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FLUX DIFFERENCE SPLITTING ON SELF-ADJUSTING GRID FOR 1-D TRANSIENT FREE SURFACE FLOWS

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SYNOPSIS

Flux difference splitting scheme of Roe is implemented on a self-adjusting grid for solving one-dimensional transient free surface flows. The finite difference grid adjusts itself by averaging the local characteristic velocities with respect to the signal amplitude. The grid adjusting procedure developed by Harten and Hyman removes the smearing of discontinuities caused by the first-order flux difference splitting of Roe. The scheme presented herein also incorporates technique to satisfy energy inequality condition in all the cases of one-dimensional transient free surface flows. Improved resolution of discontinuities by the self-adjusting grid is demonstrated through numerical examples. Results using the self-adjusting grid are also compared with the second-order accurate TVD Lax-Wendroff scheme.

INTRODUCTION

Mathematical simulation of the set of partial differential equations governing one-dimensional transient free surface flows is widely recognized as an efficient tool for solving the related flow problems. The recognition has resulted in a significant improvement in the solution techniques for open channel flow problems. Recently, advances made in the field of gas dynamics in obtaining high resolution of discontinuous flows has shifted the focus of research from classical schemes to more sophisticated high resolution, shock-capturing schemes for solving flow problems with strong discontinuities. The application of flux splitting technique (e.g. Fennema and Chaudhry (4,5), Jha et al.(8)), flux difference splitting technique (e.g. Glaister (6), Alcrudo et al. (1), Jha et al. (9)) and higher-order TVD and ENO schemes (Yang et al. (16)) to one-dimensional transient free surface flows have been reported with varying success.

Resolution of a shock has so far been achieved mainly through shock-fitting or shock-capturing techniques. The shock-fitting approach (Katopodes and Strelkoff (10), Chen and Armbruster (3)) isolates a bore and computes its propagation for one time-step independently of the computation in the two adjacent continuous regions. This approach, however, implies a prior knowledge of the occurrence of a shock. At the same time a shock must be tracked so that the Rankin-Hugoniot condition can be applied at the location of the shock. The problem is further compounded if there are many shocks appearing and disappearing as the solution proceeds in time. These difficulties have given rise to the shock-capturing technique (see Lax (11), Lax and Wendroff (12)). The shock-capturing technique does not treat a shock as a moving internal boundary and solution is obtained by integrating the governing equations in conservation form (MacCormack (13), Beam and Warming (2)). However, the shock-capturing technique might smear a shock when applied to a first-order accurate fixed grid finite difference scheme. It often becomes necessary to extend such schemes to a higher-order of accuracy for enhancing the shock resolution.

Another approach for avoiding the smearing of a shock is to track a shock and make its location coincide with a mesh point and then use a finite difference scheme capable of perfectly resolving a stationary shock. Although a shock has to be tracked in much the same way as in the case of shock-fitting approach, its treatment as a moving internal boundary is not required. Harten and Hyman (7) devised a self-adjusting grid that, when used with appropriate finite difference schemes, yields perfect shock resolution by ensuring that a shock always lies on a mesh point.

In this paper, the applications of the self-adjusting grid developed by Harten and Hyman (7) to Roe's flux difference splitting scheme for solving one-dimensional transient free surface flows are investigated. Roe's scheme is first-order accurate and achieves conservative splitting of the flux difference through a particular averaging of the flow variables. In the rest of this paper, Roe's scheme is described along with treatment for satisfying energy inequality condition. Details of the self-adjusting grid and its application to Roe's scheme are presented subsequently. Numerical examples are presented that demonstrate improved shock resolution due to the use of the self-adjusting grid. The results are also compared with the second-order accurate TVD Lax-Wendroff scheme.

