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NUMERICAL ANALYSIS OF WATER UPTAKE BY PLANT ROOTS

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SYNOPSIS

Observed daily soil moisture at Värpinge in Lund, Sweden, is analyzed using a finite difference model of Richard's equation including water uptake by plant roots. Numerical calculations are carried out both for drying and wetting. The sink term which is evaluated by matric suction is used for the numerical calculations. The hydraulic parameters, which specify the hydraulic characteristics of the unsaturated soil, are optimally estimated through sensitivity analysis. Matric suction is calculated from observed soil moisture using these hydraulic parameters. The numerical results show that the water uptake by roots is important for the evaluation of soil water transport in vegetated fields. The presented method can be used when only soil moisture observations only are at hand for vegetated fields to evaluate unsaturated hydraulic characteristics.

INTRODUCTION

Numerous researchers have carried out theoretical investigations on water movement in unsaturated soils. However only a few previous examples are available on the evaluation of hydraulic characteristics directly from field soil moisture observations. This is especially true for observations which include effects of vegetation. In this paper we present a method for evaluation of unsaturated hydraulic characteristics, only using observed soil moisture only. The observations include effects of vegetation and evaporation. No information on root or plant distribution, however, is included in the observations.

In this study, daily soil moisture data, which was measured at each 10 cm depth by neutron probe, were analyzed by the use of a finite difference model of Richard's equation including water uptake by plant roots. Closed form equations which deal with the hydraulic characteristics (eg., relationship between soil moisture and matric suction) by van Genuchten (9) were used for the calculations. To estimate the sink term caused by water uptake by roots, plant related data is usually necessary. Most standard field surveys, however, do not supply these data. In order to evaluate this sink term and vegetational effects, we combine a method of Feddes et al. (3) with van Genuchten's (9) method. In Feddes' method, the water uptake by roots is evaluated by matric suction. In turn, matric suction is calculated from the hydraulic characteristics (van Genuchten's method) using the observed soil moisture.

FIELD OBSERVATIONS

The soil moisture distribution in time and at difference depths was measured at Värpinge in Lund, Sweden, from April 23 to June 17, 1976 (Dahlblom (2), Jinno (5), Zhang and Berndtsson (10)). The soil consists mostly of sandy loam and moraine clay and the surface is covered by short grass. Daily measurements of soil moisture were taken by neutron probe at four observation points at the site as shown in Fig. 1. The measurements were carried out at every 10 cm from the surface to 100 cm depth and at every 20 cm from 100 cm to 240 cm depth. Potential evaporation was calculated by Penman's method using observations of wind velocity, humidity, temperature, and radiation.

Fig. 1 shows a vertical cross section of the soil moisture distribution for May 25 and 31. The two-dimensional diagrams were obtained by interpolation of soil moisture values at the four observation points (17 depths). Fig. 2 shows daily values of soil moisture at 10 and 20 cm depths, precipitation, and potential evaporation. Totally 15.9 mm precipitation were recorded on May 25, and soil moisture increased at every observation point (wetting). Evapotranspiration reduced the soil moisture during the period without rainfall from June 3 to June 9 (drying). The most active soil moisture region is the shallow zone from 0 to 40 cm depth. Below this depth, the soil moisture is almost constant. In this study, observed soil moisture from 10 - 40 cm depth was analyzed.

