{6}$$

$$\frac{u_{bc}}{u_{*c}} = f_c\left(\frac{u_{*c}d}{\nu}\right) \tag{7}$$

where ν = kinematic viscosity of water; and $f(u_*d/\nu)$ and $f_c(u_{*c}d/\nu)$ = velocity distribution functions for transport and critical conditions, respectively. For $f_c(u_{*c}d/\nu)$, the following well-known logarithmic law for smooth walls can be employed.

$$\frac{u}{u_*} = \frac{1}{\kappa}\ln(Y) + 5.5 \tag{8}$$

where u = local flow velocity at height y; κ = Kármán's universal constant(=0.4); and $Y = u_*y/\nu$. On the other hand, the form of $f(u_*d/\nu)$ has not been clarified. This problem is addressed later in this paper.

Now, using Eqs. 2, 5, 6 and 7, Eq. 1 can be rewritten in the following dimensionless form:

$$\frac{q_T}{u_*d} = \frac{1}{\mu_c}\frac{1}{(\mu_m/\mu_c)}(\tau_* - \tau_{*c})f\left(\frac{u_*d}{\nu}\right)\left\{1 - \sqrt{\frac{\mu_m}{\mu_c}}\sqrt{\frac{C_{Dc}}{C_{Dm}}\frac{f_c(u_{*c}d/\nu)}{f(u_*d/\nu)}}\sqrt{\frac{\tau_{*c}}{\tau_*}}\right\} \tag{9}$$

where $\tau_* = u_*^2/\{(\sigma/\rho - 1)gd\}$; and $\tau_{*c} = u_{*c}^2/\{(\sigma/\rho - 1)gd\}$. If Eq. 8 is adopted for both f and f_c, and if it is assumed that $\mu_m = \mu_c$ and $C_{Dc} = C_{Dm}$, then Eq. 9 coincides with a formula proposed by Michiue (11). However, as pointed out previously, the velocity of sediment-laden flow has different characteristics from that of clear-water flow. Moreover, it seems unreasonable to consider that the friction and drag coefficients of particles are constant for both resting and moving conditions.

In order to make Eq. 9 practicable, it is necessary to define $f(u_*d/\nu)$ and to clarify the characteristics of τ_{*c}, μ_c, μ_m/μ_c and C_{Dc}/C_{Dm}. The function $f(u_*d/\nu)$ is formulated theoretically in the next section, while the characteristics of other unknown parameters, which are too difficult to be treated theoretically, are clarified empirically by referring to existing data.

VELOCITY DISTRIBUTION OF FLOW WITH BED LOAD

Theoretical Consideration

In this section, the velocity distribution for sediment-laden flow on smooth and fixed beds is formulated based on Prandtl's Mixing Length Theory.

According to the Mixing Length Theory, the relationship between local fluid shear stress, τ, and local velocity gradient, du/dy, is expressed as

$$\tau = \rho\nu\frac{du}{dy} + \rho\ell^2\left(\frac{du}{dy}\right)^2 \tag{10}$$

where ℓ = mixing length.

To obtain an explicit solution of Eq. 10, spatial distributions of τ and ℓ were assumed as shown in Fig. 1. The whole flow field is divided into three regions, namely, region-1 where $0 \le y \le \delta_L$, region-2 where $\delta_L \le y \le \alpha d$

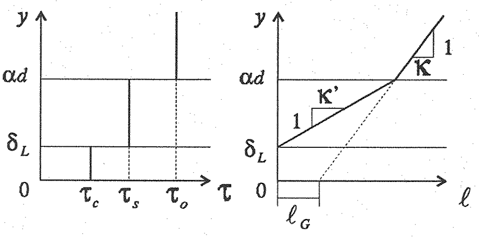

Fig. 1 Assumptions of shear stress and mixing length

and region-3 where $y \ge \alpha d$. Also, for each region, the distributions of τ and ℓ are assumed, after the model of Iwagaki (8), as follows:

