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FORMULA OF SEDIMENT DISCHARGE IN OSCILLATION-CURRENT COEXISTING FLOW

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ABSTRACT

Bed-load transport process in oscillation-current coexisting flow is numerically simulated to predict the distribution of the moving period of particles under various combination of current and oscillation flow. Integration of the proposed non-equilibrium bed-load transport formula, which is consisted by the pick-up rate and the moving period, provides the bed-load discharge. Formula of sediment discharge within a half-cycle of oscillation and that of net sediment discharge in oscillation-current coexisting flow are proposed by approximating the results of the simulation.

INTRODUCTION

Sediment discharge in unsteady flow has been usually estimated by applying the formulas derived for steady-uniform conditions under the quasi-steady assumption. When one considers the unsteadiness of sediment transport in natural environments, such as estuary and coastal environment, one should inspect the applicability and the physical background of the previous formulas of sediment discharge, and should require the formula of sediment discharge derived by treating the unsteady sediment transport process straightfowardly.

Sleath (1) measured the time series of sediment transport rate in an oscillating flow, by using an oscillating plate in still water. Shibayama & Horikawa (2) measured the change of the sediment discharge within a cycle of wave, and they found the existence of the lag between the shear stress and the sediment discharge. Sawamoto & Yamashita (3) performed experiments on the time variations of the pick-up rate and deposit rate of sand, and proposed a simple stochastic model of the dislodgment process of bed-load particles for the estimation of pick-up rate variation. In their model, however, they treated the moving process of particles deterministically, and consequently they could not succeed in explaining the experimental results of deposit rate.

Tsujimoto, Graf & Suszka (4) indicated that the unsteadiness of bed-load transport is caused by: (i) direct effect of flow unsteadiness and (ii) relaxation effect due to the slow response of the bed-load transport system to the change of flow properties. Nakagawa, Tsujimoto & Gotoh (5) proposed a non-equilibrium bed-load transport formula in unsteady flow, which was constituted by the pick-up rate and the distribution of moving period, as an analogy to Nakagawa & Tsujimoto (6)'s non-equilibrium bed-load transport formula in non-uniform flow. The pick-up rate is mainly governed by the direct effect of flow unsteadiness; while the distribution of moving period is affected primarily by the relaxation effect of the bed-load transport system.

Gotoh, Tsujimoto & Nakagawa (7) performed a numerical simulation of the dislodgment process of bed-load particles in accelerated flow, and modified the pick-up rate formula proposed by Nakagawa

& Tsujimoto (6) based on the results of the simulation by taking the flow acceleration into account. Gotoh, Tsujimoto & Nakagawa (8) executed a numerical simulation of moving process with special reference to the irregular collision among moving particles and protrusions composed by bed-material particles, and they found a good agreement between the simulation and the experiment in oscillation-current coexisting flow. Their simulation also gave a good prediction of the distribution of deposit rate due to wave action experimentally investigated by Sawamoto & Yamashita (3).

In this study, a numerical simulation of the bed-load transport (Gotoh et al. (8)) is performed under the various kinds of hydraulic conditions in oscillation-current coexisting flow, to estimate the moving period. The formula of sediment discharge within a half-cycle of oscillation and that of net sediment discharge are proposed by approximating the simulated results based on the non-equilibrium sediment transport formula (Nakagawa et al. (5)).

Strictly speaking, the motion of sediment in oscillating flow is different from that under a wave action. In this paper, the motion of sediment in oscillation-current coexisting flow, in which the effect of Stokes drift, or the net transport of water due to the wave non-linearity, is expressed by adding a current onto an oscillation, is treated as a first-order approximation of that under a wave action. Because the wave field is a spatially nonuniform, the oscillation-current coexisting flow is not sufficient for describing the detail of the whole region of wave field. In this paper, the calculated domain is limited only in the layer in the vicinity of a bed surface, hence the oscillation-current coexisting flow is used as a first-order approximation of the wave field.

OUTLINE OF SIMULATION

The basic concept of the simulation was proposed by Nakagawa, Tsujimoto & Hosokawa (9), where the bed-load transport process is assumed to be the repetition of (i) the collision process with protrusion on bed, and (ii) the moving process without collision, as illustrated in Fig.1. For the simplicity, two-dimensional motion is considered.