GOVERNING EQUATIONS

The governing equations for one-dimensional transient free surface flows are statements of the conservation of mass and momentum and can be written as

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \mathbf{S} = 0 \quad ; \qquad \mathbf{E} = \mathbf{J}\frac{\partial \mathbf{U}}{\partial x} \tag{1}$$

based on the assumption of hydrostatic pressure distribution, incompressibility of water and sufficiently small bottom slope of the channel. The vectors for a prismatic channel of arbitrary cross section are defined as

$$\mathbf{U} = \begin{bmatrix} A \\ uA \end{bmatrix} ; \qquad \mathbf{E} = \begin{bmatrix} uA \\ Au^2 + gF_h \end{bmatrix} ; \qquad \mathbf{S} = \begin{bmatrix} 0 \\ -gA(S_o - S_f) \end{bmatrix} \tag{2}$$

where A = cross-sectional area of flow; u = velocity; g = acceleration due to gravity; S_o = bed slope and S_f = friction slope, assumed to be given by a steady-state formula. F_h = hydrostatic pressure force term expressed as

$$F_h = \int_0^h (h - \eta)\, W(\eta)\, d\eta \quad ; \qquad W(\eta) = \frac{\partial A}{\partial \eta} \tag{3}$$

where h = depth of flow; η = integration variable indicating distance from channel bottom and $W(\eta)$ = channel width at distance η from the channel bottom. **J** is the Jacobian of **E** with respect to **U** and is given by

$$\mathbf{J} = \begin{bmatrix} 0 & 1 \\ gA / W(\eta) - u^2 & 2u \end{bmatrix} \tag{4}$$

The governing equations are known to be hyperbolic which implies that **J** has a complete set of independent and real eigenvectors expressed as

$$\mathbf{e}^{1,2} = \begin{bmatrix} 1 \\ u \pm c \end{bmatrix} \quad ; \quad c = \sqrt{\frac{gA}{W(h)}} \qquad (5)$$

where c = celerity. The eigenvalues of **J** are given by

$$\lambda^{1,2} = u \pm c \qquad (6)$$

ROE'S SCHEME ON FIXED GRID

Roe's first-order accurate flux difference splitting scheme for one-dimensional transient free surface flows can be written as

$$\mathbf{U}_i^{t+1} = \mathbf{U}_i^t - \gamma\left[\mathbf{F}_{i+1/2}^t - \mathbf{F}_{i-1/2}^t\right] \qquad (7)$$

where i and t = space and time indices, respectively; $\gamma = \Delta t/\Delta x$; Δt = time increment and Δx = finite difference grid size in space (Fig.1). The source term has been dropped from the present consideration. All variables are computed at known time level t, if not indicated otherwise. $\mathbf{F}_{i+1/2}$ and $\mathbf{F}_{i-1/2}$ are called numerical fluxes and are expressed as

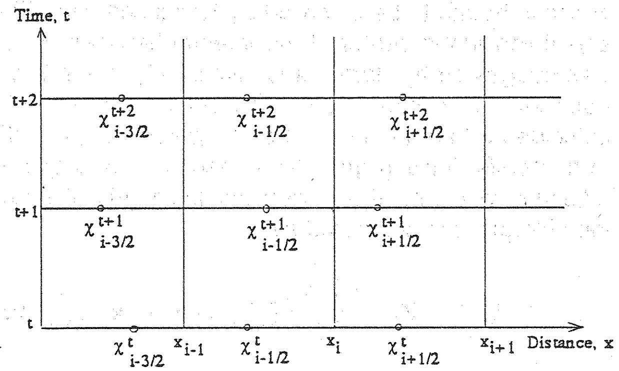


Fig.1 Fixed Grid Underlying Self-adjusting Grid.

$$\mathbf{F}_{i\pm1/2} = 0.5\left(\mathbf{E}_i + \mathbf{E}_{i\pm1}\right) - 0.5\sum_{k=1}^{2}\left|\lambda_{i\pm1/2}^k\right|\alpha_{i\pm1/2}^k\mathbf{e}_{i\pm1/2}^k \qquad (8)$$

α = wave strength, defined as

$$\alpha_{i\pm1/2}^{1,2} = \mathbf{e}_{i\pm1/2}^{-1}\Delta\mathbf{U}_{i\pm1/2} \quad ; \qquad \Delta\mathbf{U}_{i+1/2} = \mathbf{U}_{i+1} - \mathbf{U}_i \qquad (9)$$

The eigenvalues λ and the eigenvectors **e** at points ($i\pm1/2$) are computed from the following averaged variables as suggested by Roe (14)

$$u_{i\pm1/2} = \frac{A_{i+1}^{1/2}u_{i+1} + A_i^{1/2}u_i}{A_{i\pm1}^{1/2} + A_i^{1/2}} \qquad (10)$$

$$c_{i\pm1/2}^2 = g\frac{\Delta\left(F_h\right)_{i\pm1/2}}{\Delta A_{i\pm1/2}} \quad ; \qquad \Delta(\bullet)_{i+1/2} = (\bullet)_{i+1} - (\bullet)_i \qquad (11)$$

