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A NUMERICAL INVESTIGATION OF THE EFFECT OF A SLIGHT TERRAIN SLOPE ON THE URBAN HEAT ISLAND

By

Guangwei HUANG

Graduate Student, Dept.of Civil Engineering, University of Tokyo, Bunkyo-Ku, Tokyo, Japan

and

Nobuyuki TAMAI

Professor, Dept.of Civil Engineering, University of Tokyo, Japan

and

Yoshihisa KAWAHARA

Associate Professor, Dept.of Civil Engineering, University of Tokyo, Japan

SYNOPSIS

The terrain slope is a real-world feature. In this study, the effect of a slight terrain slope on the flow structure is studied in connection with the problem of the urban heat island by a proposed two-length scale $k-\varepsilon$ model. Findings indicate that even a slight terrain slope can cause appreciable change in the flow structure. The degree of alteration depends on both the slope and the temperature difference between the rural and urban area.

INTRODUCTION

The most frequently observed and best documented climatic effect of urbanization is the increase in surface temperature of the urban area. This phenomenon has come to be known as the urban heat island. It is similar to that produced by sea/land breezes in the sense that both phenomena are caused by differential heating and cooling. However, the mechanism of the former is more complex than that of the latter. Great efforts have been made to study the heat island phenomenon over the last several decades. Those studies have revealed many aspects of the heat island phenomenon. Yet the authors are aware that the effect of a slight terrain slope on the structure of heat islands has been much neglected, and believe that the study of slope effect may lead to better understanding of the urban heat island problem because many cities are not built up on flat plains. Besides, undulating structures in downtown areas bear a resemblence to up-slope or down-slope. The objective of this study is to take a look at some differences in the flow structure which might be present between sloped and unsloped cases by means of numerical simulation.

One of the difficulties in simulating the heat island phenomenon is the parameterization of turbulence. Today a wide range of turbulence models is available from simple parameterization to sophisticated Reynolds stress-equation models. The Reynolds-stress closure models seem to deal satisfactorily with many problems, but they are computationally quite expensive due to many additional equations needed for Reynolds stresses, and numerically very unstable which could lead to serious convergence problems. By contrast, the $k-\varepsilon$ model is robust, it seldom leads to convergence problems. All these considerations indicate that the use of the $k-\varepsilon$ model with modification seems justified. Following this line of reasoning, proposed herein is a two-length scale $k-\varepsilon$ model, in which two different lengths, one for mixing and one for dissipation are used. It is believed that this two-length scale model can account for the effect of stratification better than the standard $k-\varepsilon$ model. And the computing cost of this two-length scale $k-\varepsilon$ model is almost the same as the standard one.

Because the principal goal of the present study is simply to investigate the effect of a slight slope on the flow structure over a warmer surface, the flows considered in this study is non-rotating, which means that the natural phenomena of interest are small enough or fast enough for the Coriolis forces to be negligible compared with the buoyancy and inertial forces. Moreover, no phase-change is taken into consideration, and the lower boundary of the flow domain is aerodynamically smooth. Although the flow situation is idealized, it retains many of the important features of the real-world case.

GOVERNING EQUATIONS AND MODEL DESCRIPTION

Basic assumptions

In this study, it is assumed that the Boussinesq approximation is applicable, which implies that one considers the fluid to be incompressible except for the buoyancy term. Most meteorological flows with a vertical scale $H <$ 1km, a horizontal scale $L \ll$ 12km and a wind speed $V \ll$ 100m/s can be treated in this way. The concept of virtual temperature is used to introduce the specific humidity into the momentum equations.

Basic equations for momentum, heat and water vapor

In a Cartesian system of coordinates(x_i, i=1 to 3), where g_i represents the three components of gravity g, the basic equations of steady-state flow resulting from Reynolds averaging process are following:

$$\frac{\partial U_i}{\partial x_i} = 0 \tag{1}$$

$$U_j\frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_j} + \frac{\partial}{\partial x_j}(\nu\frac{\partial U_i}{\partial x_j} - \overline{u_i'u_j'}) - g_i(\frac{T}{T_\infty} - 1) - 0.61g_i(q - q_\infty) \tag{2}$$

$$U_j\frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j}(k_t\frac{\partial T}{\partial x_j} - \overline{u_j'T'}) \tag{3}$$

$$U_j\frac{\partial q}{\partial x_j} = \frac{\partial}{\partial x_j}(k_q\frac{\partial q}{\partial x_j} - \overline{u_j'q'}) \tag{4}$$

where $\vec{U} = U_i(i = 1, 2, 3)$ is the velocity, $T =$ the potential temperature, q = the specific humidity which is defined as the mass of water vapor per unit mass of moist air, ν=the kinematic molecular viscosity, k_t= the molecular diffusivity for heat in air, k_q=the molecular diffusivity for water vapor in air, T_∞=the reference temperature.