Collision process with protrusion on bed:
Collision process is idealized as a rigid sphere rolling over a step formed by a protrusion on the bed. Considering the changes of the momentum and the angular momentum during the process of collision, one can obtain the relation between the initial speed of particle and the speed of particle after a collision. The distribution of the height of bed protrusions, which governs the effect of collision on the speed of particle, is approximated by an exponential distribution after Nakagawa et al. (9).

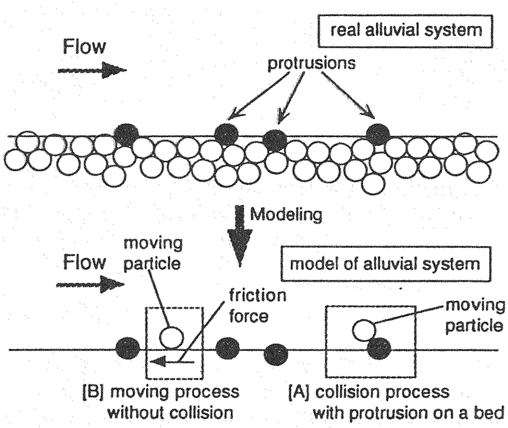

Fig. 1 Illustration of bed-load transport

Moving process without collision:
The equation of the sliding motion is adopted to simulate the moving process without collision.

$$\rho\left(\frac{\sigma}{\rho}+C_{\mathrm{M}}\right)A_3 d^3\frac{\mathrm{d}u_{\mathrm{p}}}{\mathrm{d}t}=\frac{1}{2}C_{\mathrm{D}}\rho\left|u-u_{\mathrm{p}}\right|(u-u_{\mathrm{p}})A_2 d^2 + \rho(1+C_{\mathrm{M}})A_3 d^3\frac{\mathrm{d}u}{\mathrm{d}t}-g\left(\frac{\sigma}{\rho}-1\right)A_3 d^3\mu_{\mathrm{f}} \tag{1}$$

where u_p=speed of particle in the horizontal direction; x_p=coordinate in horizontal direction; ρ= mass density of fluid; σ=mass density of particle; C_M= added mass coefficient; C_D=drag coefficient; g=gravitational acceleration; and A_2, A_3=two-and three-dimensional geometric coefficients, respectively. Nakagawa et al. (9) suggested that the coefficient of the kinetic friction, μ_f, is related to the particle's speed as follows:

$$\mu_f = \frac{1}{2} \frac{\mu_{f0}}{(u_p / \sqrt{gd})^2 + 0.5} \tag{2}$$

where μ_{f0}=coefficient of static friction. The motion of particle is traced by solving Eq. 1 with Runge-Kutta-Verner method with performing the collision assessment at every time step by comparing the position of the moving particle with the spatial distribution of protrusions on bed. When the collision occurs, the speed of particle just after the collision, u_{out}, is calculated, and the motion of the particle is traced again by Eq. 1 for a given u_{out} as an initial condition. The particle is regarded as falling to the rest on the bed, when the following condition is satisfied(assessment of the rest event after collision):

$$u_{out} \leq \beta_1 u_{in} \tag{3}$$

where β_1= an empirical constant related to the imperfect elasticity of collision; and u_{in}= the speed of particle just before the collision.

Parameters of contact between moving particle and bed surface:

Two parameters, β_1 and μ_{f0}, which represent the magnitude of the contact between moving particle and bed-material particles, depend upon the characteristics of bed-material particles: β_1 represents the imperfect elasticity of collision, and μ_{f0} is the coefficient of static friction. The former governs the magnitude of the contact between a moving particle and bed-material particles in collision process, while the latter governs the magnitude of the contact between them in moving process. In this study, these parameters are set as determined by Gotoh et al. (8): β_1=0.05 and μ_{f0}=0.4, to fit the predictions of the simulation to the experiment by Sawamoto & Yamashita (3). The other coefficients involved in this simulation are as follows: C_M=0.5, C_D=0.4, A_2=π/4; and A_3=π/6, which are frequently employed in the previous studies.