Roe and Pike (15) have shown that the above averages are unique in satisfying conservative properties and consistency with the governing equations. However, they violate energy inequality condition in case of a rarefaction wave. The most common remedy for this problem is to replace the modulus of λ in Eq. 8 by a small positive quantity δ whenever the modulus of λ is less than δ. In order to avoid trial procedure for the value of δ, the formula suggested by Harten and Hyman (7) is used which can be written as

$$\delta_{i+1/2} = \max\left[0, \lambda(\mathbf{U}_i, \mathbf{U}_{i+1}) - \lambda(\mathbf{U}_i), \lambda(\mathbf{U}_{i+1}) - \lambda(\mathbf{U}_i, \mathbf{U}_{i+1})\right] \quad (12a)$$

$$\delta_{i+1/2} = \max\left[0, \lambda(\mathbf{U}_{i-1}, \mathbf{U}_i) - \lambda(\mathbf{U}_{i-1}), \lambda(\mathbf{U}_i) - \lambda(\mathbf{U}_{i-1}, \mathbf{U}_i)\right] \quad (12b)$$

SELF-ADJUSTING GRID

Roe's scheme on a fixed grid, as described in the preceding section, can perfectly resolve a shock if that lies at the extremities of the interval between i+1/2 and i-1/2. However, if a shock lies in the interior of this interval, which may often be the case, the shock is bound to be smeared by Roe's scheme. The maximum smearing would occur if a shock lies at the center of the interval between i+1/2 and i-1/2. A solution to this problem was suggested by Harten and Hyman (7) in the form of a variable grid that adjusts itself at each step of computation. The adjustment is such that the location of a shock always coincides with a grid point, thereby resembling a stationary shock. Applying Roe's scheme to this self-adjusting grid yields resolution of a shock as a perfect discontinuity. The self-adjusting grid and the underlying fixed grid is shown in Fig. 1. The end points of the variable grid are computed as

$$\chi^{t+1}_{i\pm1/2} = X_{i\pm1/2} + \left[\sum_{m=-1}^{1}\left(\bar{\chi}^{t+1}_{i+m\pm1/2} - X_{i\pm1/2}\right)\beta_{i+m\pm1/2} f\left(\bar{\chi}^{t+1}_{i+m\pm1/2}; x_i, x_{i\pm1}\right)\right] / \varphi_{i\pm1/2} \quad (13)$$

where $X_{i\pm1/2} = 0.5\left(x_i + x_{i\pm1}\right)$.

The variable grid is derived based on the following conditions;

(i) Mesh end points $\chi_{i\pm1/2}$ coincides with the location of a single admissible discontinuity that originates at time t and ends in the half open set(x_i, $x_{i\pm1}$] at time t+1.

(ii) There is only one variable mesh point in each interval of the underlying fixed grid.

(iii) $\chi^{t+1}_{i+1/2} - \chi^{t+1}_{i-1/2} \geq R_i$,where $R_i = \min\left[x_{i+1} - x_i, x_i - x_{i-1}\right]$, for all i and t if the same is true at the initiation of the computation.

$\bar{\chi}$ is the first-guess value of the interval end points computed as

$$\bar{\chi}^{t+1}_{i\pm1/2} = \chi^{t}_{i\pm1/2} + \Delta t\, v^{t}_{i\pm1/2} \quad (14)$$

where v is the speed of a single discontinuity given by

$$v_{i\pm1/2} = \left\{\sum_{k=1}^{2}\left(\alpha^{k}_{i\pm1/2}\right)^2 \lambda^{k}_{i\pm1/2}\right\} / \beta_{i\pm1/2} \quad (15)$$

where α = wave strength and λ = eigenvalues, as defined in the preceding section. β and the function f are expressed as

$$\beta_{i\pm1/2} = \sum_{k=1}^{2}\left(\alpha^{k}_{i\pm1/2}\right)^2 \tag{16}$$

$$f\left(\chi_{i+m+1/2}; x_i, x_{i+1}\right) = 1 \quad ; x_i < \overline{\chi}_{i+m+1/2} \le x_{i+1}$$
$$= 0 \quad ; \text{otherwise} \tag{17}$$