$$(region-1)\quad \tau = \tau_c \quad ; \quad \ell = 0 \tag{11}$$

$$(region-2)\quad \tau = \tau_s = (\tau_c + \tau_o)/2 \quad ; \quad \ell = \kappa'(y - \delta_L) \tag{12}$$

$$(region-3)\quad \tau = \tau_o \quad ; \quad \ell = \kappa y + \ell_G \tag{13}$$

where δ_L = thickness of laminar layer; α = empirical constant to estimate the upper limit of bed load layer; ℓ_G = additional mixing length due to wakes behind moving particles and/or their rolling motion; κ' = virtual Kármán's constant for region-2 to be evaluated with

$$\kappa' = \frac{\kappa \cdot \alpha d + \ell_G}{\alpha d - \delta_L} \tag{14}$$

It should be noted that some part of the above assumptions for τ and ℓ are inconsistent with their actual features. However, it has been proved by experience that even such a manner of assumption may produce useful results. Although, in Fig. 1 it is assumed that $\alpha d > \delta_L$, it is possible for δ_L to exceed αd when moving particles are extremely fine. This case is discussed later to avoid confusion. When those assumptions of τ and ℓ are included, Eq. 10 is solved as follows.

For region-1, a solution of Eq. 10 with no-slip condition yields a linear velocity distribution,

$$U = U_{*c}^2 Y \tag{15}$$

where $U = u/u_*$; and $U_{*c} = u_{*c}/u_*$. In regions-2 and -3, τ is assumed to be constant and ℓ is expressed as a linear function of y. Therefore, when the relationship between ℓ and y is represented by $\ell = ay + b \quad (a, b = constants)$, the following common expression of solutions for the two regions can be obtained through transformation of the independent variable:

$$U = \frac{1}{a} F(L) + C \tag{16}$$

where

$$F(L) = \ln\left(2L + 2\sqrt{L^2 + \frac{1}{4}}\right) + \frac{1}{L}\left(\frac{1}{2} - \sqrt{L^2 + \frac{1}{4}}\right) \quad ; \quad L = \frac{\sqrt{\tau/\rho} \cdot \ell}{\nu} \tag{17}$$

; and C = integral constant. The integral constant, which being a function of u_*, u_{*c}, ℓ_G and so on, can be determined on the condition that there are no gaps in the velocity profiles. As a result, the following definite expressions of velocity distribution function have been obtained:
For region-2, denoting $U_{*s} = \sqrt{(u_{*c}^2 + u_*^2)/2}/u_*$ and $R_{*c\delta} = u_{*c}\delta_L/\nu$,

$$U = \frac{U_{*s}}{\kappa'} F(L_2) + R_{*c\delta} U_{*c} \quad ; \quad L_2 = \kappa' U_{*s}\left(Y - \frac{R_{*c\delta}}{U_{*c}}\right) \tag{18}$$

For region-3, denoting $R_{*cG} = u_{*c}\ell_G/\nu$ and $R_{*c\alpha} = u_{*c}\alpha d/\nu$,

$$U = \frac{1}{\kappa} F(L_3) + A \quad ; \quad L_3 = \kappa Y + \frac{R_{*cG}}{U_{*c}} \tag{19}$$

; and A is defined as

$$A = \frac{U_{*s}}{\kappa'} F(L_4) + R_{*c\delta} U_{*c} - \frac{1}{\kappa} F(L_5) \quad ;$$

$$L_4 = \frac{U_{*s}}{U_{*c}} \kappa' (R_{*c\alpha} - R_{*c\delta}) \quad ; \quad L_5 = \kappa \frac{R_{*c\alpha}}{U_{*c}} + \frac{R_{*cG}}{U_{*c}} \tag{20}$$