NON-EQUILIBRIUM SEDIMENT TRANSPORT FORMULA IN UNSTEADY FLOW

Convolution integral formulation of sediment transport formula:

The unsteady non-equilibrium sediment transport rate is formulated by Nakagawa et al. (5) as follows:

$$q_B(t) = \frac{A_3 d}{A_2} \int_0^{\infty} p_s(t-\tau) \cdot u_p(t|t-\tau) \int_{\tau}^{\infty} f_T(\zeta|t-\tau) \mathrm{d}\zeta \, \mathrm{d}\tau \tag{4}$$

in which $p_s(t)$=pick-up rate of bed-load particle; $f_T(\tau|t)$= probability density function of moving period of the particle picked up at time t; and $u_p(\tau|t)$=speed of the particle picked up at time t. When we assume that the speed of particle is independent of the particle's dislodging time Eq. 4 can be written as follows:

$$q_B(t) = \frac{A_3 d}{A_2} \cdot u_p(t) \int_0^{\infty} p_s(t-\tau) \int_{\tau}^{\infty} f_T(\zeta|t-\tau) \mathrm{d}\zeta \, \mathrm{d}\tau \tag{5}$$

The assumption of the independence of the particle's speed of the dislodging instant is usually appropriate. Then, one can estimate the speed of particle as follows:

$$u_p = \beta_{up} \left\{ u - \frac{2A_3 d}{A_2} \frac{(\sigma/\rho - 1) g \alpha_{up} \mu_{f0}}{C_D} \right\} \quad ; \quad \alpha_{up} = \beta_{up} = 0.8 \tag{6}$$

in which α_{up}, β_{up}= empirical constants. The coefficient α_{up} is set so that the speed of particles is equal to zero at the critical phase of particle's motion, and the coefficient β_{up} implies the decrease of the speed of particles due to the collision with protrusions on bed.

Simplified model of flow field:

Pick-up rate depends on the instantaneous hydraulic condition, while moving period depends on the hysterisis of the motion of particles. Hence, the sediment discharge is affected by the pattern of the flow during a cycle of oscillation. In this study, the shear velocity of oscillation-current coexisting flow is formulated by the simple formulation as follows:

$$u_*(t) = u_{*w} \sin\omega t + u_{*cr} \qquad (7)$$

in which u_{*w},u_{*cr}= shear velocities due to the wave (or oscillation) and current, respectively; and $\omega=2\pi/T$, T=period of the oscillation. The shear velocities are calculated by using an explicit expression of a friction coefficient for a wave-current coexisting motion proposed by Tanaka (10). The velocity of the flow surrounding the moving particle, the elevation of which is supposed that $y=d/2$, is related to the shear velocity by the logarithmic law as follows:

$$\phi_p = \frac{u}{u_*} = \frac{1}{\kappa}\ln\left(\frac{30.1\chi}{k_s}\frac{d}{2}\right) \qquad (8)$$

in which κ=Kármán constant; χ=coefficient for the modification considering the effect of grain-size Reynolds number, u_*k_s/ν; k_s= bed roughness; and it is assumed that $k_s=d$. The time derivative of flow velocity is calculated by assuming that the lag of the velocity near the bed, u, to the shear velocity, u_*, is sufficiently small ($d\phi_p/dt=0$) as follows:

$$\frac{du}{dt} = \phi_p \frac{du_*}{dt} \qquad (9)$$

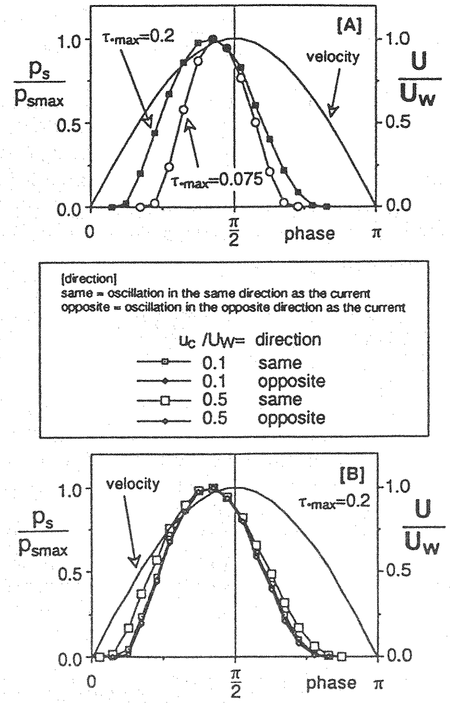

Fig. 2 Variation of pick-up rate during the course of a half oscillation cycle

Although some experimental results show the existence of the lag between a shear velocity, u_*, and the velocity of the flow near a bottom wall, u, in this study, the calculated domain of the bed-load motion is so thin, in other words, the height of the reference point of the flow velocity is so low, that the lag between u_* and u is negligible.