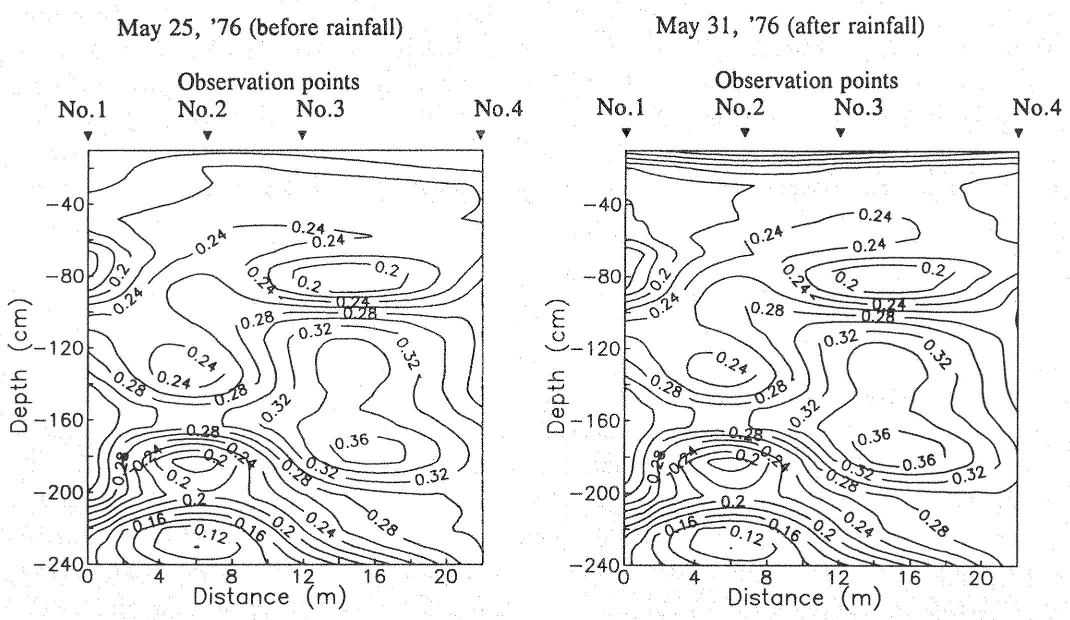

Fig. 1. Soil moisture profile in vertical two-dimensional cross section (observed). Left; May 25, 1976, before rainfall, Right; May 31, after rainfall

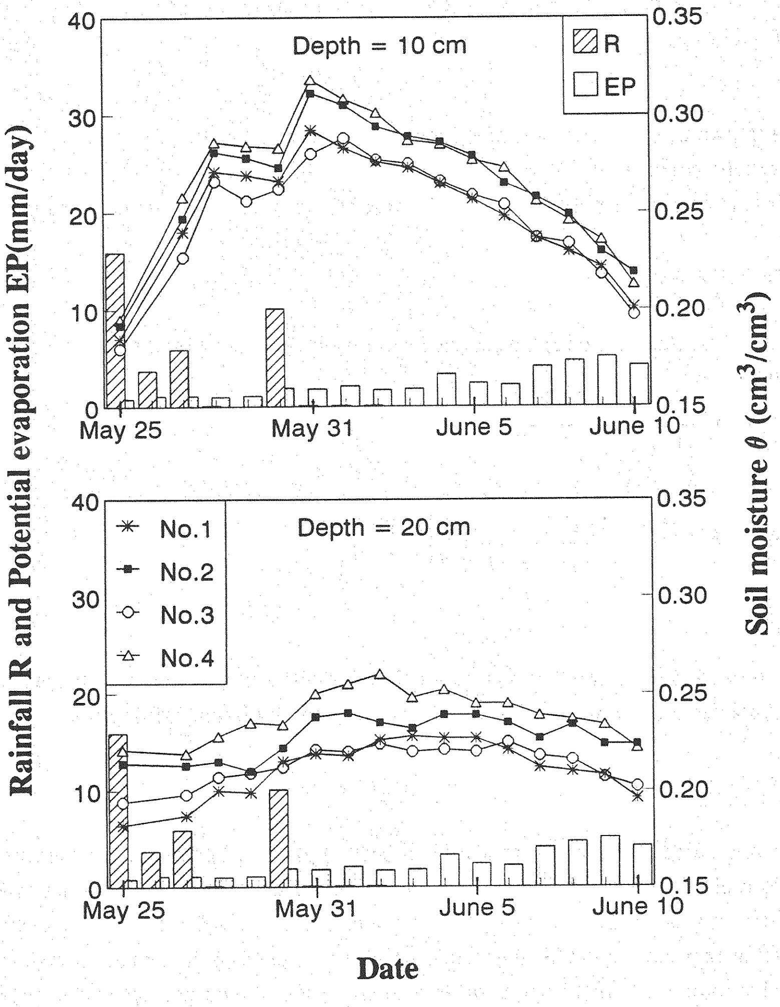

Fig. 2 Observed daily fluctuation of soil moisture θ, rainfall R and potential evaporation EP.

NUMERICAL ANALYSES

Basic equations

The one-dimensional unsaturated soil moisture transport with a sink term caused by the plant roots can be expressed by:

$$\frac{\partial \theta}{\partial t} = C\frac{\partial h}{\partial t} = \frac{\partial}{\partial y}\left[K\left(\frac{\partial h}{\partial y} - 1\right)\right] - S(h) \qquad (1)$$

where θ = soil moisture; C = specific water capacity (= $d\theta/dh$); t = time; h = matric suction; K = unsaturated hydraulic conductivity; y = depth; and $S(h)$ = sink term caused by plant roots. To solve Eq. 1, the hydraulic characteristics ($\theta(h)$ and $K(h)$ relationships) and the sink term are needed.

