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# ON THE DESIGN OF THE SELF BASING LINEAR WEIR USING THE SOLUTION OF PROPORTIONAL THREE-HALVES POWER WEIR

By

K. Keshava Murthy

Indian Institute of Science, Bangalore, India

and

G. N. Virupaksha

Indian Institute of Science, Bangalore, India

## SYNOPSIS

In this paper a new design of a self basing linear weir is presented using the solution of the proportional three-halves weir. The profile of the proportional three-halves weir has the special property of approaching the shape of rectangular weir after a small initial value. This is used as an SBL-2 type generating function to arrive at a self basing linear weir.

It is shown theoretically that the exact linear weirs with bases can be modified into a self basing linear weirs by the addition of a correcting function of the form $\alpha/(1+\beta x)^{1/2}$, where $\alpha$ and $\beta$ are constants. It is shown that the SBL weir designed in this paper has slight advantages over the SBL weir designed earlier utilizing the profile of a quadratic weir, since it has higher sensitivity and a slightly greater coefficient of discharge.

Experiments with two typical weirs confirm the theory by giving a constant average coefficient of discharge of 0.64.

## INTRODUCTION

One important outcome of the investigations carried out on the geometrically simple linear weirs namely the inverted V-notch (IVN) and Bell-Mouth (BM) weirs (1,2), is the emergence of the linear weir defined by a single profile. It should be underlined here that for the IVN and BM weirs, a portion of the profile above the weir crest, itself, acts as the base for the weirs. In other words the base weir becomes an integral part of the whole weir. Hence these weirs are appropriately called "Self Basing Linear Weirs". One of the main draw backs of the IVN and the BM weirs outlined above is that they have a limited range of head which inhibits their wide use. The exact solutions for the SBL weir was given by Stout (6) which is physically unrealizable.

Recently Keshava Murthy and Giridhar (7,8) have developed a new concept of self basing linear weir (SBL weir), where the base weir is an integral part of the whole weir. This is unlike the Sutro weir and other exact linear weirs (3,4,5) which are essentially compound weirs having a base weir and a complimentary weir defined by two separate equations. This has been achieved by utilizing the special characteristics of the complimentary weir profile of the quadratic weir (9). The significant feature of this weir is that the profile of the complimentary weir is continuously decreasing and fastly convergent and its width tends to zero correct to third decimal place after a small height. The function defining this profile is utilized as SBL-2 type generating function (7,8) to get the head-discharge function of the SBL weir. By choosing a proper co-ordinate system it is possible to obtain a graph of dQ/dH vs H (where Q = discharge at height H) in which the initial portion of the graph is non-linear for a small height d and becomes nearly constant after this height d. In other words, dQ/dH is constant for all practical purposes in the range $d \le h \le \infty$ and the discharge Q is proportional to the linear power of head H. Utilizing this special feature of the quadratic weir profile, Keshava Murthy and Giridhar (7,8) have arrived at an SBL weir defined by single profile. This weir gives discharge proportional to the linear power of head, for all flows in the range $d \le h \le \infty$ where heads are reckoned above a reference plane. It has been shown that the accuracy of the linear discharge-head relationship increases as head increases.

In this paper, another solution for SBL weir is presented by exploiting the special property of the proportional three-halves power weir (10). The proportional three-halves power weir designed with bases produce for all flows above the base, discharge proportional to the three-halves power of head reckoned above datum (see Fig.1). The significant feature of these weirs is that their complimentary profiles quickly approach the shape of a rectangular weir (i.e., the width tends to a constant value after a small height). Hence the function defining the profile of this weir, with the proper choice of a co-ordinate system can be gainfully utilized as the SBL-2 type generating function in order to get another SBL weir. It is found that, the SBL weir derived from this generating function is associated with a reference plane, above which all head are reckoned. It is proved theoretically that, the conventional linear weirs can be modified to SBL weir (designed by using the exact solutions of proportional three-halves weirs) by the addition of a simple correcting function of the form $\alpha/(1+\beta x)^{1/2}$, where $\alpha$ and $\beta$ are constants.

## 1. Special Characteristics of the Head Discharge Function of the SBL Weir

The discharge function of the SBL weir, in addition to satisfying certain basic conditions (8), should also satisfy another property namely it should tend to become linear very rapidly after a small threshold depth or base depth, d, the variation from linearity becoming smaller as the head increases. In other words, the error in replacing this head-discharge relationship by a linear one should rapidly and continuously decreases as head 'h' increases.

### 1.1 Choice of the Generating Function

It is seen from the design of the proportional three-halves power weir (10) (see Fig.1), that the complementary weir profile, quickly reaches a constant value after a certain initial value leading to two parallel straight edges as in he rectangular weir. By shifting the origin to O' (see Fig.1), it is seen that the ordinate become almost constant in the range $d \le h \le \infty$ , after an initial value of d. Hence this function can be taken as a generating function of the SBL-2 type to obtain the H-Q function of a SBL weir.

## 2. Formation of the Head - Discharge Function

The function defining the profile of the parabolic base proportional three-halves power weir (10) (see Fig.1) is,

$$f(x) = y = W\left[ \sqrt{1+\frac{x}{a}} - \sqrt{\frac{x}{a}} + \frac{1}{\sqrt{3}} \tan^{-1} \sqrt{\frac{4x}{3a}} \right] \quad (1)$$

where x and y = the vertical and horizontal co-ordinates, W = half width of the weir at the origin, a = the base weir depth. It is seen that the above function rapidly approaches a constant value after a small depth. By shifting the origin to O' (see Fig.1a), the profile can be expressed as,

