Journal of Hydroscience and Hydraulic Engineering
Vol. 11, No. 2, January, 1994, 69-82

AN IMPLICIT MODEL BASED ON CONSERVATIVE FLUX SPLITTING TECHNIQUE FOR ONE DIMENSIONAL UNSTEADY FLOW

By

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SYNOPSIS

To simulate one-dimensional unsteady free surface flows an implicit mathematical model is developed which includes such feature as the conservative flux splitting technique. The concept of directional property of signal propagation in case of the hyperbolic partial differential equations is implemented by split flux technique. Automatic switching of space difference operators through signs of characteristic directions enables the proposed model to appropriately handle the simultaneous presence of supercritical and subcritical flows. The concept of approximate Jacobian is used for the conservative splitting of flux vectors. Details of the governing equations and the model are presented. One-dimensional dam-break flood wave and reflected shock wave propagations are analysed to demonstrate the applicability and the validity of the proposed model.

INTRODUCTION

The need for the solution of the partial differential equations governing one-dimensional unsteady free surface flows has been well appreciated. The governing equations do not lend themselves to analytical solution except for the cases simplified to the extent of losing their practical usefulness although the analytical solutions (e.g. Stoker (17)) are useful for examining validity of numerical solutions. The limitation of analytical approach coupled with time and efficiency problem associated with physical models make the mathematical modelling more feasible as an alternative means to solve the governing equations. So much literature exists on the mathematical modelling of one dimensional unsteady flow that their review can not be accommodated here and they may be referred to any textbook (e.g. Abbott (1), Cunge et al. (5), Anderson et al. (3)). However, the work on the numerical modelling of one-dimensional unsteady flow continues because there still does not exist a model which can be claimed as giving the best result in all possible cases.

The main problems in developing a mathematical model for one dimensional unsteady flow may be resulting from the non-linearity of the equations and occurrence of discontinuous flow in the form of shocks or bores. Numerical treatment of the flow situations where both supercritical and subcritical flows are present simultaneously or in sequence is also often complicated. In view of the practical requirements it is desirable to develop a mathematical model which is simple to formulate and program, handles natural channel geometry and treats shocks and bores to a reasonable extent.

The conservation form of the governing partial differential equations is suitable if shock or bores are expected to develop in the solution (e.g. Lax (11), Lax and Wendroff (12)). It is also essential that the finite difference equivalent of the conservative governing equations is conservatively expressed. The MacCormack scheme uses conservative form of the governing equations for its explicit predictor-corrector type algorithm. It captures the shock to some extent (e.g. Fennema and Chaudhry (6)). However, the MacCormack scheme does not take into account the direction of signal propagation which is different in subcritical and supercritical flows (e.g. Moretti (15)) and as a result it fails in such severe situation as when the ratio of tailwater depth to reservoir depth is very small in a dam-break problem (e.g. Alcrudo et al. (2)). Split flux technique may be a means to handle the direction of signal propagation in case of hyperbolic partial differential equations. Several schemes based on flux splitting technique have been developed (e.g. Beam and Warming (4), Moretti (15), Gabutti (7)). The Moretti's λ-scheme and its improved version, the Gabutti scheme, are second order accurate predictor-corrector type explicit schemes based on split flux technique. However, in absence of conservative flux splitting technique for shallow water equations, these schemes were based on non-conservative form of the governing equations. Besides, the second order accuracy of the MacCormack, λ- and the Gabutti schemes results in unacceptable oscillations near the shock and bore (e.g. Fennema and Chaudhry (6)). The Beam and Warming scheme is implicit scheme with first order accuracy in space and second order accuracy in time. Although the Beam and Warming scheme is based on the governing equations in conservation form, some of the terms in the finite difference equation are evaluated non-conservatively, thereby loosing some of the conservative properties. Jha et al. (10) developed an implicit scheme based on split flux technique. The continuity equation was evaluated conservatively but the non-conservative form of the momentum equation was used. When applied to some of the severe cases of dam-break problem, this scheme gives slower front celerity and higher front height.

In this paper an implicit model is developed for one dimensional unsteady open channel flows. Automatic switching of the space difference operators through splitting of flux enables the proposed model to handle the flow situations with simultaneous presence of supercritical and subcritical flows. The conservative splitting of flux vector is achieved through the concept of approximate Jacobian. We first present development of the governing equations in conservation form beginning with the Saint Venant equations. Non-conservative split flux form, the corrections based on Roe's approximate Jacobian (16) for obtaining conservative scheme and finite difference equations are presented subsequently. Finally, some dam-break problems and propagation of shock wave are analysed to demonstrate the applicability of the model.

GOVERNING EQUATIONS

The Saint Venant equations expressed for a prismatic channel of arbitrary cross section are employed as the governing equations for one-dimensional unsteady free-surface flows (see Mahmood and Yevjevich (14))

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + \frac{A}{B}\frac{\partial u}{\partial x} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial h}{\partial x} - g(S_o - S_f) = 0 \tag{2}$$

where h = flow depth; u = flow velocity; A = cross-sectional area; B = top width of flow at height h from channel bottom; g = acceleration due to gravity; S_o= bed slope; S_f = friction slope; x = distance along the channel; and t = time.