Turbulence closure by a two-length scale model

For turbulent fluxes of momentum, heat and water vapor, a gradient diffusion approach is taken:

$$-\overline{u_i'u_j'} = \nu_t(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}) - \frac{2}{3}k\delta_{ij} \tag{5}$$

$$-\overline{u_j'T'} = \frac{\nu_t}{\sigma_t}\frac{\partial T}{\partial x_j} \tag{6}$$

$$-\overline{u_j'q'} = \frac{\nu_t}{\sigma_q}\frac{\partial q}{\partial x_j} \tag{7}$$

where ν_t = the turbulent viscosity. k = the kinetic energy of fluctuating motion, and is a measure of the intensity of turbulence. σ_t, σ_q are the turbulent Prandtl and Schmidt number respectively, and both are taken to be 0.9 in this study. According to the Prandtl-Kolmogorov hypothesis:

$$\nu_t = C_k l_k k^{1/2} \tag{8}$$

where l_k is the mixing length scale of energy-containing eddies, and C_k is a numerical constant. In the meantime, we have the Kolmogorov relation which expresses the dissipation rate of turbulence energy ε as a function of k, and the dissipation length scale l_ε:

$$\varepsilon = C_\varepsilon k^{3/2}/l_\varepsilon \tag{9}$$

In the standard $k - \varepsilon$ model, it is assumed that a common length scale can be used for mixing as well as for dissipation, i.e, $l_k = l_\varepsilon$. But if the buoyancy is present, the two length-scales may behave differently with increasing instability.

As buoyancy effects become strong, large difference appears between vertical and horizontal eddy motions. The vertical mixing must be dependent upon the general characteristics of the vertical components of the turbulent eddies, i.e, the mean eddy size and the amount of turbulent energy. Because these characteristics may be described by the energy spectrum of the vertical fluctuations of the wind speed, we hypothesize that the mixing length l_k is proportional to the vertical component of the turbulent kinetic energy. Therefore, it can be expected that:

$$\frac{l_k}{l_\varepsilon} = C_1(\overline{w'^2}/k) \tag{10}$$

In the particular case of neutral stratification,

$$l_k = l_\varepsilon \tag{11}$$

Then, $C_1 = (\overline{w'^2}/k)^{-1}_{neutral}$, which can be determined from experimental data[12]. In order to find out a functional relation between the two lengths, let us consider the simplified rate equation for $\overline{w'^2}$ as below:

$$\frac{\partial \overline{w'^2}}{\partial t} = -\frac{\partial \overline{w'^3}}{\partial z} + 2\beta\overline{w'T_v'} - C_4\frac{\varepsilon}{k}(\overline{w'^2} - \frac{2}{3}k) - \frac{2}{3}\varepsilon \tag{12}$$

where $\beta = g/T_\infty$ is the buoyancy parameter. T_v = the virtual temperature. This simplified rate equation is derived from the exact rate equation in the following way. One first neglects the advection terms and molecular diffusions, then, the pressure redistribution term which appears in the exact rate equation is modeled as "return-to-isotropy" terms following Rotta's suggestion. The mean-strain and buoyancy contributions to the pressure strain term is absorbed into the Rotta's "return-to-isotropy" term by suitably increasing the magnitude of C_4. The value of C_4,

c_ε	$c_{1\varepsilon}$	$c_{2\varepsilon}$	c_k	σ_k	σ_ε
0.166	0.144	1.92	0.55	1.0	1.3

Table 1: Values of the constants in the two-length scale k-ε model

in the present work, is taken to be 4 as proposed by Rotta. In addition, use of boundary-layer approximation has been made in arriving at the above form.

