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DISLODGMENT PROCESS OF SEDIMENT PARTICLE ON BED AT AN UNSTEADY FLOW

By

Hitoshi GOTOH

Research Associate, Department of Civil Engineering, Kyoto University
Yoshida Hon-machi, Sakyo-ku, Kyoto, 606, Japan

Tetsuro TSUJIMOTO

Associate Professor, Department of Civil Engineering, Kanazawa University
2-40-20, Kodatsuno, Kanazawa, 920, Japan

and

Hiroji NAKAGAWA

Professor, Department of Global Environmental Engineering, Kyoto University
Yoshida Hon-machi, Sakyo-ku, Kyoto, 606, Japan

ABSTRACT

Fluvial system is often non-uniform and unsteady, and therefore, the non-equilibrium sediment transport model should be developed in a time-and-space field, in order to describe the fluvial phenomena precisely. In this study, a convolution-integral formed sediment transport model is developed along a time axis. The sediment pick-up rate, which is one of the fundamental parameters of the model, is estimated by a numerical simulation of dislodgment process of a particle on a bed by rolling under the effects of flow unsteadiness. The result of the simulation is compared with the experimental data of sediment pick-up rate in an oscillation-current coexisting flow field to confirm the applicability of the method proposed in this paper.

INTRODUCTION

Since the stochastic sediment transport model was constructed by Einstein (1) and Paintal (9) enthrough the laboratory experiments, many theoretical and experimental studies have been attempted to develop it. But most of the studies adopted a Lagrangian approach and therefore it was difficult to express the local condition of the flow. Nakagawa & Tsujimoto (4) deduced an Eulerian stochastic model, which was a convolution-integral formulation constituted by pick-up rate and step length. Their non-equilibrium bed-load transport model along a spatial axis has been a useful means to describe several fluvial phenomena.

Einstein (1) introduced the concept of the "exchange time" which was defined as the ratio of a particle's diameter to a settling velocity on formulating the pick-up rate. But the reason why the settling velocity was chosen as a velocity scale of dislodgment was ambiguous in his theory. The dislodging velocity of a particle should be defined related to an appropriate velocity scale. Nakagawa & Tsujimoto (3) estimated a physically reasonable scale of the velocity on the basis of the equation of motion by introducing an approximation instead of solving it directly. From more precise point of view, the velocity should be obtained by integrating the equation of motion, because the dislodgment process is non-uniformly accelerated process, and especially under the unsteady flow condition, the non-uniformity of acceleration is important to be taken into account.

In this paper, the convolution-integral formed sediment transport model is developed along a time axis. And then, the numerical simulation of the dislodgment process is executed by a numerical integration of the equation of rolling motion of a particle dislodging from a bed. Modification of the Nakagawa & Tsujimoto's formula is performed considering the effect of flow acceleration, with the aid of the numerical simulation. Finally, the modified formula of the pick-up rate is compared with the experimental data in an oscillation-current coexisting flow to confirm the applicability of the formula.

NON-EQUILIBRIUM SEDIMENT TRANSPORT AT AN UNSTEADY-UNIFORM FLOW

The formulation of bed-load transport along a time axis is proposed , which is a modification of spatially non-equilibrium bed-load transport model established by Nakagawa & Tsujimoto (4).

$$q_B(t) = \frac{A_3 d}{A_2}\int_0^{\infty} p_s(t-\tau)\cdot u_g(t|t-\tau)\int_{\tau}^{\infty} f_T(\zeta|t-\tau)d\zeta\, d\tau \qquad (1)$$

in which q_B=bed-load transport rate (substantial volume per unit time per unit width); A_2, A_3= geometric coefficients of sediment particle; d=diameter of sand particle; $p_s(t)$= pick-up rate at time t; $f_T(\zeta|t)$= probability density that the moving period of the particle picked up at time t is equal to ζ; $u_g(t|t-\tau)$= velocity of a bed-load particle at time t which is picked up at time $(t-\tau)$ and still keeps moving at time t. Eq.1 can be rewritten as follows:

$$q_B(t) = \frac{A_3 d}{A_2}\int_0^{\infty} p_s(t-\tau)\cdot U_T(\tau|t-\tau)d\tau \qquad (2)$$

where U_T is an apparent particle velocity which is defined as

$$U_T(\tau|t-\tau) \equiv u_g(t|t-\tau)\int_0^{\infty} f_T(\zeta|t-\tau)d\zeta \qquad (3)$$

$U_T(\tau|t)$ is a modified velocity which is obtained by considering an existence of the particle which does not continue to move during τ. $U_T(\tau|t)$ is the impulse response of the bed-load transport system in the time axis.