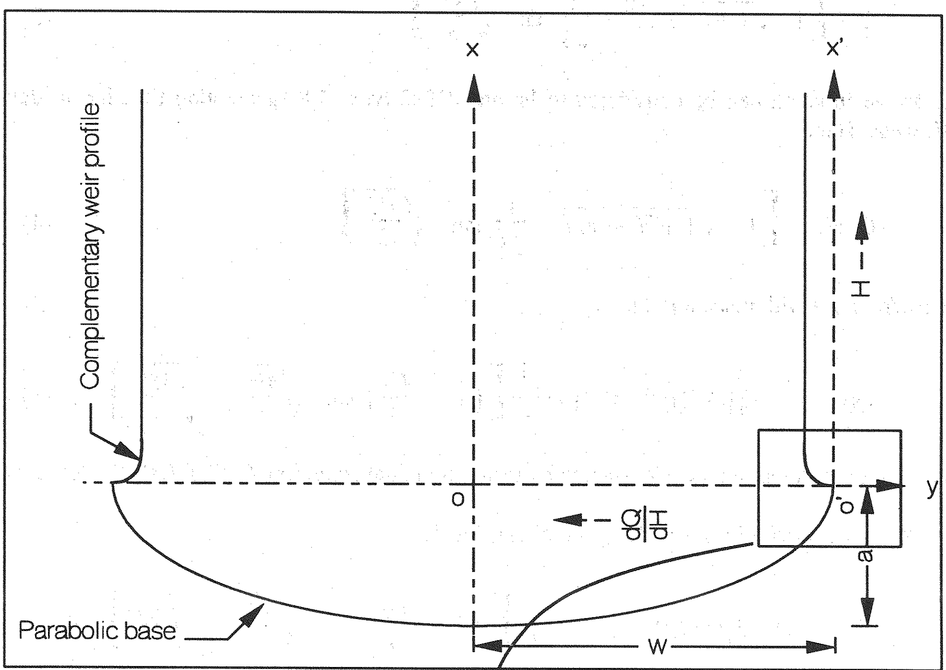


Fig.1a Typical Parabolic based Proportional three-halves power weir

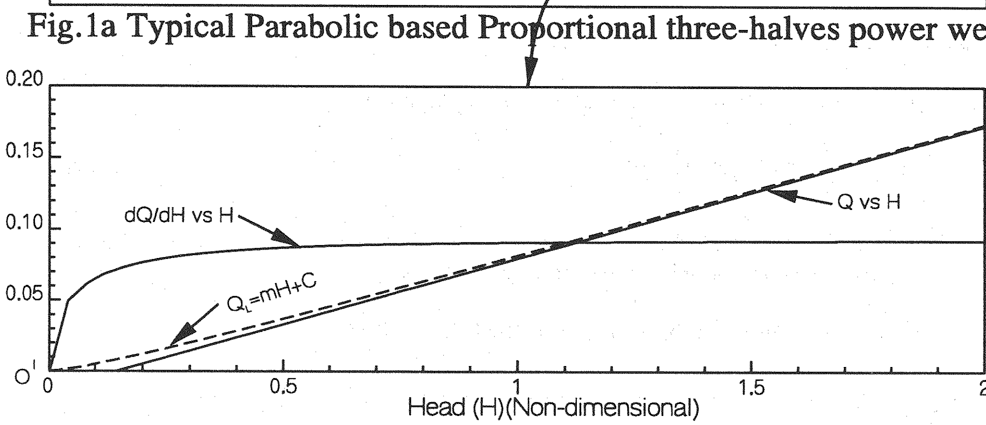


Fig. 1b Plot showing Q and dQ/dH vs H

$$y' = w\left[ 1 - \sqrt{1+\frac{x'}{a}} + \sqrt{\frac{x'}{a}} - \frac{1}{\sqrt{3}} \tan^{-1} \sqrt{\frac{4x'}{3a}} \right] \quad (2)$$

Non-dimensionlizing Eq.2 by taking Y=y'/W and X=x'/a, we get

$$Y = \left[ 1 - \sqrt{1+X} + \sqrt{X} - \frac{1}{\sqrt{3}} \tan^{-1} \sqrt{\frac{4X}{3}} \right] \quad (3)$$

The above function can be considered to be an SBL-2 type (7,8) generating function to develop the SBL weir. Hence

$$Q'(H) = \left[ 1 - \sqrt{1+H} + \sqrt{H} - \frac{1}{\sqrt{3}} \tan^{-1} \sqrt{\frac{4H}{3}} \right] \quad (4)$$

Integrating Eq.4 with respect to H,

$$Q(H) = H - \frac{2}{3}[(1+H)^{3/2} - H^{3/2}] - \frac{\sqrt{3}}{4}\left[ \left(1 + \frac{4H}{3}\right) \tan^{-1} \sqrt{\frac{4H}{3}} - \sqrt{\frac{4H}{3}} \right] + c \quad (5)$$

The constant of integration c is evaluated by using the initial condition, H=0, Q(H)=0. Hence c=2/3.

Therefore, the required primary H-Q function Q(H) is,

$$Q(H) = H - \frac{2}{3}[(1+H)^{3/2} - H^{3/2}] - \frac{\sqrt{3}}{4}\left[ \left(1 + \frac{4H}{3}\right) \tan^{-1} \sqrt{\frac{4H}{3}} - \sqrt{\frac{4H}{3}} \right] + \frac{2}{3} \quad (6)$$

The above discharge curve is shown in Fig.1b.

Expanding the terms in right hand side of Eq.6 and simplifying, we get,

$$Q(H) = \frac{2}{9} H^{3/2} - 2H^2 - \frac{8}{45} H^{5/2} + \dots \quad (7)$$

From Eq.7, it is observed that, the least power of head H in the H-Q function is 3/2 and hence the designed SBL weir will have a finite, non-zero crest width (7,8).

2.1 Derivation of the Function $f_s(X)$ Defining the Self Basing Profile

The basic discharge equation for the flow through the self basing weir (see Fig.2) is

$$q = 2\, C_d \sqrt{2g} \int_0^h \sqrt{h-x}\, f_s(x)\,dx \quad (8)$$

where, q = discharge or rate of flow, h = head over the weir crest, g = gravitational acceleration and $C_d$ = coefficient of discharge.

The coefficient of discharge $C_d$ is assumed to be constant for sharp-crested weirs and streamlined flows. $C_d$ is a function of several parameters, including the head and crest height from the bottom of the channel. It has been determined from experiments that the variation of $C_d$ will generally be well within ±1% of the average $C_d$. Non-dimensionalizing Eq.8, we have,

$$Q(H) = \int_0^H \sqrt{H-X}\, f_s(X) dX \quad (9)$$

where, $f_s(X) = f_s(x)/W$, $H = h/a$, $X = x/a$, $Q = q/2WC_d \sqrt{2g}a^{3/2}$, W = half top width of the base weir of the proportional three-halves power weir and a = base weir depth of the proportional three-halves power weir.