The friction slope is assumed to be given by Manning's formula expressed as

$$S_f = \frac{u|u|n^2}{R^{4/3}} \tag{3}$$

wherein n = Manning's roughness coefficient; R = A/P = hydraulic radius; and P = wetted perimeter.

The basic assumptions behind the governing equations are: (1) water is incompressible, (2) pressure is hydrostatic, (3) bottom slope of the channel is sufficiently small, and (4) geostrophic effects and wind stresses are negligible.

Development of the split flux form is rather simple if the governing equations are written in conservation form and a vector notation is introduced. Eqs. 1 and 2 can be transformed into conservation law form by the following manipulations. Multiplication of Eq.1 by B and simplification yields the conservation form of the continuity equation as

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{4}$$

where Q = discharge in the channel.

Multiplying Eq. 2 by A and Eq. 3 by u and adding the results give the following equation

$$\frac{\partial Q}{\partial t} + \frac{\partial (Q^2/A)}{\partial x} + gA\frac{\partial h}{\partial x} - gA(S_o - S_f) = 0 \tag{5}$$

Further derivation from Eq. 5 requires hydrostatic pressure force term given by

$$F_h = g\int_0^h (h - \eta)W(\eta)\, d\eta \tag{6}$$

where η = integration variable indicating distance from channel bottom and $W(\eta)$ is the channel width at distance η from the channel bottom and is expressed as

$$W(\eta) = \frac{\partial A}{\partial \eta} \tag{7}$$

On differentiation with respect to x, Eq. 6 gives

$$\frac{\partial F_h}{\partial x} = gA\frac{\partial h}{\partial x} \tag{8}$$

Inserting Eq. 8 into Eq. 5 yields the conservation form of the momentum equation as

$$\frac{\partial Q}{\partial t} + \frac{\partial (Q^2/A + F_h)}{\partial x} - gA(S_o - S_f) = 0 \tag{9}$$

The governing equations in conservation form , Eqs. 4 and 9, may be expressed in vector form as

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \mathbf{S} = 0 \tag{10}$$

where

$$\mathbf{U} = \begin{bmatrix} A \\ Q \end{bmatrix} \quad ; \ \mathbf{E} = \begin{bmatrix} Q \\ \frac{Q^2}{A} + F_h \end{bmatrix} ; \qquad \mathbf{S} = \begin{bmatrix} 0 \\ -\ gA(S_o\text{-}S_f) \end{bmatrix} \tag{11}$$

To account for variations in the channel width at expansions and contractions, the source term of Eq. 11 may be modified as

$$\mathbf{S} = \begin{bmatrix} 0 \\ -\ gI - gA(S_o\text{-}S_f) \end{bmatrix} \tag{12}$$

where I is the force exerted by channel walls due to irregularity of the channel cross-section and is given by

$$I = g\int_0^h (h - \eta)\frac{\partial W(\eta)}{\partial x}\, d\eta \tag{13}$$

SPLIT FLUX FORM

The hyperbolic nature of the governing partial differential equation has inherent information on the direction of signal propagation. Some flow information come from the upstream direction while the others come from downstream direction. In physical terms information is transmitted only from upstream in supercritical flows while in subcritical flows information comes from both upstream as well as from downstream directions. This property must be taken into account in the development of the finite difference techniques. If the flow is known to be always supercritical then the problem is easily handled. However, in subcritical flows it can not be predetermined that what information comes from upstream and what information comes from downstream. The whole problem is further complicated when both subcritical and supercritical flows are present simultaneously or in sequence. The flux splitting technique provides a method by which information coming from upstream and downstream directions of the flow can be expressed as separate terms in the equation. Thereafter, different direction of space differences can be used for different space derivative terms. The split flux form of Eq. 10 can be obtained by noting that vector **E** is related to its Jacobian, **M**, and the flow variable, **U**, as

$$\frac{\partial \mathbf{E}}{\partial x} = \mathbf{M}.\frac{\partial \mathbf{U}}{\partial x} \tag{14}$$

where

$$\mathbf{M} = \begin{bmatrix} 0 & 1 \\ -(Q/A)^2 + gA/B & 2Q/A \end{bmatrix} \tag{15}$$