Among the dimensionless parameters defined above, $R_{*c\delta}$ can be related with the other ones as explained hereafter. When $L \gg 1/2$, $F(L)$ may be approximated as

$$F(L) \cong F'(L) = \ln(4L) - 1 \tag{21}$$

Furthermore, when $u_* = u_{*c}$, because there are no sediment particles moving or settling on fixed beds, L_3 may be regarded as $L_3 \cong L_3' = \kappa Y$ in the range where $y \gg \delta_L$. Thus, for a sufficiently high range of a flow in critical condition of particle motion, Eq. 19 can be reduced to

$$U = \frac{1}{\kappa} F'(L_3') + A' \quad ; \quad A' = R_{*c\delta}\left\{1 - \frac{1}{R_{*c\alpha}} \frac{1}{\kappa} F(\kappa R_{*c\alpha})\right\} \tag{22}$$

Finally, by considering the equivalency between Eqs. 8 and 22, the following expression of $R_{*c\delta}$ is obtained:

$$R_{*c\delta} = 6.825 \frac{R_{*c\alpha}}{R_{*c\alpha} - (1/\kappa) F(\kappa \cdot R_{*c\alpha})} \tag{23}$$

As pointed out previously, the above solution has been obtained for the cases where $\alpha d \geq \delta_L$. When $\alpha d \leq \delta_L$, the assumptions of τ and ℓ should be changed and, consequently, a different solution is yielded as follows. In this case, as the bed load layer is included in a laminar flow layer, it is sufficient for the flow to be divided into only two regions, namely, laminar region where $0 \leq y \leq \delta_L$ and turbulent region where $y \geq \delta_L$. For the laminar region, solution of Eq. 10 yields the same form as Eq. 15 because $\ell = 0$ and $\tau = \tau_c$. For the turbulent region, assuming that $\ell = \kappa y$, $\tau = \tau_o$ and that the term of laminar shear stress in Eq. 10 is negligibly small, the following solution is obtained:

$$U = \frac{1}{\kappa}\ln(Y) + \left\{R_{*c\delta}U_{*c} - \frac{1}{\kappa}\, ln\left(\frac{R_{*c\delta}}{U_{*c}}\right)\right\} \tag{24}$$

However, the expression of $R_{*c\delta}$ is not the same form as Eq. 23. It is given as $R_{*c\delta} = 11.64$ from the condition that Eqs. 8 and 24 should be equivalent when $u_* = u_{*c}$.

Verification Based on Flume Observations

In this section, performance of the above velocity distribution function is examined based on flume data. Although data collected by Ishihara et al. (7) is available, it should be noted that all of their experiments were conducted under the condition of thin sheet flow. Therefore, the authors carried out some experiments in order to extend the source of available data for observations of relatively deep flows.

Experimental apparatus and measurements: A flume, 14.6m long, 0.6m wide and 0.4m deep was used for the experiments. The bed surface of the flume was finished to be hydraulically smooth in lacquer paint. An automatic sand feeder, that supplies test sand continuously at a constant rate, was installed near the upstream end of the flume.

Seven experiments were conducted with two kinds of uniform test sand. The conditions of the experiments are summarized in Table. 1. In each experiment, a steady and uniform flow was established under the condition that the flow transported the maximum rate of test sand without deposition. And then, the vertical distribution of flow velocity was measured by using a brass Pitot tube with a mouth diameter of 1.0mm.

Table. 1 Characteristic data of experimental runs

RUN	d (cm)	bed slope ($-$)	h (cm)	u_*d/ν ($-$)	τ_*/τ_{*c} ($-$)	q_T (cm^2/s)	flow discharge (ℓ/s)
A-1	0.085	0.0025	2.248	19.75	1.5	0.010	4.28
A-2	0.085	0.0025	2.475	20.73	1.6	0.023	5.70
A-3	0.085	0.0025	4.830	28.95	3.0	0.080	14.35
B-1	0.170	0.0025	1.915	36.46	1.2	0.006	3.23
B-2	0.170	0.0025	2.300	39.96	1.6	0.020	5.38
B-3	0.170	0.0025	3.750	51.02	2.8	0.074	10.53
B-4	0.170	0.0025	4.780	57.60	3.6	0.097	14.82