NUMERICAL SIMULATION ON DISLODGMENT PROCESS OF SEDIMENT PARTICLE ON BED

Sediment pick-up rate is defined as a probability density of particle dislodgment per unit time. Therefore, the time scale of dislodgment is necessary to be estimated before evaluating a pick-up rate, and study on the investigation of dislodgment process is performed based on the equation of motion instead of the static balance of the acting forces. Nakagawa & Tsujimoto (4) proposed a formula of pick-up rate on the basis of the mechanism of the particle motion, considering the probabilistic characteristics of the hydrodynamic force. In their study, first, the dislodging velocity of the particle was estimated by considering the equation of motion. Secondly, the time required for a particle to be dislodged was evaluated from the dislodging velocity and the length scale of dislodgment which is in the same order as the particle's diameter. In the dislodgment process by a rolling motion, the angular velocity of the particle and the angle of escape were adopted as the governing parameters instead of the dislodging velocity and the length scale, respectively. Then, the sediment pick-up rate was estimated as a reciprocal of the time required for a particle to be dislodged.

In Nakagawa & Tsujimoto's study, the dislodging velocity was defined as the averaged value on analyzing the equation of motion. In the process of the dislodgment, however, using the averaged value as a velocity scale brings some errors from the precise viewpoint, because the effect of acceleration is not negligible. From this point of view, in this study, the process of dislodgment is investigated by a numerical integration of the equation of rolling motion without any approximation on the dislodging velocity.

The fluctuation of the hydrodynamic force acting on a particle is simulated with the aid of the Monte-Calro method. As shown in Fig. 1, a spherical particle (A) resting on a bed is dislodged with rolling around the particle (B) neighboring in the downstream direction. The equation of rolling motion is written as

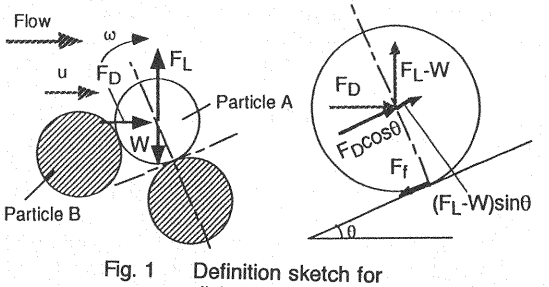

Fig. 1 Definition sketch for dislodgment process

$$M\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = (\varepsilon F_\mathrm{L} - W)\sin\theta + \varepsilon F_\mathrm{D}\cos\theta - F_\mathrm{f} \qquad (4)$$

where x=longitudinal coordinate; θ=angle of contact; M=virtual mass of the particle; F_L=lift force; F_D=drag force; W=submerged weight; F_f=frictional force between the particle A and B. Coefficient ε represents the sheltering effect of the particle neighboring in the upstream direction on the drag force.

Considering the relation $x=\theta d$, one can rewrite Eq.4 as follows:

$$Md\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = (\varepsilon F_\mathrm{L} - W)\sin\theta + \varepsilon F_\mathrm{D}\cos\theta - F_\mathrm{f} \qquad (5)$$

The moment equation around the gravity center of the particle A is given by

$$I_\mathrm{G}\frac{\mathrm{d}\omega}{\mathrm{d}t} = F_\mathrm{f}\frac{d}{2} \qquad (6)$$

in which I_G= moment of inertia; and ω= angular velocity of particle. If the particle A rolls perfectly around the particle B without slip,

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = \frac{1}{2}\frac{\mathrm{d}\omega}{\mathrm{d}t} \qquad (7)$$

From Eqs. 6 and 7, the frictional force F_f can be written as

$$F_\mathrm{f} = \frac{4I_\mathrm{G}}{d}\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} \qquad (8)$$