Since the governing equations are hyperbolic, **M** can be written in diagonalized form as

$$\mathbf{M} = \frac{1}{2c}\begin{bmatrix} 1 & -1 \\ u+c & -(u-c) \end{bmatrix}\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}\begin{bmatrix} -(u-c) & 1 \\ -(u+c) & 1 \end{bmatrix} \tag{16}$$

where $c = (gA/B)^{1/2}$; $u = Q/A$; and λ_i's = eigenvalues of **M** giving the characteristic directions. The eigenvalues are given by

$$\lambda_1 = u + c \qquad \text{and} \qquad \lambda_2 = u - c \tag{17}$$

Since the eigenvalues expressed in Eq. 17 give the characteristic directions the terms associated with each characteristic represent the information passed along that characteristic. By eliminating the information passed along one characteristic a new Jacobian matrix may be obtained that contains only the information carried along one characteristic. In this way matrix **M** can be split into two components, positive and negative. This is achieved by testing sign of the eigenvalues for positive and negative components of **M** as follows,

$$\mathbf{M} = \mathbf{M}^+ + \mathbf{M}^- \; ; \quad \lambda_i^+ = \max(\lambda_i, 0) \; ; \quad \lambda_i^- = \lambda_i - \lambda_i^+ \tag{18}$$

Thus Eq. 10 can be written in split flux form as

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{M}^+ . \frac{\partial \mathbf{U}}{\partial x} + \mathbf{M}^- . \frac{\partial \mathbf{U}}{\partial x} + \mathbf{S} = 0 \tag{19}$$

The space derivative associated with the positive component of the Jacobian matrix represents the information carried along the positive characteristic coming from upstream of the flow and it may be approximated by a backward space difference. Likewise, the space derivative associated with the negative component of **M** may be evaluated by a forward space difference. Therefore, the scheme based on Eq. 19 will appropriately handle the directional property of signal propagation. However, the Jacobians of the flux appear outside of the space derivative in Eq. 19. Consequently Eq. 19 does not remain in conservation form. The correction necessary to retain the conservative properties while using Eq. 19 is explained next.

CONSERVATIVE SPLITTING

For Euler equations Roe (16) developed a technique for constructing approximate Jacobians ensuring conservation. The idea was subsequently applied to shallow water equations by Glaister (8) and Alcrudo et al. (2). Roe's technique uses mean value theorem. Following Roe's approach an approximate Jacobian of flux is constructed for every pair of adjacent nodes which satisfies conservative properties and is consistent with the governing equations. In other words the approximate Jacobian must satisfy the following conditions:

i) Provide a linear mapping from the vector space **U** to the vector space **E**.
ii) As the values of the flow variable **U** at points i-1/2 and i+1/2 approach the value of flow variable at point i, the approximate Jacobian approaches the value of the Jacobian at point i. Mathematically, for any node i,

$$\text{As } \mathbf{U}_{i-1/2} \rightarrow \mathbf{U}_{i+1/2} \rightarrow \mathbf{U}_i \; ; \; \widetilde{\mathbf{M}}(\mathbf{U}_{i-1/2}, \mathbf{U}_{i+1/2}) \rightarrow \mathbf{M}(\mathbf{U}_i)$$

where $\mathbf{M} = \dfrac{\partial \mathbf{E}}{\partial \mathbf{U}}$; $\widetilde{\mathbf{M}}$ = Approximate Jacobian.

iii) For any $\mathbf{U}_{i-1/2}$ and $\mathbf{U}_{i+1/2}$

$$\widetilde{\mathbf{M}} \, \Delta \mathbf{U} = \Delta \mathbf{E}$$

iv) The approximate Jacobian has real eigenvalues and a complete set of linearly independent eigenvectors.

We utilise this concept of approximate Jacobian and re-define Eq. 14 as

$$\frac{\partial \mathbf{E}}{\partial x} = \widetilde{\mathbf{M}}\frac{\partial \mathbf{U}}{\partial x} = \widetilde{\mathbf{M}}^{+}_{i-1/2}\frac{\partial \mathbf{U}}{\partial x} + \widetilde{\mathbf{M}}^{-}_{i+1/2}\frac{\partial \mathbf{U}}{\partial x} \tag{20}$$

where

$$\widetilde{\mathbf{M}}_{i\pm 1/2} = \widetilde{\mathbf{M}}(\mathbf{U}_{i\pm 1/2}) = \mathbf{M}(\mathbf{U}_i, \mathbf{U}_{i\pm 1}) \tag{21}$$

The problem has now reduced to defining the arguments of approximate Jacobian, i.e. $\mathbf{U}_{i+1/2}$ and $\mathbf{U}_{i-1/2}$ in Eq. 21. These arguments of approximate Jacobian are fully defined if celerities and velocities at points (i-1/2) and (i+1/2) are defined. Glaister (8) and Alcrudo et al. (2) in their separate applications of Roe's technique (16) to shallow water equations defined the velocities at half grid points as the square root averages, identical to the one defined by Roe (16). In the present work also the velocities at half grid points are defined in an identical way. Therefore, for any node i the velocities at half grid points are

$$u_{i-1/2} = \frac{\sqrt{Q_i u_i} + \sqrt{Q_{i-1} u_{i-1}}}{\sqrt{A_i} + \sqrt{A_{i-1}}} \tag{22}$$

$$u_{i+1/2} = \frac{\sqrt{Q_i u_i} + \sqrt{Q_{i+1} u_{i+1}}}{\sqrt{A_i} + \sqrt{A_{i+1}}} \tag{23}$$