It should be noted that, when the total pressure tube of a Pitot tube is made with a pipe of diameter, ϕ_p, local flow velocities in the range $0 < y < \phi_p/2$ can not be measured. However, as shown in Fig. 2, if the total pressure tube is successively lifted up from a flume bed at a small interval, Δy, the total pressure, P_L, at the height, y_L, lower than that of the center of tube mouth, can be evaluated approximately with the following relationships:

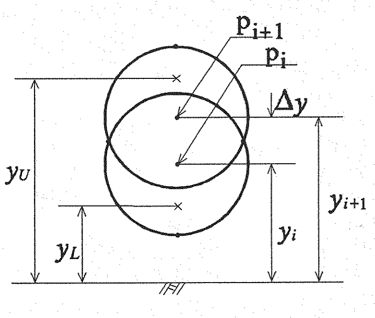

Fig. 2 Schematic diagram of a mouth of Pitot tube

$$y_L \cong y_i - \phi_p/2 + \Delta y/2 + (4-\pi)\phi_p/8 \tag{25}$$

$$P_L = P_U - \frac{\phi_p}{\Delta y}(P_{i+1} - P_i) \tag{26}$$

where $y_i = \phi_p/2 + i \cdot \Delta y$, with i = step number of measurement ($i = 0, 1, 2, \cdots$); P_U = total pressure at height, $y_U \cong y_{i+1} + \phi_p/2 - \Delta y/2 - (4-\pi)\phi_p/8$; and P_i = out-put of total pressure tube obtained by setting the center of tube mouth at height y_i. Although the procedure of the deduction of Eqs. (25) and (26) has not been explained in detail, they were preliminarily verified to some extent through their application to velocity measurements in laminar sub-layers of clear-water flows.

Application and discussion: Figs. 3 and 4 show some examples of comparison between velocity distributions calculated by the proposed equations and those observed in the experiments performed by the authors. In these

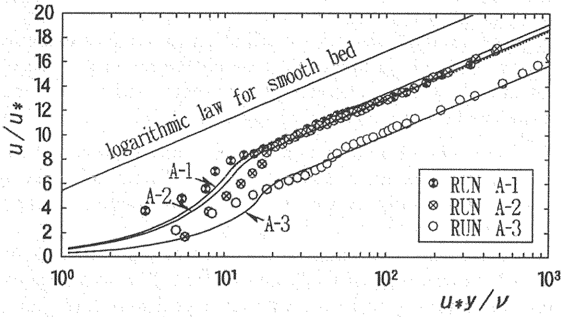

Fig. 3 Comparison between velocity observations collected by authors and velocity distribution curves calculated with present equations(A-series)

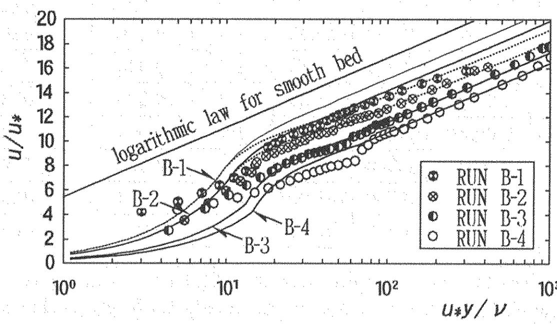

Fig. 4 Comparison between velocity observations collected by authors and velocity distribution curves calculated with present equations(B-series)

figures, plots indicate observations, while solid and broken lines indicate calculation results based on different assumptions of ℓ_G.

