Journal of Hydroscience and Hydraulic Engineering
Vol. 12, No. 1, May, 1994, 67-78

OPTIMIZATION OF COEFFICIENTS IN RUNOFF-EROSION MODELING BY STANDARDIZED POWELL METHOD

By

C. A. G. Santos, K. Suzuki and M. Watanabe

Department of Civil and Ocean Engineering, Ehime University
3 Bunkyo-cho, Matsuyama, 790 Japan

and

V. S. Srinivasan

Department of Civil Engineering, Federal University of Paraíba
882-Aprígio Veloso, 58100 Campina Grande, PB, Brazil

SYNOPSIS

Standardized Powell method (SP method) for finding the minimum of a non-linear function with many variables is applied for the optimization of coefficients in a runoff-erosion model, which is based on the kinematic wave assumption for both overland flow and river channel flow. Four parameters to be optimized in the model are the channel erosion parameter, the soil detachability factor, the sediment entrainment parameter by rainfall impact and the initial moisture-tension parameter. Data of runoff discharge and sediment yield corresponding to heavy rainfall events in 1987 and 1988 were obtained in a test field with the area of 5200 m^2 in a typically semiarid region of the northeastern Brazil. The sediment yield was found to increase remarkably when the total rainfall depth in a continuous rainfall event exceeds 10 mm and to be directly connected with runoff discharge, i.e., about 5% of the total runoff in weight. These data proved that the SP method is useful for the determination of the four coefficients in the runoff-erosion model.

INTRODUCTION

The sediment yield phenomenon is generally divided into two categories, sheet erosion by overland flows and sediment transport in river channels. Many types of sediment yield models have been discussed with the mathematical formulation of the pertinent processes (1, 9). However, construction of a runoff-erosion process modeling requires more attention to calibrate and verify the model with observed data. Runoff and sediment yield data obtained in an experimental basin installed near Sumé in the semiarid region of Paraíba in the northeastern Brazil are presented and simulated with a physically-based, event-oriented, numerical model for soil erosion and sediment transport which is summarized by Lopes (4). The model has four major parameters to be determined and they are the channel erosion parameter, the soil detachability factor, the sediment entrainment parameter by rainfall impact and the initial moisture-tension parameter, in which the former three are constant for the specific place under consideration and the last one depends largely on the preceding rainfall. Conjugate direction method proposed by Powell (7) is used for the determination of these four parameters. The conjugate direction method has been proved to be useful for finding the minimum of a non-linear function with many variables, for example the determination of coefficients in Tank model of runoff analysis. This Powell method was modified by Nagai et. al (5, 6) and called Standardized Powell method (SP method) in which coefficients of the model are standardized by their initial values. The SP method is applied for the optimization of the four parameters in the runoff-erosion modeling for the small test basin in the northeastern Brazil, in which the parameters are determined so that the differences between the estimated sediment discharges and the observed ones are minimized.

RUNOFF-EROSION MODELING AND PARAMETERS

Basic Equations for Surface Runoff

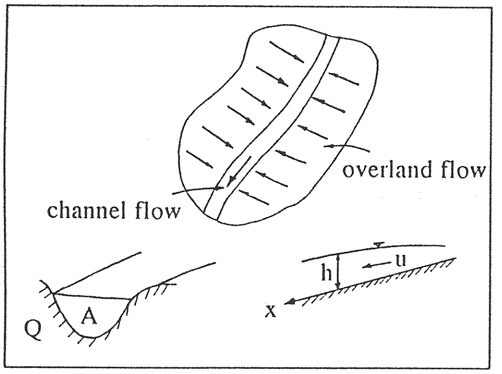

Fig. 1 Overland and channel flows.

Erosion of land surface soil and sediment transport are caused by overland and channel flows as shown in Fig. 1. These flows are usually analyzed based on the kinematic wave approximation.

Continuity and momentum equations are given as follows, respectively:

$$\frac{\partial h}{\partial t}+\frac{\partial (uh)}{\partial x}=r \quad (1); \qquad u=\alpha h^{m-1} \quad (2) \qquad \text{(for the overland flow)}$$

$$\frac{\partial A}{\partial t}+\frac{dQ}{dA}\frac{\partial A}{\partial x}=q_A \quad (3); \qquad Q=\alpha A R_H^{m-1}=\frac{\alpha A^m}{S^{m-1}} \quad (4) \qquad \text{(for the channel flow)}$$

where h = local flow depth; u = local mean flow velocity; r = lateral inflow rate per unit area; t = time; x = distance in the flow direction; and α, m are constants; Q = local flow discharge; A = section area; q_A = lateral inflow per unit length of channel; R = hydraulic radius; and S = wetted perimeter, r(x,t) in Eq. 1 is given by the rainfall intensity i(t) and the infiltration rate f(t) as follows (3):