For shallow water equation Alcrudo et al. (2) and Glaister (8) defined the celerities at half grid points as the arithmetic averages as given below

$$c_{i\pm 1/2} = \frac{c_i + c_{i\pm 1}}{2} \tag{24}$$

Eq. 24 works if the discontinuity in the depth is not very large. For dam-break problem if the difference between reservoir depth and tailwater depth is large then the celerity as defined by Eq. 24 fails to give a useful solution. Alcrudo et al.'s (2) extension of the scheme to second order of accuracy removes this problem. Glaister (8) defined the depths at half grid points, in addition to the celerities as defined in Eq. 24, as

$$h_{i\pm 1/2} = \sqrt{h_i\, h_{i\pm 1}} \tag{25}$$

In the present study the depths at half grid points are defined as in Eq. 25 and the corresponding celerities are computed as

$$c^2_{i\pm 1/2} = g\sqrt{h_i\, h_{i\pm 1}} \tag{26}$$

FINITE DIFFERENCE SCHEME

The time derivatives are approximated by a forward time difference. Therefore, the time derivative at any node i is approximated as

$$\left[\frac{\partial \mathbf{U}}{\partial t}\right]^t_i = \frac{\mathbf{U}^{t+1}_i - \mathbf{U}^t_i}{\Delta t} \tag{27}$$

For the space derivatives a time weighting factor is used which may be varied to obtain the optimum scheme between a fully explicit and a fully implicit schemes. With the time derivatives expressed as in Eq. 27 and the time weighting employed to the space derivatives, the following one-parameter scheme is used to advance the solution in time:

$$\mathbf{U}_i^{t+1} + \Delta t\theta\left[\mathbf{M}_{i-1/2}^{+t}\frac{\partial \mathbf{U}^{t+1}}{\partial x} + \mathbf{M}_{i+1/2}^{-t}\frac{\partial \mathbf{U}^{t+1}}{\partial x}\right]$$

$$= \mathbf{U}_i^t + \Delta t(1-\theta)\left[\mathbf{M}_{i-1/2}^{+t}\frac{\partial \mathbf{U}^{t}}{\partial x} + \mathbf{M}_{i+1/2}^{-t}\frac{\partial \mathbf{U}^{t}}{\partial x}\right] + \Delta t\mathbf{S}_i^t \qquad (28)$$

where superscripts t and t+1 = known and higher time levels, respectively; subscript i = grid location; Δt = time step (Fig. 1); and θ = time weighting factor. $\theta = 1$ gives fully implicit scheme and $\theta = 0$ results in fully explicit scheme.

The space derivatives in Eq. 28 are replaced by either a backward or forward difference depending on whether they are associated with the positive or the negative components of **M**, respectively. Therefore, the following first order space differences are defined for node i

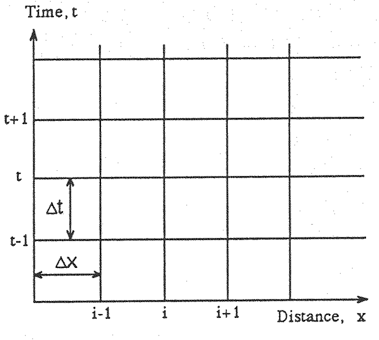

Fig.1 Finite difference grid

$$\mathbf{M}_{i-1/2}^{+t}\frac{\partial \mathbf{U}}{\partial x} = \mathbf{M}_{i-1/2}^{+t}\frac{\mathbf{U}_i - \mathbf{U}_{i-1}}{\Delta x} = \nabla_x \mathbf{U}_i \qquad (29)$$

$$\mathbf{M}_{i+1/2}^{-t}\frac{\partial \mathbf{U}}{\partial x} = \mathbf{M}_{i+1/2}^{-t}\frac{\mathbf{U}_{i+1} - \mathbf{U}_i}{\Delta x} = \Delta_x \mathbf{U}_i \qquad (30)$$

Replacing the space derivatives of the Eq. 28 by the difference operators defined in Eqs. 29 and 30, the complete finite difference equation is obtained as

$$\mathbf{U}_i^{t+1} + \alpha\theta\left\{\mathbf{M}_{i-1/2}^{+}\nabla_x\left[\mathbf{U}_i^{t+1}\right] + \mathbf{M}_{i+1/2}^{-}\Delta_x\left[\mathbf{U}_i^{t+1}\right]\right\} =$$

$$\mathbf{U}_i^t + \alpha(1-\theta)\left\{\mathbf{M}_{i-1/2}^{+}\nabla_x\left[\mathbf{U}_i^t\right] + \mathbf{M}_{i+1/2}^{-}\Delta_x\left[\mathbf{U}_i^t\right]\right\} + \mathbf{S}_i^t \qquad (31)$$

where $\alpha = \Delta x/\Delta t$; and Δx = grid interval in space.