Substituting for Q(H) from Eq.6

$$\int_0^H \sqrt{H-X}\, f_s(X) dX = H - \frac{2}{3}[(1+H)^{3/2} - H^{3/2}] - \frac{\sqrt{3}}{4}\left[\left(1+\frac{4H}{3}\right) \tan^{-1}\sqrt{\frac{4H}{3}} - \sqrt{\frac{4H}{3}}\right] + \frac{2}{3} \quad (10)$$

Differentiating Eq.10 with respect to H using Leibnitz's rule (11),

$$\int_0^H \frac{f_s(X)}{\sqrt{H-X}} dX = 2\left[1 - \sqrt{1+H} + \sqrt{H} - \frac{1}{\sqrt{3}} \tan^{-1}\sqrt{\frac{4H}{3}}\right] = Q'(H) \quad (11)$$

Eq.11 is in the form of Abel's integral equation and the solution of which is given by (11),

$$f_s(X) = \frac{2}{\pi}\int_0^X \frac{Q''(H)}{\sqrt{X-H}} dH \quad (12)$$

Substituting Q"(H) from Eq.11 and integrating, we get,

$$f_s(X) = 1 - \frac{2}{\pi} \tan^{-1} \sqrt{X} - \frac{2}{\sqrt{9+12X}} \quad (13)$$

$$f_s(X) = W\left[1 - \frac{2}{\pi} \tan^{-1} \sqrt{\frac{x}{a}} - \frac{2}{\sqrt{9+12x/a}}\right] \quad (14)$$

Eq.14 defines the profile of the SBL weir, which has Q(H) as its H-Q relationship which is almost linear after a small base depth or threshold depth. The accuracy to which it is linear can be best explained by comparing this theoretical H-Q relationship with an exact linear H-Q relationship. The profile given by Eq.14 is shown in Fig.2.

## 3. The Exact Linear H-Q Relationship of the Designed SBL Weir

It is found that after a small threshold depth d, the H-Q relationship is almost linear. This linear H-Q relationship can be replaced by the exact linear H-Q relationship. The theoretical H-Q graph approximates the asymptote and the deviation of the value given by the tangent at infinity from that given by the h-q graph becomes increasingly negligible (see Fig.3).

Let, $Q_L$ represent the asymptote.

$$\text{Let } Q_L = mH + C \quad (15)$$

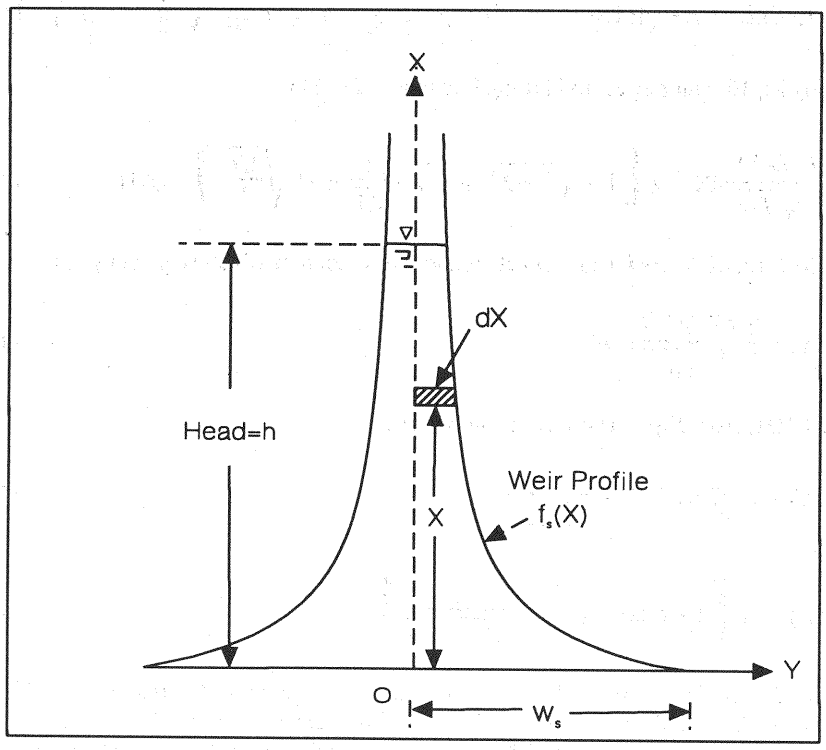


Fig.2 Definition Sketch of the Proposed Weir

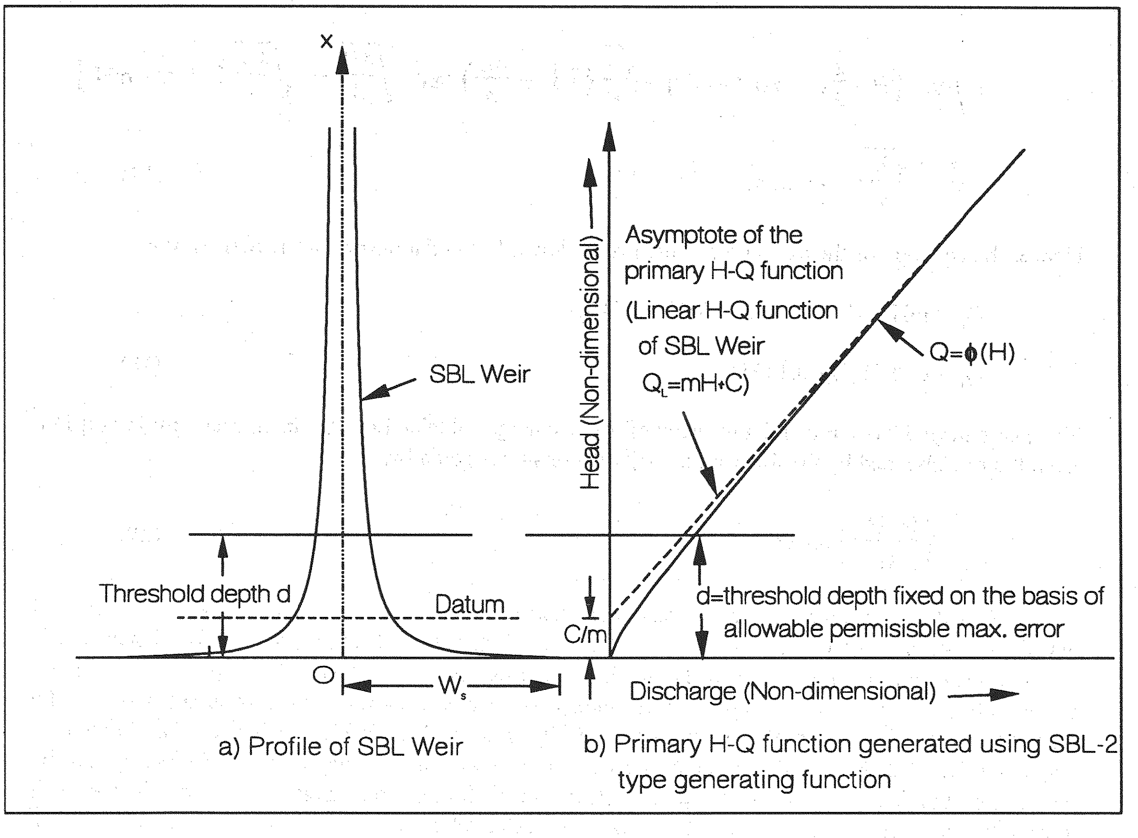


**Fig.3 Linear Characteristics of the SBL weir**

represent the asymptote, where m and C are constants. The constants m and C can be found out as,

