MOVEMENT OF WATER VAPOR IN
SAND COLUMN AND MECHANISM OF EVAPORATION
BY

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ABSTRACT

This paper presents an experimental investigation on the movement of water vapor in a sand column, which is comprised of dry, capillary and saturated zones, and the relationship between the vapor pressure profile and evaporation velocity. The vapor pressure is obtained from the simultaneous measurements of temperature and relative humidity. It became clear that the evaporation actively occurred near a top of the capillary fringe and most of the evaporated water vapor moved toward the atmosphere and the remaining water vapor moved downward and then condensed. The experimental results for the vapor pressure profiles agreed with the theoretical results based on the one way diffusion theory. A comparison between the theoretical and experimental results demonstrated the importance of the effect of the vapor pressure gradient across the dry zone on the evaporation velocity.
Keywords : evaporation, drying, heat and mass transfer, groundwater flow.

INTRODUCTION

Evaporation from soils is influenced by the external evaporativity related to meteorological factors (for example, air temperature, humidity and wind velocity) and by internal factors such as the physical properties of unsaturated soils (the relationship of the hydraulic conductivity and matric suction to the moisture content) and the height of the groundwater table. Evaporation is of two types, one being the evaporation from the soil surface. It happens when the groundwater table is shallow. The second is the evaporation occurring in soils, then there exists dry or capillary zones above the groundwater table. A particular problem is that the behavior of water vapor moving through pores in soils is not yet fully understood. This is one of the main reasons why it is still difficult to estimate the evaporation rate.

Suzuki (6) and Shinjyo (5) investigated that the movement of the water vapor in closed vessels under the temperature gradient and concluded that the water vapor moves from a warm

part towards a cool part. The movement of the water vapor in an open column, however, is much more complicate and is dominated by the combination of the external and internal conditions said above. The vapor pressure and vapor density in pores are significant factors to assess the value of the water vapor flux moving in sands. We have already established an experimental method to obtain these two factors along the sand column (1, 2, 3, 4).

The purpose of this paper is to describe the characteristics of the behavior of water vapor evaporated in the sand column and the relationship between evaporation rate and vapor pressure distribution.

HEAT ENERGY EQUATION

Considering the accuracy of the experiment, a heat loss through the sand column with a heat insulator has to be introduced into the heat energy equations. Since the evaporation mainly takes place around the interface between dry and unsaturated zones (3), the heat energy equations in steady state are given for the following zones or part.

(A) Dry zone (zone 1)

Heat transfer in the dry zone is generated by the heat conduction and the sensible heat by the movement of the water vapor, but the sensible heat can be omitted usually. Thus the energy equation becomes

$$\lambda_1 \frac{\partial^2 T_1}{\partial z^2} - \alpha_c (T_1 - T_a) = 0 \tag{1}$$

where λ is the thermal conductivity, T is the temperature, α_c is the heat loss coefficient, z is the coordinate in the vertical direction and T_a is the air temperature. Suffix 1 expresses the dry zone.

(B) Unsaturated zone (zone 2)

Heat flux in an unsaturated zone mainly depends on the heat conduction because the sensible heat by liquid water movement is negligible (2). As λ is a function of the volumetric water content θ, λ is no longer constant in this zone (3). Therefore, the heat energy equation takes the form,

$$\frac{\partial}{\partial z} \left(\lambda_2 \frac{\partial T_2}{\partial z} \right) - \alpha_c (T_2 - T_a) = 0 \tag{2}$$

Suffix 2 means the unsaturated or capillary zone.

(C) Interface

The latent heat by the evaporation plays a major role in determining the heat energy budget at the interface between dry and capillary zone, and it can be expressed as,

$$\lambda_1 \left(\frac{\partial T_1}{\partial z} \right) - \lambda_2 \left(\frac{\partial T_2}{\partial z} \right) = \Delta h_v M_v \tag{3}$$

where Δh_v is the latent heat of vaporization and M_v is the evaporation flux density. Moreover , the temperature must continue at the interface $(z = -D)$. It follows that

$$T_1 = T_2 = T_D \tag{4}$$

in which T_D expresses the temperature at the interface. Eqs.(1) and (2) are numerically solved by the Finite Differential Method. For further details of the computation flow, refer to Fukuhara et al. (2, 3).

