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APPLICATION OF THE LINEAR PERTURBATION MODEL FOR RIVER FLOW ESTIMATION

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SYNOPSIS

The linear perturbation model (LPM) is applied firstly to estimate the runoff from rainfall in the Fuji catchment in Japan and the Nan catchment in Thailand; and secondly for flood routing in the Nan catchment. For the latter, two cases are considered: one with single input - single output; the other with multiple inputs - single output. Both nonparametric(unit hydrograph) and parametric (linear transfer function) forms of the models are examined. Good agreements between observed and estimated discharges are obtained for the flood routing context with the model efficiency R^2 being 84% or more in both nonparametric and parametric forms. In the context of rainfall-runoff process, the applicability of the LPM is found to be unsatisfactory either for the Fuji or the Nan catchments with R^2 being 59% and 69%, respectively.

INTRODUCTION

River flow estimation is one of the most important aspects of water resources management. Presently, there are a number of river flow estimation techniques available. Selection among these techniques depends on the purpose and data availability. One of the simplest techniques is the linear input-output system. The input can be considered as either rainfall in the catchment or upstream discharge or the combination of both while the output is the discharge at the point of interest. Usually, estimation of river flow in the head water catchment requires an elaborate rainfall-runoff modelling that can reflect the complexity of processes occurring in the soil mantle. In most of the rainfall-runoff modelling, two parts are considered: the first computes the "generated runoff" which is produced by the interaction among the rainfall, soil moisture, potential evaporation as well as the rate of infiltration; the latter computes the routing part or damping effect where the generated runoff is used as the input. In the first part which is usually nonlinear, the model can be expressed either in simple or complex nonlinear mathematical formulations. So, it seems that the linear model (LM) is not suitable as long as it is applied directly to the phenomena. Nash and Foley (8) applied the time invariant linear model to the Grendon Underwood catchment in the United Kingdom and found that the results were not good. However, there may exist some transformations by which the linear relation of the rainfall-runoff can be assumed. Nash and Barsi (9) developed the linear perturbation model (LPM) for rainfall-runoff process which is originally referred to as the "hybrid model" proposed by Barsi (1). They considered that the departures from the seasonal mean occurring in the rainfall and runoff series could be assumed as linear. Kachroo et al. (4) further applied the LPM to many catchments of the world with different topography, vegetation covering, and climatic conditions. Better results were found on catchments where there existed high periodicity.

For river flow estimation in the channel reach where upstream discharge is available, one may use flood routing techniques since the rainfall-runoff process has already been integrated in the hydrograph at the upstream point. In this aspect, the linear model(LM) without any transformation had been applied successfully in many catchments (e.g., Dooge (5); O'Connor (12); Natale and Todini, (11)). Based on the previous work of Nash and Barsi (9), the LPM had been extended by Liang (6) in the context of flood routing for multiple inputs-single output system. The inputs can be the discharge on the main stream and/or on the tributaries or combined with the rainfall in the intermediate catchment. Further applications had been successfully made for the large catchments such as in China by Liang & Nash (7) and in the Mekong river by Kachroo et al. (3).

The purpose of this paper is to investigate the applicability of the linear model (LM) and the linear perturbation model(LPM) to a humid catchment in Japan and on the humid and monsoon area in Thailand. For the context of the rainfall-runoff process, both catchments were tested, while for the context of flood routing, only the catchment in Thailand was studied utilizing the information of upstream discharges and the rainfall in the intermediate catchment as the inputs to estimate the inflow to the Sirikit reservoir.

THE LINEAR PERTURBATION MODEL(LPM)

The concept of the LPM is that *in any year in which the rainfall or other input exactly follows the seasonal expectation, the output would similarly follows its seasonal expectation, and in other years the departures(or perturbations) from the seasonal expectation occurring in the input and output would be linearly related.* It is expected that on highly periodic catchments, the subtraction of the seasonal mean from the original series would remove large part of the nonlinear effects in the system and, consequently, the assumption of a linear relationship between the departures would be better compared with using the assumption for the total actual input and output series. This may be, as stated by Kachroo et al.(3), an analogous treatment of subtraction of base flow and rain loss series in the classical unit hydrograph approach. The structure of the LPM can be schematized as shown in Fig. 1 where I_T=daily input; I_S=seasonal mean daily input; I_D=daily input departures; Q_T=daily output; Q_S=seasonal mean daily output; and Q_D=daily output departures.

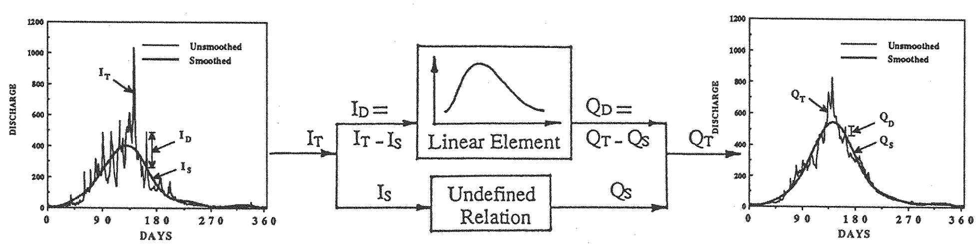


Fig. 1 Schematic representation of the structure of the LPM

The seasonal mean for daily input and output series, I_S and Q_S are obtained by the seasonal model (SM) which uses the mean daily values and then smoothened by the discrete Fourier series analysis (detailed description may be found from Salas et al. (13)) throughout the period of calibration. The SM is considered as a "naive model" and might give good estimation of discharge if the catchment exhibits strong periodicity. It should be noted that I_S and Q_S shall be repeated every year throughout the period of consideration. Once I_S and Q_S are known, the daily input departures I_D can be expressed as

$$I_D = I_T - I_S \qquad (1)$$

which is the difference between the corresponding I_S and I_T series. Similarly, the daily output departures can be given by

$$Q_D = Q_T - Q_S \qquad (2)$$

Then the linear relationship between I_D and Q_D (as shown in the upper box of Fig. 1) can be assumed. Once the daily output departures are computed from the adopted linear relationship, they are superimposed to the seasonal mean of the original output series in order to obtain the computed total output. This implies that the LPM takes into account the information of output series that has already described by the seasonal pattern of the catchment. There are two forms of solutions applied in this paper to estimate this linear relationship and consequently to estimate the discharges. One is the nonparametric(unit hydrograph) form, another is the parametric(constrained) form of transfer function.