In the calculation, the parameters, τ_{*c}, α, and ℓ_G were given in the following manner. With respect to τ_{*c}, Maione (10), Michiue (11) and others have proposed some empirical equations for its evaluation. However, definitions of the critical condition adopted for the derivation of the above mentioned equations are not unified or clarified. In the present research, therefore, an attempt was made to derive a new equation of τ_{*c} based on a definite criterion of the critical condition. At first, the dependence of q_T on τ_* under a condition of very weak particle motion was examined through extrapolation of existing flume data, and the criterion for τ_{*c} was determined as $q_T/u_{*c}d = 10^{-3}$. In the following, τ_{*c} was found to be closely related with $u_{*c}d/\nu$, as shown in Fig. 5, and was formulated as

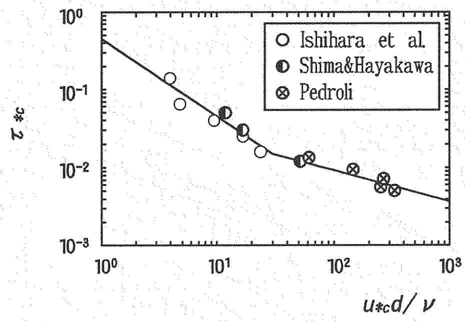

Fig. 5 Relationship between τ_{*c} and $u_{*c}d/\nu$

$$d < 0.1548cm \quad : \quad \tau_{*c} = \frac{0.45}{(u_{*c}d/\nu)} \tag{27}$$

$$d > 0.1548cm \quad : \quad \tau_{*c} = \frac{0.0585}{(u_{*c}d/\nu)^{0.4}} \tag{28}$$

The second parameter, α, was assumed to be 1.2 because, firstly, the particles of test sand were transported mostly in the manner of sliding and/or rolling, and because, secondly, as seen in Figs. 3 and 4, most of the measured velocity profiles show sudden increases in the velocity gradients around the points where $y \cong 1.2d$; refer to Table. 1 for u_*d/ν. The last parameter, ℓ_G, whose characteristic having not yet been clarified, was given as $\ell_G = 0$ for the first step.

In Figs. 3 and 4, where solid lines indicating the calculation results obtained by assuming $\ell_G = 0$, the level of agreement between calculated and observed results differs depending on whether the rate, τ_*/τ_{*c}, is larger or smaller than 2. In cases where $\tau_*/\tau_{*c} > 2$, namely Runs A-2, A-3, B-3 and B-4, the calculated and measured velocity profiles agree acceptably well over wide ranges of flow corresponding to regions-2 and -3. In the ranges corresponding especially to region-1, however, a prominent disagreement is recognized between the two kinds of results. Particularly, the actual flows appear to keep non-zero velocities even at points extremely close to the flume bed, while the present theory assumes that non-slipping laminar flows exist on fixed beds. The relatively higher velocities in the actual flows are thought to be produced by vertical momentum transfer due to rolling motion of the particles as well as by the accompanying of bed-contacting water with moving particles. However, the authors have not succeeded in correcting their theoretical model from such a point of view.

In the cases of Runs A-1, B-1 and B-2 where $1 < \tau_*/\tau_{*c} < 2$, the calculated results agree pretty well with the observations in the ranges of region-1, whereas a remarkable disagreement can be seen in the ranges of region-2 and -3. This disagreement is mostly due to underestimation of velocity gradients in region-2. In these experiments, the particles were observed to move mostly in a sliding manner at considerably lower velocities than those of surrounding flows. Regarding this observation result, it is supposed that the relatively gentler gradients of the observed flow velocities are due to the effect of wakes occurring behind the particles. Hence, for a trial, velocity profiles of Runs A-1, B-1 and B-2 were recalculated by assuming as ℓ_G/d=0.07, 0.15 and 0.15 respectively, and are indicated by the broken lines. A fine improvement in the degree of agreement has been achieved. At present, however, the influence of wakes on ℓ_G has not been thoroughly clarified.