EVAPORATION RATE AND VAPOR PRESSURE

Let's consider the water vapor flux density due to the evaporation from the interface between air and soil particles in pores. The vapor flux density, M'_v, due to the diffusion is written as,

$$M'_v = -\frac{\xi D_{atm}}{R_v T}\frac{\partial P_v}{\partial z} + \frac{\xi D_{atm} P_v}{R_v T^2}\frac{\partial T}{\partial z} \tag{5}$$

where P_v is the water vapor pressure component in the gas pressure, R_v is the gas constant of water vapor and ξ is the ratio of the contact surface area between pores and soil particles to an apparent contact surface area (unit area). D_{atm} denotes the molecular diffusion coefficient of the water vapor in pores and is given by

$$D_{atm} = \varepsilon \zeta D^*_{atm} \tag{6}$$

in which, ε is the porosity, ζ is the tortuosity and D^*_{atm} is the molecular diffusion coefficient of the water vapor in the atmosphere. Considering the mass transfer with the convective velocity of air, V, the total evaporation mass flux density, M_v, is obtained by

$$M_v = M'_v + \rho_v V \tag{7}$$

If the air does not contribute to the mass transfer and the mass transfer due to the gas pressure-gradient is neglected (this is confirmed from Figure 7 in this paper), the convective velocity, V, can be calculated. Thus, the vapor flux density, M_v, becomes

$$M_v = -\frac{\xi D_{atm}}{R_v T}\frac{P}{(P - P_v)}\frac{\partial P_v}{\partial z} \tag{8}$$

where P is the gas pressure in the soil pores.

After all, Eq. (8) is apparently the same as the expression of M_v, which is induced by the one way diffusion theory under the constant temperature.

As shown in Figure 5, the vapor pressure distribution has a negative gradient through the dry zone and a positive gradient in the capillary zone. This means that the water vapor moves upward from the interface towards the soil surface and finally is released to the atmosphere. From this result, Eq. (8) should be applied to the dry zone. The analytic solution of Eq. (1) is described as follows:

$$T_1 = (C_1 + C_2 z)\, exp\left[\left(\frac{\alpha_c}{\lambda_1}z\right)\right] + T_a \tag{9}$$

The temperature, T_0, at the soil surface ($z = 0$) and the temperature of the groundwater, T_w ($z = -L$), are given as the boundary conditions. Since the temperature at the interface ($z = -D$), T_D, can be calculated from Eqs. (1) and (2), the integral constants C_1 and C_2 take the forms,

$$C_1 = T_0 - T_a \tag{10}$$

$$C_2 = -\frac{T_D - T_a - C_1 exp\left[-\left(\alpha_c/\lambda_1\right)D\right]}{D exp\left[-\left(\alpha_c/\lambda_1\right)D\right]} \tag{11}$$

Substituting T_1 in Eq. (9) into T in Eq. (8) and then Eq. (8) is integrated from $z = 0$ to $z = -D$, i.e. over the whole dry region. Thus, M_v is expressed as

$$M_v = \frac{\xi D_{atm} P}{R_v F(-D)}ln\left(\frac{P - P_{v0}}{P - P_{vD}}\right) \tag{12}$$

$$
\begin{aligned}
F(z) = &\left[-\frac{C_1}{\alpha_c/\lambda_1} + \frac{C_2}{\left(\alpha_c/\lambda_1\right)^2} + exp\left[\left(\frac{\alpha_c}{\lambda_1}z\right)\right]\left(\frac{C_1 + C_2 z}{\alpha_c/\lambda_1}\right)\right.\\
&\left. - \frac{C_2 exp\left[\left(\alpha_c/\lambda_1\right)z\right]}{\left(\alpha_c/\lambda_1\right)^2} + (T_a + 273.15)\,z\right]
\end{aligned}
\tag{13}
$$