Nonparametric Form

For single input - single output system, the linear element shown in the upper box of Fig. 1 can be expressed in the form which defined by the convolution summation as follows:

$$y_t = \sum_{j=1}^{m} x_{t-j+1} h_j + e_t \qquad (3)$$

where x_t=input(departure) series; y_t=output(departure) series; h_j and m =ordinates and memory length of the pulse response; and e_t=error term. It should be noted that x_t and y_t are equivalent to either I_T and Q_T for the linear model(LM) or I_D and Q_D for the linear perturbation model(LPM). In case of multiple inputs - single output system, Eq. 3 can be rewritten as

$$y_t = \sum_{k=1}^{L} \sum_{j=1}^{m(k)} x_{t-j+1}^{(k)} h_j^{(k)} + e_t \qquad (4)$$

where k=each input up to L inputs. L=1, if the input x_t is a single lumped input(i.e. rainfall or upstream discharge) and $L \geq 2$ if x_t is a multiple lumped inputs vector (i.e. combination of upstream discharges and rainfall in the intermediate catchment). Eq. 4 for a series of N time intervals can be written in matrix form as

$$Y = X^{(1)}H^{(1)} + X^{(2)}H^{(2)} + \cdots\cdots + X^{(L)}H^{(L)} + E \qquad (5)$$

where Y is an (N,1) column vector of the output series such that

$$Y = [y_1 \;\; y_2 \;\; \cdots\cdots \;\; y_N]^T \qquad (6)$$

where T indicates transpose operation.

$X^{(k)}$ is an [N,m(k)] matrix of the kth input series

$$X^{(k)} = \begin{bmatrix} x_1^{(k)} & x_0^{(k)} & \cdots\cdots & x_{1-m(k)+1}^{(k)} \\ x_2^{(k)} & x_1^{(k)} & \cdots\cdots & x_{2-m(k)+1}^{(k)} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ x_{m(k)}^{(k)} & x_{m(k)-1}^{(k)} & \cdots\cdots & x_1^{(k)} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ x_N^{(k)} & x_{N-1}^{(k)} & \cdots\cdots & x_{N-m(k)+1}^{(k)} \end{bmatrix} \tag{7}$$

$H^{(k)}$ is an [m(k),1] column vector of the pulse response ordinates corresponding to the kth input series

$$H^{(k)} = [h_1^{(k)} \; h_2^{(k)} \; \ldots\ldots \; h_{m(k)}^{(k)}]^T \tag{8}$$

E is an (N,1) column vector of error terms $[e_1 \;\; e_2 \; \ldots . \; e_N]^T$. Eq. 5 may be written as

$$Y = XH + E \tag{9}$$

where X is an [N,M] matrix

$$X = [X^{(1)} \; X^{(2)} \; \ldots\ldots \; X^{(L)}] \tag{10}$$

where

$$M = \sum_{k=1}^{L} m(k);$$

and H is an [M,1] column vector

$$H = [H^{(1)T} \; H^{(2)T} \; \ldots\ldots \; H^{(L)T}]^T \tag{11}$$

If $\hat{H}$ denotes an estimate of H, then Eq. 9 can be rewritten as

$$Y = X\hat{H} + E \tag{12}$$

If the objective is to minimize the sum of squares of the model output errors, then the optimal value of H can be determined directly by the method of ordinary least squares(OLS) as

$$\hat{H} = [X^TX]^{-1} X^TY \tag{13}$$

The variance of the vector $\hat{H}$ can be expressed as

$$Var(\hat{H}) = \sigma^2(X^TX)^{-1} \tag{14}$$

where σ^2=the variance of error term e_t, being assumed stationary; and its unbiased estimator of σ^2 is given by

$$s^2 = \frac{1}{N-M}\sum_{t=1}^{N} e_t^2 \tag{15}$$

The variance of each ordinate $h_j^{(k)}$ of $\widehat{H}^{(k)}$, [j=1,2,....,m(k)] may be obtained by taking the corresponding terms of principal diagonal of $(X^TX)^{-1}$ and multiplying by σ^2. Then the standard error(SE) of each corresponding ordinate $h_j^{(k)}$ of $\widehat{H}^{(k)}$ can be obtained by

$$SE(h_j^{(k)}) = \sqrt{Var(h_j^{(k)})} \quad (16)$$

In order to choose the memory length (m) of each input, it might be first approximated from the cross correlation function. Practically, the memory length (m) can be obtained by trial and error choosing the longer m wherein the last ordinate h_m is still above its standard error obtained by Eq. 16.

Since $h_j^{(k)}$ of Eq. 4 is an unrestricted series to be obtained by least squares solution (Eq. 13), the model of Eq. 4 and its $h_j^{(k)}$ is referred to as "nonparametric". In practice, one may encounter the problem of unrealistic shape of the pulse responses such as a W-shape, especially when there exists high serial and cross correlation among the inputs. In such a case, some known functions might be applied on $h_j^{(k)}$, and the corresponding solution is called "parametric".

Parametric Form

The parametric linear model to be used here is the Linear Transfer Function (LTF) model proposed by Box and Jenkins (2). This model has been demonstrated to be a parsimonious model (capable of achieving a high degree of generality with relatively few parameters). The linear transfer function (LTF) of order (r,b,s) is defined as

$$(1-\delta_1 B- \ldots -\delta_r B^r)y_t = (\omega_0+\omega_1 B+ \ldots +\omega_{s-1}B^{s-1})x_{t-b} \quad (17)$$

where δ_r =the autoregressive parameters of order r; ω_s=the moving average parameters of order s; and b=time delay between input and output. B is a backward shift operator such as

$$Bx_t = x_{t-1} \quad \text{and in general}$$

$$B^n x_t = x_{t-n}$$

By introducing the error term e_t, Eq. 17 may be written as

$$\delta(B)y_t = \omega(B)B^b x_t + e_t \quad (18)$$

or

$$y_t = \sum_{j=1}^{r} \delta_j y_{t-j} + \sum_{j=1}^{s} \omega_j x_{t-j-b+1} + e_t \quad (19)$$

where

$$\delta(B) = (1-\delta_1 B- \ldots -\delta_r B^r)$$
$$\omega(B) = (\omega_0+\omega_1 B+ \ldots +\omega_{s-1}B^{s-1})$$

Eq. 19 is used for single input - single output system. For the multiple inputs - single output system, Eq. 19 can be written as

$$y_t = \sum_{j=1}^{r} \delta_j y_{t-j} + \sum_{k=1}^{L} \sum_{j=1}^{s(k)} \omega_j^{(k)} x_{t-j-b(k)+1}^{(k)} + e_t \tag{20}$$

where L=number of inputs; and k=the kth input. Eq. 20 can be written in matrix notation in a similar manner as in the nonparametric case and can then be solved by the ordinary least squares method. The pulse responses for each input k can be derived from the transfer function parameters (Box and Jenkins (2)) as follows:

$$\begin{aligned} &h_j = 0 && j<b \\ &h_j=\delta_1 h_{j-1}+\delta_2 h_{j-2}+\cdots+\delta_r h_{j-r}+\omega_{j-b+1} && j=b, b+1, b+2, \ldots, b+s-1 \\ &h_j=\delta_1 h_{j-1}+\delta_2 h_{j-2}+\cdots+\delta_r h_{j-r} && j>b+s-1 \end{aligned} \tag{21}$$

Note that unlike the pulse response h_j's in Eq. 4 of the nonparametric form, the h_j's in Eq. 21 are restricted. That is why this case (Eq. 20) is called the parametric form. The main difficulty in application of the LTF is how to choose the order of the model. This is based largely on the judgement of the analyst. Nevertheless, two important criteria must be borne in mind: principle of parsimony in using parameters and the physically realistic pulse response function.