Modified formula of pick-up rate - effect of flow unsteadiness:

Gotoh et al. (7) modified the pick-up rate formula by Nakagawa & Tsujimoto (6) by taking account of the flow unsteadiness as follows:

$$p_{s*} = p_s\sqrt{\frac{d}{(\sigma/\rho-1)g}} = F_0\tau_*\left(1-\frac{k_2\tau_{*c0}}{\tau_*}\right)^{k_3 m} \qquad (10)$$

in which k_2 and k_3 are coefficients which are sensitive to the effect of flow acceleration; and m and F_0 are empirical constants (m=3.0, F_0=0.078). The coefficients, k_2 and k_3, are the functions of the dimensionless time derivative of shear stress, $d\tau_*/dt_*$ ($t_* = t\sqrt{(\sigma/\rho-1)g/d}$).

Figure 2 shows the change of the pick-up rate under the sinusoidal flow velocity given by Eqs. 9 and 10. Figure 2 [A] shows the effects of the amplitude of shear stress τ_{*max} on the pick-up rate; while Figure 2 [B] shows the effects of the relative current-strength to the oscillation (u_c/U_w) on the pick-up rate. Although the time variation of pick-up rate should be symmetric around the phase $\pi/2$; the flow acceleration brings about an asymmetry of the time variation: pick-up process is promoted during 0-$\pi/2$, and is suppressed during $\pi/2$-π. Consequently the phase corresponding to the peak of the pick-up rate comes earlier than that of flow velocity. The distributing range of the time variation of pick-up rate increases with the increase of shear stress (see Fig. 2 [A]). The increase of shear stress makes the dislodging phase come earlier; and makes the depositing phase come later, hence the distributing range becomes wider. The change in the distributing range due to the relative current-strength to the oscillation (u_c/U_w) is smaller than that of flow acceleration (compare Fig. 2 [A] and [B]).

Time variations of moving period and sediment discharge:

Figure 3 shows the change of mean moving period against the dislodging phase of particle. The effects of the amplitude of shear stress τ_{*max} on the mean moving period appears in Fig. 3 [A]; while the effects of the relative current-strength to the oscillation (u_c/U_w) on the mean moving period, T_m, is shown in Fig. 3 [B]. Mean moving period is less significantly affected by the flow condition so that the approximation of mean moving period is expressed as follows independently of the flow condition:

$$\frac{T_m}{T} = \left(0.45 - \frac{t}{T}\right) \cdot \Xi_M + 0.28(1 - \Xi_M) \tag{11}$$

$$\Xi_M = 1 - \exp\left(-\frac{2}{B_M}\frac{t}{T}\right) \quad ; \quad B_M = 0.3 \tag{12}$$

In this study, the simulation is performed under the condition that the current velocity is smaller than the velocity amplitude of oscillating velocity (u_c/U_w<1), and consequently the motion of sediment particle must be governed mainly by the oscillation. This is the reason why the mean moving period in various hydraulic condition can be approximated by a unified equation.

Figure 4 shows the responding change of sediment discharge to the sinusoidal change of the flow velocity. Figure 4 [A] shows the effects of the amplitude of shear stress τ_{*max} on the sediment discharge; while Fig. 4 [B] shows the effects of the relative current-strength to the oscillation (u_c/U_w) on the sediment discharge. Although the time variation pattern of sediment discharge is asymmetric similarly to that of pick-up rate, the skewness of the pattern is different each other: in the pattern of pick-up rate variation, the rising limb is steeper than the falling one, while in the pattern of sediment discharge variation, the rising limb is milder than the falling one. Figure 4 [B] indicates that the sediment discharge distribution is less affected by the current strength.