$$m = \lim_{H\to\infty} \frac{dQ}{dH}$$

$$m = \lim_{H\to\infty} \left[1 - \sqrt{1+H} + \sqrt{H} - \frac{1}{\sqrt{3}} \tan^{-1} \sqrt{\frac{4H}{3}}\right]$$

$$= 1 - \frac{\pi}{2\sqrt{3}} = 0.0931 \qquad (16)$$

and $C = \lim_{H\to\infty} (Q - mH)$

$$= \lim_{H\to\infty} \left[H - \frac{2}{3}\{(1+H)^{3/2} - H^{3/2}\} - \frac{\sqrt{3}}{4}\left\{\left(1 + \frac{4H}{3}\right) \tan^{-1} \sqrt{\frac{4H}{3}} - \sqrt{\frac{4H}{3}}\right\} + \frac{2}{3} - mH\right]$$

$$= \frac{2}{3} - \frac{\sqrt{3}\pi}{8} = -0.0135 \qquad (17)$$

Hence, the equation of the asymptote, which is the linear H-Q relationship of the SBL weir is,

$$Q_L = mH + C \qquad = 0.0931H - 0.0135$$

$$Q_L = 0.0931(H - 0.1451) \qquad (18)$$

The percentage deviation or relative error of the discharge, obtained by the linear relationship (Eq.18), from the one obtained by the theoretical H-Q relationship is given by,

$$e = \left|\frac{Q - Q_L}{Q}\right| \text{X } 100 \qquad (19)$$

This deviation continuously decreases as head increases (see Fig.4) and further, it is observed that the deviation is nearly 0.1% at H = 4.7. Beyond this for al practical purposes, the designed weir is as accurate as an exact linear weir. If the maximum permissible error taken as 1.5% (usually ±2 percent is normally allowed (12)), the depth corresponding to maximum permissible error is analogous to the flow through the base weir of any existing exact linear weir. This depth is called the 'base depth' or 'threshold depth' d. For all flows above the threshold depth, the theoretical H-Q relationship can be replaced by the linear H-Q relationship. Therefore, for all flows in the range $d \le h \le \infty$ , the weir will have a discharge proportional to the head measured above reference plane or datum situated at '0.145a' above the weir crest (see Fig.4b). The linearity law is valid for al heads $h \ge d$.

Five other SBL weirs were designed using the same method as discussed above using different proportional three halves power weir functions. Table 1 gives, the details of the profiles of these weirs along with their linear characteristics.

## 4. Universalisation of the Co-ordinates of Designed SBL Weir

It would be advantageous to derive the profile equation of the SBL weir, having universal co-ordinates with respect to its dimensional parameters namely its crest width "$2W_s$" and threshold depth "d". These parameters are related to the top width of the base weir "2W" and the depth of the