According to the work of Andre[2)], stationarity can be assumed for $\overline{w'^2}$ during daytime. If we further neglect the diffusion term in Eq.(12), we then get:

$$\frac{\overline{w'^2}}{k} = \frac{2}{3}\left(\frac{C_4 - 1}{C_4}\right) + 2\frac{\beta\overline{w'T'_v}}{C_4\varepsilon} \tag{13}$$

Inserting the above expression into eq.(10), and by mathematical manipulation, finally yields:

$$l_k = \left(1 - \frac{R_f}{1 - R_f}\right) l_\varepsilon \tag{14}$$

where R_f = the flux Richardson number,i.e the ration of the rate of removal of energy by buoyancy to the production by shear. It can be easily seen that $l_k > l_\varepsilon$ under conditions of unstable stratification, and $l_k < l_\varepsilon$ under conditions of stable stratification. This tendency is reasonable because the turbulent energy in the vertical direction is amplified by unstable stratification and vice versa in the stable case. In this study, l_ε is determined from eq.(9), and then l_k is calculated from the deduced relation (14) between l_k and l_ε.

The transport equations for k and ε are written as follows:

$$\begin{aligned} U_j\frac{\partial k}{\partial x_j} &= \frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_j}\right) + \underbrace{\nu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j}}_{\text{P}} \\ &+ \underbrace{\frac{g_j}{T_\infty}\frac{\nu_t}{\sigma_t}\frac{\partial (T + 0.61T_\infty q)}{\partial x_j}}_{\text{G}} - \varepsilon \end{aligned} \tag{15}$$

$$U_j\frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_j}\right) + \underbrace{c_{1\varepsilon}\frac{\varepsilon}{k}(P + G)(1 + c_{3\varepsilon}R_f) - c_{2\varepsilon}\frac{\varepsilon^2}{k}}_{\text{generation-destruction}} \tag{16}$$

The values of the constants in this two-length scale $k - \varepsilon$ model, except $c_{3\varepsilon}$, are shown in Table 1. The $c_{3\varepsilon}$ constant is chosen from Viollet[11)].

$$c_{3\varepsilon} = \begin{cases} 1, & \text{if stable} \\ 0, & \text{if unstable} \end{cases}$$

SLOPING EFFECTS ON THE URBAN HEAT ISLAND

Analysis

We consider a flat surface, sloped at a small angle α to the horizontal as sketched in Fig.1. The vertical convective scale H is of order 1km. The stratification is assumed to be neutral in the

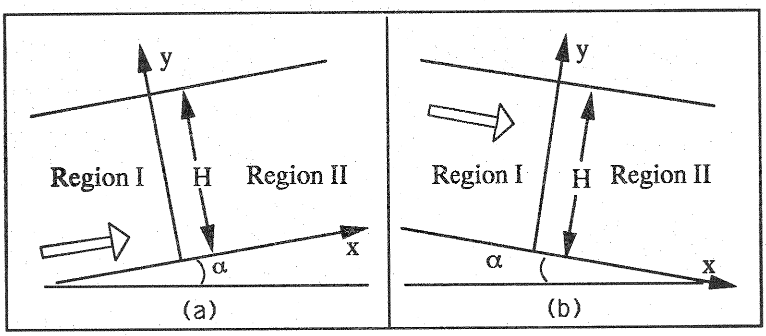

Figure 1: Flow configuration and coordinate system

region I and unstable or stable in the region II in which our interest lies. In the case of upward slope(Fig.1a), the component of the buoyancy force along x direction is:

$$\beta \Delta T \sin \alpha$$

It is positive for unstable stratifications and negative for stable stratifications. Consequently, the x component of the buoyancy force will produce an acceleration or a deceleration to the mean flow according to the stratification. In the case of downward slope(Fig.1b), the opposite is true. In other words, it tends to minimize wind shear and turbulence production near the surface if the stratification is unstable. In this study, the attention is focused on the upward terrain slope under unstable conditions in the region II.