$$r=i-f \quad (5); \qquad f(t)=\frac{dF}{dt}=K_s\left(1+\frac{N_s}{F(t)}\right) \quad (6)$$

where F(t) = cumulative depth of infiltrated water; N_s = soil moisture-tension parameter ;and K_s = saturated hydraulic conductivity. These basic equations for A, Q, u and h can be solved under the conditions: $h(0,t)=0$ for $t \geq 0$, $h(x,0)=0$ for $x \geq 0$, $Q(0,t)=Q_o(t)$ for $t \geq 0$ and $Q(x,0)=0$ for $x \geq 0$.

Basic Equations of Erosion and Deposition of Sediment

Sediment continuity equations are given by:

$$\frac{\partial (ch)}{\partial t}+\frac{\partial (cq)}{\partial x}=e_L+e_I-d_L \quad (7) \qquad \text{(for the overland flow)}$$

$$\frac{\partial (AC)}{\partial t}+\frac{\partial (CQ)}{\partial x}=e_R+q_s-d_R \quad (8) \qquad \text{(for the channel flow)}$$

where c and C = concentration of sediment in transport; q = uh; $e_L(x,t)$ = the rate of sediment entrainment by bed shear stress; d_L = rate of sediment deposition; and $e_I(x,t)$ = rate of sediment entrainment by rainfall impact; e_R = the rate of sediment entrainment by channel flow; q_s = lateral sediment inflow from adjacent flow planes; and d_R = rate of sediment deposition. When $\tau(x,t)$ = average effective bed shear stress; r_e = rainfall excess which assumed to be equal to r; and w_o = particle fall velocity; e_L, e_I which is assumed to be proportional to the product of i and r_e, and d_L are assumed to be given as follows, respectively:

$$e_L=K_L\tau^n \quad (9); \qquad e_I=K_I\, i\, r_e \quad (10); \qquad d_L=\beta\, w_o c \quad (11)$$

where K_L = soil detachability factor for shear stress; K_I =coefficient to measure soil detachability by rainfall impact; β = coefficient depending on the soil and fluid properties; and d_R is assumed to be given by the same form as d_L expressed by Eq. 11, and e_R is given by:

$$e_R = a(\tau - \tau_c)^n \quad \text{for} \quad \tau \geq \tau_c; \qquad e_R = 0 \quad \text{for} \quad \tau < \tau_c \tag{12}$$

where a and n = coefficients. The average shear stress $\tau(x,t)$ is given by $\tau = \rho gRI$ and the average critical shear stress τ_c for the representative particle size d is expressed by $\tau_c = \delta\,(\sigma - \rho)gd$, where I = the friction slope; g = the acceleration due to gravity; δ = constant; and σ, ρ = the densities of sediment and water, respectively.

In the calculation u and h for the overland flow are calculated first with Eqs. 1 and 2, from which q_A in Eq. 3 can be given, and Q and A are calculated. Then c and C can be calculated with Eq. 7 and Eq. 8, respectively.

Parameters to be Determined

There are many parameters to be given in the above mentioned model. Some of them may be assumed to be universal as m=5/3, n=3/2, δ = 0.047 and so on. The constant α is given if Manning's roughness coefficient is estimated and β can also be assumed. However, parameters such as moisture-tension parameter N_s and saturated hydraulic conductivity K_s in Eq. 6 should be given for the specific basin under consideration. The channel erosion parameter a ($kg{\cdot}m^2/N^{1.5}{\cdot}s$), the parameter for sediment entrainment by bed shear stress of overland flow K_L ($kg{\cdot}m/N^{1.5}{\cdot}s$) and the parameter to measure soil detachability by rainfall impact K_I ($kg{\cdot}s/m^4$) are also parameters to be determined for the specific field.

OBSERVED DATA AND MODELING OF TEST BASIN

Observed Data

An experimental basin was installed for the measurement of sediment yield from the basin due to rainfall in the semiarid region in the northeastern Brazil. Brown non calcic "vertic" soil covers more than 85% of the basin and this soil is typical of most of the semiarid regions of northeastern Brazil (8). Fig. 2 shows the topography of the test basin, which lies within experimental basin that is one of sub basins of Umburana River, and one of the test basins that have been studied since 1982. The basin with a mean slope of 7.1% has no vegetation with bare soil and its area and perimeter are 5200 m^2 and 302 m, respectively. At the outlet (point A in Fig. 2) of the basin, a rectangular collector for the measurement of sediment discharge is settled, terminating with a 90° triangular weir for the measurement of flow discharges. The collector holds all the surface runoff and sediment discharges from most of the low to medium rainfall events, thereby providing a means for accurate runoff and sediment measurement. A recording rain gauge was installed close to the basin to provide the necessary precipitation data.