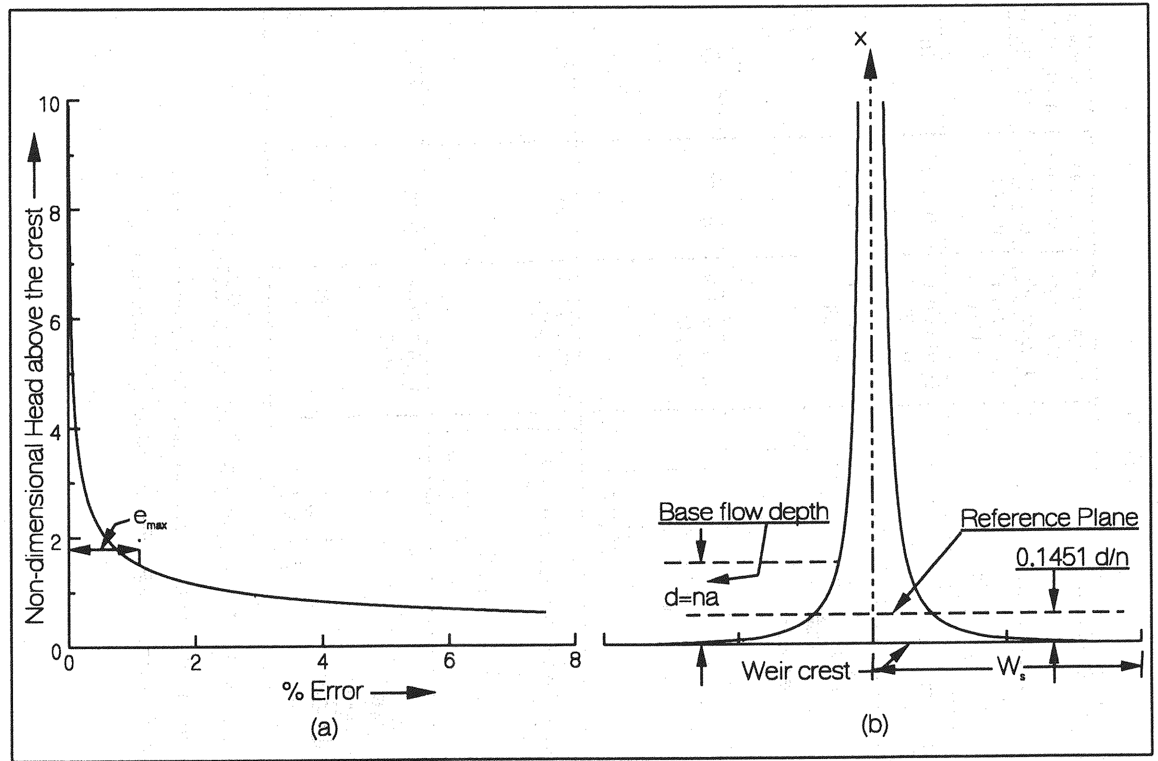


**Fig.4 Parameters of the designed SBL weir**

**Table 1.**

| Sl. no. | SBL Weir Profile Equation. | Linear Head-Discharge Relationship | Datum above the weir Crest | Threshold depth (1.5% Max.Error) | 0.1% Error at |
|---|---|---|---|---|---|
| 1. | $Y = 1-\frac{2}{\pi}\tan^{-1}\sqrt{X}-\frac{2}{\sqrt{9+12X}}$ | $Q_L$=0.0931(H-0.1451) | 0.1451 | 1.35 | 4.70 |
| 2. | $Y = \frac{4}{\pi}\left[\sqrt{1+X}-\sqrt{X}-\frac{5}{3\sqrt{3}\sqrt{3+5X}}\right]$ | $Q_L$=0.1393(H-0.1177) | 0.1177 | 1.10 | 3.55 |
| 3. | $Y_1 = \frac{2}{\pi}\left[\frac{4}{3}\left(1-\frac{2}{\pi}\tan^{-1}\sqrt{X}\right)+\frac{2}{3\sqrt{3}\sqrt{3+2X}}-\frac{1}{\sqrt{1+X}}\right]$ | $Q_{1L}$=0.1209(H-0.2138) | 0.2138 | 1.95 | 6.30 |
| 4. | $Y_1 = \frac{2}{\pi}\left[\frac{5}{3}\sqrt{1+X}-\sqrt{X}+\frac{8}{9\sqrt{3}\sqrt{3+2X}}-\frac{1}{\sqrt{1+X}}\right]$ | $Q_{1L}$=0.1962(H-0.1988) | 0.1988 | 1.85 | 5.95 |
| 5. | $Y_1 = \frac{2}{\pi}\left[\frac{1}{\sqrt{3}\sqrt{3+2X}}+2(1+X)(1-\tan^{-1}\sqrt{X})-\frac{4\sqrt{X}}{\pi}-\frac{1}{\sqrt{1+X}}\right]$ | $Q_{1L}$=0.2570(H-0.1871) | 0.1871 | 1.75 | 5.90 |
| 6. | $Y_1 = \frac{2}{\pi}\left[\frac{16}{15\sqrt{3}\sqrt{3+2X}}+\frac{7(1+X)^{3/2}}{3}-\frac{7\sqrt{X}(3+2X)}{6}-\frac{1}{\sqrt{1+X}}\right]$ | $Q_{1L}$=0.3104(H-0.1776) | 0.1776 | 1.70 | 6.90 |

**Note :** $Y=y/W$, $X=x/a$, $Y_1=y/W_1$, $Q_L=q/2C_d\sqrt{2g}\,Wa^{3/2}$, $Q_{1L}=q/2C_d\sqrt{2g}\,W_1a^{3/2}$;

$W_1$= Half Bottom width of Base weir of Proportional three-halves power weir

W = Half Top width of Base weir of Proportional three-halves power weir base weir "a" of the proportional three halves power weir, whose complementary weir profile equation is used as the generating function. From Eq.14, the half crest width $W_s = f_s(0) = W/3$. The threshold depth (d) depends on the maximum permissible value of percentage error that would be fixed (this value should not exceed ±2 percent). Taking this depth d = na (n is a constant) (see Fig.4b), the equation defining the profile of the SBL weir can be written in the non-dimensional form as,

$$Y = 3\left[1 - \frac{2}{\pi}\tan^{-1}\sqrt{nX} - \frac{2}{\sqrt{9+12nX}}\right] \qquad (20)$$

where $Y = y/W_s$ and $X = x/d$

The corresponding non-dimensional linear H-Q relationship is,

$$Q_L = \frac{0.2793}{n^{3/2}}(nH - 0.1451) \qquad (21)$$

where $Q_L = \dfrac{q}{2C_d\sqrt{2g}\,W_s d^{3/2}}$ and $H = h/d$

5. Sensitivity of the Designed Weir

Sensitivity ($\sigma$) of the weir is defined as the elementary increase in head over the elementary increase in the rate of flow (12). The sensitivity of the earlier designed weir (Keshava Murthy and Giridhar) is unity. For the designed weir $\sigma$ is obtained from Eq.18 as

$$\sigma = \frac{dQ}{dH} = \frac{1}{m} = 10.74 \qquad (22)$$

In general the sensitivity for any other SBL weir designed using the profile of the proportional three-halves weir given in the Table 1 can be obtained as $\sigma = 1/m$. (23)

Hence it is seen that the sensitivity of the designed weir is very high relative to the earlier designed SBL weirs using the quadratic weir profiles as the generating function (Keshava Murthy and Giridhar).

6. Theorem on Modifying Conventional, Linear Weirs and Bases into SBL Weirs

In this paper, it is shown that the exact linear weirs can be modified into a SBL weir (designed by suing the complementary weir functions of proportional three halves power weir) by the addition of a simple correcting function of the form $\alpha/(1+\beta x)^{1/2}$, where $\alpha$ and $\beta$ are constants. This is shown by the following theorem and proved.

Theorem

"An exact linear weir designed over a base of the form $y_1 = W(x_1/a)^M$ ($M = 0, 1/2, 1, 3/2$, and $[0 \le x_1 \le a]$), can be modified into a SBL weir, whose ordinates are proportional to the sum of the ordinates of the linear weir (with base) and a correcting function in the form of $\alpha/(1+\beta x)^{1/2}$".

Proof

The profile equation of an exact linear weir (3,4) designed over a finite orifice of depth 'a' defined by $y_1 = W(x_1/a)^M$ ($0 \le x_1 \le a$, $M = 0, 1/2, 1, \ldots$) in non-dimensional form is,

$$f_L^*(X) = \frac{2\Gamma(M+1)}{\sqrt{\pi}\,\Gamma(M+\frac{1}{2})}\int_0^l \frac{d}{dX} H_1^{\left(m-\frac{1}{2}\right)} \sqrt{X+1-H_1}\, dH_1 \tag{24}$$

where, $\Gamma$ refers to the well known Gamma function, $H_1 = H+1$, $H = h/a$ and $X = x/a$.

Let $f_L(X)$ and $f_T(X)$ be the linear weir and proportional three halves power weir profiles of any given base respectively and Q(H) the discharge through the base weir (acting as orifice) for all heads above the base weir. The discharge through the linear and proportional three halves power weirs are given by,

$$Q(H) + \int_0^H f_L(X)\sqrt{H-X}\, dX = B_L(H+\lambda_L) \tag{25}$$

and

$$Q(H) + \int_0^H f_T(X)\sqrt{H-X}\, dX = B_T(H+\lambda_L)^{3/2} \tag{26}$$

where $B_L$ and $B_T$ = the proportionality constants, and $\lambda_L$ and $\lambda_T$ = the datum constants respectively. These constants are evaluated using the discharge-continuity and the slope discharge continuity theorems (9).