The governing equation of rolling motion is derived by substituting Eq.8 into Eq.5.

$$\left(Md + \frac{4I_\mathrm{G}}{d}\right)\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = (\varepsilon F_\mathrm{L} - W)\sin\theta + \varepsilon F_\mathrm{D}\cos\theta \qquad (9)$$

The virtual mass, the moment of inertia and the acting forces are expressed as follows:

$$\left.\begin{aligned} &M = \rho\left(\frac{\sigma}{\rho} + C_\mathrm{M}\right)A_3 d^3 \quad ; \quad W = \rho\left(\frac{\sigma}{\rho} - 1\right)gA_3 d^3 \\ &F_\mathrm{L} = \frac{1}{2}C_\mathrm{L}\rho\left(u - d\cdot\cos\theta\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2 A_2 d^2 \quad ; \quad I_\mathrm{G} = \frac{2}{5}\left(\frac{d}{2}\right)^2 M \\ &F_\mathrm{D} = \frac{1}{2}C_\mathrm{D}\rho\left(u - d\cdot\cos\theta\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2 A_2 d^2 + \rho(1+C_\mathrm{M})A_3 d^3\frac{\mathrm{d}u}{\mathrm{d}t} \end{aligned}\right\} \qquad (10)$$

where σ, ρ=mass density of fluid and sand particle, respectively; and C_M=added mass coefficient. The value of coefficients used in this simulation are: C_M=0.5, C_D=C_L=0.4, $A_2=\pi/4$ and $A_3=\pi/6$. The dislodgment process corresponds to the changing process of the angle θ in numerical integration of Eq.9. The effect of the acceleration is taken into account by the unsteady drag term in Eq.10.

The initial condition is that $\theta=\beta$ at t=0. The time which an individual particle needs to finish the dislodgment process, T_d is defined as

$$\theta(T_\mathrm{d}) = 0 \qquad (11)$$

On the basis of the definition of dislodgment by Eq.11, the pick-up rate is given as the reciprocal of T_d.

$$p_\mathrm{s} = \frac{k_\mathrm{d}}{T_\mathrm{d}} \qquad (12)$$

where k_d=empirical constant.

In order to simulate the fluctuating hydrodynamic force acting on the particle, the following assumptions are introduced: Fig. 2 is a schematic expression of flow field in the vicinity of the bed. Mean velocity profile is given by the logarithmic law as follows:

$$\frac{u}{u_*} = \frac{1}{\kappa}\ln\left(\frac{30.1\chi y}{k_s}\right) \qquad (13)$$

in which κ=Kármán constant; k_s=equivalent sand roughness ($k_s=\alpha d$); α=1.0 and χ=modification coefficient considering the effect of the roughness Reynolds number. The mean bed level is to coincide with the average height of the gravity center of the particle B. The sheltering coefficient ε is assumed to be expressed as follows with reference to the study on the effect of the expossure on the drag(5):

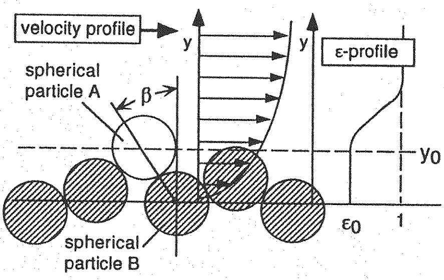

Fig. 2 Schematic figure for velocity profile and sheltering coefficient

$$\varepsilon = \begin{cases} \varepsilon_0 & \text{for} \quad y \le y_0 \\ \varepsilon_0 + (1-\varepsilon_0)\exp\left\{-\dfrac{9}{2}\left(\dfrac{y-y_0}{d}-1\right)^2\right\} & \text{for} \quad y > y_0 \end{cases} \qquad (14)$$

Eq.14 means that the sheltering effect grows smaller as a particle dislodgment process. The value of ε_0 is set 0.4 (the most commonly used value (6)), when the particle is resting on the bed.

The velocity fluctuation in the vicinity of a bed is also assumed as follows: The relation between the variation coefficient of a shear velocity η and that of a bed shear stress η_0 is given by

$$\eta = \sqrt{\eta_0 + 1} - 1 \qquad (15)$$

According to Einstein (1), η_0=0.5, then η=0.225 from Eq.15. Supposing that the fluctuation of a shear velocity follows a normal distribution, one can obtain the time series of the fluctuating shear velocity simulated by Monte-Calro method. The time step of the present simulation is Δt=1/200 s. By adopting this time step, one period of the rocking motion under τ_*=0.1 is divided into about ten steps.