For subcritical flows, Eq. 31 implements central space differencing with the appropriate weighting governed by the eigenvalues. For supercritical flows, the scheme given by Eq. 31 automatically switches to full upwind because the negative component of **M** are removed by Eq. 18. Similar sets of finite difference equations can be written for all nodes along a channel. The resulting system of equation can be arranged in the form of a block tri-diagonal matrix with each block of size (2 x 2). This block tri-diagonal system can be solved by any suitable algorithm. In this study it has been solved by the algorithm given by S.R. Chakravarthy (see Anderson et al. (3)).

Boundary and initial conditions must be correctly specified and incorporated into the scheme to obtain correct results. In the split flux algorithm boundary and initial conditions are easily incorporated. The compatibility equation valid along each characteristic are obtained and the appropriate one is replaced by the specified boundary condition. For example, in case of a closed upstream boundary, the characteristic coming from the wall may be replaced by the condition of zero mass flux through the wall, i.e. Q = 0 and the area may be computed by the compatibility equation along the characteristic coming from inside the domain.

MODEL APPLICATIONS AND RESULTS

The model proposed herein is applied to several unsteady free surface flow cases to examine the applicability and the validity of the model. All examples consider horizontal and frictionless rectangular channels. External boundaries for all the examples are evaluated by compatibility equations based on the method of characteristics. When the time weighting factor is less than 0.5 the scheme must satisfy the Courant stability criteria. For time weighting factor between 0.5 and 1.0 the proposed scheme remains unconditionally stable. In the range of implicit time weighting factor the value of $\theta = 0.6$ was found from the initial test runs of the model to be giving the best results. Therefore, $\theta = 0.6$ is used in the numerical simulations. The stability analysis procedure for non-linear system of equations are not available. For the case of linear wave equation the proposed model has similar phase and stability properties as that of the Euler implicit scheme. Their is no constraint on the time step so long as the stability is concerned. The accuracy, however, may have to be compromised if the scheme is used with a higher Courant number. In this study Courant number equal to unity is used for the proposed model.

Results obtained by the proposed model for all the applications are compared with analytical solutions as well as with results by the MacCormack (13) and the Gabutti (7) schemes. The MacCormack and the Gabutti schemes are explicit schemes with second order accuracy in space and time. Therefore, they exhibit large oscillations near such discontinuities as shock fronts. Artificial diffusion had to be added to the MacCormack and the Gabutti schemes in order to damp out the oscillations. Jameson et al. (9) method, explained in Appendix, was used for adding artificial diffusion because it adds diffusion only to the regions of steep gradient and leaves smooth regions unaffected. However, the exact amount of artificial diffusion required can not be known a priori and a number of trials must be performed to find appropriate amount of diffusion for each case. Consequently, the results by the MacCormack and the Gabutti schemes presented in this paper are obtained by several trial simulations. Stability requirements dictate that the Courant number must be less than one in the MacCormack and the Gabutti schemes. However, a very low value of the Courant number gives smaller time increment and requires longer computer time. It is, thus, preferred to keep the Courant number close to, but less than unity. Therefore, these two schemes were run at a Courant number of 0.95.

The problem of instantaneous collapse of dam and resulting flood wave is one of the most severe hydraulic phenomenon that a finite difference scheme for unsteady free surface flow may be expected to handle. At the initiation of dam collapse there is a steep discontinuity in depth at the breach section. The flow upstream of the breach remains largely subcritical while the flow downstream of the breach is highly supercritical. This flow situation presents the kind of problem that causes many finite difference schemes to fail. Another severe hydraulic phenomenon is the propagation of shock resulting from sudden closure of gate in a channel. The practical example of such situation may be the closure of the downstream gate in a power channel on sudden load rejection by the power plant. The discharge at the closed gate suddenly reduces to zero. A shock is formed which travels upstream. The proposed model is applied to these exacting hydraulic problems to investigate the applicability and validity of the model.

Dam-Break Flood Wave Propagation

Two different depths of water in a channel are separated by a dam (Fig. 2) which is removed instantaneously to simulate sudden dam failure. The ratio of tailwater depth, h_t, to reservoir depth, h_r, hereafter called the 'depth ratio', $DR = h_t/h_r$, is an important parameter to investigate the applicability of the model for dam-break problem. Keeping the water depth in reservoir as 10m, tailwater depth is varied to run the models at depth ratios of 0.5, 0.05 and 0.005. The depth and velocity profiles along the channel at time (50 + Δt) seconds for three depth ratios are shown in Figs. 3(a)-5(a) and 3(b)-5(b) respectively.

The analytical solutions are obtained by the Stoker (17) method. Several existing models fail to simulate when the depth ratio becomes too small. The MacCormack and the Gabutti schemes failed to work for the depth ratio less than 0.05 until sufficient artificial diffusion was added. The Gabutti scheme did not work at all for the depth ratio 0.005. The proposed model has no limitation on depth ratio as long as the depth ratio remains finite. For the depth ratio higher than 0.5, the flow in the channel, upstream as well as downstream of the breach, remains subcritical. When compared with analytical solutions all the models give quite good results for this case as can be seen from the depth profile of Fig. 3(a) and velocity profile of Fig. 3(b).