From Eqs.25 and 26,

$$\int_0^H [f_L(X) - f_L(X)]\sqrt{H-X}\, dX = B_L(H+\lambda_L) - B_T(H+\lambda_T)^{3/2} \tag{27}$$

Differentiating Eq.27 with respect to H using Leibnitz's rule,

$$\int_0^H \frac{(f_L(X) - f_T(X))}{\sqrt{H-X}} dX = 2\,[B_L - \frac{3}{2}\, B_T\, (H+\lambda_T)^{1/2}] = \psi(H) \tag{28}$$

Eq.28 is in form of Abel's integral equation and the solution of which is,

$$[f_T(X) - f_L(X)] = \frac{1}{\pi}\int_0^X \frac{\psi'(H)}{\sqrt{X-H}}\, dH \tag{29}$$

Substituting for $\psi'(H)$ from Eq.28 and integrating,

$$f_L(X) = f_T(X) + \alpha' \tan^{-1} (\beta' X)^{1/2} \tag{30}$$

where $\alpha'$ and $\beta'$ are constants which depend on the shape of the base weir.

Eq.30 defines the complementary weir profile of the proportional three halves power weir, designed over a base $y_1 = W(x_1/a)^N$. This can be used as an SBL-2 type generating function to arrive at a SBL weir. Choosing,

$$Q'(H) = 1 - f_L(X) + \alpha' \tan^{-1} (\beta' X)^{1/2} \tag{31}$$

using Eq.24,

$$f_L(H) = \frac{2\Gamma(N+1)}{\sqrt{\pi}\,\Gamma(N+\frac{1}{2})}\int_0^l \frac{d}{dH}\left[X_1^{\left(N-\frac{1}{2}\right)} \sqrt{H+1-X_1}\, dX_1\right] \tag{32}$$

Substituting Eq.32 in Eq.31, we have,

$$Q'(H) = 1 - \frac{2\Gamma(N+1)}{\sqrt{\pi}\,\Gamma(N+\frac{1}{2})}\int_0^l \frac{d}{dH}[X_1^{\left(N-\frac{1}{2}\right)}\sqrt{H+1-X_1}\,dX_1] - \alpha' \tan^{-1}\sqrt{\beta' H} \qquad (33)$$

Integrating Eq.33 with respect to H, we get the primary H-Q function of SBL weir,

$$Q(H) = H - \frac{2\Gamma(N+1)}{\sqrt{\pi}\,\Gamma(N+\frac{1}{2})}\int_0^l \frac{d}{dH}[X_1^{\left(N-\frac{1}{2}\right)}\sqrt{H+1-X_1}\,dX_1]$$
$$- \frac{\alpha'}{\beta'}[(1+\beta H)\tan^{-1}\sqrt{\beta H} - \sqrt{\beta H}] + c \qquad (34)$$

Evaluating the constant of integration 'c' in the above expression using the initial condition Q(0) = 0, it is seen that,

$$Q(H) = H - \frac{2\Gamma(M+\frac{3}{2})}{\sqrt{\pi}\,\Gamma(M+1)}\int_0^l \frac{d}{dH}X_1^M\sqrt{H+1-X_1}\,dX_1]$$
$$- \frac{\alpha'}{\beta'}(1+\beta H)\tan^{-1}\sqrt{\beta H} - \sqrt{\beta H}] + \frac{2}{(2M+3)} \qquad (35)$$

where, M = N - 1/2.

For an exact linear weir designed over a base weir $Y_1 = X_1^M$, the discharge equation is,

$$\int_0^l X_1^M\sqrt{H+1-X_1}\,dX_1 + \int_0^H f_L^*(X)\sqrt{H-X}\,dX = B_L(H+\lambda_L) \qquad (36)$$

$f_L^*(X)$ is the linear weir designed over the base weir $Y_1 = W(X_1)^M$. Using the initial condition and the slope-discharge continuity theorem, and simplifying,

$$B_L = \frac{\sqrt{\pi}\,\Gamma(M+1)}{2\Gamma(M+\frac{3}{2})} \text{ and } \lambda_L = \frac{2}{2M+3} \qquad (37)$$

Let, $\eta = \dfrac{2\Gamma(M+\frac{3}{2})}{\sqrt{\pi}\,\Gamma M+1} = \dfrac{1}{B_L}$

Comparing Eqs. 35 and 36, the primary H-Q function can be expressed as,

$$Q(H) = \eta\psi_L(H) - \frac{\alpha'}{\beta'}[(1+\beta H)\tan^{-1}\sqrt{\beta' H} - \sqrt{\beta' H}] \qquad (38)$$

where,

$$\psi_L(H) = B_L(H+\lambda_L) - \int_0^L X_1^M\sqrt{H+1-X_1}\,dX_1 \qquad (39)$$

Solving Eq.36, using Eq.39, we get

$$f_L^*(X) = \frac{2}{\pi}\int_0^X \frac{\psi_L''(H)}{\sqrt{X-H}}\,dH \tag{40}$$

From Eq.35 and Eq.37, the discharge equation of SBL weir is (see Fig.2)

$$\int_0^H f_s(X)\sqrt{H-X}\,dX = \eta\psi_L(H) - \frac{\alpha'}{\beta'}(1+\beta H)\tan^{-1}\sqrt{\beta' H} - \sqrt{\beta' H}\,] \tag{41}$$

Solving the above integral equation by converting it into Abel's form,

$$f_s(X) = \frac{2}{\pi}\left\{ \eta\int_0^X \frac{\psi_L(H)}{\sqrt{X-H}}\,dH - \frac{\alpha'}{\beta'}\int_0^X \frac{d^2}{dH^2}\left[\frac{[(1+\beta' H)\tan^{-1}\sqrt{\beta' H} - \sqrt{\beta' H}\,]}{\sqrt{X-H}}\right] dH \right\} \tag{42}$$

Using Eq.40 and evaluating the second integral in the above equation, we get,

$$f_s(X) = \left[\eta f_L^*(X) - \frac{\alpha'\sqrt{\beta'}}{\sqrt{1+\beta' X}}\right] \tag{43}$$

$$f_s(X) = \eta\left[f_L^*(X) - \frac{\alpha}{\sqrt{1+\beta X}}\right] \tag{44}$$

where $\alpha$ and $\beta$ are constants. This proves the theorem.