Based on the flume experiments carried out by the authors, several problems have been pointed out with respect to the performance of the proposed velocity distribution function. However, let us recall here that the function is defined, in the procedure for formulating the bed load capacity, to estimate the representative velocity of fluid drag acting on moving particles. As far as evaluation of the bed load capacity is concerned, it may be sufficient for the present function to be accurate merely at the elevation of particle heads. If examined again from such a point of view, it could be regarded as being considerably useful, because some of the prominent disagreements which have been pointed out with respect to the ranges of regions-1 and -3 could be ignored.

In addition, the performance of the present function is examined also in Fig. 6, based on experiments conducted by Ishihara et al. (7). The calculated results, indicated by solid lines, are obtained by assuming $\alpha = 1.2$ and $\ell_G = 0$, and are compared with the experimental results indicated by plots. It should be noted that τ_*/τ_{*c} was always greater than 2 in these experiments. The results of the experiments and calculations show an acceptable level of correlation except for the case where $d = 0.052cm$. In the calculation for this exceptional case, δ_L was resultantly estimated as $\delta_L \leq \alpha d$. However, it may be possible to assume that the particles were involved in laminar layer due to their slender shapes. Based on this assumption, for a trial, the velocity distribution was recalculated by using Eqs. 15 and 24 and the new result is indicated by a broken line in Fig. 6. The correlation between the calculated and observed results has been improved to a sufficient level.

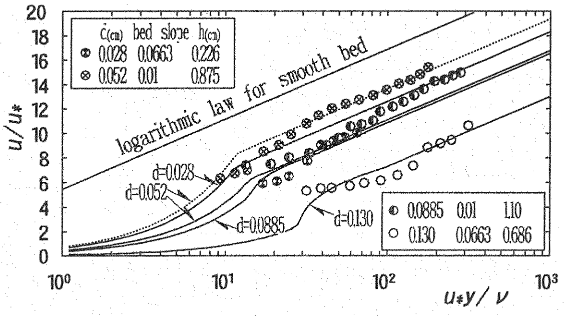

Fig. 6 Comparison between velocity observations collected by Ishihara et al. and velocity distribution curves calculated with present equations

ESTIMATION OF PARAMETERS INVOLVED IN PROPOSED BED LOAD CAPACITY EQUATION

In order to put Eq. 9 into practice, it is necessary to quantitatively estimate the three unknown parameters C_{Dc}/C_{Dm}, μ_c and μ_m/μ_c.

Several authors, including Aksoy (1), Kanda & Suzuki (9), investigated the property of drag coefficient of spheres, C_{Ds}, under the condition that they were settling on flume beds. As a result, it has been found that the dependence of C_{Ds} on the Reynolds number does not essentially differ between turbulent shear flows and infinitive flows with uniform velocity. In other words, C_{Ds} is not closely dependent on local velocity profiles and tends to decrease with increase in the Reynolds number. Furthermore, in the experiments performed by Ohnuki et al. (12), a sphere rolling down at constant velocities on a slope set at various angles in a still water showed nearly 25% greater values of C_{Ds} in comparison with the cases where it was merely sliding down on the slope. On the other hand, unfortunately, little is known about the drag coefficient of actual sediment particles. Referring to existing knowledge of spheres, it seems that C_{Dc}/C_{Dm} is a little greater than unity since the Reynolds number, based on the relative velocity between fluid and particles, appear to be greater for moving particles than for those in critical condition of motion. However, the rolling motion of particles, which becomes more apparent as τ_* increases, may cause an increase in C_{Dm} thus a decrease in C_{Dc}/C_{Dm}. As a result of these considerations, it seems reasonable to assume C_{Dc}/C_{Dm} to be unity.

In relation to μ_c, Tubaki et al. (15) reported that a critical velocity of a crushed-stone particle calculated by assuming $\mu_c = 0.38$ took a nearly 25% smaller value than the observation result. Hence, it seems rather appropriate to consider that μ_c is as large as 0.5.