The Model Gain Factor

The model gain factor (also called as a scale or an amplification factor) may be defined as the sum of the elements in the least squares solution of vector h. For the kth input, the gain factor can be expressed as

$$g^{(k)} = \sum_{j=1}^{m(k)} h_j^{(k)} \tag{22}$$

In case of the LTF model, the gain factor for the kth input can be expressed as

$$g^{(k)} = \frac{\omega_1^{(k)}+\omega_2^{(k)}+\cdots\cdots+\omega_{s(k)}^{(k)}}{1-\delta_1-\delta_2-\cdots\cdots\cdots-\delta_r} \tag{23}$$

Estimation of Discharges

For the nonparametric form, once the pulse response $h_j^{(k)}$ and the gain factor $g^{(k)}$ are obtained, the unit hydrograph response $uh_j^{(k)}$ by normalizing the ordinates of $h_j^{(k)}$ as

$$uh_j^{(k)} = \frac{h_j^{(k)}}{g^{(k)}} \qquad \text{for } j = 1, \ldots, m(k) \tag{24}$$

Then, from the convolution form of Eq. 4, the estimated discharge $\hat{y}_t$ can be computed as

$$\hat{y}_t = \sum_{k=1}^{L} g^{(k)} \sum_{j=1}^{m(k)} x_{t-j+1}^{(k)}\, uh_j^{(k)} \tag{25}$$

For the LTF, once the autoregressive parameters δ_j of order r and the moving average parameters ω_j of order s are determined, the estimated discharge $\hat{y}_t$ based on Eq. 20 can be computed as

$$\hat{y}_t = \sum_{j=1}^{r} \delta_j y_{t-j} + \sum_{k=1}^{L} \sum_{j=1}^{s(k)} \omega_j^{(k)} x_{t-j-b(k)+1}^{(k)} \qquad (26)$$

It should be noted that Eq. 26 can be used in two modes. One is the updating mode when the terms y_{t-j} are the observed values in the past, another is the non-updating mode when the terms y_{t-j} are replaced by the estimated values $\hat{y}_{t-j}$ from the previous calculations. For the latter case, if the order of the model [r,b,s] of 1,0,2 are chosen, then Eq. 26 is exactly the same as the classical Muskingum Routing model.

MODEL EFFICIENCY ASSESSMENT CRITERIA

In order to evaluate the efficiency of various models, Nash & Sutcliffe (10) defined the model efficiency R^2 as

$$R^2 = \frac{(F_0 - F)}{F_0} \qquad (27)$$

where

$$F_0 = \sum (y_t - \bar{y}_t)^2$$
$$F = \sum (y_t - \hat{y}_t)^2$$

F=overall disagreement between the estimated discharges $\hat{y}_t$ and the observed discharges y_t; F_0=initial variance; $\bar{y}_t$ =mean observed discharges for the calibration period. The R^2 defined above is analogous to the coefficient of determination in the regression analysis.

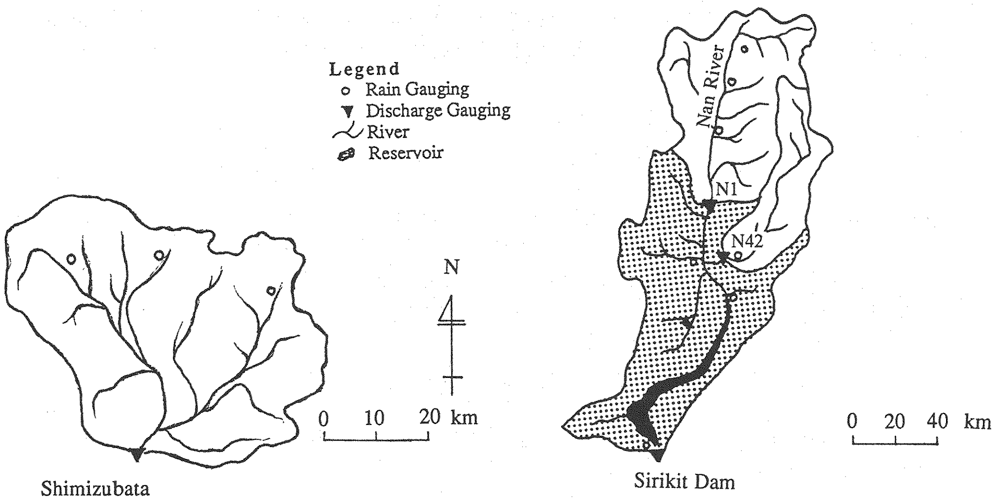
APPLICATION

Both the LM and the LPM are applied to the catchment of the Fuji river in Japan and the Nan river in Thailand. For rainfall-runoff process, only a single lumped input of the areal rainfall is applied. In case of flood routing context, applied to the Nan catchment, both single input and multiple inputs are investigated. Both the nonparametric(unit hydrograph) and parametric(LTF) forms are applied for the LM and the LPM, respectively.

The Fuji Catchment

The Fuji River Basin(3,990 sq.km) shown in Fig. 2(a) is located in the central part of Japan flowing out to the Pacific Ocean. The catchment is characterized as the valley with the average altitude at the central area about +260 m(MSL) and surrounded by the high ranges of mountains of altitudes up to +3,000 m(MSL). There is a discharge gauging station located at Shimizubata covering the catchment area of 2,121 sq.km. Rainfall over the catchment occurs almost all year around with the average of about 1,236 mm/y (1975-1984). Fig. 3(a) and (b) show the seasonal patterns of the rainfall and discharge of the catchment, respectively.

Both the LM and the LPM are applied in the context of rainfall-runoff process to estimate the discharge at Shimizubata. Seven years of record from 1975 to 1981 are selected for calibration. Unfortunately, the discharges record in 1982 are missing due to a severe flood, therefore only two years of record from 1983 to 1984 are used for verification.