φ is a weighted amplitude of the waves at time t+1 and is expressed as

$$\varphi_{i+1/2} = \sum_{m=-1}^{1} \beta_{i+m+1/2} f\left(\chi_{i+m+1/2}; x_i, x_{i+1}\right) \tag{18}$$

The interval end points computed by Eq. 13 satisfy first two of the three conditions listed above. It may still violate the third condition. The following treatment overcomes this last problem.

$$\chi^{t+1}_{i\pm1/2} = \widetilde{\chi}^{t+1}_{i\pm1/2} \quad ; d^{t+1}_i > 0.5R_i$$
$$= \widetilde{\chi}^{t+1}_{i\pm1/2} + 0.5R_i\varphi_{i\mp1/2} / \left(\varphi_{i+1/2} + \varphi_{i-1/2}\right) \quad ; \text{otherwise} \tag{19}$$

where

$$d^{t+1}_i = \widetilde{\chi}^{t+1}_{i+1/2} - \widetilde{\chi}^{t+1}_{i-1/2} \tag{20}$$

At each step of computation, new interval end points are computed and the grid is automatically adjusted according to evolving solutions.

ROE'S SCHEME ON SELF-ADJUSTING GRID

A conservative scheme on a variable mesh can be written as (Harten and Hyman(7))

$$\mathbf{U}^{t+1}_i = \left[(\Delta\chi)^t \mathbf{U}^t_i - \Delta t(\widetilde{\mathbf{F}}_{i+1/2} - \widetilde{\mathbf{F}}_{i-1/2})\right] / (\Delta\chi)^{t+1} \quad ; \quad \Delta\chi = \chi_{i+1/2} - \chi_{i-1/2} \tag{21}$$

and the numerical flux for Roe's scheme on the self-adjusting grid is expressed as

$$\widetilde{\mathbf{F}}_{i\pm1/2} = 0.5\,(\widetilde{\mathbf{E}}_i + \widetilde{\mathbf{E}}_{i\pm1}) - 0.5\sum_{k=1}^{2}\left|\widetilde{\lambda}^{k}_{i\pm1/2}\right|\alpha^{k}_{i\pm1/2}\mathbf{e}^{k}_{i\pm1/2} \tag{22}$$

where

$$\widetilde{\mathbf{E}} = \mathbf{E} - v\mathbf{U} \tag{23}$$

$$\widetilde{\lambda}_{i\pm1/2} = \lambda_{i\pm1/2} - v_{i\pm1/2} \tag{24}$$

The corrections for satisfying energy inequality condition is implemented in an identical way as in the case of Roe's scheme on a fixed grid. Eq. 21 yields integral solution of the problem which may be significantly different from the pointwise values at the finite difference nodes in the case of rarefaction waves. Consequently, plotting of these integral values as pointwise values may indicate the existence of several constant states which is entirely a problem of plotting algorithm. The problem can be avoided to some extent by plotting the pointwise values at $\chi_{i+1/2}$ obtained by the following averaging (Harten and Hyman (7))

$$h_{i+1/2} = \left[(\chi_{i+1/2} - x_i)\, h_{i+1} + (x_{i+1} - \chi_{i+1/2})\, h_i\right] / (x_{i+1} + x_i) \qquad (25)$$

NUMERICAL EXAMPLES

All examples consider a 4000 m long rectangular, horizontal and frictionless channel. The initial conditions have been selected so as to generate severe discontinuous flow situations for examining the shock resolution by the method presented herein. Grid size in space for the underlying fixed mesh was set to 10 m, giving a total of 401 nodes along the channel, and the time increment was computed at each step of computation. The Courant number for Roe's scheme on fixed grid and for the TVD Lax-Wendroff scheme was 0.95. However, a Courant number of 0.6 was used for Roe's scheme on the self-adjusting grid as oscillations in the shock front region are not damped out at higher Courant numbers. The details of the TVD Lax-Wendroff scheme can be referred to Alcrudo et al.(1) and Jha et al. (9). For the first two examples, plotted values in the case of the self-adjusting grid are obtained by Eq. 25.