In this simulation, the angle of escape is set β=π/4. It is the one of the most important factors that governs the simulation. The particle's arrangement is so complicated that its characteristics is difficult to be expressed by a simple model. The measurement of β by Tsujimoto (13) suggested that it was more narrowly distributed on a loose bed than on a fixed bed. This experimental result would support the assumption that β is constant.

Figure 3 shows the calculated results of the angle of contact, θ, and the angular velocity of the particle, ω. The bed shear stress in Fig. 3 (a) is set τ_*=0.1 that is sufficiently larger than the critical bed shear stress. The variance of the time required for particle's dislodgment is not so large as shown in Fig. 3 (a). This sug-

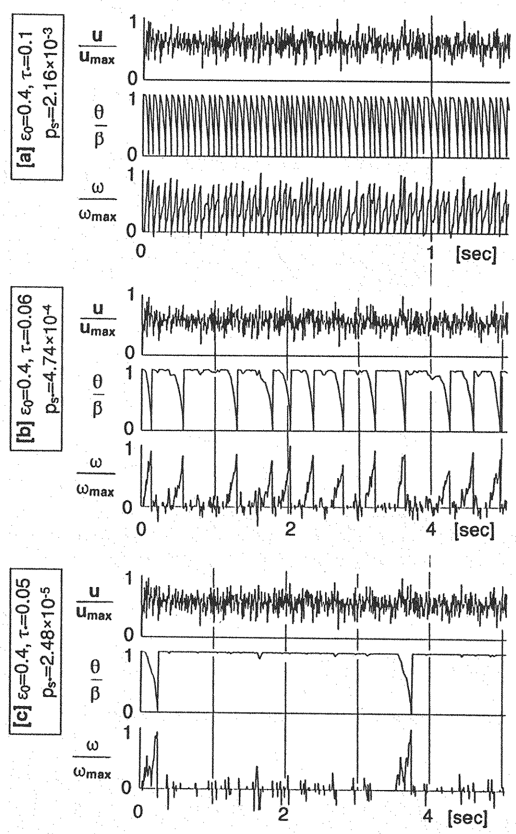

Fig. 3 Time series of u, θ and ω

gests that the dislodgment process is mainly governed by the mean bed shear stress, and that the contribution of the fluctuation of the bed shear stress is less significant. In Fig.3 (b), the bed shear stress is set τ_*=0.06 that is slightly larger than the critical bed shear stress. In this case, the dislodgment process is affected not only by the mean bed shear stress but also by the fluctuation of the bed shear stress. The time series of the angle of contact shows the occurrence of the rocking motion before the incipient motion. And it can be seen that the particle keeps rocking motion with an irregular duration until the large momentum enough to dislodge the particle is supplied.

Tsuchiya, Ueda & Oshimo (12) investigated the rocking motion under the periodic change of the hydrodynamic force. They focussed on the characteristics of the self-induced rocking motion, and deduced the governing equation of motion which is a nonlinear ordinary-differential equation similar to the Mathieu equation. And they concluded that the dislodgment corresponds to an unstable solution of it. While in this paper, the rocking motion caused by the fluctuating hydrodynamic forces are simulated. In case of dislodgment process under the wave action, both types of rocking motion are not negligible, but, when a periodic change of the hydrodynamic force hardly plays an important role, for instance in a fluvial streams, the rocking motion caused by the fluctuating hydrodynamic forces is dominant.

The bed shear stress in Fig.3 (c) is set just the critical bed shear stress; τ_*=0.05. The time required for a particle to be dislodged in Fig.3 (c) is longer than that in Fig.3 (b), thus the frequency of the dislodgment in Fig.3 (c) is smaller than that in Fig.3 (b).