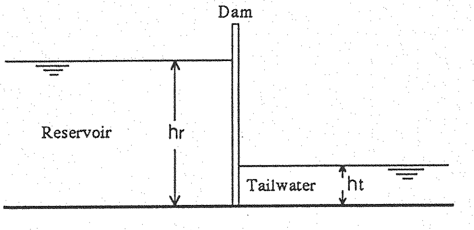

Fig. 2 Dam-break problem

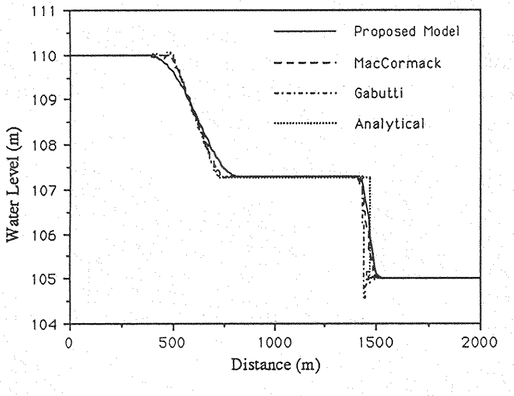

Fig. 3(a) Water surface profile along the channel for depth ratio 0.5.

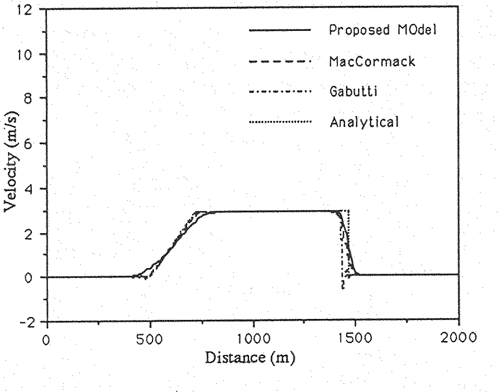

Fig. 3(b) Velocity profile along the channel for depth ratio 0.5.

The coefficient of artificial diffusion used for the MacCormack and the Gabutti schemes were 0.5 and 0.6, respectively. The Gabutti scheme still gives the depth below the tailwater level and some oscillations near upper end of the negative wave (Fig. 3(a)).

When the depth ratio is less than 0.5 the flow downstream of the breach becomes supercritical although upstream of the breach still maintains subcritical flow. For the depth ratio of 0.05 the depth and velocity profiles along the channel are shown in Figs. 4(a) and 4(b), respectively. The Gabutti and the MacCormack schemes do not work for this depth ratio. The addition of artificial diffusion, diffusion coefficient equal to 0.75, to the MacCormack scheme improves the results substantially which are almost identical to the present results. The Gabutti scheme works with an artificial diffusion coefficient of 0.95. However, the results of the Gabutti scheme are inferior to those of the proposed model or the MacCormack scheme. The predicted front height is the highest and celerity the lowest by the Gabutti scheme. When the depth ratio is further reduced to 0.005 the Gabutti scheme completely blows up, with or without artificial diffusion. The MacCormack scheme continues to work if necessary artificial diffusion is added, in this case the diffusion coefficient is equal to 0.90. Results for this case are shown in Figs. 5(a) and 5(b).

The MacCormack scheme, although more diffused than the proposed model, gives slightly better results for the front part than the proposed model.

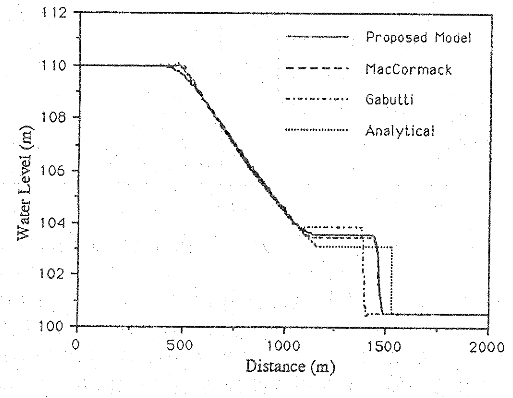

Fig. 4(a) Water surface profile along the channel for depth ratio 0.05.

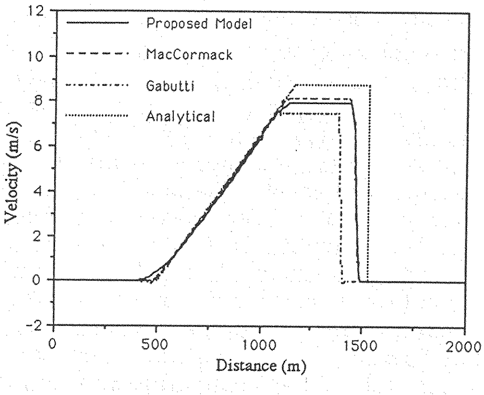

Fig. 4(b) Velocity profile along the channel for depth ratio 0.05.

Considering the fact that for the smaller depth ratios the MacCormack and the Gabutti schemes do not work without artificial diffusion, and the amount of artificial diffusion is not known a priori, the proposed model may be better suited to simulating dam-break flood wave propagation.