Hydraulic characteristics of the soil

The hydraulic characteristics of an unsaturated soil may be evaluated by the following relationships (van Genuchten (9)):

$$\theta = \theta_r + \frac{\theta_s - \theta_r}{[1 + (a|h|)^n]^m} \qquad (2)$$

$$K(h) = K_s \frac{[1 - (a|h|)^{n-1}\{1 + (a|h|)^n\}^{-m}]^2}{[1 + (a|h|)^n]^{m/2}} \qquad (3)$$

where θ_s = saturated soil moisture; θ_r = residual soil moisture; a, n, m = empirical parameters depending on characteristics(m = 1 - 1/n); and K_s = saturated hydraulic conductivity.

Sink term caused by plant roots

There are many models for sink term and have been applied to laboratory experiments and field observations. Herkelrath et al. (4) developed *the root contact model.* This model contains a sink term which is proportional to a product of the hydraulic gradient and the volumetric soil moisture and has a proper physical background. On the other hand, when it is applied to field data, the parameters (eg., the root length distribution per unit volume of the soil and a root membrane permeability)are required in the calculations. Since information of root distribution usually is not at hand, it is difficult to apply these types of models to the computations. The sink term was evaluated by a generalized model of Feddes et al. (3). The sink term in Feddes' model is given by,

$$S(h) = \alpha(h)\, S_{\max} \qquad (4)$$

where $\alpha(h)$ = water extraction function ($0 < \alpha(h) < 1$); and S_{max} = maximum possible water uptake rate by the roots.

We assumed a uniform root distribution for the calculations due to the lack of data for actual root distribution. The parameter $\alpha(h)$ of Eq. 4 is:

$$\begin{aligned}
\alpha(h) &= \frac{h}{h_1} & 0>h\geq h_1 \\
\alpha(h) &= 1 & h_1>h\geq h_2 \\
\alpha(h) &= \frac{h_3 - h}{h_3 - h_2} & h_2>h\geq h_3 \\
\alpha(h) &= 0 & h_3>h
\end{aligned} \tag{5}$$

Feddes et al. (3) suggested -30 cm and -20,000 cm as values of h_1 and h_3, respectively, and -500 cm to -1000 cm for h_2. Marker and Mein (6) conducted laboratory experiments and showed good agreement for those parameters in their results.

Water uptake by the roots was calculated as:

$$T = \int_{L_{r1}}^{L_{r2}} S(h)\, dy \tag{6}$$

where T = transpiration at time t; L_{r1} = upper edge of roots; and L_{r2} = lower edge of roots.

The parameter S_{max} of Eq. 4 was obtained as:

$$S_{\max} = \frac{T_v}{L_{r2} - L_{r1}} \tag{7}$$

The parameter $\alpha(h)$ is equal to 1 when the transpiration T is equivalent to the potential evapotranspiration T_v.

Difference scheme

Eq. 1 was solved using an implicit difference scheme as follows:

$$C_i^j \frac{h_i^{j+1} - h_i^j}{\Delta t}$$

$$= \frac{K_{i+1/2}^j [\frac{(h_{i+1}^{j+1} - h_i^{j+1})}{\Delta y} - 1] - K_{i-1/2}^j [\frac{h_i^{j+1} - h_{i-1}^{j+1}}{\Delta y} - 1]}{\Delta y} - S_i^j \quad (8)$$

where i, j = node of the difference scheme.

The ratio of transpiration to potential evapotranspiration is necessary as an upper boundary condition. In this study, 90 % of potential evapotranspiration were assumed to be root water uptake, and 10 % were assumed to evaporate at the upper boundary of soil surface (Campbell (1)). The soil moisture was assumed to be constant at the lower boundary of 40 cm depth, since observed soil moisture below 40 cm was almost constant. The above methodology has been used before with good results by Momii (7) using observed data (Protopapas and Bras (8)).