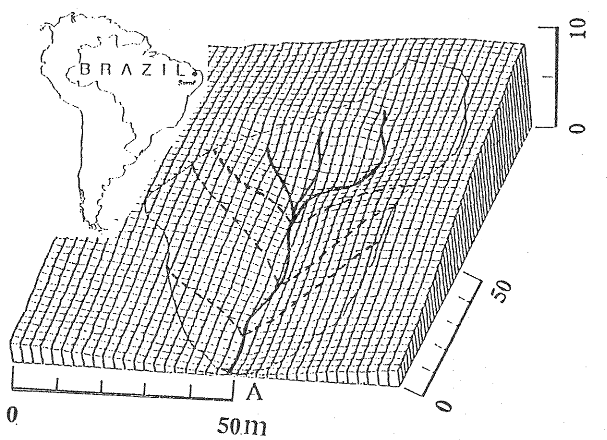

Fig. 2 Aeroview of test basin.

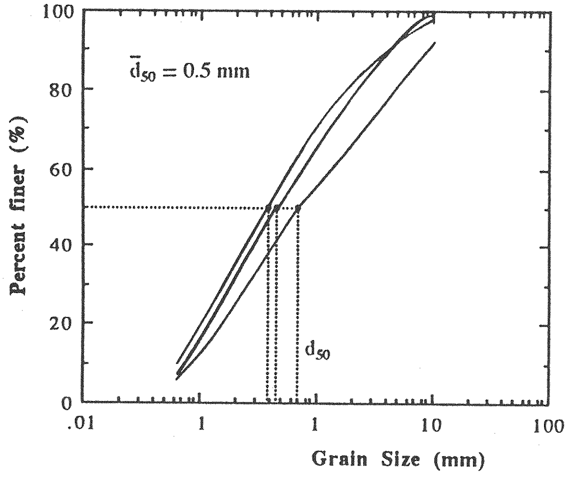

Fig. 3 Grain size distribution for three different places in the test basin.

Grading curves from three different points in the basin are shown in Fig. 3. The 50% finer diameter is about 0.5 mm, which is taken here as the representative grain diameter.

Rainfall intensity and flow discharge have been measured every minute and the total sediment discharges have been measured at the end of every continuous rainfall. In this study the period between 1987 and 1988 was chosen because the vegetation was controlled to keep the soil bared, then 21 events without measure error were selected. Fig. 4 shows the relationship between the total depth of a continuous rainfall R_o and that of the corresponding discharge L_o, where observed data are plotted according to antecedent days without rainfall in four groups. The runoff discharge rate is very small when that total rainfall depth is less than about 10 mm because of large infiltration capacity of the soil and large evaporation in the semiarid area. Except for a few events, the runoff depth is influenced by the antecedent days without rainfall as well as by the total rainfall depth. Runoff coefficient for the total rainfall depth more than 10 mm ranges widely from about 0.2 to 0.7, depending on the antecedent days without rainfall and characteristics of the rainfall intensity and duration. Fig. 5 shows the relationship between total rainfall depth R_o and total sediment yield E_o. Sediment yield is also negligible for the total rainfall less than about 10 mm, but soil seems to be actively eroded when the total rainfall becomes more than 10 mm. Sediment discharge seems to be directly connected with the runoff discharge as shown in Fig. 6 ($\gamma = 0.86$) rather than the rainfall depth more than 10 mm in Fig. 5 ($\gamma = 0.64$). From Fig. 6, the sediment yield from the test basin can be roughly estimated as 5% of the total runoff discharges in weight. This large rate of sediment production is due to the steep slopes without vegetation as well as the soil fragility.

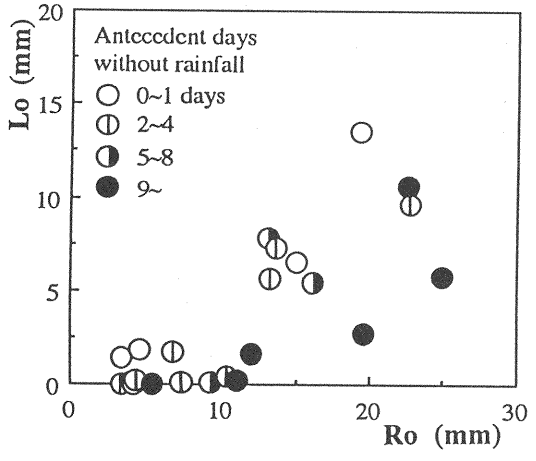

Fig. 4 Total rainfall depth R_o and runoff depth L_o.