(a) The Fuji Catchment

(b) The Nan Catchment

Fig. 2 Location map of (a) The Fuji catchment in Japan and (b) The Nan catchment in Thailand

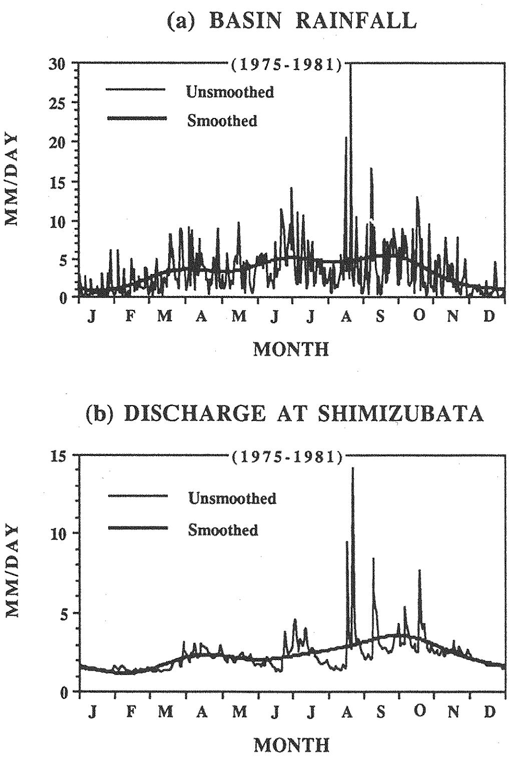


Fig. 3 Seasonal pattern of rainfall and discharges for the Fuji catchment

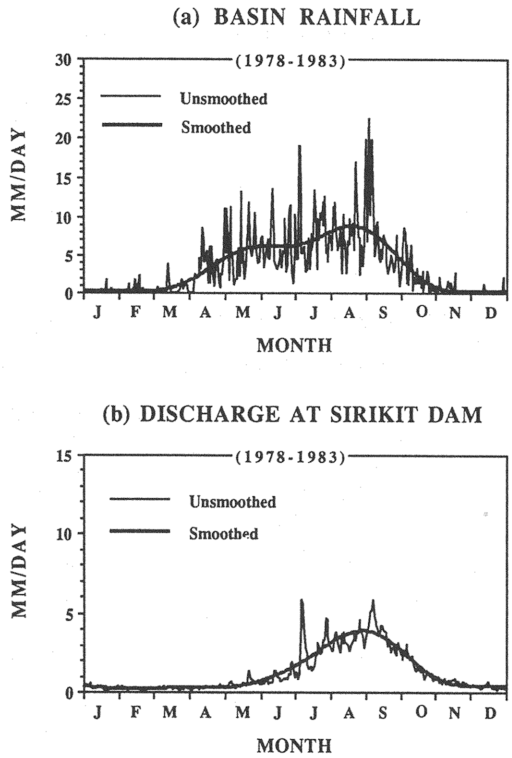


Fig. 4 Seasonal pattern of rainfall and discharges for the Nan catchment

The Nan Catchment

The Nan River Basin(34,330 sq.km) shown in Fig. 2(b) is located in the northern part of Thailand next to the border between Thailand and Laos. The altitude ranges from about +200 m(MSL) in the lower part to +2,080 m(MSL) in the upper part of the basin. The Nan river, one of the major tributaries of the Chao Phraya river, originates from the high mountainous area in the upper part flowing southward to the Sirikit reservoir. This reservoir is located in the lower part of the basin covering the catchment area of 13,130 sq.km. Rainfall over the catchment is influenced by the southwest monsoon from May to October with occasional occurrence of depression storms. The average annual rainfall is about 1,248 mm (1978-1986). Distinction between wet and dry seasons is obvious as shown in Fig. 4(a) and (b). See Takeuchi (15) for more information about the Chao Phraya river.

There exist two discharge gauging stations namely N1 and N42 covering the catchment area of 4,609 and 2,107 sq.km, respectively. These two stations are located 155 and 120 km further upstream of the Sirikit dam, respectively. Six years of records from 1978 to 1983 are used for calibration while the records from 1984 to 1986 are used for verification. Both the LM and the LPM are applied; firstly in the context of rainfall-runoff analyses; secondly in the context of flood routing. In the latter case, both single and multiple inputs are performed.

RESULTS AND DISCUSSIONS

Rainfall-Runoff Analyses

Figs. 5 and 6 show the sample results of rainfall-runoff estimation by the nonparametric form of the LPM for the Fuji and the Nan catchments as compared with the observations. In both catchments, the goodness of fit is better in the calibration stage and poorer in the verification stage. In calibration stage, the estimated seems quite well simulating the observed, but in the verification stage the estimation errors are large, both in high flows and low flows in both catchments. The model efficiencies R^2 obtained from both the LM and the LPM are summarized in Table 1. For the Fuji catchment, the R^2 obtained by the LPM from the nonparametric(UH) form are 62.50% (for calibration period) and 58.26% (for verification period) which are slightly better than those obtained by the LM which are 55.92% (calibration) and 56.22% (verification), respectively. Also for the parametric(LTF) form, the results obtained by the LPM are slightly better than those obtained from the LM for all cases. But in the Nan catchment, the improvements introduced by the LPM are significant over the LM for all cases both in the calibration and the verification periods, namely, the R^2 of LM by the UH are 60.52% (calibration) and 51.10% (verification) whereas those of LPM are 73.38% (calibration) and 68.93% (verification).

Such a remarkable contrast is not surprising because even though both catchments are in the humid area, their periodic behaviours are different. The periodic behaviour may be indicated by the model efficiency R^2 obtained by the seasonal model (SM) in each catchment tabulated in Table 2. From this table, the R^2 for runoff are very low for the Fuji catchment which are 8.05 % in the calibration period and 2.81 % in the verification period , while for the Nan catchment, the R^2 are higher, 42.57 % and 51.74 % in the calibration and the verification periods, respectively. This can be seen in Figs. 3 and 4 that the runoff in the Fuji catchment is prevailed by the departure component, while in the Nan catchment, it is prevailed by the periodic component (seasonal mean). Since the periodic component in the Fuji catchment is very low, little part of nonlinearity can be explained by periodic component.