Dam-Break Problem

Two different depths of water are assumed to be separated by a dam placed at 2000 m from either end of the channel. The depth in the reservoir portion is 10 m and that in the tailwater portion is 0.05 m, giving a tailwater depth to reservoir depth ratio of 0.005. The

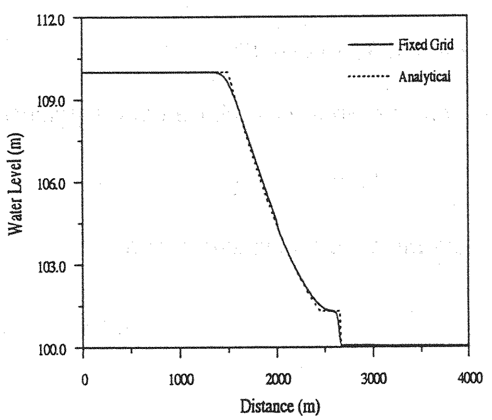


Fig.2 Dam-Break Problem by Roe Scheme on Fixed Grid.

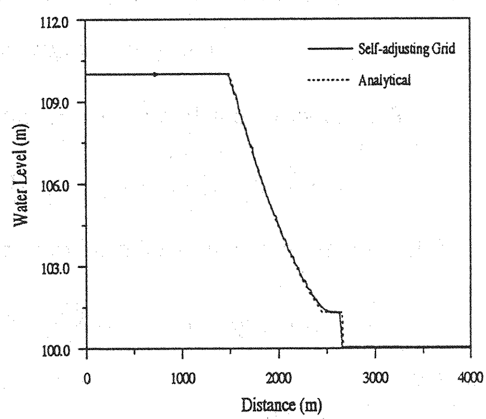


Fig.3 Dam-Break Problem by Roe Scheme on Self-adjusting Grid.

discontinuity in the depth at the middle of the channel is specified as the initial condition which simulates sudden and complete collapse of the dam. The flow downstream of the breach section is very rapid with a vertical shock front leading the flow.

This problem was simulated by Roe's scheme on the fixed grid and on the self-adjusting grid and the results were obtained at 50 seconds. Figs. 2 and 3 show the water surface profiles along the channel obtained using the fixed grid and the self-adjusting grid, respectively, in comparison with the analytical solution. It can be seen from Fig. 2 that Roe's scheme on the fixed grid causes diffusion both in the shock front and in the negative wave. Roe's scheme on the self-adjusting grid captures both the negative wave and the shock front significantly better than Roe's scheme on the fixed grid (Fig.3). The negative wave shows almost no diffusion and the shock front is resolved within one grid. The water surface profile is not very smooth due to plotting of the averaged value of the integral solution.

Collision of Shock Waves

A simulated head-on collision of shock waves is presented in this example. The initial conditions in the channel are as given below

$$h(x,0) = 100\text{ m} \; ; \quad 0 \le x \le 1000\text{ m}$$
$$= 1\text{ m} \; ; \quad 1000 < x \le 3000\text{ m}$$
$$= 50\text{ m} \; ; \quad 3000 < x \le 4000\text{ m}$$

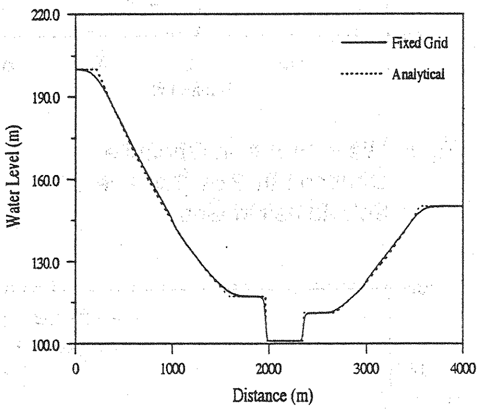


Fig.4 Wave Fronts in Opposite Direction by Roe Scheme on Fixed Grid.

Fig. 4-6 show the results at 25 seconds by Roe's scheme on fixed grid, on self-adjusting grid and the TVD Lax-Wendroff scheme, respectively, along with the analytical solution. At this time the shock fronts moving in the opposite directions are facing each other. Roe's scheme on fixed grid diffuses the negative wave and the shock front while Roe's scheme on the self-adjusting grid captures the shock with high resolution. Although first-order accurate, Roe's scheme yields results comparable to the second-order accurate TVD Lax-Wendroff scheme, both in terms of accuracy and shock resolution.