Generally speaking, the friction coefficient between a body and solid surface is smaller in the dynamic state than in the static state. Hence, it can be suggested that $\mu_m/\mu_c < 1$. Furthermore, it should be noted that Tubaki et al. (15), for instance, employed a smaller value of $\mu_m (= 0.3)$ than that of μ_c for the calculation of bed load capacity. However, the characteristic of μ_m, especially its dependence of τ_*, has not yet been thoroughly clarified, although it may be significant.

Now, if the values of C_{Dc}/C_{Dm} and μ_c are given, μ_m/μ_c can be evaluated from existing experimental data of bed load capacity and it becomes possible to discuss its property empirically. Fig. 7 shows an example of the relationship between μ_m/μ_c and τ_*/τ_{*c} which was obtained on the assumption that $C_{Dc}/C_{Dm} = 1$ and $\mu_c = 0.5$, and by using observations presented by Ishihara et al. (7), Pedroli (13) and Shima & Hayakawa (14). It is seen that the resultant values of μ_m/μ_c are less than unity, as suggested above, with some exceptions. However, significant dependence of μ_m/μ_c on τ_*/τ_{*c} can not be observed. Hence, from a practical point of view, it may be considered that $\sqrt{\mu_m/\mu_c} = 0.8$.

Finally, in Fig. 8, dimensionless bed load capacities calculated by using the present formula with the above values of model parameters, C_{Dc}/C_{Dm}, μ_c and μ_m/μ_c, are compared with the observations. Sufficiently close correlation is noticed between the calculated and observed results except for one of the cases of Ishihara et al. (7) in which $d = 0.052cm$. The cause of this exception is currently being investigated.

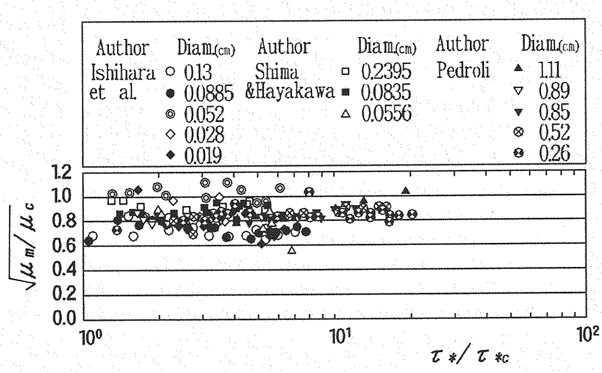

Fig. 7 Relationship between $\sqrt{\mu_m/\mu_c}$ and τ_*/τ_{*c}

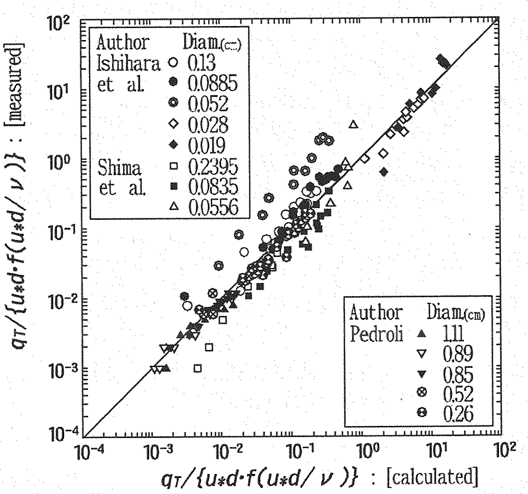

Fig. 8 Verification of proposed formula for bed load capacity

SUMMARY AND CONCLUSION

This paper has presented a formula for calculating "bed load capacity" of flows on smooth and fixed beds. The bed load capacity is defined as the maximum rate of bed load that can be transported by a flow on a fixed bed under the condition of non settling particles.

The bed load capacity was initially expressed as a product of the volume of moving particles per unit area of bed surface and their mean advancing velocity. Then, both factors were formulated to yield a practical form of the bed load capacity formula, as i.e. Eq. 9, by referring to Ashida & Michiue's (3) theory. In this procedure, the influence of moving particles on velocity distribution of water flow, difference in friction coefficient of sediment particles between their static and dynamic states, and variation in the drag coefficient were all taken into account.