THE FORMULA OF PICK-UP RATE AT AN UNSTEADY FLOW

The unsteady effect in general consists of two factors: (A) the effect of the acceleration; and (B) the effect of the hysteresis. On the spatial axis, the pick-up rate is dominated by a local hydraulic conditions, while the step length is influenced by the upstream flow condition. From a view-point of analogy, the pick-up rate on the time axis is dominated mainly by instantaneous flow condition, therefore, only the factor (A) should be considered on estimating the pick-up rate. In this chapter, the effect of acceleration on a pick-up rate is investigated with the aid of numerical simulation explained in the previous chapter.

The relation between the local velocity u at $y=d/2$ and the shear velocity u_* is given by

$$\phi_p = \frac{u}{u_*} = \frac{1}{\kappa}\ln\left(\frac{30.1\chi y}{k_s}\right) \tag{16}$$

The temporal derivative of the local velocity is written as follows:

$$\frac{du}{dt} = \phi_p \frac{du_*}{dt} + u_* \frac{d\phi_p}{dt} \tag{17}$$

The phase lag between the shear velocity and the local velocity in the vicinity of the bed is small enough that $d\phi_p/dt$ is negligible. The shear velocity and its time derivative are expressed as a function of dimensionless shear stress τ_* as follows:

$$u_* = \sqrt{\tau_*\left(\frac{\sigma}{\rho}-1\right)gd} \tag{18}$$

$$\frac{du_*}{dt} = \frac{1}{2}\sqrt{\left(\frac{\sigma}{\rho}-1\right)gd}\,\frac{1}{\sqrt{\tau_*}}\frac{d\tau_*}{dt} \tag{19}$$

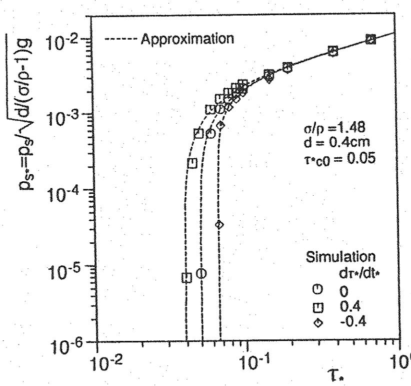

Fig. 4 Effect of acceleration on sediment pick-up rate

If the dimensionless shear stress and its time derivative are known, the velocity and its derivative in the vicinity of a bed can be calculated by Eqs. 16, 17, 18 and 19. Then the motion of the particle can be simulated by calculating based on Eq. 9. And then the pick-up rate at unstedy flow is obtained. In Fig. 4, the result is depicted and it is to be approximated by the follwing qeuation.

$$p_{s*} = p_s\sqrt{\frac{d}{(\sigma/\rho-1)g}} = F_0\tau_*^{1/2}\left(1-\frac{k_2\tau_{*c0}}{\tau_*}\right)^{k_3 m} \tag{20}$$

where F_0=empirical constant (=0.011); k_2, k_3=coefficients for the modification which indicate the effect of the acceleration; and τ_{*c0}= critical bed shear stress at an steady flow. Fig.4 suggests that the effect of acceleration is predominant only under the condition around the critical bed shear stress.

In order to evaluate the coefficients, k_2 and k_3, the simulations were executed by changing the magnitude of the acceleration. Fig. 5 shows the effect of the acceleration on the values of the coefficients k_2 and k_3. The curves in this figure are approximated formulations written as follows:

$$k_2 = 1 + \alpha_{c2}\frac{d\tau_*}{dt_*} + \beta_{c2}\Gamma_{c2} \tag{21}$$

$$\Gamma_{c2} = \begin{cases} 1 - \exp\left\{\dfrac{1}{B_2}\left[\dfrac{d\tau_*}{dt_*} - \left(\dfrac{d\tau_*}{dt_*}\right)_1\right]\right\} & \text{for } \dfrac{d\tau_*}{dt_*} < \left(\dfrac{d\tau_*}{dt_*}\right)_1 \\ 0 & \text{for } \dfrac{d\tau_*}{dt_*} \geq \left(\dfrac{d\tau_*}{dt_*}\right)_1 \end{cases} \tag{22}$$

$$k_3 = 1 + (\alpha_{c3} + \beta_{c3}\Gamma_{c3})\frac{d\tau_*}{dt_*} \tag{23}$$

$$\Gamma_{c3} = \begin{cases} 1 - \exp\left(-\dfrac{1}{B_3}\dfrac{d\tau_*}{dt_*}\right) & \text{for } \dfrac{d\tau_*}{dt_*} > 0 \\ 0 & \text{for } \dfrac{d\tau_*}{dt_*} \leq 0 \end{cases} \tag{24}$$

where $t_* \equiv t\sqrt{(\sigma/\rho - 1)g/d}$. The relative density of the sand was set 1.48 in this computation, to compare the present simulation with the experimental data explained in the following chapter.