where P_{v0} and P_{vD} are the water vapor pressure components in the gas pressure at $z = 0$ and $z = -D$, respectively. From Eq. (12), the vapor pressure distribution over the dry zone is given as follows :

$$P_v = P - (P - P_{v0}) \, exp \left[\frac{-M_v R_v F(z)}{\xi D_{atm} P} \right] \tag{14}$$

EXPERIMENTAL EQUIPMENT AND METHOD

Figure 1 shows the experimental apparatus and measurement system. Experiments are conducted in a sand column $0.1 \ m \times 0.1 \ m$ and $0.6 \ m$ high, which is made of acrylic resin. The sand column is surrounded by a heat insulator, which is composed of high density-glass wool with a width of $25 \ mm$. The heat loss coefficient, α_c is obtained as $6.28 \times 10^{-4} \ (KWm^2/K)$ by a preliminary test. The soil used in this experiment is a uniform sand of mean diameter $0.2 \ mm$. The groundwater level is controlled by an adjusting tank with overflow equipment. A $350 \ W$ heat lamp shines downward towards the sand surface , so that the temperature gradient is formed along the sand column. All experiments starts at the same time the heat lamp begins working.

The temperature, T, the relative humidity in the soil pores, RH and the differential pressure are measured by the thermo-hygrometer (VISALA Ltd.) and the differential pressure gage (YAMAMOTO ELEC. Corp.), respectively. The thermo-hygrometer is inserted into a guide tube with an inside diameter of $13 \ mm$ and the differential pressure gage is connected with a bulb of guide tube . It takes about 30 minutes to measure T and RH per each tube. The evaporation rate is obtained by the decrease in the water level of the adjusting tank. A distilled water is used in this experiment and is supplied into the sand column to keep the groundwater level constant several times a day. The volumetric water content, θ is measured by using a soil moisture sensor (NORTH HIGHTECH Ltd.). All experiments are carried out in a constant temperature and humidity room.

Experimental cases and conditions are summarized in Table 1. The experimental cases are classified on the basis of the radiation strength and groundwater level. The groundwater level is located at $z = -0.55 \ m$ and $-0.44 \ m$, and the lamp is set at a height of $0.3 \ m$ or $0.5 \ m$ from the soil surface.
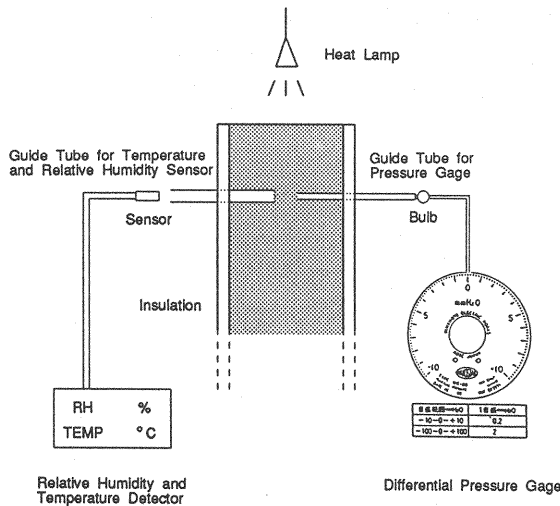

Figure 1   Schematic view of experimental apparatus and measurement system

Table 1   Experimental conditions

Experimental case		CASE – 1	CASE – 2	CASE – 3	CASE – 4
Density of sand	(Kg/m³)	2.462×10^3	2.462×10^3	2.462×10^3	2.462×10^3
Duration	(Day)	7	7	7	7
Groundwater level	(m)	-0.55	-0.55	-0.40	-0.40
Porosity	(-)	0.42	0.42	0.42	0.42
Evaporation rate	(m/s)	1.80×10^{-9}	2.31×10^{-9}	2.31×10^{-8}	2.86×10^{-8}
Lamp height	(m)	0.5	0.3	0.5	0.3