Numerical procedure

A slope-aligned (x,y,z) coordinate is used as shown in Fig.1. The slope is assumed to be two-dimensional so that all gradients in z direction are zero. Computational conditions in all runs are compiled in Table 2. The subscripts w1,w2 in the Table 2 refer to the surface conditions in the region I and II respectively and the subscript *in* refers to the free-stream condition. In order to represent the real urban characteristics, in all runs, the surface temperature in the region II is given higher value than that in region I while the surface value of specific humidity in region II is lower than that in region I. This is because that urban areas are covered by a large percentage of asphalt and concrete, which are usually less moist surfaces with higher heat capacity than the surrounding countryside. The computational fetch on the surface is of order 1km while the depth of convective boundary layer is of order 0.5km. The mean flow and turbulence model equations are solved with a conservative finite volume method on a non-uniform and staggered grid system. A grid system comprising 50×50 grid lines is employed in this study. Near the solid wall, the wall-function technique is adopted which relates the streamwise velocity U, the kinetic energy k and the dissipation rate ε at the first grid point to the local friction velocity U_τ by

$$u_+ = \frac{U}{U_\tau} = \frac{1}{\kappa}\ln(Ey^+); \quad k = \frac{U_\tau^2}{\sqrt{c_\mu}}; \quad \varepsilon = \frac{U_\tau^3}{\kappa y}; \quad y^+ = \frac{U_\tau y}{\nu} \tag{17}$$

$$\phi_+ = \sigma_t(u_+ + P^*) \tag{18}$$

$$P^* = 9.24[(\frac{\sigma}{\sigma_t})^{3/4} - 1](1 + 0.28e^{-0.007\frac{\sigma}{\sigma_t}}) \tag{19}$$

where κ = the von Karman constant, E = a friction parameter. In the present study, $\kappa = 0.42, E = 9.8$. ϕ stands for temperature and humidity. When it refers to humidity, σ_t means the turbulent Schmidt number.

Run	α	U_{in}	T_{w1}	T_{w2}	T_{in}	q_{w1}	q_{w2}	q_{in}
1	0.0(radian)	1.0(m/s)	300K	310K	300K	0.04	0.02	0.0
2	0.002	1.0(m/s)	300k	310K	300K	0.04	0.02	0.0
3	0.003	1.0(m/s)	300K	310K	300K	0.04	0.02	0.0
4	0.003	1.0(m/s)	300K	320K	300K	0.04	0.02	0.0
5	0.0	1.0(m/s)	300K	320K	300K	0.04	0.02	0.0

Table 2: Computation Conditions

Results and discussion

The velocity distributions on different slopes have been calculated from the two-length scale model. They are compared with each other in Figs.2, 3 and 4. Fig.2 shows the velocity distributions near the rural-urban interface in the region II over different inclined surfaces. Fig.3 shows the velocity distributions at downstream station. It can be concluded from these comparisons that the slope has a significant effect on the mean velocity profiles even if the surface is only slightly inclined. This is because the slope-induced term in the momentum equation of x direction, when added to the pressure gradient term, gives an effective pressure gradient near the ground that speeds up the air flow. As the slope increases, the velocity profile becomes more flattened. Finally, a low-level jet may develop.

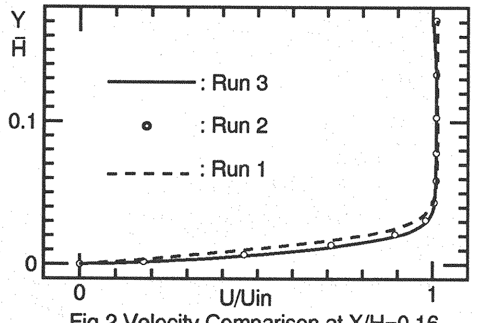

Fig.2 Velocity Comparison at X/H=0.16

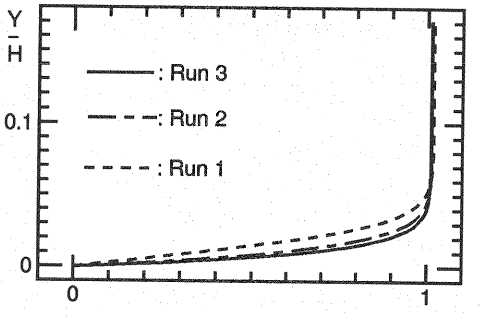

Fig.3 Velocity Comparison at X/H=1.8

Fig.4 provides a comparison of velocity profiles over different slopes under the condition of larger temperature difference between the region I and region II. By comparing Fig.4 with Fig.1, one can realize that the slope-induced modification is increased by the larger temperature difference between the two regions. The temperature profiles, made dimensionless by the surface temperature, is shown in Fig.5. The solid curve in Fig.5 corresponds to the unsloped case while the mark of plus corresponds to the sloped case. It can be observed that the presence of a slight terrain slope leads to a slight reduction in temperature. The decrease in temperature is attributable to the increase in wind speed by sloping. It should be mentioned that the comparisons presented in this study is obtained from steady-state calculation. If the simulation of diurnal variation is performed, a larger reduction in temperature due to sloping effect might be expected.