In both catchments, the R^2 obtained from both the LM and the LPM are not high. One reason is , as mentioned above, the seasonal mean does not explain the every year daily pattern very much. But there still be other reasons. One of them might be due to seasonal differences in the response functions that are not considered in this paper. Figs. 7 and 8 are the scatter plot diagrams between the observed and the residual errors of discharges for the Fuji and the Nan catchments, respectively. From these figures, overestimating discharges during low flow and underestimating during high flow are found in both catchments. This is because the linear

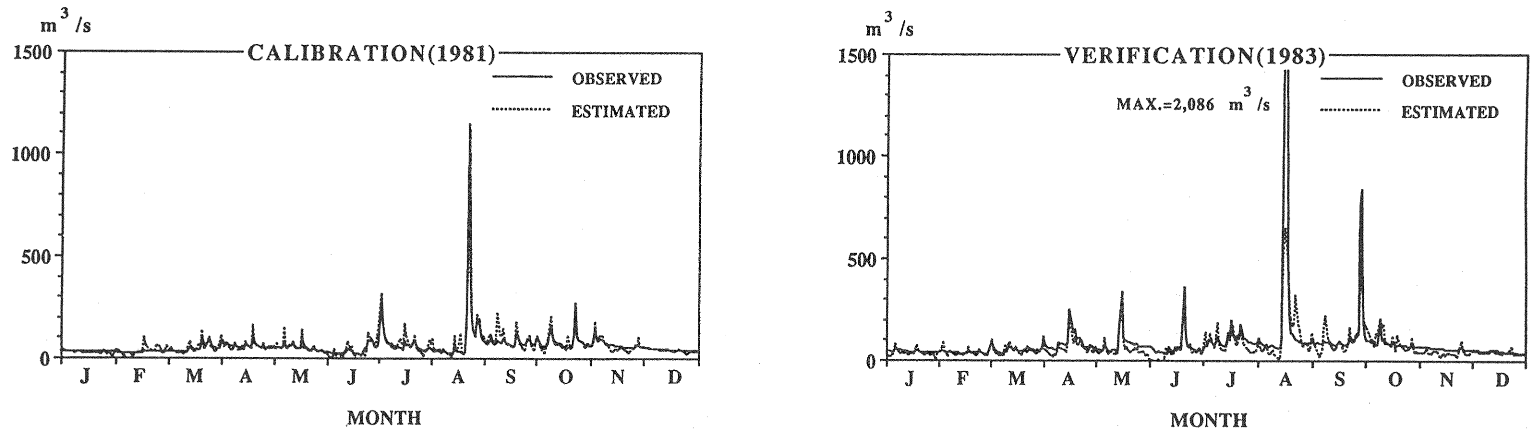


Fig. 5 Comparison between observed and estimated discharges for the Fuji catchment obtained from rainfall-runoff process(LPM, nonparametric form), calibration period: 1975-1981, verification period: 1983-1984

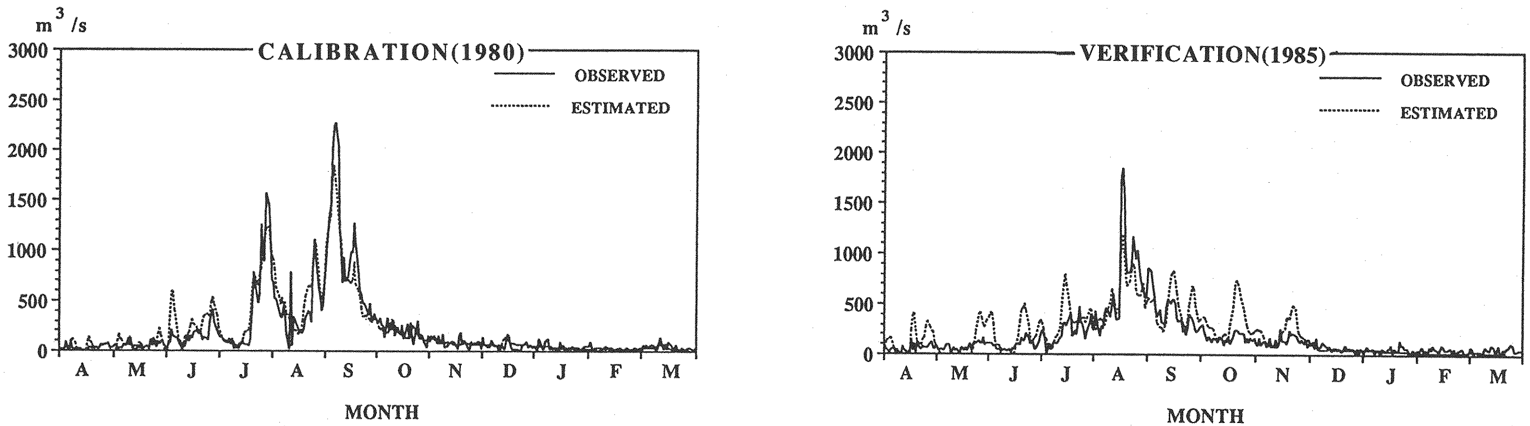


Fig. 6 Comparison between observed and estimated discharges for the Nan catchment obtained from rainfall-runoff process(LPM, nonparametric form), calibration period: 1978-1983, verification period: 1984-1986

Table 1 Summary Results of The Model Efficiency R^2

Number of Inputs	Input	Memory Length (Days)	Model Efficiency R^2,%														Remarks
			Linear Model(LM)							**Linear Perturbation Model(LPM)**							
			Nonparametric (UH)		Linear Transfer Function(LTF)					Nonparametric (UH)		Linear Transfer Function(LTF)					
					Updating		Non−updating*		[r,b,s]			Updating		Non−updating*		[r,b,s]	
			Cal.	Ver.	Cal.	Ver.	Cal.	Ver.		Cal.	Ver.	Cal.	Ver.	Cal.	Ver.		
The Fuji Catchment																	
1	Rainfall	20	55.92	56.22	60.84	65.11	48.77	56.73	[2,0,2]	62.50	58.26	64.21	67.32	59.28	58.80	[2,0,2]	(a)
The Nan Catchment																	
1	Rainfall	15	60.52	51.10	87.21	83.33	59.02	50.38	[1,0,4]	73.38	68.93	87.99	84.10	72.23	68.66	[1,0,4]	(a)
1	Runoff at N1	3	87.67	82.71	92.79	88.25	87.43	82.41	[2,0,3]	88.29	84.64	92.85	88.43	88.01	83.91	[2,0,3]	(b)
1	Runoff at N42	3	87.85	87.24	92.90	89.32	87.80	87.52	[2,0,3]	89.52	88.64	93.11	89.59	89.37	88.72	[2,0,3]	(b)
2	Runoff at N1	3	92.29	89.39	94.36	90.96	92.16	89.04	[2,0,3]	92.93	90.57	94.49	91.27	92.80	90.23	[2,0,3]	(c)
	Runoff at N42	2							[2,0,3]							[2,0,3]	
3	Runoff at N1	3							[2,0,3]							[2,0,3]	(c)
	Runoff at N42	2	93.28	90.77	94.75	91.39	92.99	90.32	[2,0,3]	94.07	91.67	94.92	91.73	93.67	91.28	[2,0,3]	
	Rainfall	15							[2,0,4]							[2,0,4]	

Note : Cal. = calibration, Calibration Period: 1975−81 for the Fuji River and 1978−83 for the Nan River
Ver. = verification, Verification Period: 1983−84 for the Fuji River and 1984−86 for the Nan River
* = the non−updating procedure is referred to the simulation mode where the observed past discharges in the autoregressive terms in Eq. 26 are replaced by the estimated ones

Remarks : (a) for rainfall−runoff process; (b) for multiple inputs−single output flood routing; (c) is the same as (b) incorporated with the rainfall in the intermediate catchment.