EXPERIMENTAL RESULTS AND DISCUSSIONS

Figure 2 shows the temperature profiles. The temperature and evaporation rate, calculated by Eqs. (1), (2) and (12), are also presented in Figure 2. Comparing of the present computational results with our previous one (2), which did not consider the heat loss, the former becomes better, especially near the soil surface. The computed evaporation rate also agrees with the evaporation rate measured in the experiments. From these results, it is understood that once the external and internal conditions are given, there exists only one set of the steady temperature profile and evaporation flux density, according to these conditions.

Next, in order to understand the heat transfer at the interface, three kinds of heat energy flux densities, which contribute to $T(z)$, are extracted by the computation. The first is the heat flux density, Q_{T1}, due to the heat conduction, the second is Q_{T2}, i.e. the latent heat flux density due to the evaporation and the third is Q_{T3} due to the heat loss. Results are as follows : Q_{T1} : Q_{T2} : $Q_{T3} = 1 : 1 : 0.13$ in CASE – 1, Q_{T1} : Q_{T2} : $Q_{T3} = 1 : 1.29 : 0.1$ in CASE – 2, Q_{T1} : Q_{T2} : $Q_{T3} = 1 : 1.83 : 0.07$ in CASE – 3 and Q_{T1} : Q_{T2} : $Q_{T3} = 1 : 2.33 : 0.08$ in CASE – 4. The contribution from Q_{T2} gets larger as the evaporation rate increases.

Figure 3 presents the volumetric water content profiles. It is known that the dry layer is thicker in CASE – 1 and CASE – 2 than in CASE – 3 and CASE – 4.
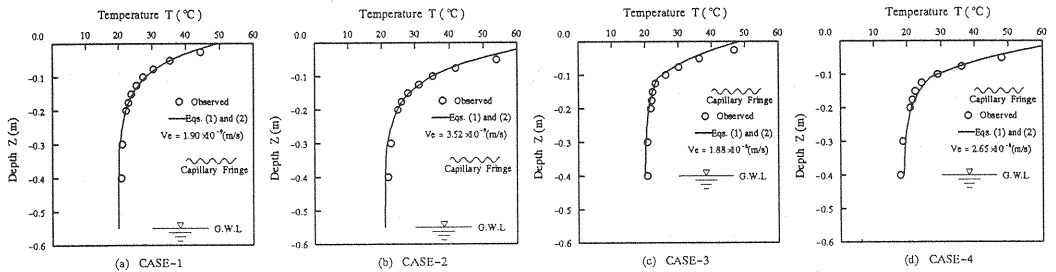

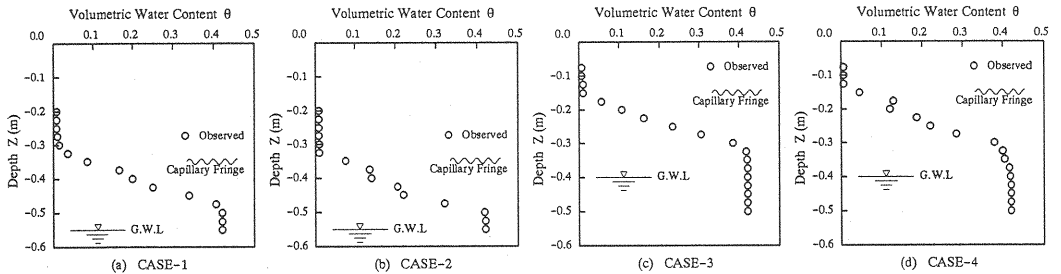Figure 2   Temperature distributions

Figure 3   Volumetric water content profiles

Figure 4 shows the relative humidity distributions, $RH(z)$. In CASE $-$ 3 and CASE $-$ 4, RH are nearly 100 % over the large portion of the capillary zone and a sudden decrease is observed around the interface. The values of RH at the upper part of the capillary zone in CASE $-$ 1 and CASE $-$ 2, however, are lower than 100 % .