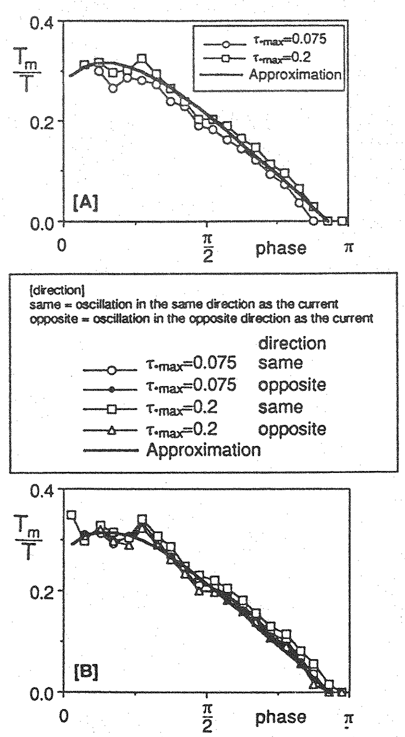

Fig. 3 Mean moving period

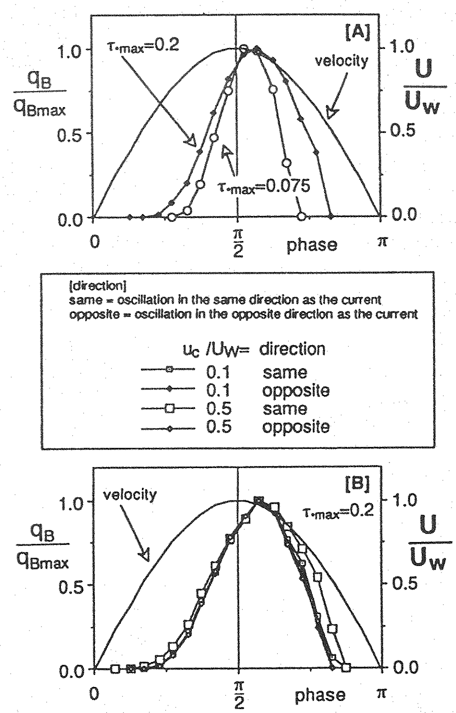

Fig. 4 Variation of sediment discharge during the course of a half oscillation cycle

APPROXIMATED FORMULA OF SEDIMENT DISCHARGE IN UNSTEADY FLOW

Practical use, such as the prediction of beach profile, needs the formula of sediment discharge for a half-cycle of oscillation and that of net sediment discharge. The approximated formulae are proposed based on the results of numerical simulation. Hence the proposed formulae have somewhat physical background expressed by the present simulation.

Sediment transport formula for half cycle of oscillation:

Figure 5 shows the change of the simulated sediment discharge in a half cycle of oscillation against the amplitude of shear stress τ_{*max}. Two kinds of particulate materials, the diameter of which are 1 mm and 0.1 mm, were used to get a so wide range of shear stress as realistic range of flow velocity: extremely large flow velocity is required to set the shear stress sufficiently large for the 1 mm-diameter sand. In this case, the current velocity is set zero (oscillation only). The results of the simulation are approximated well by the following equation.

$$\overline{q_{B*}} = \frac{\overline{q_B}}{\sqrt{(\sigma/\rho - 1)gd^3}} = 4.0\tau_*^{3/2}\left(1 - \frac{\tau_{*c0}}{\tau_*}\right)^{m_B} \tag{13}$$

When τ_{*max} is sufficiently large, sediment discharge is proportional to the (3/2)-th power of τ_{*max}. This tendency is consistent to the formula proposed by Meyer-Peter & Müller (11) for the sediment discharge in uni-directional flow; and the formula proposed by Sleath (1) for the sediment discharge due to the wave action.