Table 2 Results of The Seasonal Model(SM) Fitting

Catchment Name	Model Efficiency R^2,%			
	Raifall		Runoff	
	Calibration	Verification	Calibration	Verification
Fuji	8.05	2.81	3.58	3.18
Nan	42.57	51.74	18.52	17.43

Table 3 Summary Results of Seasonal Mean Used in The LPM for The Fuji Catchment

Seasonal Mean Used	Model Efficiency R^2,%			
	Seasonal Mean		LPM(nonparametric)	
	Calibration	Verification	Calibration	Verification
Constant mean	0.00	−0.17	61.78	54.77
31−day moving average	8.61	2.55	64.14	56.16
11−day moving average	13.71	2.56	66.12	55.47
7−day moving average	13.79	3.10	66.54	55.82
Unsmoothened daily mean	22.42	0.53	71.02	53.38

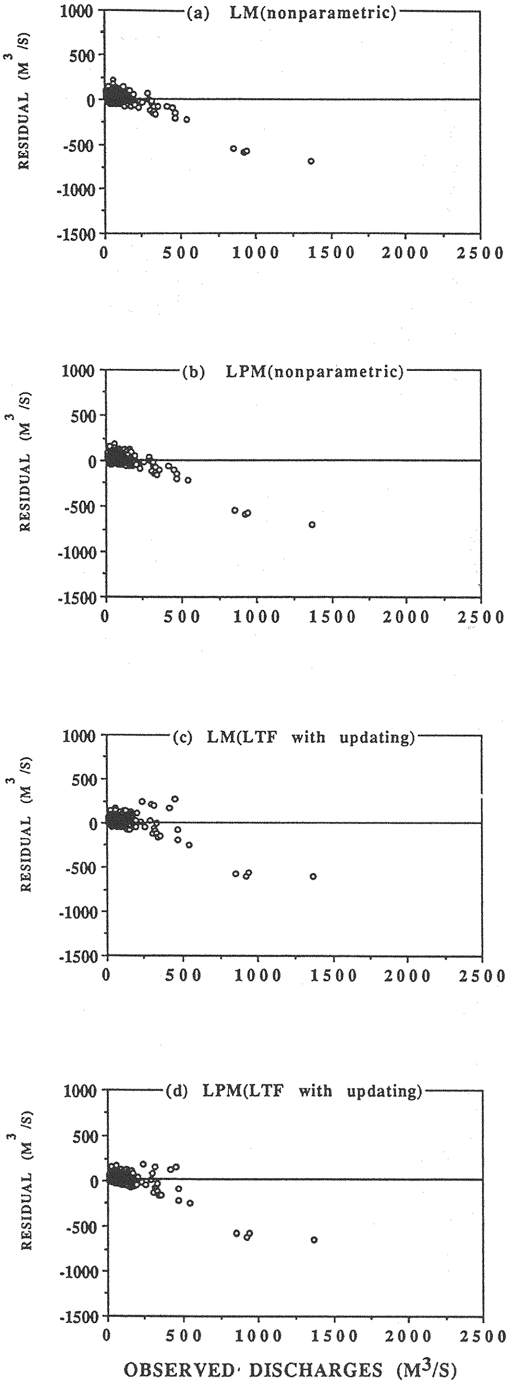


Fig. 7 Scatter plot between residuals and observed discharges at Shimizubata, the Fuji river, both by the LM and the LPM for rainfall-runoff process

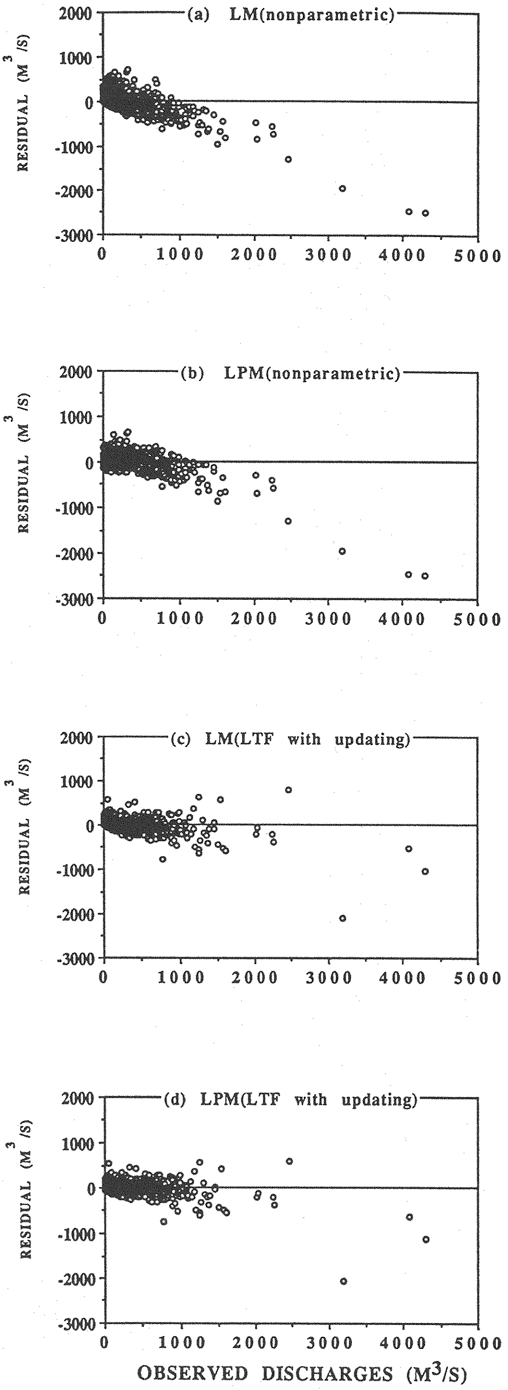


Fig. 8 Scatter plot between residuals and observed discharges at Sirikit dam, the Nan river, both by the LM and the LPM for rainfall-runoff process

models cannot simulate the nonlinearity of the rainfall-runoff relationship. In the dry season, the runoff percentage (gain factor in this paper) is rather low according to the deficit in soil moisture, and in the wet season, especially when heavy rainfall occurs (mostly from typhoons and depression storms in both catchments), higher runoff percentage should exist due to the saturated soil moisture. This seasonality may be taken into account by applying different response functions for different seasons.

As the model efficiency R^2 of the SM applied for the rainfall and the runoff of the Fuji River shows, the SM does not well represent its seasonality. Several other formula of calculating the seasonal mean I_s and Q_s are therefore examined for the LPM. Those include constant mean, moving average mean and unsmoothened daily mean. Table 3 summarizes the model efficiencies R^2 obtained from those seasonal means and the LPM. Since the modified seasonal means still produce poor results in terms of R^2, the LPM consequently cannot produce much different results compared to that obtained from the original model except that employing the unsmoothened daily mean. But in the latter case, the R^2 is higher in the calibration period only. In the verification period, the R^2 are relatively poor compared with any other cases. This indicates that the extraction of seasonal components may not be very sensitive to the choice of formula for calculating the seasonal mean.