On the other hand, It is found from computations that the effect of water vapor on the mean velocity profiles over a warmer surface is enhanced when the surface is inclined. For example, the maximum relative deviation of velocity profiles caused by the presence of water vapor in sloped case may double that in unsloped case.

CONCLUSIONS

A two-length $k-\varepsilon$ closure method, replacing the eddy viscosity formula in the standard $k-\varepsilon$ model, $\nu_t = c_\mu \frac{k^2}{\varepsilon}$ by eq.(8), has been applied to the numerical study of the effect of terrain

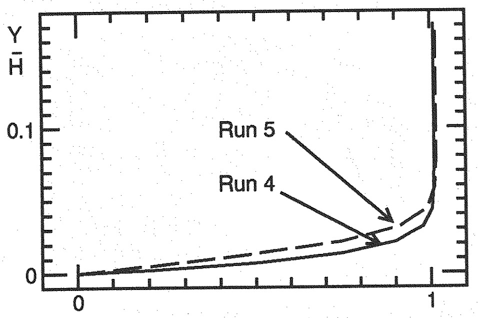

Fig.4 Velocity Comparison at X/H=0.16

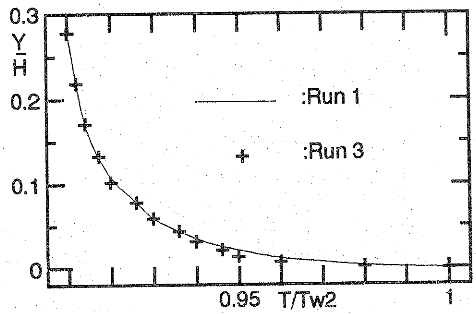

Fig.5 Temperature Comparison at X/H=1.5

slope on the flow structure over a heat island. It provides useful insight into the topographical influence on the urban heat island. As revealed by numerical results, even slight terrain slopes can cause appreciable change in the flow structure. Under certain conditions studied, a slight up-slope accelerates the flow near the surface, which is beneficial to human being. However, the real situation is much more complicated than that in this model study. An up-slope may play negative role under different circumstances. Therefore, further in-depth study is needed to clarify the slope-buoyancy effects. On the basis of obtained results, we would like to hypothesize that some discrepancies between the computational results and the observation data, reported in the literature, may partly be explained by the influence of a slightly inclined surface. Finally, a remark on the two-length scale $k-\varepsilon$ model should be made. The introduction of two scales is just a step toward improving the preditability of the $k-\varepsilon$ model for buoyant shear flow by considering more of physics of turbulent process. However, the present two-scale $k-\varepsilon$ model still does not resolve the problem of isotropic normal stresses common to eddy-viscosity closures. The further refinement of the $k-\varepsilon$ model can be envisaged in various directions. One feasible improvement which may achieve better prediction of all Reynolds stresses would be to develop a two-scale nonlinear $k-\varepsilon$ model. Work in this direction is currently in progress.

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APPENDIX - NOTATION

k	= turbulent kinetic energy;
k_q	= molecular diffusivity for water vapor in air;
k_t	= molecular diffusivity for heat in air;
l_k	= mixing length scale;
l_ε	= dissipation length scale;
q	= mean specific humidity;
q'	= fluctuating specific humidity;
R_f	= flux Richardson number;
T	= mean temperature;
T'	= fluctuating temperature;
u_i	= fluctuating velocity component in x_i direction;
U_i	= mean velocity component in x_i direction;
U_τ	= friction velocity;
$\overline{w'^2}$	= vertical component of turbulent kinetic energy;
ν	= kinematic molecular viscosity;
ν_t	= turbulent eddy viscosity;
ε	= dissipation rate of k;
σ	= molecular Prandtl/Schmidt number;
σ_t, σ_q	= turbulent Prandtl and Schmidt number;

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