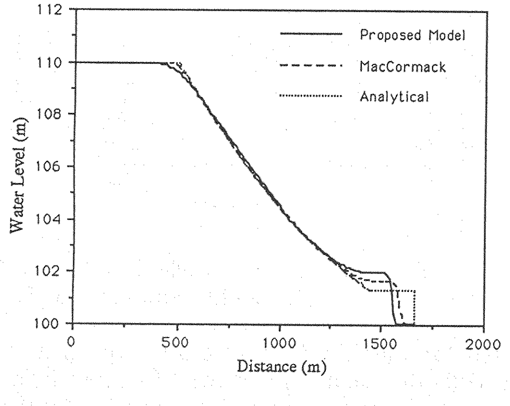

Fig. 5(a) Water surface profile along the channel for depth ratio 0.005.

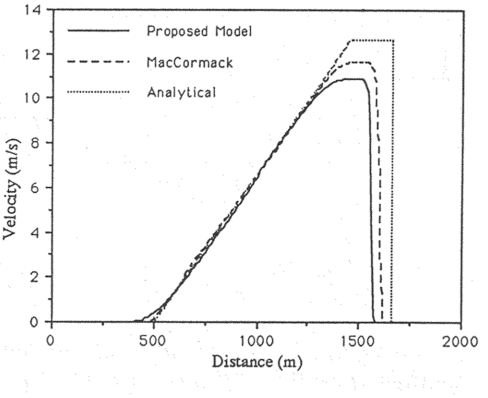

Fig. 5(b) Velocity profile along the channel for depth ratio 0.005.

Instantaneous Closure of Gate

The propagation of shock in a rectangular channel resulting from instantaneous closure of gate is simulated by the proposed model as well as by the MacCormack and the Gabutti schemes. The problem is sketched in Fig. 6. A 10m deep steady, uniform flow with a velocity of 4.952m/s is specified as the initial condition in a 2000m long horizontal frictionless channel. At time t=0, the discharge at the downstream end is set to zero which simulates instantaneous and complete closure of gate. A shock is formed which travels upstream leaving still water behind. The results of numerical simulation given in Fig. 7 along with the analytical solution are obtained at time (111 + Δt) seconds. Although optimum artificial diffusion is again added to the MacCormack and the Gabutti schemes (diffusion coefficients is equal to 0.95 and 2.0 for the MacCormack and the Gabutti schemes respectively) to damp out the oscillations, some oscillations still remain. The Gabutti scheme is seen to give the lowest celerity while the MacCormack and the proposed models compare reasonably well with the analytical solution.

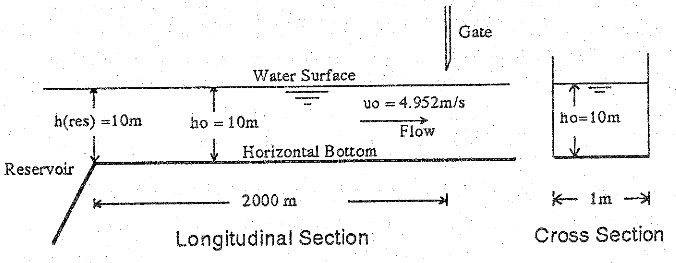

Fig. 6 Data for surge propagation due to sudden gate closure.

Mass balance was checked for all the examples presented above. The mass balance error for the proposed model is of the order of 10^{-4}%. The Gabutti scheme looses 3% to 5% of mass whereas mass balance error for the MacCormack scheme remains less than 1%.

The response of the model to the changes in the Manning's roughness coefficient is examined for the dam-break problem with a depth ratio of 0.05. The water surface profiles for three Manning's roughness coefficients at time (50 + Δt) seconds are shown in Fig. 8.

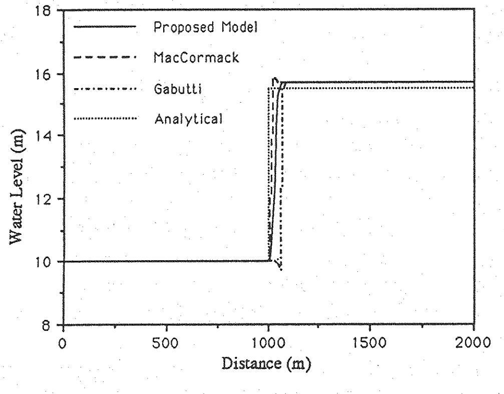

Fig. 7 Propagation of surge due sudden gate closure.

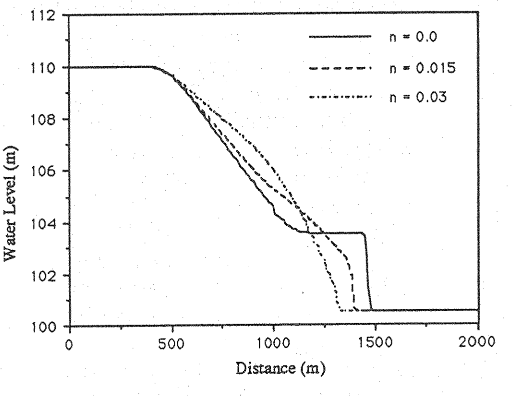

Fig. 8 Water surface profiles for different Manning's n for depth ratio 0.05.