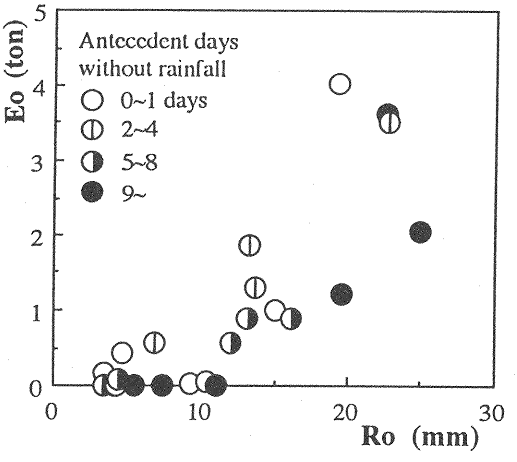

Fig. 5 Total rainfall depth R_o and sediment yield E_o.

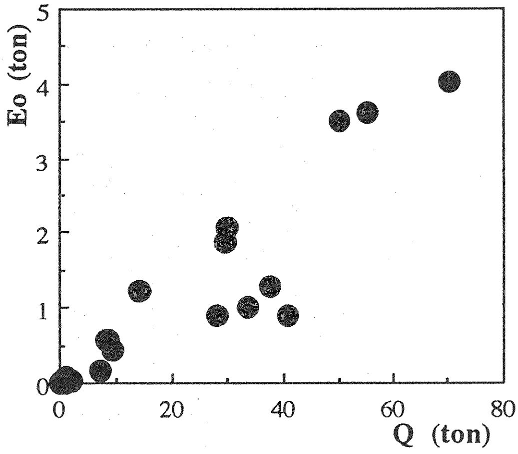

Fig. 6 Total flow discharge Q and sediment yield E_o.

Modeling of the Basin

The test basin in Fig. 2 is divided into 10 elements as shown in Fig. 7 for the runoff-erosion modeling explained above. Overland flows are assumed in the elements No. 1, 2, 3, 5, 6, 8 and No. 9, and channel flows in the elements No. 4, 7 and No. 10. The definition of planes were based on soil, slopes and surface cover characteristics. The plane boundaries are either streamlines or contour lines. During the discretization process an attempt was made to minimize geometric distortion by preserving the areas and lengths of flow paths for each plane element. Dimensions of the model elements are given in Table 1. The shape of the cross section of channel elements is assumed to be triangular and the lateral slopes are also given in the table.

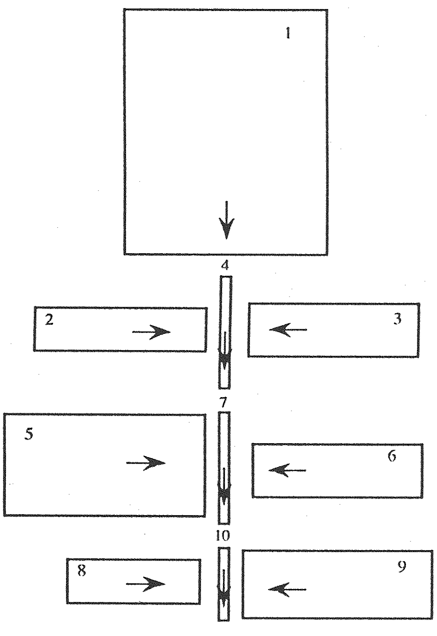

Fig. 7 Modeling of test basin.

Table 1 Dimensions of modeled elements.

Element	Area (m^2)	Length (m)	Width (m)	Slope	Lateral Slope
1	2166.66	41.18	52.60	0.0896	–
2	349.12	34.87	10.02	0.0936	–
3	434.45	34.67	12.52	0.0998	–
4	–	23.04	–	0.0554	0.25 : 1
5	931.36	41.33	22.54	0.0806	–
6	447.00	35.67	12.52	0.0903	–
7	–	23.04	–	0.0466	0.25 : 1
8	278.85	27.85	10.02	0.0791	–
9	592.56	39.43	15.03	0.0878	–
10	–	16.53	–	0.0665	0.25 : 1

DETERMINATION OF PARAMETERS BY SP METHOD

Summary of Standardized Powell Method (SP method)

Powell (4) proposed a new method to find values of n parameters $x_1, x_2, ..., x_n$, so that a function of these parameters, $J(x_1, x_2, ..., x_n)$, is a minimum.