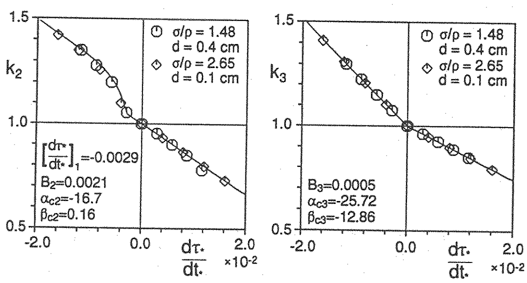

Fig. 5 Coefficients of flow acceleration

Figure 6 shows the application of Eq. 20 to the natural sand (σ/ρ=2.65). The Nakagawa & Tsujimoto's formula (3) is also shown in this figure with the experimental data obtained by Nakagawa & Tsujimoto (3), Shinohara & Tsubaki (10), Takahashi (11), and Yano, Tsuchiya & Michiue (14). The discrepancy of Eq. 20 from Nakagawa & Tsujimoto's formula which coincides well with the experimental data is summarized as follows: (i) the pick-up rate calculated by Eq. 20 increases more rapidly than the curve by Nakagawa & Tsujimoto in the vicinity of the critical bed shear stress, and (ii) the gradient of Eq. 20 is smaller than that of the Nakagawa & Tsujimoto's. (i) indicates that the fluctuation-dominant region of Eq. 20 is smaller than that of the Nakagawa & Tsujimoto's; while (ii) indicates that the increasing tendency of the pick-up rate by Eq. 20 is weaker than of Nakagawa & Tsujimoto's.

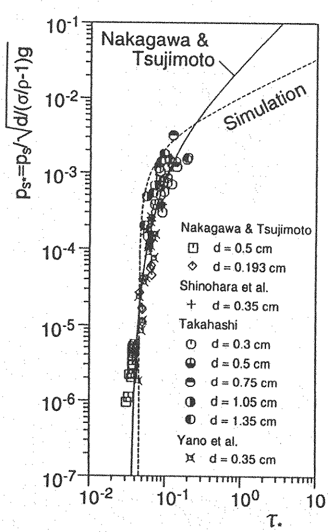

Fig. 6 Sediment pick-up rate

Because the effect of the acceleration is affected mainly by the deterministic factors and is included in the model of dislodgment, the effect of the acceleration can be reasonably estimated by this simulation. And the approximated formula of this simulation has the similar formation to Nakagawa & Tsujimoto's. In order to establish the formula of the pick-up rate which has a good coincidence with the experimental data at an steady flow and simultaneously includes the effect of the acceleration, Nakagawa & Tsujimoto's formula is modified as follows:

$$p_{s*} = p_s\sqrt{\frac{d}{(\sigma/\rho-1)g}} = F_0\tau_*\left(1-\frac{k_2\tau_{*c0}}{\tau_*}\right)^{k_3 m} \tag{25}$$

where empirical constant m=3 and F_0=0.03 for the natural sand. In this study, the coefficients k_2, k_3 are determined for the particle, whose relative density is 1.48 and 2.65.

According to Fig. 4, the effect of acceleration on a sediment pick-up rate is small for the sufficiently large bed shear stress. Therefore, pick-up rate formula should be modified for the small bed shear stress. When they are devide by the respective estimation for sufficiently large bed shear stress $p_{s*\infty}$, they can be expressed by the same equation as follows.

$$\frac{p_{s*}}{p_{s*\infty}} = \left(1-\frac{k_2\tau_{*c0}}{\tau_*}\right)^{k_3 m} \tag{26}$$

therefore, the modification by eq. 25 is reasonable.