Figures 5 and 6 show the vapor pressure profiles, $P_v(z)$ and the vapor density profiles, $\rho_v(z)$, respectively. Figure 5 also presents the calculated vapor pressure profiles in the dry region from Eq.(14). The calculated results coincide with the experimental results and $P_v(z)$ in the dry region is nearly linear. The shapes of $P_v(z)$ and $\rho_v(z)$ along the soil column are quite similar each other. In CASE $-$ 3 and CASE $-$ 4, $P_v(z)$ and $\rho_v(z)$ have their maximums at the interface, i.e. $z = -D$, and decrease towards the soil surface and groundwater surface. Thus the pressure gradient, dP_v/dz, is negative over the dry zone and positive over the capillary zone. That is, the water vapor moves upward towards the soil surface in the dry zone and downward towards the groundwater surface in the capillary zone.

This means that the liquid water, pulled up to the interface by the matric potential, evaporates actively around there. This phenomenon causes the salt accumulation occurring at the interface, observed by Sato et al. (4). Furthermore, it is supposed that the water vapor moving downward towards the groundwater surface may condense in the capillary zone. The value of P_v in CASE $-$ 1 and CASE $-$ 2 monotonously decreases from the capillary zone toward the dry zone,
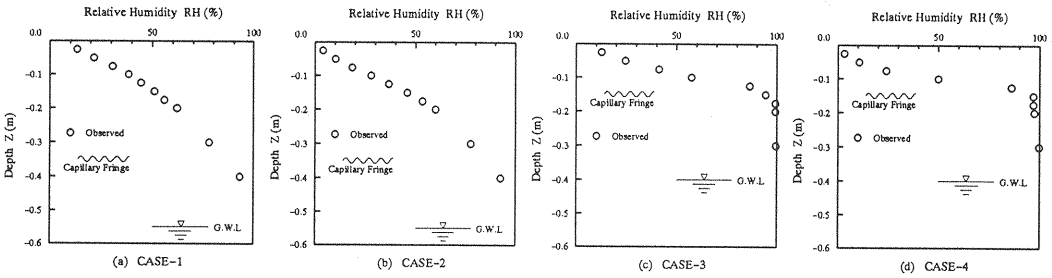

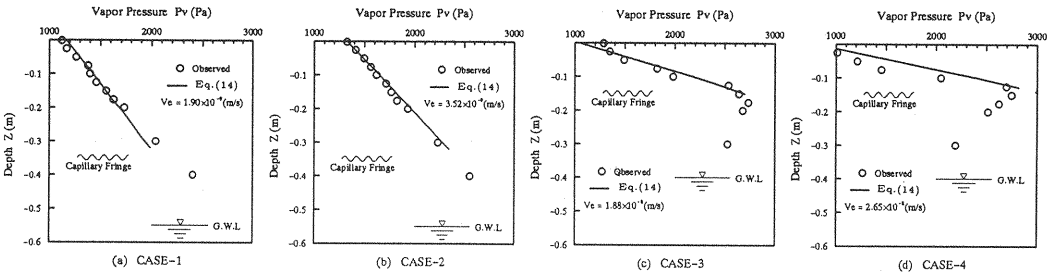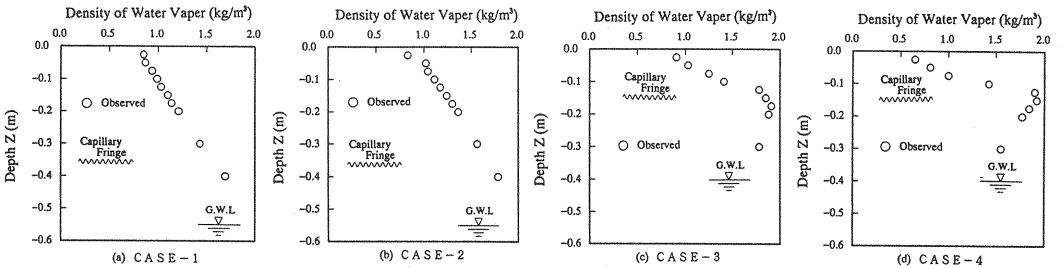Figure 4   Relative humidity distributions