Parameter estimation

To use Eq. 8, parameter values of Eqs. 2 and 3, ie., θ_s, θ_r, K_s, a, and n need to be estimated. The values θ_s and θ_r were assumed to be 0.45 and 0.10, respectively. The parameters K_s, a, and n were estimated by an error minimizing method. One parameter was fixed and the other two parameters were evaluated in a one-dimensional model. The estimation error of soil moisture was calculated using an objective function according to:

$$J = \frac{1}{N} \sum_{f=1}^{N} [\frac{\sum_{e=1}^{M} (\theta_e^{*f} - \theta_e^f)^2}{\sum_{e=1}^{M} (\theta_e^{*f})^2}]^{1/2} \quad (9)$$

where θ_e^{*f} = observed soil moisture; θ_e^f = calculated soil moisture; e = node of the difference scheme at depth 10 and 20 cm; f = time step of observations; M = number of observation points used for the estimation; and N = number of observation events during the time period modeled. The parameter values which minimized the above objective function were chosen as optimum values.

WETTING

The unknown parameters of Eqs. 2 and 3, K_s, a, and n, were estimated for the wetting process at observation point no. 1 from May 25 to 28. Soil moisture distribution on May 25 (Fig. 2) was given as initial conditions for the calculations, and measurement values at 10 cm and 20 cm depths were used for θ^* in Eq. 9. Fig. 3 (a) shows values of the objective function J for K_s, a, and fixed n (= 1.75 cm^{-1}) for point no.1. The minimum value of J was obtained for K_s = 0.12 cm/hr and a = 0.04. For a fixed value of a (= 0.04), the minimum value of J was obtained for K_s = 0.12 cm/hr and n = 1.75. Since parameters K_s = 0.12 cm/hr, a = 0.04 and n = 1.75 cm^{-1} gave a minimum value of J, those values were used as optimum parameters at no.1 in this study.

Fig. 3 (b) shows J values for various K_s, n, and fixed a (= 0.04) at observation point no. 4. A minimum value for J was obtained for K_s = 0.08 cm/hr and a = 1.75 cm^{-1}. The optimum parameters at the four observation points are shown in Table 1. Since the optimum parameters at the four observation points showed a large variation, especially for the saturated hydraulic conductivity K_s, further studies on two-dimensional parameter distribution are required.

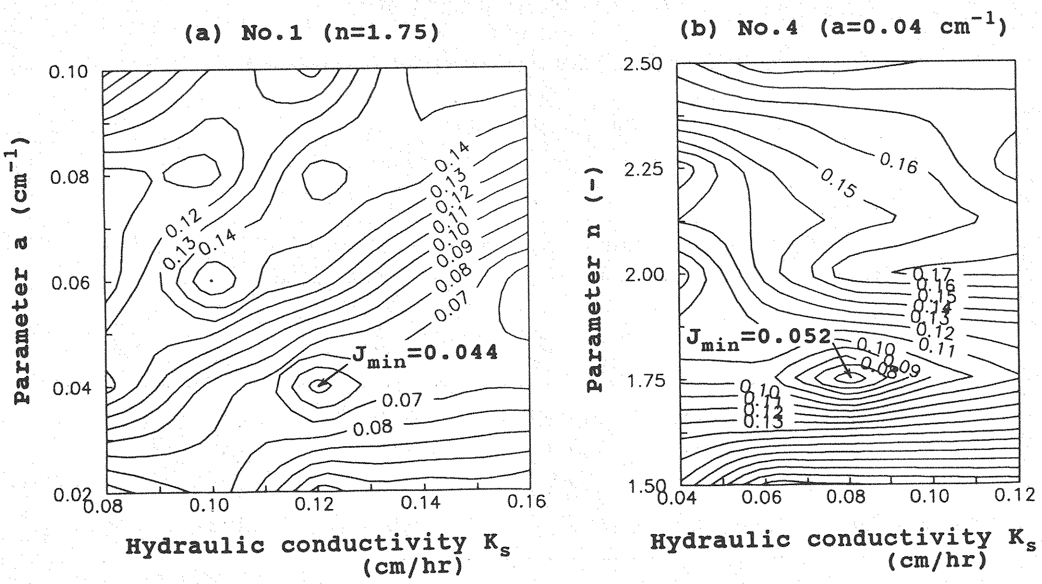

Fig. 3 Sensitivity of the objective function J for the parameters K_s, a and n.

Fig. 4 shows the one-dimensional calculation at observation point no. 2 (May 28) using the optimum parameters. Good agreement can be observed for these optimum parameters.