## 7. Experiments

Fig.5 shows the set-up used to conduct experiments on two typical weirs having crest widths of 15 cm and 20 cm. The weirs were fixed at the end of a rectangular channel 18.5 cm long, 1.2 m wide and 1.1 m deep, with crest in each case set 20 cm above the channel bed. The channel had adequate stilling arrangements made using small cement concrete blocks, stone chips, wire meshes and baffle walls. Water was fed through a head tank measuring 2.25 m x 2.25 m x 1.8 m, to which water was supplied by means of a pump with a maximum discharge capacity of 112 lps. This supply was augmented with another pump of 78.4 lps capacity, whenever necessary. The weirs were fabricated in 6.5 mm thick mild steel plates. After carefully marking the weir profile on the plate by a scratch-awl the opening was first cut roughly by a band saw machine and then filed accurately to the final exact dimensions. Dimensions of the prepared weir models were checked at various sections by means of a dial gauge vernier calliper and found to be in very good agreement with calculated values within a maximum deviation being as small as ±0.2mm. The head over the weir was measured using a point gauge, having a least count of 0.025mm, fixed 4 m upstream of the weir section. The discharges were measured volumetrically in a measuring tank 4.52 m x 4.52m x 1.5 m through readings in a perspex tube of 20 mm internal diameter connected to the bottom of the tank at one end. The discharges were determined by finding the time taken for the water level to rise between the two indicator fixed in prespex tube exactly separated by a distance of 50 cm. The indicators were connected to the leads of an electronic timer through a start and stop mechanism. At least 20 minutes were allowed between two consecutive sets of readings to allow for the water level to stabilise. Fig.6 shows the photographic view of the weir discharging into the tank and Fig.7 shows the plot of actual discharge versus head. It is seen that the experiments are in excellent agreement with the theory by giving a constant average coefficient of discharges of 0.641 and 0.635 respectively. Fig.8 shows the variation of $C_d$ with respect to head.