For the *non-updating mode* of the parametric(LTF) form, the results yield slightly lower R^2 in both catchments. This is to be expected when the pulse responses are constrained. Fig. 9 shows the pulse responses of both catchments obtained from the LPM for both nonparametric(UH) and the parametric(LTF) forms. It can be seen that the pulse responses obtained from the LTF form are smoother than that obtained from the UH form for both catchments especially in the recession. In case of the *updating mode* of the LTF form, considerable improvement of R^2 from the LM to the LPM are found in the Nan catchment, but not in the Fuji. This is due to the fact that the runoff in the Nan catchment has a long memory and depends considerably on previous runoff which have already reflected the rainfall-runoff transformation in the past. For the Fuji catchment, no significant improvement is found since it has a short memory and the autocorrelation of runoff is not high.

Even though the application of the LPM to these two catchments is not appreciated, one of the advantages of the LPM is that the estimated volume of discharge is better preserved than that from the LM, since the seasonal means of input and output are removed first before computing the relation between the departures and added back again to obtain the total series. Also, it should be note that in large catchment where periodic behaviour is remarkably high and the process of rainfall-runoff transformation is rather slow, the LPM gives better results as performed by Sivaarthitkul (14) and Kachroo et al. (4).

Flood Routing

Application is made for the Nan catchment only since there exist two upstream discharge gauging stations, namely, at N1 and N42. *Single input case* : For the single input - single output system, each input at N1 and N42 is applied independently. The R^2 summarized in Table 1 shows that there is no significant difference between the LM and the LPM. *Two inputs case* : Similar manner is found for the case of multiple inputs (combination of two inputs at N1 and N42). This implies that, in the flood routing context, the relationship between input(s) and output in this catchment is almost linear. In case of two inputs (N1 and N42), the R^2 is rather high (above 90%) and can be acceptable for both in the form of UH and LTF. For the LTF form, the graphical plots between observed and estimated discharge hydrographs for both calibration and verification periods are shown in Fig. 10. Fig. 11 shows the scatter diagrams of the residual errors for both the LM and the LPM where it can be seen that the residual errors during the peak floods are considerably reduced as compared with the case of rainfall-runoff process. Fig. 12 also shows the pulse responses at the Sirikit dam corresponding to each input. It shows that the pulse response of two days delay has significant weights, which implies that it is possible to make

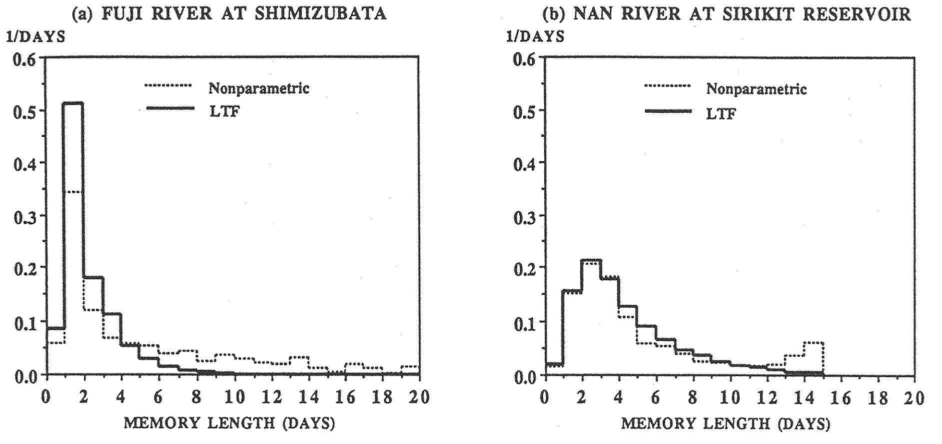


Fig. 9 Pulse responses from rainfall-runoff process by the LPM

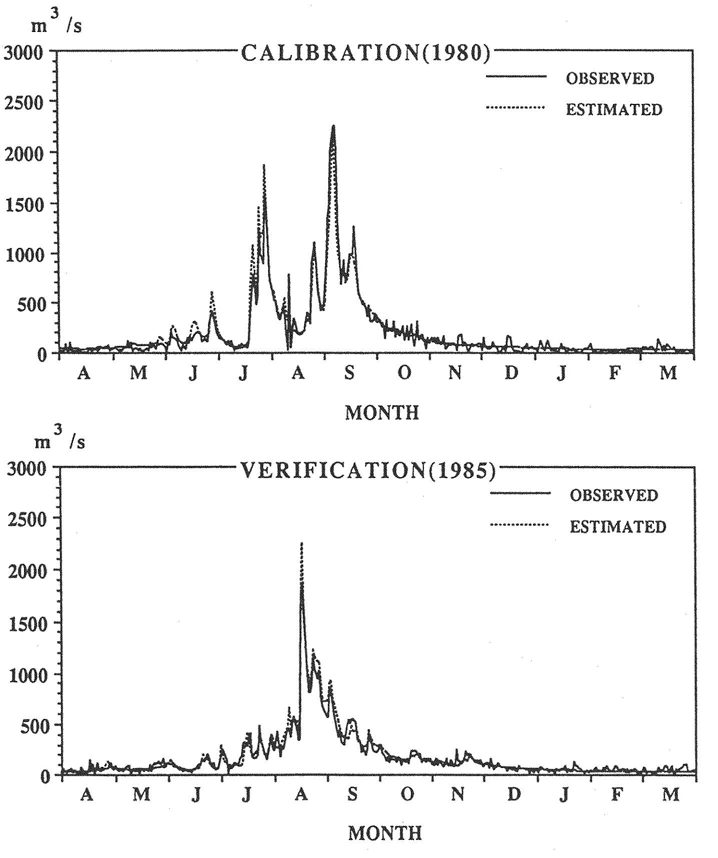


Fig. 10 Comparison between observed and estimated discharges for the Nan catchment obtained from flood routing from N1 and N42 (LPM, using LTF form (non-updating)) calibration period : 1978-1983; verification period : 1984-1986