The method of minimization which changes one variable at a time is modified to find the minimum of a quadratic form in a finite number of steps. Each iteration of the procedure commences with a search down n linearly independent directions $d_1, d_2, ..., d_n$, starting from the best known approximation to the minimum, p_o. These directions are chosen to be the coordinate directions initially, so the start of the first iteration is identical to an iteration of the method which changes one parameter at a time. This latter method is modified to generate conjugate directions by making each iteration define a new direction, d, and choosing the linearly independent direction for the next iteration to be $d_2, d_3, ..., d_n$, d. After n iterations all the directions are mutually conjugate and in consequence the exact minimum of the quadratic is found. Fig. 8 shows an example for conjugate directions for two dimensional space. Function J is assumed to be expressed by a quadratic.

Nagai et. al (5, 6) standardized each model parameter divided by its initial values, which makes the calculation effective even if orders of parameters to be determined are different.

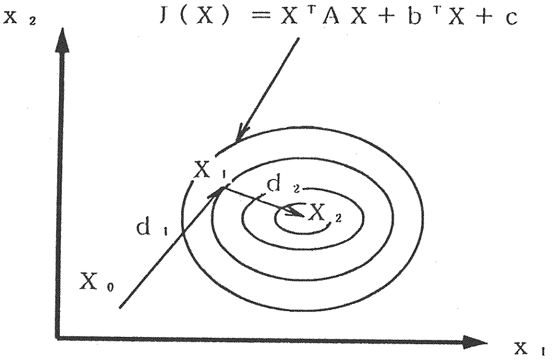

Fig. 8 Conjugate directions for two dimensional space of a quadratic function J(x).

Optimization of Parameters in Runoff-Erosion Model

Major parameters to be determined in the runoff-erosion model (WESP (Watershed Erosion Simulation Program) by Lopes (4)) are N_s, a, K_L and K_I, where N_s is the moisture-tension parameter, a is the channel erosion parameter, K_L is the soil detachability factor and K_I is sediment entrainment parameter by rainfall impact. The parameter N_s depends on the moisture condition at the beginning of rainfall, but the other three parameters can be assumed to be constant for the specific basin.

These parameters are standardized by their initial values as

$$x_1=\frac{N_s}{N_{s0}}, \qquad x_2=\frac{a}{a_0}, \qquad x_3=\frac{K_L}{K_{L0}}, \qquad x_4=\frac{K_I}{K_{I0}} \tag{13}$$

where suffix 0 means the initial values. When these four variables are given, runoff discharge L and sediment yield E can be calculated by the runoff-erosion model. The function J to be minimized is defined as

$$J=\left|\frac{L_o-L_c}{L_o}\right|+\left|\frac{E_o-E_c}{E_o}\right| \tag{14}$$

where suffix o means observed data and suffix c means calculated values. L_c and E_c are functions of x_1, x_2, x_3 and x_4, and therefore so is J. These four parameters are to be optimized by the SP method so that the evaluation function J becomes a minimum. All the parameters should be positive, and if some of the parameters become negative, a penalty function V is added to J so that the evaluation function becomes excessively large. The following function is introduced here:

$$V=\sum_{i=1}^{4} V_i \tag{15}$$

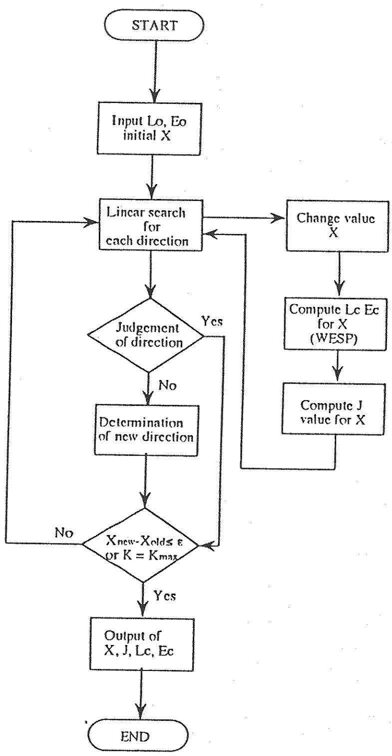

Fig. 9 Flow chart of calculation by SP method.

where $V_i = 0$ when $x_i \geq 0$, and $V_i = (x_i - e_i)^2$ when $x_i < 0$ and $e_i = 1$.
Fig. 9 is the flow chart of calculation by the SP method combined with the runoff-erosion model (WESP).

Twelve rainfall events, which caused more than 100 kg of sediment yield, are selected from 21 events for determination of the parameters, and the other 9 rainfall events are used to confirm the adequacy of the determined parameters.