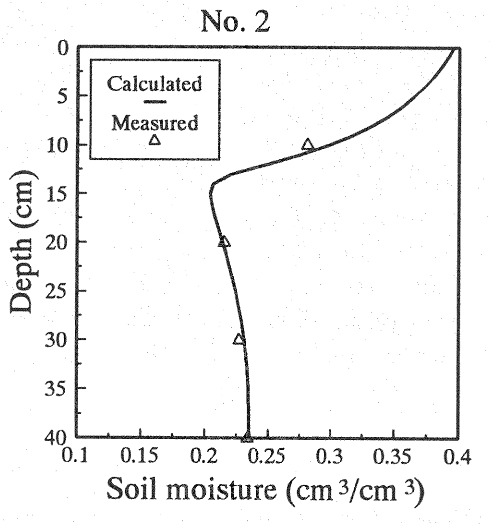

Fig. 4 Comparison between calculated and observed soil moisture during wetting (observation point no.2, May 28)

Table 2. Estimated parameters for wetting.

	n	a (cm^{-1})	K_s (cm/hr)	*error* J_{min}
No.1	1.75	0.04	0.12	0.044
No.2	1.75	0.03	0.075	0.053
No.3	1.75	0.05	0.19	0.045
No.4	1.75	0.04	0.08	0.052

DRYING

The unknown parameters a and n were estimated by using observed soil moisture at 10 cm and 20 cm depths for June 3 to 7. Optimum values of the saturated hydraulic conductivity K_s at each observation point during wetting were used for these calculations. In Eq. 7, L_{r1} was set to -1 cm and L_{r2} was set to -20 cm. In Eq. 5, h_2 was set to -500 cm (Marker and Mein (6)). Table 2 shows the optimum parameters for the drying process obtained by the same procedure as for wetting. These parameters can be used as hydraulic characteristics in the case of a drying soil.

Fig. 5 (a) and (b) show the vertical soil moisture distribution at observation point no. 1. The soil moisture distribution in Fig. 5 (a) was calculated using the optimum parameter values during drying. The initial conditions shown in the figure (symbol □) coincide with the values on June 3 of Fig. 2. The numerical calculation shows good agreement with observed data. For a comparison, the same parameters were estimated and used in the numerical calculation, neglecting the water uptake by roots. In this case, the potential evapotranspiration only was used as flux boundary condition at ground surface. The result is shown in Fig. 5 (b). The figure shows big discrepancy between observed and calculated soil moisture, especially for the 10 cm depth. Consequently, it is very important to consider root water uptake, when estimating the soil moisture transport.

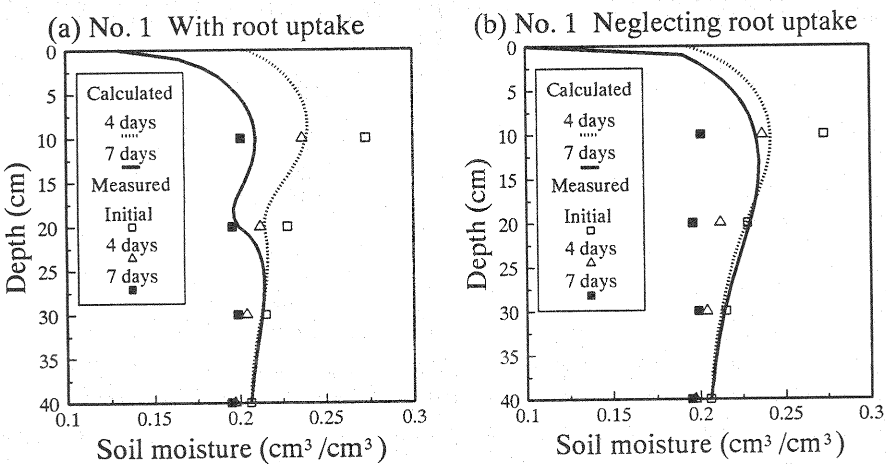

Fig. 5 Comparison between calculated and observed soil moisture during drying (observation no. 1). (a) With root uptake. (b) Neglecting root uptake.