Figure 6 shows a comparison between Eq. 13 and experimental data (Abou-Seida (12), Kalkanis (13), Sawamoto & Yamashita (3)). Data in the sheetflow condition, which were included in the data of Abou-Seida and Kalkanis, are excluded in this figure, according to the study of Sawamoto & Yamashita. Equation 13 agrees fairly well with the experimental data. Figure 7 shows the dimensionless sediment discharge q_B/w_0d (w_0=settling velocity of sand), which is frequently used in the formula of sediment discharge due to wave action. The present formula is shown by a solid curve; with the formulae proposed by Madean & Grant (14) and Sleath (1) by dashed curves. Though Sleath employed $q_B/\omega_0 d^2$ as the dimensionless discharge in his formula, it was converted to the dimensionless parameter q_B/w_0d, where the period of oscillation T is set T=1.0s by considering the range of Sleath's experiment (0.51s<T<2.47s); and the diameter of the sand is set d=1mm by taking range of the previous experiment into account. When one would plot Eq.13 on the figure, the dimensionless parameter $q_{B*}(=q_B/\text{r}((\sigma/\rho-1)gd^3))$ should be transformed into q_B/w_0d, and the following procedure is adopted.

$$\frac{\overline{q_B}}{w_0 d} = \overline{q_{B*}} \cdot \sqrt{\frac{4C_D}{3}} \tag{14}$$

$$C_D = \frac{24}{R_e} + 0.4 \quad ; \quad R_e = \frac{u_* d}{\nu} \tag{15}$$

Although any formula passes through the scattering range of the experimental data, the meaning of the present formula is different from the other two formulae: it is the approximation of the results of numerical simulation on the basis of the moving mechanism of sand particle on a flat bed. It demonstrates the proportionality of sediment discharge to the (3/2)-th power of dimensionless shear stress τ_{*max} for the sufficiently large τ_{*max}, if the moving mechanism considered in the present model is valid in a large τ_{*max} region. In the high τ_{*max} region, the bed surface is not flat,

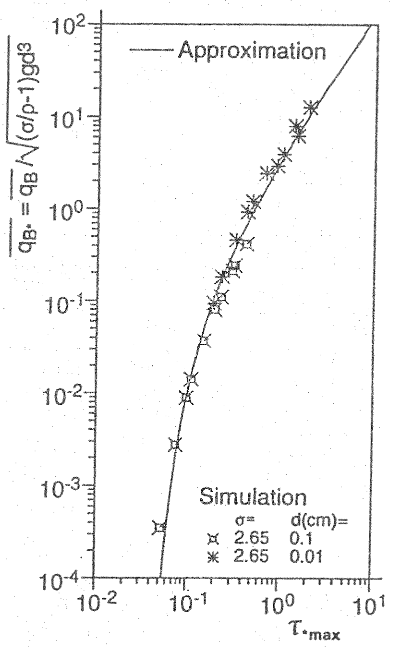

Fig. 5 Sediment discharge within a half oscillation cycle: results of simulation

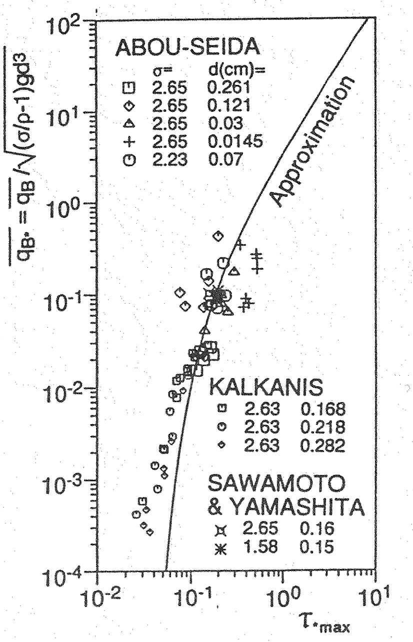

Fig. 6 Sediment discharge within a half oscillation cycle: comparison between experiment and simulation

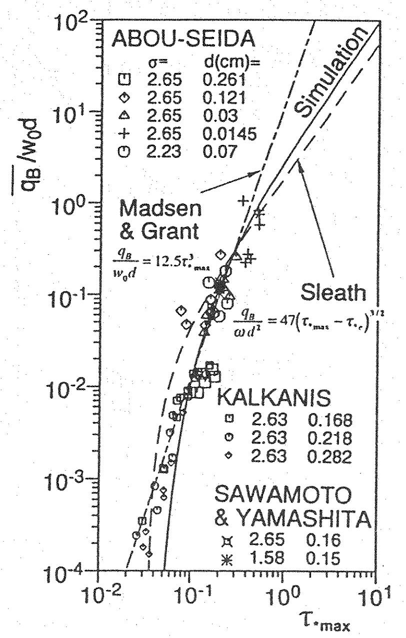

Fig. 7 Formula of sediment discharge within a half oscillation cycle

hence the effect of the ripples and the difference in the moving mechanism, or the sheetflow, should be taken into account, to improve the accuracy of the simulation.