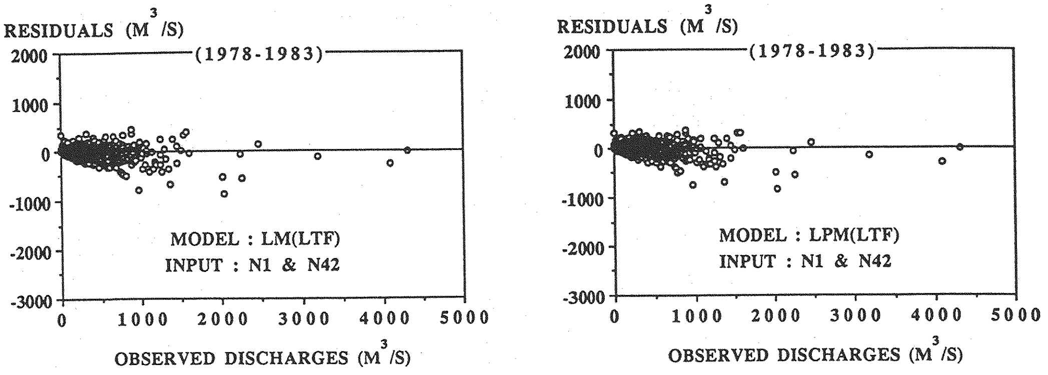


Fig. 11 Scatter plot between residuals and observed discharges at Sirikit dam by the LM and the LPM using LTF form (non-updating)

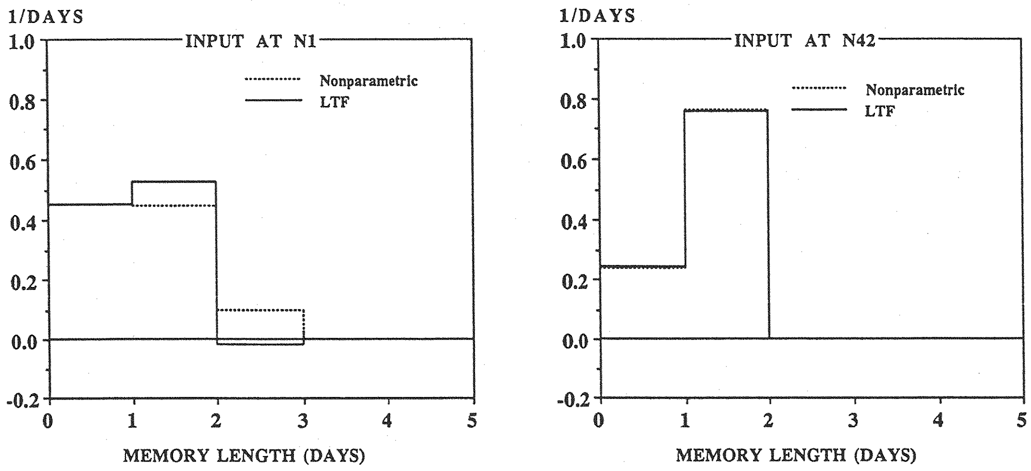


Fig. 12 Pulse responses at Sirikit dam from two inputs at N1 and N42 by the LPM

forecast in downstream after observing the upstream discharge without having the forecast of upstream discharge in advance.

Three inputs case : Further investigation has also been made by taking into account the rainfall in the intermediate catchment (shaded area in Fig. 2(b)) as an additional input. For the case of the LPM, the R^2 of 94.07% and 91.67% for the UH form and 93.67% and 91.28% for the LTF form are obtained for the periods of calibration and verification, respectively. These R^2 are slightly better than those of the two inputs case, using only the upstream discharges at N1 and N42 as the inputs. This might be due to the hydrological phenomena in the intermediate catchment are similar to that in the catchment in upstream. In forecasting practice, including more information especially the rainfall in the intermediate catchment will, of course, be a question of investment cost. From an economical point of view, the model that requires less information yet provides the most satisfactory results would be best desired. Therefore, the models that use only two upstream discharges as inputs are preferable. For operational forecasting, the LTF form is recommended since it can be operated in an updating mode. The R^2 from this updating mode are rather high, but they should be compared with those obtained from other updating techniques rather than those from the non-updating mode in this study.

CONCLUSIONS

For rainfall-runoff analyses:

(1) The results from both the LM and the LPM are not so good in terms of model efficiency R^2 for both catchments of the Fuji river in Japan and the Nan river in Thailand.
(2) The assumption of linearity between departure series of rainfall and runoff poorly holds and the LPM performs unsatisfactorily, especially in the Fuji river (the model efficiency R^2 = 59%) which is in the humid temperate zone and the periodic behaviour is low. For the Nan catchment which is in the tropical monsoon zone where there are the distinct wet and dry seasons and considerable periodic behaviour exists, the R^2 by the LPM is a little larger (69%) than that for the Fuji; and the LPM performs somewhat better than the LM.
(3) The LPM should be applicable for highly periodic catchment, since the average pattern of rainfall and runoff takes care of the nonlinearity between those two series.
(4) The updating parametric (LTF) form which includes the new information of recent past records gives higher R^2 (84%) in the catchment where the autocorrelation is high like in the Nan catchment.
(5) In general, the pulse response obtained from the LTF form is smoother than that obtained from the nonparametric(UH) form for any case especially in the recession.
(6) One of the advantages of the LPM is that the estimated volume of discharges is better preserved than that from the LM.

For flood routing:

(1) Good results($R^2 \gtrsim 84\%$)are obtained in both the LM and the LPM. The reason is considered that the system is almost linear.
(2) The two upstream discharge inputs yield acceptable model efficiency for both the nonparametric(UH) and the parametric(LTF) forms even without incorporating the rainfall in the intermediate catchment.
(3) For operational forecasting, the LTF form is recommended since it can be operated in the updating mode.

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APPENDIX - NOTATION

The following symbols are used in this paper:

b = time delay;

B = backward shift operator;

e_t = error terms;

E = column vector of error terms;

F = initial variance;

F_0 = residual variance;

g = model gain factor;

$h_j^{(k)}$ = pulse response ordinate j of input k;

H = column vector of the pulse response(s);

$\hat{H}$ = column vector of the estimated pulse response(s);

I_D = daily input departure series;

I_S = seasonal mean daily input;

I_T = total daily input series;

k = the kth input;

L = number of inputs;

L = number of inputs;
LM = Linear Model;
LPM = Linear Perturbation Model;
LTF = Linear Transfer Function;
m = memory length;
M = summation of the memory length of each input;
n = order of the backward shift operator;
N = number of days in a series;
Q_D = daily output departure series;
Q_S = seasonal mean daily output;
Q_T = total daily output series;
r = order of autoregressive terms;
R^2 = model efficiency;
s = order of moving average terms;
s^2 = unbiased variance;
SM = Seasonal Model;
T = transpose matrix;
$uh_j^{(k)}$ = unit hydrograph ordinate j of input k;
UH = Unit Hydrograph;
x_t = daily input (departure) series:
X = matrix of input series;
y_t = daily output (departure) series;
$\overline{y}_t$ = mean observed discharges for calibration period;
$\hat{y}_t$ = estimated discharges;
Y = column vector of output series;
δ_r = autoregressive parameters;
ω_s = moving average parameters; and
σ^2 = biased variance.

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