At about 31 seconds the two shock fronts collide into each other and form a complex flow structure with a widening water column around the location of collision. Considering the solution by the TVD Lax-Wendroff scheme to be the most accurate, a comparison is made in Figs.7 and 8 with solutions by Roe's scheme on fixed grid and on the self-adjusting grid, respectively. Slight diffusion is evident in case of Roe's scheme on the fixed grid (Fig.7). Solution by Roe's scheme on the self-adjusting grid, with perfect shock resolution (Fig.8), is almost indistinguishable from the TVD Lax-Wendroff scheme.

Bore Propagation

Propagation of bore on still water and one bore propagating over the other is analyzed in this example. The initial condition in the channel is specified as 1 m deep still water throughout the channel. The downstream end of the channel is kept closed. Two successive bores are introduced from the upstream end with the following upstream boundary conditions.

$$Q(0,t) = 50\ m^3 / s \text{ per unit width} \quad ;\ 0 \le t \le 100 \text{ seconds}$$
$$= 100\ m^3 / s \text{ per unit width} \quad ;\ t > 100 \text{ seconds}$$

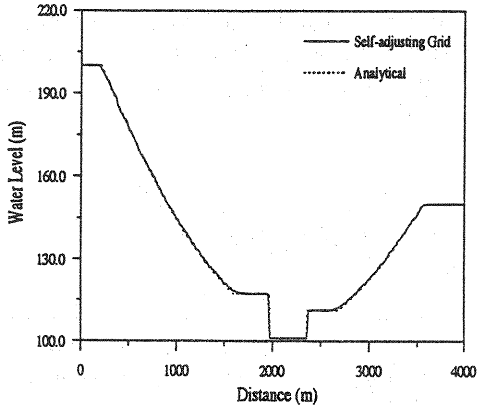


Fig.5 Wave Fronts in Opposite Direction by Roe Scheme on Self-adjusting Grid.

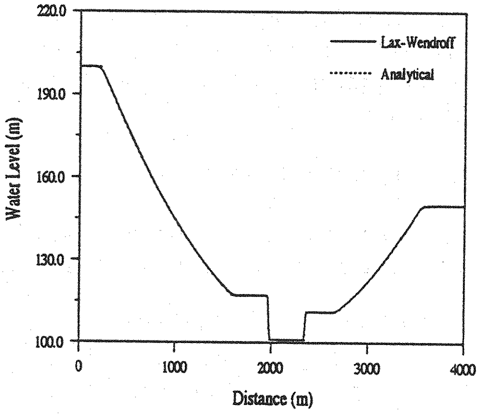


Fig.6 Wave Fronts in Opposite Direction by TVD Lax-Wendroff Scheme.

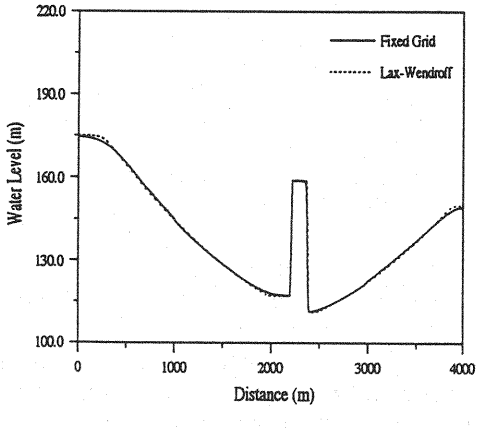


Fig.7 Collision of Wave: Roe's Scheme on Fixed Grid.

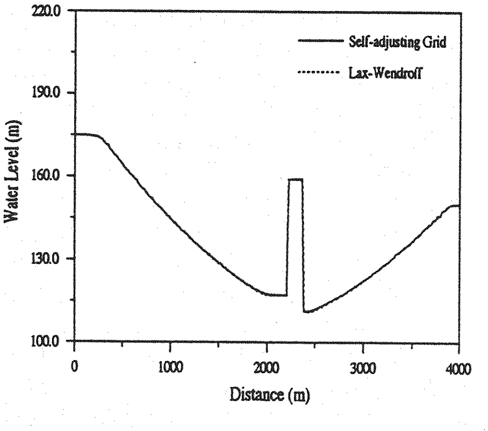
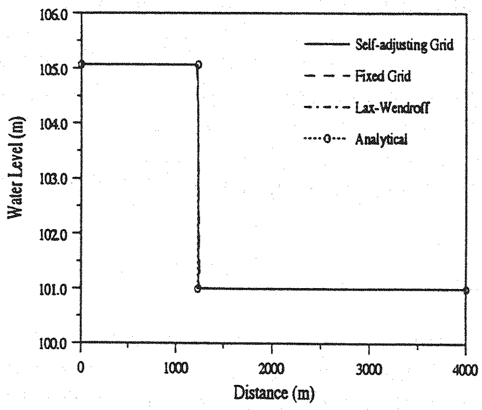