Initial values of the parameters are arbitrary and given with reference to the previous work (2) about other test basins near this basin as follows: $N_{s0} = 40$ mm ($x_1 = 1$), a_0=0.014 kg·m^2/N$^{1.5}$·s ($x_2 = 1$), K_{L0}= 2.17 kg·m/N$^{1.5}$·s ($x_3 = 1$), K_{I0} = 5.0x10^8 kg·s/m^4 ($x_4 = 1$).

The infiltration is essential in this semiarid area when the preceding rainfall is small as considered here and K_s is determined as K_s = 5.0 mm/h for convenience with reference to the results of infiltration test on constant load (2). Manning's coefficient in α is assumed with reference to Woolhisen (10) to be 0.020 for the plane and 0.030 for the channel, and the coefficient for sediment deposition β is 0.50 for the plane and 1.0 for the channel.

Results of Optimization

Fig. 10 shows examples of variation of the evaluation function J with one parameter keeping the other three parameters fixed for rainfall event No. 5 after one step of the optimizing iteration. It is seen that the change of the moisture-tension parameter N_s has a great influence on the evaluation function J, although the change of the parameter for sediment entrainment by rain drop impact K_I of 10^8 order has little influence on the change of J.

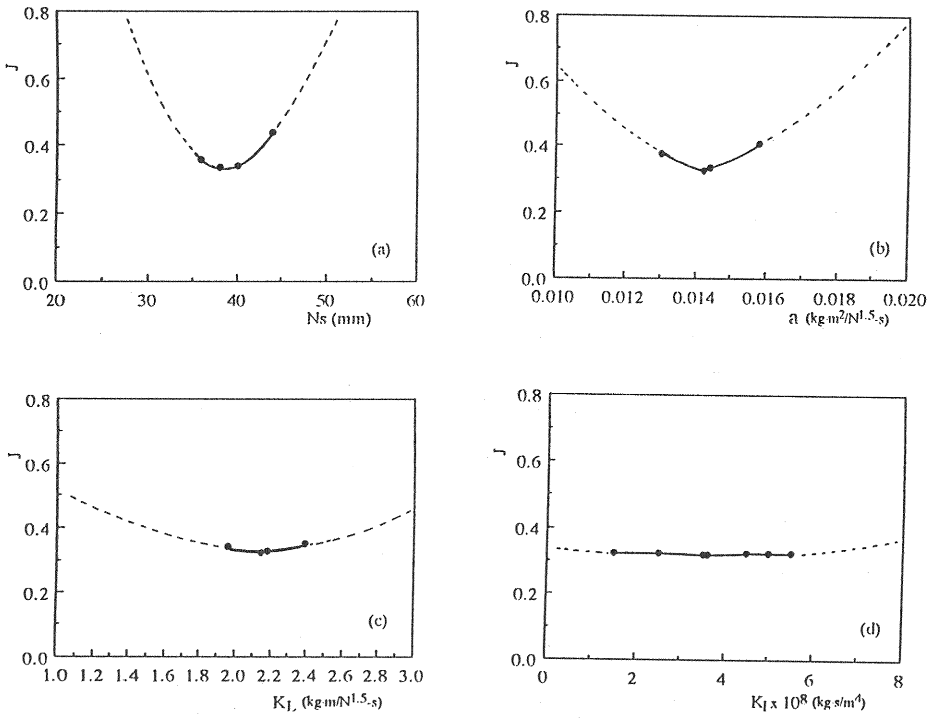

Fig.10 Examples of variation of the evaluation function J after one step of the optimizing iteration (rainfall event No. 14).

Table 2 shows the optimized parameters N_s, a, K_L and K_I, and observed and calculated runoff depth L and sediment yield E. Three parameters a, K_L and K_I should be constant for all rainfall events because they are characterized by the sand and soil in the test basin. The orders of these parameters for

Table 2 Optimized parameters and calculated values for data of E_o > 100 kg.

Rainfall Number	N_s (m)	a ($kg \cdot m^2/N^{1.5} \cdot s$)	K_L ($kg \cdot m/N^{1.5} \cdot s$)	$K_I \times 10^8$ ($kg \cdot s/m^4$)	E_o (kg)	E_c (kg)	L_o (mm)	L_c (mm)
4	0.02899	0.0215	2.412	4.8	1214.6100	1214.6116	2.7410	2.7412
6	0.00406	0.0138	1.937	1.5	168.1800	168.2022	1.3710	0.7407
8	0.09180	0.0128	2.098	3.4	2061.8601	2061.9255	5.7120	5.885
9	0.00745	0.0189	2.760	1.5	568.4900	568.7043	1.5770	1.6846
11	0.00825	0.0139	2.119	2.0	1875.5300	1875.5878	5.6690	4.6410
12	0.00585	0.0147	2.363	1.5	580.0700	580.1601	1.6890	1.7164
13	0.05370	0.0144	2.176	5.1	4019.0400	4018.5918	13.5210	6.3088
14	0.03860	0.0142	2.145	3.6	3615.4000	3615.0056	10.6000	7.2291
15	0.04520	0.0139	2.136	3.1	1286.6500	1286.7074	7.2310	3.1380
16	0.01550	0.0136	2.194	6.7	3504.5500	3504.3662	9.6330	8.8865
17	0.05700	0.0139	2.034	6.9	887.4700	887.3751	5.3610	2.6507
18	0.03690	0.0145	2.232	8.1	898.4700	898.5058	7.8190	2.7904