APPLICATION OF THE PICK-UP RATE FORMULA TO THE OSCILLATION-CURRENT COEXISTING FLOW

The oscillation-current coexisting flow was experimentally generated by a U-tube type oscillating water tunnel illustrated in Fig. 7. The equipment was made of acrylic resin, and the working section of the water tunnel was 180cm long, 10cm high and 40cm wide. The access to the working section was provided by the haches located above the central part of the water tunnel. The motion of the sediment particles was recorded by a CCD video camera over the working section. The experimental condition is shown in Table 1. The detail of the experiment was provided by another paper by Nakagawa, Tsujimoto & Gotoh (8).

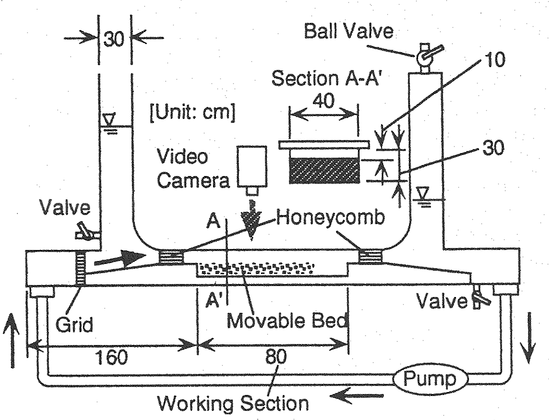

Fig. 7 Experimental apparatus

Figure 8 shows the comparison between the modified Nakagawa & Tsujimoto's formula (Eq. 25) and the experimented data. The coefficient F_0 is calculated by taking account of the effect of the particle's density as follows:

$$\frac{F_{0m}}{F_0} = \left(\frac{\sigma/\rho-1}{\sigma/\rho+C_M}\right)_m \bigg/ \left(\frac{\sigma/\rho-1}{\sigma/\rho+C_M}\right) \tag{27}$$

in which the subscript ns means natural sand (σ/ρ=2.65) and the subscript m means the modification considering the particle's density. Letting the Nakagawa & Tsujimoto's value for a natural sand (σ/ρ=2.65) F_0=0.03, into Eq. 27, the value for the particle employed in the experiment (σ/ρ=1.48) is calculated as F_{0m}=0.014. However, when one would minimize the discrepancy of Eq. 25 from the experimental data, the value of F_{0m} should be set 0.036: it is 2.6 times larger than the value

Table 1 Experimental condition

	Run 1	Run 2
Diameter of the particle d(cm)	0.4	
Relative density of the particle σ/ρ	1.48	
Amplitude of the oscillating velocity U_w(cm/sec) at y=d/2	41.01	43.74
Mean velocity of the current u_c (cm/sec)	18.0	18.0
Ratio of U_w to u_c (U_w/u_c)	2.28	2.43
Period of the oscillation T(sec)	4.0	4.0

modified by taking account of the effect of the particle's density (0.014). This difference in the value F_0 is allowable, when one considers the degree of scattering of the data of pick-up rate even at an steady flow. In the following, it is assumed that F_{0m}=0.036.

The range of the distribution of the pick-up rate is estimated fairly well by both the modified and non-modified Nakagawa & Tsujimoto's formula. The peak of the modified formula appears earlier than that of the non-modified formula. For the case (A), where the ratio of the amplitude of the velocity in oscillation (U_w) to the current velocity (u_c) is small, the magnitude of the peak obtained by the modified formula agrees well with the experimental data, but the peak-phase is underestimated by the modified formula. On the other hand, for the case (B), where the ratio of U_w to u_c is large, the peak-phase estimated by the modified formula shows a fairly good agreement with the experimental data, but its peak-magnitude is overestimated by the modified formula.

Another feature is an asymmetry of the distribution. The experimental data are skew: the rising limb is steeper than the falling one. The modified formula reproduces the experimental data with a clear tendency. The difference between the modified and the non-modified formulas is whether the effect of the acceleration is taken into account or not. The effect of the acceleration appears as the promotion of dislodgment in accelerated phase and its suppression in decelerated phase. The modified formula has a point of inflection in the falling limb, and this tendency is also recognized in the experimental data.