Fig.8 Collision of Wave: Roe's Scheme on Self-adjusting Grid.

Fig. 9(a) shows the solutions at 100 seconds, just before another bore is introduced at the upstream end. At this time a single bore is seen traveling over the still water. In order to examine the shock resolution by various schemes more clearly, an enlarged view of the region near the bore front is presented in Fig. 9(b). As expected, Roe's scheme on fixed grid exhibits maximum diffusion. It is interesting to see that Roe's scheme on the self-adjusting grid captures the front with higher resolution than the second-order accurate TVD Lax-Wendroff scheme. Roe's scheme on self-adjusting grid resolves the front within one grid.



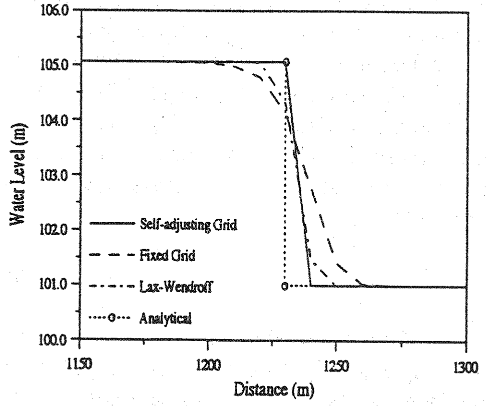
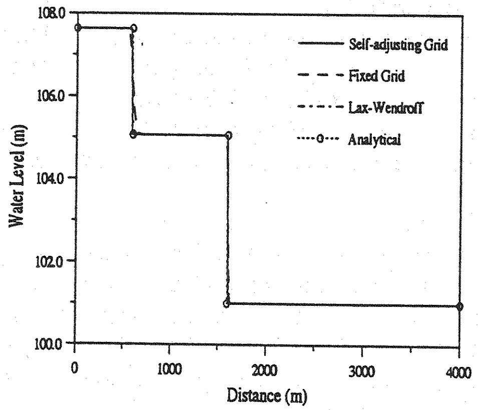


Fig.9(a) Bore Propagation on Still Water.

9(b) Front Details.



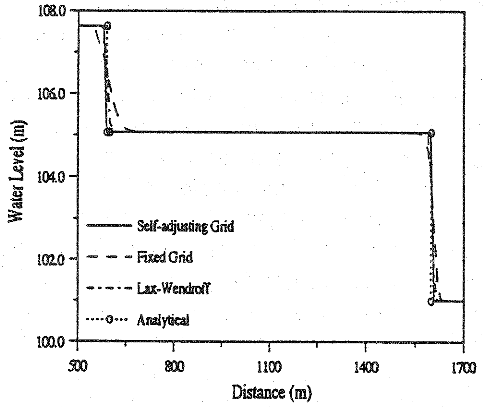
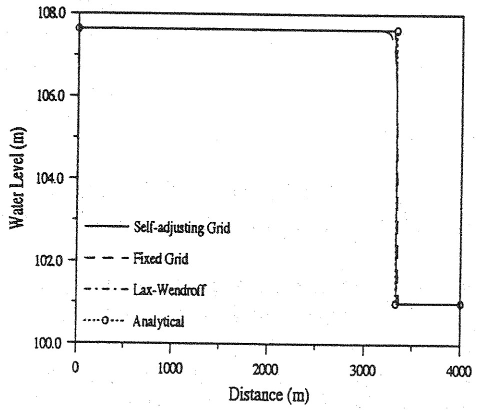


Fig.10(a) Bore Propagation Over Another Bore.