Net sediment discharge:

Figure 8 shows the sediment discharge during half-cycle of oscillation against relative current-strength to the oscillation (u_c/U_w) under the condition of the same current velocity. The definition of τ_{*max} for the same and the opposite direction of the oscillation is expressed schematically in Fig. 9. The solid line in the figure represents Eq. 13 under the condition in which oscillation only exists. The effect of current on the results is sufficiently small for the small shear stress, hence Eq. 13 is appropriate for the estimation of the net sediment discharge under the oscillation regardless of the existence of current. While, Eq. 13 cannot be adopted to the estimation of the sediment discharge for the large shear stress, especially under the existence of the current velocity in the opposite direction of the oscillation.

The net sediment discharge formulated as a function of the relative current-strength to the oscillation (u_c/U_w) is convenient in practical use. Figure 10 shows the change of the net sediment discharge against the change of τ_{*max}. In the figure, τ_{*max} is defined as the sum of the shear stress caused by current and the amplitude of the shear stress of the oscillation (see Fig. 9). τ_{*max} is equal to the maximum value of shear stress under the existence of the current in the same direction of oscillating flow. When $u_c/U_w>0.5$, Eq. 13 becomes a good approximation of the results of the simulation, because the sediment discharge under the existence of the current in the opposite direction of oscillating flow is negligible comparing to that in the same direction of oscillating flow. The net sediment discharge decreases with the decrease of u_c/U_w, because the difference of the sediment discharge in the same direction of current from that in the opposite direction of current decreases with the decreasing u_c/U_w. This tendency, which is clearly found for $u_c/U_w<0.5$, is expressed by introducing the coefficients K_{B1}, k_{B2}, k_{B3} in the Eq. 13 as follows:

$$q_{B*net} = 4K_{B1}\tau_*^{(3/2)k_{B2}}\left(1-\frac{\tau_{*c0}}{\tau_*}\right)^{k_{B3}m_B} \quad \text{for} \quad u_c/U_w \geq 0.1 \tag{16}$$