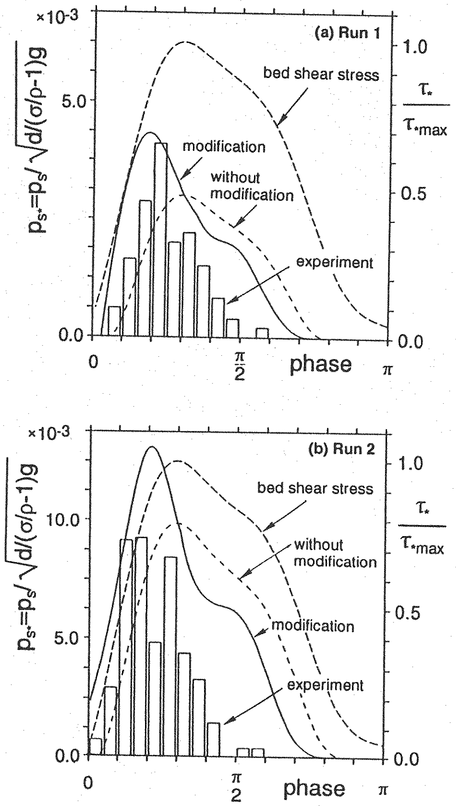

Fig. 8 Sediment pick-up rate in oscillation-current coexisting flow

CONCLUSIONS

The results obtained in this paper are summarized below:

(1) In this study, the convolution-integral formed sediment-transport model was developed along a time axis.

(2) In order to estimate the sediment pick-up rate, the numerical simulation of dislodgment process executed on the basis of the equation of the rolling motion under the fulactuating hydrodynamic force simulated by Monte-Carlo method. From the result of the simulation, the features of the rocking motion caused by the fluctuating hydrodynamic force can be recognized. This fact supports the reliability of the present simulation.

(3) With the aid of the numerical simulation, the effects of flow unsteadiness is taken into account on incipient motion of bed particles. The modification of the pick-up rate formula was represented by the two coefficients, the change of which due to flow unsteadiness were clarified by the numerical simulation.

(4) The result of the simulation agrees well with the tendency of the experimental data of sediment pick-up rate by oscillation-current coexisting flow, and the applicability of the method proposed in this paper was confirmed.

Most of the contents of this study was already published in Japanese (7), but revised herein.

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APPENDIX-NOTATION

The following symbols are used in this paper:

A_2, A_3 = two- and three-dimensional geometrical coefficients of particles respectively;
C_M = added mass coefficient;
C_D, C_L = drag coefficient and lift coefficient;
d = diameter of particle;
F_0 = empirical constant in pick-up rate formula;
$f_T(\zeta|t)$ = probability density function of the moving period of the particle picked up at time t;
F_D = drag force;
F_f = frictional force between the dislodging particle and the resting particle;
F_L = lift force;
g = gravitational acceleration;
I_G = moment of inertia;
k_2, k_3 = coefficients of modification in pick-up rate formula;
k_d = empirical constant;
k_s = equivalent sand roughness in logarithmic law of velocity;
m = empirical constant in pick-up rate formula;
M = virtual mass of the particle;
t_* = dimensionless time scale;
T_d = the duration of the individual particle's dislodgment;

u	= flow velocity;
u_*	= shear velocity;
$u_g(\tau\|t)$	= speed of the particle picked up at a time t;
$U_T(\tau\|t)$	= the impulse response of the bed-load transport system in temporal axis;
$p_s(t)$	= pick-up rate of bed-material particle;
$q_B(t)$	= bed-load transport rate;
W	= submerged weight of particle;
x	= longitudinal coordinate;
α	= empirical constant;
β	= angle of escape;
$\varepsilon, \varepsilon_0$	= sheltering coefficient and its value for the resting particle on the bed;
η, η_0	= variation coefficients of shear velocity and bed shear stress;
χ	= coefficient to express the effect of the grain-size Reynolds number in logarithmic law of velocity;
θ	= angle of contact;
ω	=aungular velocity of the particle;
ϕ_p	= ratio of the flow velocity to the shear velocity;
ρ	= mass density of fluid;
σ	= mass density of bed-material particle;
τ_*	= dimensionless bed shear stress; *and*
τ_{*c0}	= dimensionless critical shear stress at an steady flow.

Subscripts

m	= modification for mass density; *and*
ns	= natural sand.

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