The negative wave is least affected by the changes in the Manning's roughness coefficient because of the higher depth in this region. The wave front is retarded as the Manning's coefficient is increased. As a result, the depths increase. Fig. 8 indicates that the model's response to the changes in the Manning's roughness coefficient is quite reasonable.

CONCLUSIONS

An implicit model has been developed to simulate one-dimensional unsteady open channel flows. Automatic switching of the space differences through flux splitting technique enables correct handling of supercritical and subcritical flows. The conservative properties have been achieved by incorporating the concept of approximate Jacobian proposed by Roe (16). The boundary conditions are incorporated in to the model by method of characteristics. The proposed model is simple and straightforward to formulate and to program. It works for any dam-break problem with finite tailwater to reservoir depth ratio. It has been shown to give reasonably good result for a depth ratio of up to 0.005 for which many existing schemes, such as the Gabutti and the MacCormack schemes, fail. The proposed model also gives good result when applied to simulate propagation of shock resulting from the sudden closure of gate.

For all the considered cases the proposed model gives better results than the Gabutti scheme and similar results when compared with the MacCormack scheme. Because of the second order of accuracy the MacCormack and the Gabutti schemes require artificial diffusion to damp out oscillations near the surge or bore. The required amount of artificial diffusion has to be determined by several trial simulations which may not be desirable for real life applications. In contrast, the proposed model does not require any artificial diffusion and gives reasonably accurate results. The mass balance error for the proposed model is 1000 times less than that for the Gabutti and the MacCormack schemes. The proposed model may be confidently used to simulate such unsteady open channel flow problems as presented in this study.

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APPENDIX

Here we briefly explain the Jameson (9) method which has been used in the present study for introducing artificial viscosity in MacCormack and Gabutti schemes. First, a parameter β is computed from a normalized form of one variable, say the depth, h. Therefore, at any node i one has

$$\beta_i = \frac{|h_{i+1} - 2h_i + h_{i-1}|}{|h_{i+1}| + 2|h_i| + |h_{i-1}|} \quad \text{(A.1)}$$

if node i is a boundary node then

$$\beta_i = \frac{|h_{i\pm 1} - h_i|}{|h_{i\pm 1}| + |h_i|} \quad \text{(A.2)}$$

Plus and minus signs in Eq. A.2 are used at upstream and downstream boundaries, respectively. Similarly, β_{i+1} and β_{i-1} are also computed. The parameter β is then defined at half grid points as

$$\beta_{i\pm 1/2} = K \max\left(\beta_{i\pm 1}, \beta_i\right) \quad \text{(A.3)}$$

K is referred to as the diffusion coefficient and is varied to determine the amount of artificial viscosity added to a numerical scheme. The flow variables computed by the mathematical model are finally modified at the end of each time step as

$$\mathbf{U}_i^{cor} = \mathbf{U}_i + \beta(\mathbf{U}_{i+1} - \mathbf{U}_i) - \beta(\mathbf{U}_i - \mathbf{U}_{i-1}) \quad \text{(A.4)}$$

where $\mathbf{U}^{cor}$ = corrected flow variables.

APPENDIX - NOTATIONS

The following symbols are used in this paper:

A	= cross-sectional area of flow;
B	= top width of flow at height h from channel bottom;
c	= celerity;
E	= flux matrix;
F_h	= hydrostatic pressure force;
g	= acceleration due to gravity;
h	= flow depth;
h_0	= initial depth in the channel;
h_f	= height of the wave front;

h_r = initial water depth in reservoir;
h_t = initial tailwater depth;
I = force exerted by channel walls due to irregularity of the channel cross-section;
i = subscript for grid location in space;
K = diffusion coefficient;
$\mathbf{M}$ = Jacobian of $\mathbf{E}$ with respect to $\mathbf{U}$;
$\widetilde{\mathbf{M}}$ = approximate Jacobian;
n = Manning's roughness coefficient;
0 = subscript for initial values;
P = wetted perimeter;
Q = discharge;
R = hydraulic radius;
$\mathbf{S}$ = matrix containing source terms;
S_f = friction slope;
S_o = bed slope;
t = superscript for time;
$\mathbf{U}$ = vector for flow variables;
u = velocity;
u_0 = initial velocity in the channel;
$W(\eta)$ = channel width at distance h from the channel bottom;
x = distance along the channel;
Δ_x, ∇_x = forward and backward space difference operators, respectively;
α = $\Delta x/\Delta t$;
β = parameter for artificial diffusion;
Δt = time step;
Δx = grid interval in space;
h = integration variable indicating distance from channel bottom;
λ = eigenvalues of $\mathbf{A}$; and
θ = time weighting factor.

(Received April 12, 1993; revised October 19, 1993)