all the rainfall events can be assumed to be equal, and the average values of the parameters over the events can become the values for the specific test field. That is: $a = 0.015\ kg \cdot m^2/N^{1.5} \cdot s$, $K_L = 2.22\ kg \cdot m/N^{1.5} \cdot s$, $K_I = 4.0 \times 10^8\ kg \cdot s/m^4$.

The initial moisture-tension parameter N_s changes largely with each rainfall event, because N_s is directly related to the moisture in the soil, therefore to the antecedent rainfall condition before the event. Fig. 11 shows the relationship between N_s and antecedent days without rainfall, where data for E_o < 100 kg are optimized values using the above averaged value of a, K_L and K_I. The fitting curve in Fig. 11 can be used to estimate N_s, and it is very convenient because the values of N_s depend not only on the antecedent days without rainfall but also the antecedent rainfall intensity and other conditions.

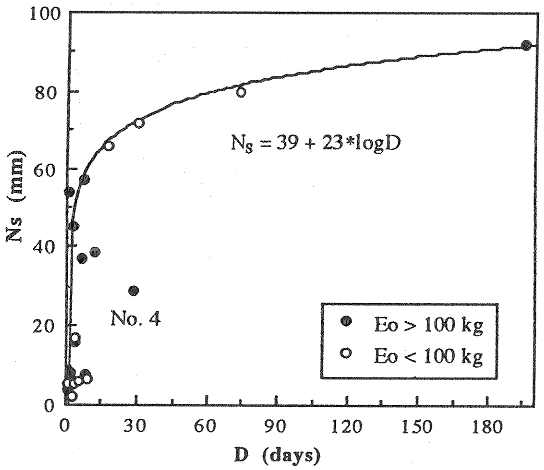

Fig. 11 Optimized moisture-tension parameter Ns and number of days between consecutive storms D.

Fig. 12 and Fig. 13 show the comparison between observed and calculated runoff depth L and sediment yield E for all the 21 rainfall events, using the above average values of a, K_L, K_I and the fitting curve for N_s. The calculated values for L_c and E_c seem to follow the observed values for almost every rainfall event from weak to heavy rainfall with few exceptions such as rainfall number 4 for which the fitting curve for N_s gives a much large value than the optimized values as in Fig. 11. However, calculated L_c is less accurate than E_c, which may be attributed to the direct response of L_c to the deviation of the N_s fitting curve from the optimum N_s value for each event.

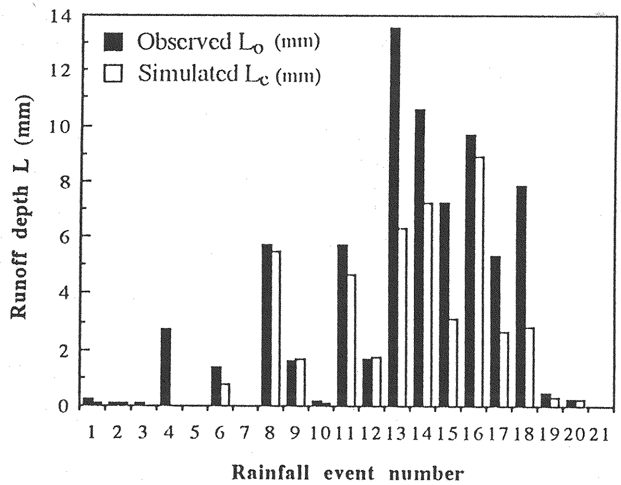

Fig. 12 Observed total runoff depths Lo and simuleted one Lc.