The velocity distribution function of the sediment laden flow, which is included in Eq. 9, was deduced on the bases of Prandtl's Mixing Length Theory. The whole flow field was divided into three regions and the flow in each region was solved using simple models for spatial distributions of shear stress and mixing length. Applicability of the proposed velocity distributicn function was verified based on flume observations collected by the authors and Ishihara et al (7).

The parameters introduced in Eq. 9 were estimated empirically. As a result, the variation in drag coefficient was regarded as insignificant. The friction coefficient of sediment particles in the static state was estimated to be as great as 0.5. The ratio of the friction coefficient of sediment particles in a dynamic state to that in a static state was proved to be independent of the dimensionless tractive stress, and the appropriate value of the square of the ratio was estimated to be 0.8.

The performance of the proposed bed load capacity formula was examined based on flume observations collected by several researchers. The calculated and observed results correlated significantly thus verifing the validity of the present formula. It should be noted, however, that the achievement of this study is concerned with uniform sediment only, although sediment existing particularly in mountain streams is usually composed of particles with a wide distribution of sizes. Further investigations are necessary in order to solve the problems of mixed sediment.

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APPENDIX - NOTATION

The following symbols are used in this paper:

a	= constant;
A, A'	= dimensionless parameters defined as Eqs. 20 and 22;
b	= constant;
C	= integral constant;
C_{Dc}, C_{Dm}	= drag coefficients of particles at rest and in motion;

C_{Ds}	= drag coefficient of sphere;
d	= normal diameter of sediment;
f, f_c	= velocity distribution functions for transport and critical conditions;
F, F'	= functions defined as Eqs. 17 and 21;
g	= gravitational acceleration;
h	= flow depth;
i	= step number of setting the center of tube mouth;
K_2, K_3	= coefficients of the projected area and particle volume;
ℓ	= Prandtl's mixing length;
ℓ_G	= additional mixing length due to wakes and/or rolling motion;
L	= dimensionless mixing length;
L_2, L_3, L'_3, L_4, L_5	= dimensionless expressions of mixing length;
P_i	= out-put of total pressure tube at height y_i;
P_L, P_U	= total pressure at height y_L and y_U;
q_T	= bed load capacity;
R_{*cG}, $R_{*c\alpha}$, $R_{*c\delta}$	= Reynolds number defined with u_{*c} and ℓ_G, with u_{*c} and αd and with u_{*c} and δ_L;
u	= local flow velocity;
u_b, u_{bc}	= representative velocities of fluid drag in transport and critical conditions;
u_*,u_{*c}	= apparent shear velocity and critical shear velocity;
U	= dimensionless local flow velocity;
U_*, U_{*c}	= dimensionless apparent shear velocity and dimensionless critical shear velocity;
U_{*s}	= dimensionless shear velocity($= \sqrt{(u_{*c}^2 + u_*^2)/2}/u_*$);
v_s	= mean velocity of particles;
V	= volume of moving particles per unit area of bed surface;
y	= height from bed surface;
y_i	= height of center of total tube in i-th step of measurement;
y_L, y_U	= height of objective point to get out-put of total pressure;
Y	= dimensionless height from bed surface;
α	= empirical constant to estimate the upper limit of bed load layer;
δ_L	= thickness of laminar layer;
Δy	= small interval for total pressure measurement;
κ, κ'	= universal and virtual Kármán's constants (Eq. 14);
μ_c, μ_m	= coefficients of friction force in static and dynamic states of particles;
ν	= kinematic viscosity of water;
ρ	= density of water;
σ	= density of sediment;
τ	= local fluid shear stress;
τ_c	= critical shear stress;
τ_G	= tractive bed shear stress for sediment transport;
τ_o	= apparent bed shear stress;
τ_*, τ_{*c}	= dimensionless apparent bed shear stress and dimensionless critical shear stress;
ϕ_p	= diameter of total pressure tube mouth.

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