10(b) Front Details



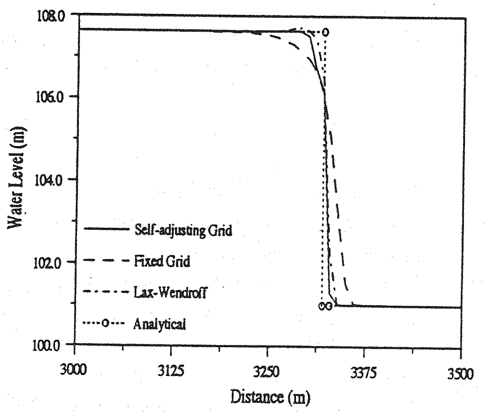


Fig.11(a) Overtaking of Bore.

11(b) Front Details

Fig. 10(a) shows the results at 130 seconds. The second bore is seen traveling over the first bore. A close-up of the bore front is shown in Fig. 10(b). The comparative resolution of different methods used remain the same as before, Roe's scheme with the self-adjusting grid providing the best shock resolution. The second bore moves faster than the first bore due to its higher celerity and eventually overtakes the first bore. Thereafter it travels as a single bore. Fig. 11(a) shows results at 270 seconds, soon after the second bore overtakes the first bore. An enlarged view of the front region is shown in Fig. 11(b) which indicates similar comparative features as in the case of results at 100 and 130 seconds.

CONCLUSIONS

Roe's flux difference splitting scheme on the self-adjusting grid developed by Harten and Hyman (7) has been investigated for applications to one-dimensional transient free surface flows. The grid adjusting procedure, that proceeds by averaging the local characteristic velocities with respect to the signal amplitude, reduces the diffusion caused by averaging procedure when Roe's scheme is used with fixed grid. The treatment for satisfying the energy condition has also been incorporated.

Several test problems have been analyzed for examining the effect of self-adjusting grid. Numerical examples indicate that the use of the self-adjusting grid significantly improves the shock resolution of Roe's scheme. In many cases, Roe's scheme with the self-adjusting grid yields shock resolution better than the second-order TVD Lax-Wendroff scheme. The overall accuracy of Roe's scheme is comparable to the TVD Lax-Wendroff scheme in all cases considered.

The underlying fixed grid does not necessarily have to be uniform. It may be adjusted according to the evolving solution to obtain enhanced resolution, if severity of a problem so demands. On the other hand, better resolution can also be achieved by incorporating the self-adjusting grid into a second-order accurate scheme. However, such improvements would require more computation time and for most practical problems of one-dimensional transient free surface flows, the accuracy provided by the methods presented in this paper may be sufficient.

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APPENDIX - NOTATION

The following symbols are used in this paper :

A = cross-sectional area of flow;
c = celerity;
d = grid size on self-adjusting grid;
$\mathbf{E}$ = flux matrix on fixed grid;
$\tilde{\mathbf{E}}$ = flux matrix for Roe's scheme on self-adjusting grid;
$\mathbf{e}$ = matrix of eigenvalues;
$\mathbf{F}$ = numerical flux on fixed grid;
$\tilde{\mathbf{F}}$ = numerical flux for Roe's scheme on self-adjusting grid;
f = function of χ;
F_h = hydrostatic pressure force term;

g	= acceleration due to gravity;
h	= flow depth;
i	= grid location in space;
J	= Jacobian of **E** with respect to **U**;
k	= wave number;
Q	= discharge;
R	= minimum of two adjacent intervals on fixed grid;
S	= matrix containing source terms;
S_f	= friction slope;
S_o	= bed slope;
t	= index for time;
U	= vector for flow variables;
u	= velocity;
v	= speed of single discontinuity;
$W(\eta)$	= channel width at distance η from channel bottom;
x	= distance along channel;
α	= wave strength;
β	= sum of square of α;
χ, $\tilde{\chi}$	= interval end-points on self-adjusting grid;
$\bar{\chi}$	= first guess value of interval end-points on self-adjusting grid;
Δ	= operator, i.e. $\Delta f_{i+1/2} = f_{i+1} - f_i$;
δ	= small positive quantity;
γ	= $\Delta t/\Delta x$;
η	= integration variable indicating distance from channel bottom;
φ	= weighted amplitude of the waves;
λ	= eigenvalues of **J**; and
$\tilde{\lambda}$	= eigenvalues of **J** for Roe's scheme on self-adjusting grid.