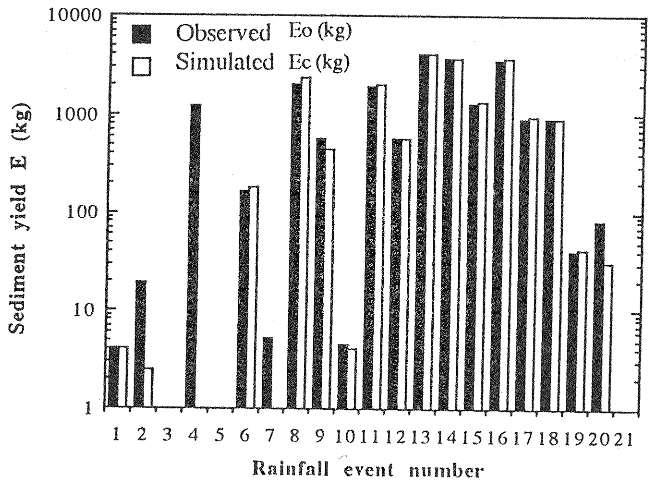

Fig. 13 Observed total sediment yield Eo and simulated one Ec.

CONCLUSIONS

Standardized Powell method for finding the minimum of a non-linear function with many variables is applied for the optimization of parameters in a runoff-erosion modeling, using data of runoff and sediment yield observed in a test field with the area of 5200 m^2 in a typically semiarid region of the northeastern Brazil. The following conclusions are obtained:

1) Runoff-erosion modeling based on the kinematic wave approximation both for overland flow and channel flow can give a good estimation of the runoff discharge and sediment yield with appropriate values of the parameters in the model.

2) The Standardized Powell method is useful for the optimization of four parameters in the runoff-erosion modeling.

3) The channel erosion parameter a, the soil detachability factor K_L and sediment entrainment parameter by rainfall impact K_I are obtained as constant for almost all rainfall events, i.e., a = 0.015 $kg{\cdot}m^2/N^{1.5}{\cdot}s$, K_L = 2.22 $kg{\cdot}m/N^{1.5}{\cdot}s$, $K_I = 4.0x10^8$ $kg{\cdot}s/m^4$ for the test basin.

4) The moisture-tension parameter N_s is proved to depend mainly on the number of days D between the consecutive storms, and the relationship between N_s and D is determined for the basin, for example $N_s \approx 90$ mm for D > 50 days and N_s varies from 0 to 60 mm within few antecedent days without rainfall.

ACKNOWLEDGMENT

The writers wish to thank Dr. Lopes of Superior School of Agriculture at Mossoró for the WESP (program), and SUDENE (Brazil) and ORSTOM (France) for the field data.

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APPENDIX - NOTATION

The following symbols are used in this paper:

a	= the channel erosion parameter;
a_0	= the initial value of a;
A	= the section area;
c, C	= the concentration of sediment in transport;
$d_1, d_2, ..., d_n$	= the linearly independent directions;
d_L	= the rate of sediment deposition;
d_R	= the rate of sediment deposition;
D	= the number of days between the consecutive storms;
$e_I(x,t)$	= the rate of sediment entrainment by rainfall impact;
$e_L(x,t)$	= the rate of sediment entrainment by bed shear stress;
e_R	= the rate of sediment entrainment by channel flow;
E	= the sediment yield;
E_c	= the calculated sediment yield;
E_o	= the observed sediment yield;
$f(t)$	= the infiltration rate;
$F(t)$	= the cumulative depth of infiltrated water;
g	= the acceleration due to gravity;
h	= the local flow depth;
$i(t)$	= the rainfall intensity;

I	= the friction slope;
$J(x_1, x_2, ..., x_n)$	= a function;
K_I	= a coefficient to measure soil detachability by rainfall impact;
K_L	= a soil detachability factor for shear stress;
K_{I0}	= the initial value of K_I;
K_{L0}	= the initial value of K_L;
K_s	= the saturated hydraulic conductivity;
L	= the runoff discharge;
L_c	= the calculated runoff discharge;
L_o	= the observed runoff discharge;
m	= constant;
n	= coefficient;
N_s	= the soil moisture-tension parameter;
N_{s0}	= the initial value of N_s;
p_o	= the minimum approximation;
q_A	= the lateral inflow per unit length of channel;
q_s	= the lateral sediment inflow from adjacent flow planes;
Q	= the local flow discharge;
r	= the lateral inflow rate per unit area;
r_e	= the rainfall excess;
R	= the hydraulic radius;
S	= the wetted perimeter;
t	= the time;
u	= the local mean flow velocity;
V	= the penalty function;
w_o	= the particle fall velocity;
x	= the distance in the flow direction;
$x_1, x_2, ..., x_n$	= parameters;
α	= constant;
β	= a coefficient depending on the soil and fluid properties;
δ	= a constant;
ρ	= the density of water;
σ	= the density of sediment;
$\tau(x,t)$	= the average effective bed shear stress;
τ_c	= the average critical shear stress for the representative particle size; and
γ	= correlation coefficient.

(Received June 10, 1993 : revised February 22, 1994)