Table 2. Estimated parameters for drying.

	n	a (cm^{-1})	K_s (cm/hr)	error J_{min}
No.1	2.25	0.08	0.12	0.047
No.2	3.20	0.08	0.075	0.060
No.3	2.4	0.1	0.19	0.050
No.4	3.25	0.08	0.08	0.062

Fig. 6 (a) and (b) show daily fluctuations of soil moisture at 10 and 20 cm depths at observation point no. 1. Fig. 6 (a) shows calculation with the sink term and Fig. 6 (b) shows calculation without the sink term. When including the sink term, the agreement between calculated and observed data is good (Fig. 6 (a)). When the sink term is not included in the calculations, the agreement is poor. Therefore the sink term is very important for the evaluation of soil moisture transport in vegetated fields. For more detailed evaluation of the soil moisture transport in vegetated fields, plant-related data should be collected.

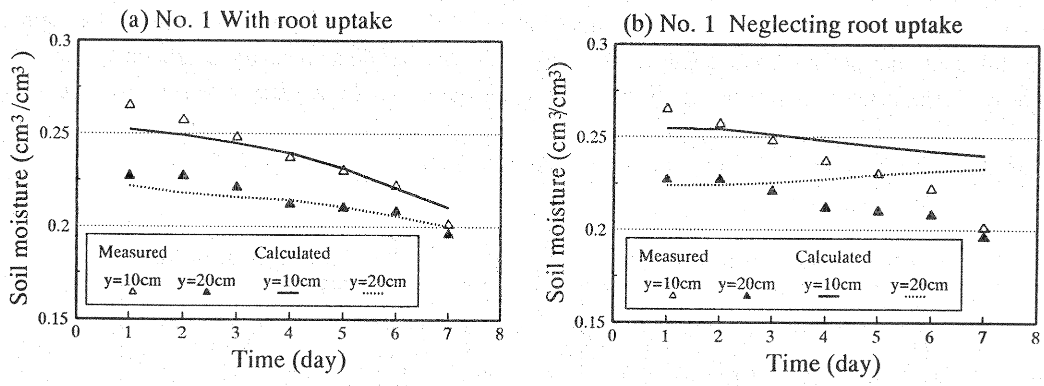

Fig. 6 Comparison between calculated and observed soil moisture during drying.

CONCLUSION

Daily observations of soil moisture at 10, 20, 30, and 40 cm depths were used to estimate the unsaturated hydraulic characteristics of the soil. Numerical calculations indicated that the hydraulic characteristics during drying is different from that of wetting because of the hysteresis phenomenon.

Though plant related data are necessary for detailed evaluation of the water uptake by plant roots, most field surveys supply soil moisture observations only. In this study, soil moisture was used to calculate matric suction using hydraulic parameters which were optimally estimated through a sensitivity analysis. In turn, the sink term was evaluated by matric suction. This sink term is necessary for numerical calculations of soil moisture transport in vegetated fields during drying. If the sink term is ignored, soil moisture transport during drying cannot be calculated correctly.

To carry out detailed calculation of unsaturated water transport for vegetated surfaces, the spatial distribution of roots and soil parameters (h_1, h_2, and h_3 etc.) should be estimated. In this study, however, the spatial distribution of roots was taken to be uniform and results of previous studies were applied for parameter estimation. Further studies are necessary to evaluate these parameters for a more detailed simulation. Even though the model of this study is simple, it can be roughly used to evaluate soil moisture transport in vegetated fields.

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APPENDIX-NOTATION

The following symbols are used in this paper:

a	=	empirical parameters due to soil characteristics;
C	=	specific water capacity (= $d\theta/dh$);
e	=	node of the difference scheme at depth 10 and 20 cm;
f	=	time step of observations;
h	=	matric suction;
i,j	=	node of the difference scheme;
K	=	unsaturated hydraulic conductivity;
K_s	=	saturated hydraulic conductivity;
L_{r1}	=	upper edge of roots;
L_{r2}	=	lower edge of roots;
M	=	number of observation points used for the estimation;
n	=	empirical parameters due to soil characteristics;
N	=	*number of observation events during the time period modeled;*
$S(h)$	=	sink term caused by plant roots;
S_{max}	=	maximum possible water uptake rate to the roots;
t	=	time;
T	=	*transpiration at time t*;
y	=	depth;
$\alpha(h)$	=	water extraction function for sink term ($0<\alpha(h)<1$);
θ	=	soil moisture;
θ_e^{*f}	=	observed soil moisture;
θ_e^f	=	calculated soil moisture;
θ_r	=	residual soil moisture; and
θ_s	=	saturated soil moisture.

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