Figure 5   Vapor pressure profiles

Figure 6   Vapor density profiles

i.e. $dP_v/dz < 0$. The value of dP_v/dz, however, changes at the interface as well as CASE – 3 and CASE – 4, and dP_v/dz in the dry zone is smaller than that in the capillary zone.

Following the diffusion theory of water vapor, the evaporation rate depends on the gradient of the vapor pressure or vapor density. In the case of evaporation occurring in a sand layer, the gradients of P_v and ρ_v over the dry zone describe the evaporation rate, V_e, so that V_e decreases in inverse proportion to the thickness of the dry zone, D, becomes large. Since P_v distribution over the dry zone is almost linear as shown in Figure 4, the difference of P_v between at the soil surface and interface, i.e. ($P_{vD} - P_{v0}$) and D represent the pressure gradient over the dry zone. As P_{v0} and P_{vD} change with the air temperature and relative humidity, radiation density and groundwater level, it is known that the evaporation rate is determined by not only the external conditions but also the internal conditions.

The coefficient relating to the increase in contact surface between pores and sand particles, ξ varies from 1.5 to 4.0. The effect of ξ emphasizes that the loss of soil moisture by the evaporation is ξ times larger than the previous expression concerning the water vapor diffusion without ξ.

Figure 7 shows the differential pressure distributions and P_{atm} denotes the atmospheric pressure. The differential pressures are almost equal to zero for every case and it is not necessary to consider the mass transfer due to the gas pressure gradient in the dry zone.

According to the results obtained in this study, Figure 8 illustrates a schematic model on the movement of the liquid water and water vapor in the sand layer. This model explains that the liquid water, pulled up by the capillary pressure, reaches the interface and evaporates there, and most of the water vapor moves towards the soil surface. Finally, the water vapor is released to the atmosphere by the external evaporativity. The water vapor remaining at the interface moves downward towards the groundwater surface again and finally condenses in the capillary zone. From this, it can be inferred that the evaporation rate, evaluated based on the mass flux density of the liquid water, is not appropriate in the strict sense.
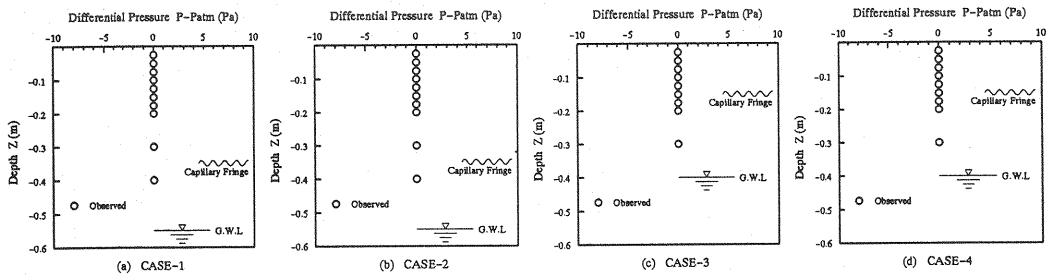
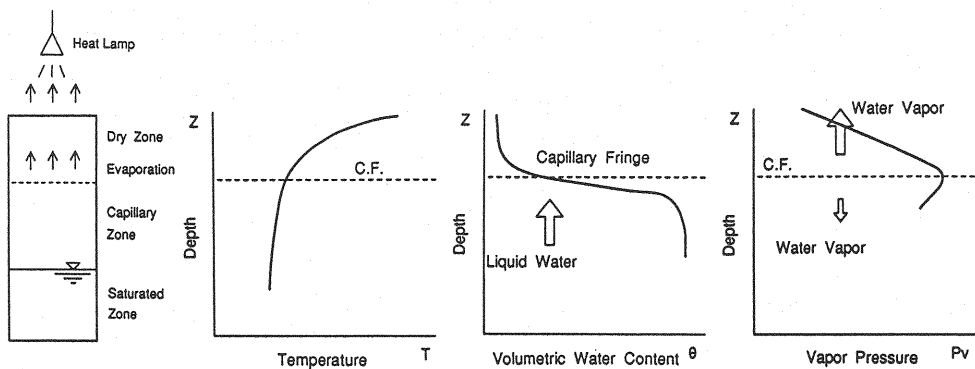Figure 7   Differential pressure distributions