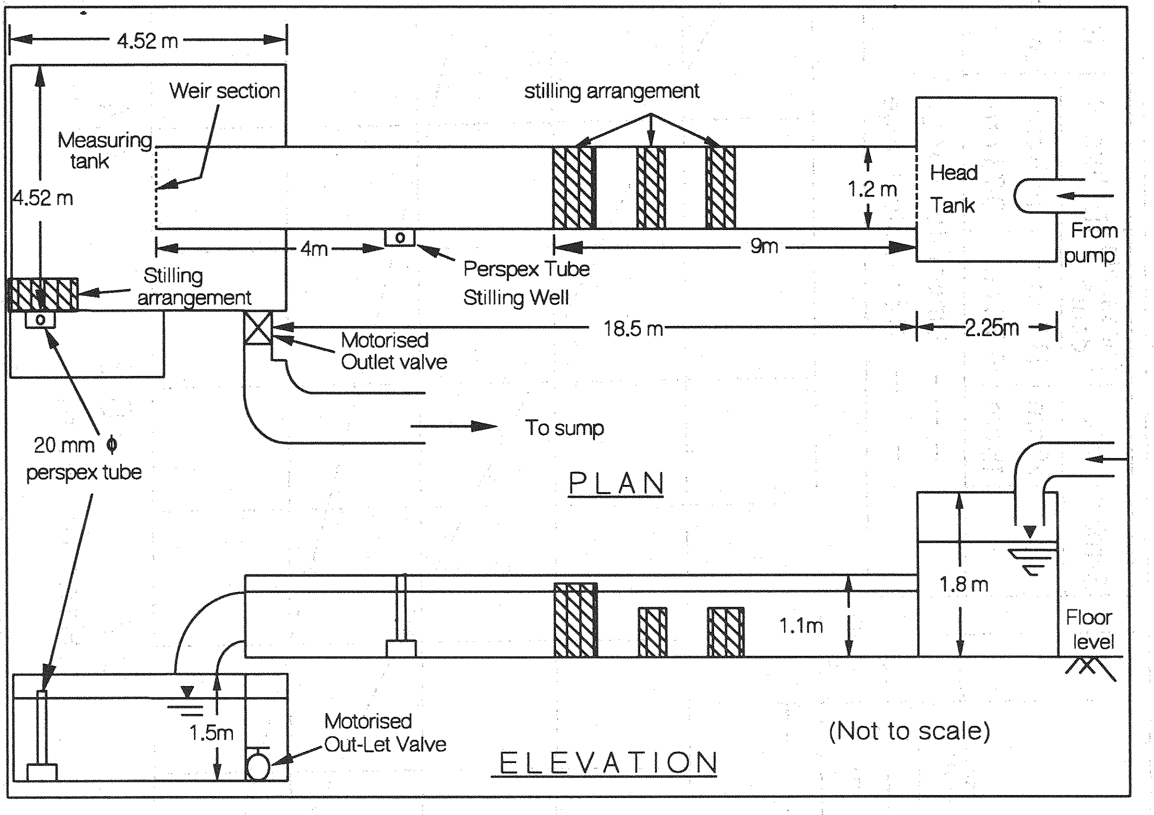


**Fig.5 Experimental Setup**

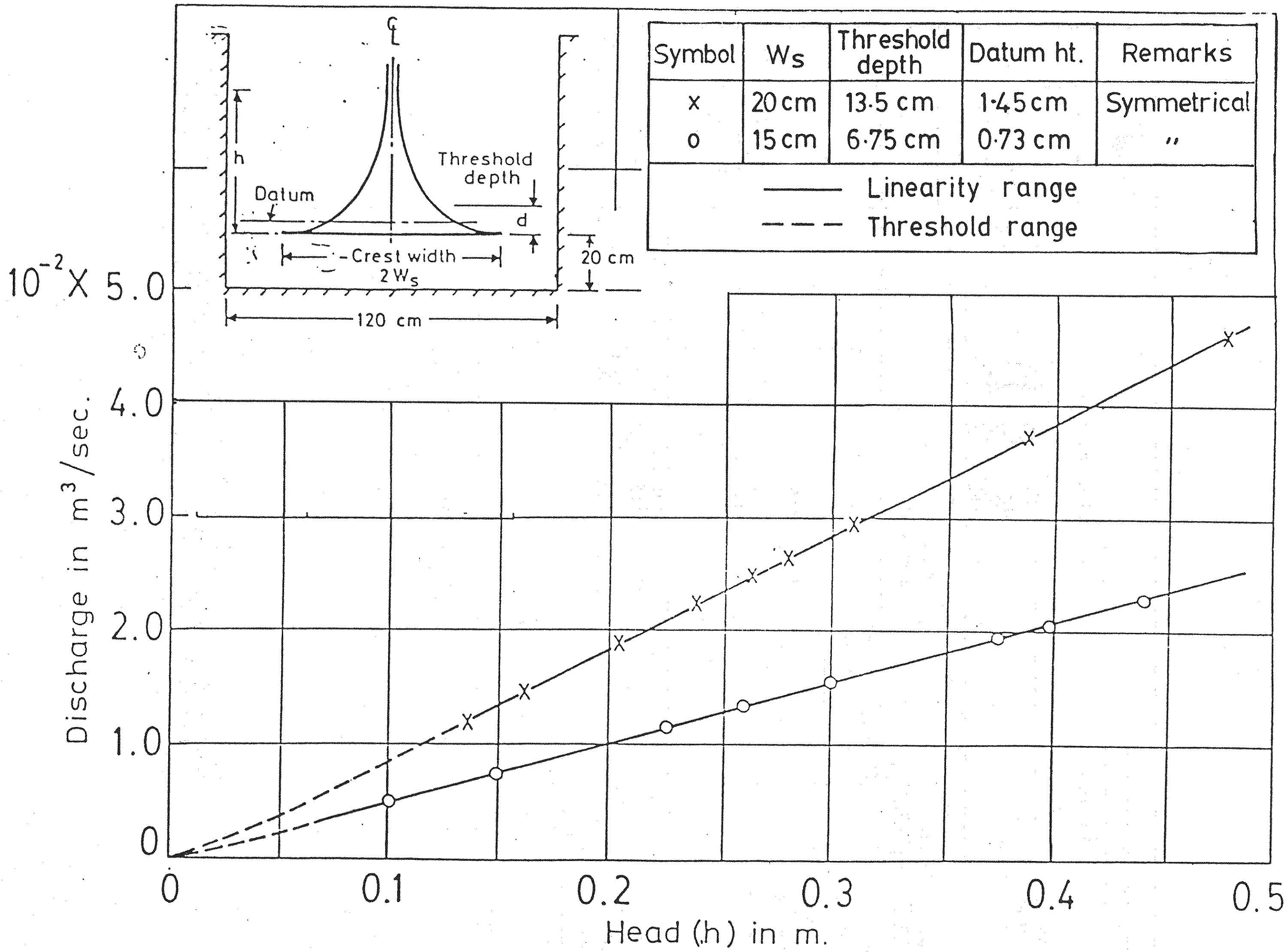


Fig. 7 . Plot of Actual Discharge Vs. Head (Experimental).

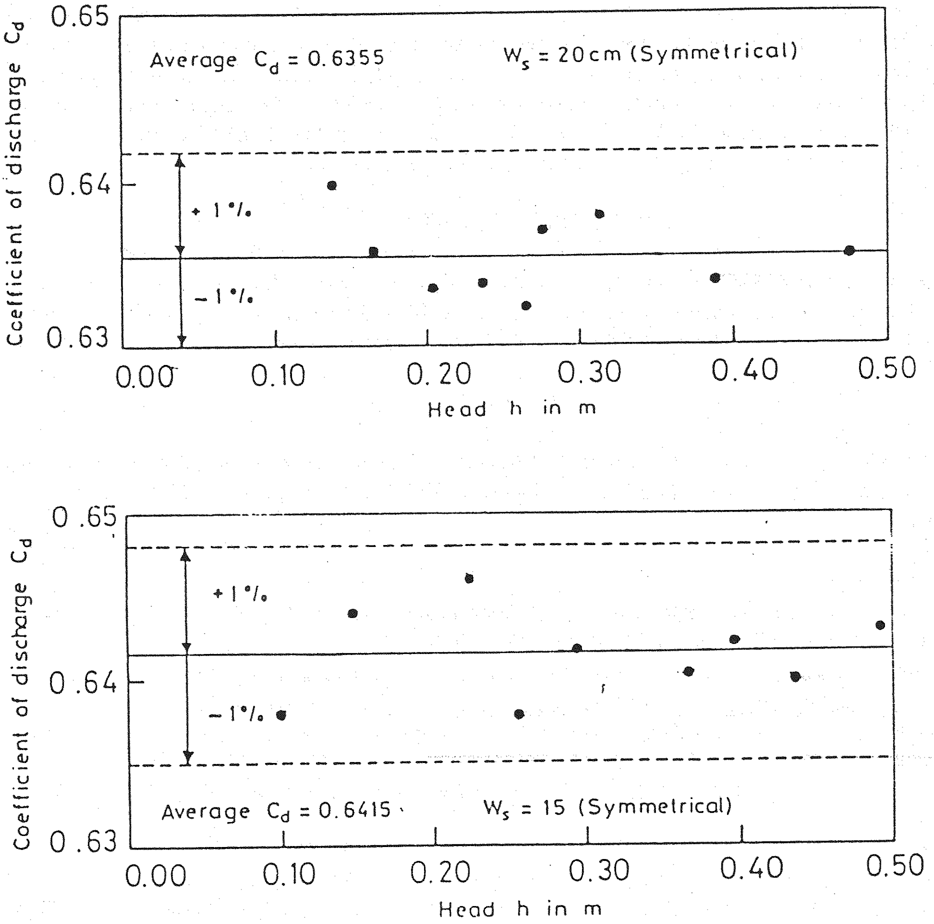


Fig. 8 : Variation of $C_d$ with head h .

## CONCLUSIONS

An interesting solution for a SBL weir utilizing the significant property of the complementary weir profile of 'Proportional three halves power weir' as a SBL-2 type generating function is given. It is shown that the discharge through it, for all flows above a threshold depth (base depth) is proportional to the head measured above a certain datum, to a high degree of accuracy, and this accuracy increases with head. It is further shown that, the conventional linear weirs with bases can be modified into SBL weirs, by the addition of a simple correcting function of the form $\alpha/(1+\beta X)^{1/2}$, where $\alpha$ and $\beta$ are constants. This is enunciated in the form of a theorem and proved therein. It is observed that the SBL weir designed herein has the advantages in having higher sensitivity and a slight higher coefficient of discharge $C_d$ over the SBL weir designed earlier by using a Quadratic weir profile. Experiments with two symmetric weirs of crest widths 30 cm and 40 cm confirm the theory by giving a constant coefficient of discharge of 0.641 and 0.635 for the 2 weirs respectively.

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