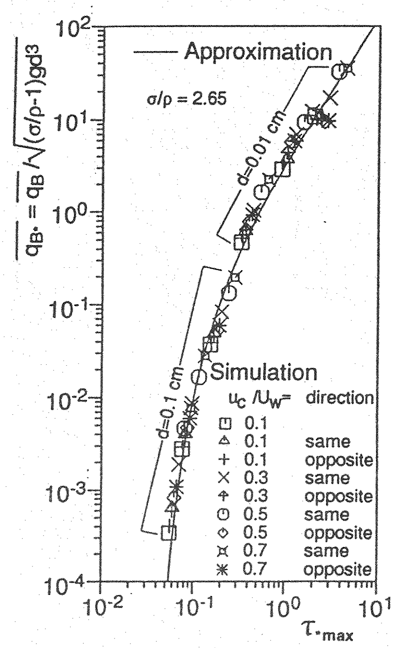

Fig. 8 Effect of current strength on the sediment discharge

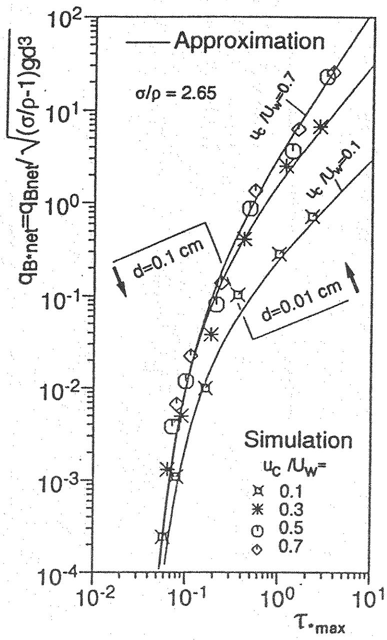

Fig. 10 Net sediment discharge in oscil-lation-current coexisting flow

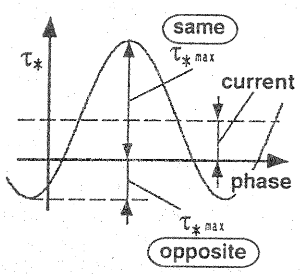

Fig. 9 Definition of τ_{*max}

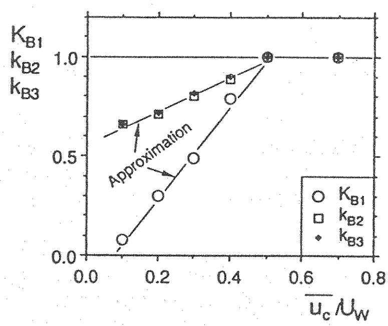

Fig. 11 Coefficients of approximated equation of net sediment discharge

The relation between the coefficients and u_c/U_w is shown in Fig. 11, and approximated as follows:

$$K_{B1} = \begin{cases} 2.31\left(\dfrac{u_c}{U_w}\right) - 0.168 & \text{for} \quad u_c/U_w < 0.5 \\ 1.0 & \text{for} \quad u_c/U_w \geq 0.5 \end{cases} \tag{17}$$

$$k_{B2} = k_{B3} = \begin{cases} 0.85\left(\dfrac{u_c}{U_w}\right) + 0.56 & \text{for} \quad u_c/U_w < 0.5 \\ 1.0 & \text{for} \quad u_c/U_w \geq 0.5 \end{cases} \tag{18}$$

The net sediment discharge for given u_c/U_W can be estimated by Eqs. (16),(17) and (18).

CONCLUSION

Numerical simulation of bed-load transport process is performed by a stochastic model of bed-load motion, which describes reasonably the probabilistic varieties of the motion of bed-load particle due to the irregular contact between moving particles and the bed surface. The results of the simulation are summarized into an approximated formula of bed load discharge during a half-cycle of oscillation. It also gives a formula of the net sediment discharge in oscillation-current coexisting flow, by which the net sediment discharge is expressed as a function of the relative strength of the current u_c/U_W.

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APPENDIX-NOTATION

The following symbols are used in this paper:

A_2, A_3	= two-and three-dimensional geometrical coefficients of particle, respectively;
B_M	= coefficient in the approximated equation of mean moving period;
C_M	= added mass coefficient;
C_D	= drag coefficient;
d	= diameter of particle;
$f_T(\zeta \mid t)$	= probability density function of the moving period of the particle picked up at a time t;

F_0 = constant in pick-up rate equation;
k_s = equivalent sand roughness;
k_2, k_3 = coefficients in pick-up rate equation;
K_{B1},k_{B2},k_{B3} = coefficients in sediment discharge equation;
m_B = constant in sediment discharge equation;
g = gravitational acceleration;
T = period of oscillation;
t_* = dimensionless time scale;
U_w = velocity amplitude of oscillation;
u = flow velocity;
u_{in} = speed of the particle just before the collision;
u_{out} = speed of the particle after the collision;
u_p = speed of the particle;
u_* = dimensionless shear velocity;
u_{*w}, u_{*cr} = wave and current components of dimensionless shear velocity;
u_c = current velocity;
p_s,p_{s*} = pick-up rate of bed-material particle and its dimensionless form;
q_B,q_{B*} = sediment discharge and its dimensionless form;
q_{B*net} = dimensionless net sediment discharge;
T_m = mean moving period;
w_0 = settling velocity of sand;
α_x = variation coefficient of interval of neighboring protrusions;
β_1 = coefficient of the imperfectly elastic collision;
β_{up} = empirical constant in the equation with respect to the particle's speed equation;
κ = Karman constant;
χ = coefficient to express the effect of grain-size Rynolds number in logarithmic law of velocity;
ν = kinematic viscosity;
ϕ_p = ratio of flow velocity to the shear velocity;
μ_f, μ_{f0} = coefficients of kinetic- and static friction, respectively;
ρ = mass density of fluid;
σ = mass density of bed-material particle;
Ξ_M = coefficient in mean moving period equation;
τ_{*max} = amplitude of dimensionless shear stress; *and*
τ_{*c0} = dimensionless critical shear stress at an steady flow.

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