Figure 8   Schematic model on movement of liquid water and water vapor in sand

CONCLUSION

The mechanism of the evaporation in the sand column is discussed and the mathematical model to evaluate the evaporation rate is proposed in this paper.

Main conclusions are as follows:

1. The gas pressure of pores in the sand is equal to the atmospheric pressure, and the mass transfer by the gas pressure gradient can be ignored.

2. In the case of the relatively high groundwater level (CASE – 3 or CASE – 4), the relative humidity, RH is almost 100 % over the capillary zone and decreases suddenly from the interface towards the soil surface. In the case of the relatively low groundwater level (CASE – 1 or CASE – 2), RH decreases monotonously toward the soil surface and does not reach 100

3. The maximum values of the vapor pressure and vapor density take place at the interface together. The vapor pressure gradient is negative over the dry zone and positive in the capillary zone in CASE – 3 and CASE – 4. The vapor pressure gradient in CASE – 1 and CASE – 2, however, is negative over the dry and capillary zones but their gradients change at the interface. The vapor pressure gradient is smaller in the dry zone than the capillary zone.

4. The evaporation rate depends on the vapor pressure gradient in the dry zone, i.e. the vapor pressure difference between at sand surface and interface and the thickness of the dry zone. Then the increase of contact surface between pores and sand particles has to be considered into the water vapor diffusion theory.

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APPENDIX – NOTATION

The following symbols are used in this paper :

C_1 = integral constant
C_2 = integral constant
D = thickness of dry zone
D_{atm} = molecular diffusion coefficient of the water vapor in the soil pores
D_{atm}^* = molecular diffusion of the water vapor in atmosphere
M_v = total vapor flux density (evaporation mass flux density)
M_v' = vapor flux density due to diffusion
P = gas pressure in the soil pores
P_{atm} = atmospheric pressure
P_v = water vapor pressure component in gas pressure
P_{v0} = water vapor pressure components in the gas pressure at soil surface ($z = 0$)
P_{vD} = water vapor pressure components in the gas pressure at interface ($z = -D$)
Q_{T1} = heat flux density due to the heat conduction
Q_{T2} = latent heat flux density due to evaporation
Q_{T3} = heat flux density due to heat loss
RH = relative humidity
R_v = gas constant of water vapor
T_0 = temperature at soil surface ($z = 0$)
T_1 = temperature in dry zone
T_2 = temperature in unsaturated zone
T_a = air temperature
T_D = temperature at interface ($z = -D$)
T_w = temperature of the groundwater ($z = -L$)
V = convective velocity
V_e = evaporation rate
z = coordinate in vertical direction
α_c = heat loss coefficient
Δh_v = latent heat of vaporization
ε = porosity
ζ = tortuosity
θ = volumetric water content
λ_1 = thermal conductivity
ξ = ratio of the contact surface area between pores and soil particles to an apparent contact surface area (unit area)
ρ_v = vapor density