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SAND WAVE DEVELOPMENT WITH SEDIMENT SORTING

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SYNOPSIS

Spatially cyclic patterns of sediment sorting, such as longitudinal alternation of the coarser and finer parts of the bed surface, are observed in stream beds with sand waves as well as without them. Such a phenomenon is related to the development and propagation of sand waves. The interaction between sediment sorting and formation of sand waves on the basis of flume tests is described. Experimental results show that sediment coarsening on the front surface of sand waves produced by the sediment sorting on upstream surfaces affects wave height. A reference grain size for the determination of wave height is proposed, according to which the reference grain size is 2 times larger than the mean diameter of the graded sediment used.

INTRODUCTION

Many studies have been made on the relation between sediment transport and the formation of sand waves. Most of them treat phenomena associated with a uniform sediment. Suzuki and Michiue (11) reported that wave heights decreased to 60% of their original values when coarse sand was added to a bed with fully developed sand waves composed of uniform sediment. The supply rate was 5% of the original sediment discharge rate, and the mean diameter of the coarse sand was 3 times larger than the mean diameter of the uniform sediment. Snishchenko et al. (10) reported that the wave heights of sand waves constituted of graded sediment were less than those constituted of uniform sediment under the same initial hydraulic conditions of mean diameter, water discharge, and bed gradient. Guy et al. (2) and Kuhnle and Southard (5) reported such phenomena as mentioned above, but did not discuss the mechanisms. Yamamoto (14) reported that the sediment of the bed surface around a trough was coarser than the original sediment and that it was finer around a crest; whereas, the sediment of the bed-load was finer at the troughs and coarser at the crests. Guy et al. (2) and Ikeda and Iseya (4) described the same phenomena as these for sediment of bed surfaces.

These results show formation of sand waves and sediment sorting have remarkable effect on each other. It is expected that coarse grains affect the development of sand waves, and thus, the interaction between the development of sand waves and sediment sorting needs to be investigated. In this paper, the natures of sand waves, such as the properties of the shape and migration of sand waves, as well as sediment sorting, are investigated, using graded sediment. The relation between the development of sand waves and sediment sorting is also considered. Finally, a reference grain size for determining of wave height is proposed, and a discussion on the difference in wave heights for graded and uniform sediments is made on the basis of the previous study.

EXPERIMENTAL SETUP AND PROCEDURE

Experiments were conducted in a straight rectangular open channel, 6.5 m long, 0.2 m wide, and 0.3 m deep. Three kinds of sediment were used. Their grain size distributions are shown in Fig.1, in which the symbol U indicates the nearly uniform sediment. The mean sediment diameter, d_m, is 0.06 cm, and the geometric standard deviation, s_g, defined by $\sqrt{d_{84}/d_{16}}$, is 1.30. M_A and M_B are graded sediments of d_m=0.07 cm and s_g=1.70, and d_m=0.08 cm and s_g=1.90, respectively. These were made by mixing two kinds of uniform sediments. A natural white fine material was used as the fine sediment in M_A for convenience of observing sediment sorting. The specific gravities of the two materials were 2.66. The hydraulic conditions are given in Table 1, in which each value is in a stable state of the bed. In this table, symbols are defined as follows: q_w = water discharge per unit width of the channel; h_m = mean flow depth; F_r = Froude number; I_e = energy slope; u_* = mean shear velocity; L = wave length; and Δ = wave height. Experiments with the symbol "*" were conducted to investigate the development of sand waves, and the sediment sorting for graded sediment. In these experiments, the wave lengths, the wave heights, and the grain size distributions of the sediments of the bed surface and subsurface layers were measured several times until the sand waves had fully developed. In Runs U-1, U-2, U-3, M-1, and M-2, measurements were made only at the stable state of

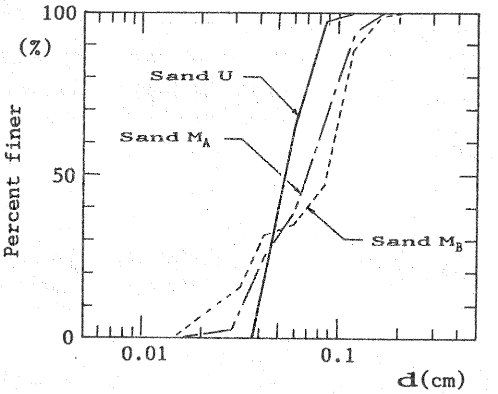

Fig.1 Grain size distributions of the sediments used.

Table 1 Experimental conditions.

Run No.	qw (cm²/sec)	hm (cm)	Fr	Ie (×10⁻³)	U* (cm/sec)	L (cm)	Δ (cm)	Sand
U-1	275	7.06	0.47	2.02	2.87	34.8	1.28	U
U-2	350	8.96	0.42	2.61	3.48	53.0	2.53	U
U-3	400	9.51	0.44	2.78	3.64	42.8	2.21	U
U-1*	400	9.75	0.42	3.25	3.96	44.6	2.40	U
U-2*	450	9.63	0.48	2.26	3.30	38.0	1.51	U
M-1	350	8.10	0.48	3.13	3.72	50.0	0.98	M_B
M-2	400	8.96	0.48	2.78	3.59	46.7	1.11	M_B
M-1*	350	8.21	0.48	2.48	3.31	30.7	0.67	M_A
M-2*	350	7.89	0.50	2.50	3.29	39.5	0.83	M_B
M-3*	400	8.87	0.48	2.95	3.69	54.0	1.16	M_A

the sand waves.

In the experiments, the sediment was transported only as bed-load. Water surface elevations were measured at intervals of 10 cm in the flow direction just before stopping the water supply, using a point gauge. Bed elevations were measured at the same intervals, and in addition at the troughs and crests along 3 lines (2 cm from each side wall and the center of the channel) after stopping the water supply. Temporal and spatial variations in the water surface and bed configurations were measured, and the mean flow depth were obtained. Wave lengths and wave heights were measured by the trough-to-trough method (12). The bed-load transport rate was specified by catching the sediment during a passing time of one-wave length at the downstream edge of the channel. The mean migration velocity of the sand waves was calculated from the time-location relation of their crests. In addition, the bed material was sampled from both the bed surface layer and the subsurface layers of one sand wave. A discussion of sediment sorting with sand wave development is made on the basis of the grain size distributions in these layers.

DEVELOPMENT OF SAND WAVES WITH SEDIMENT SORTING

Shape and Migration Properties of Sand Waves

Temporal variations in the wave lengths of sand waves for the uniform and graded sediments are shown in Fig.2. The experiments were conducted under the same initial hydraulic conditions (Run U-1* and Run M-3*). The wave length for graded sediment is longer than

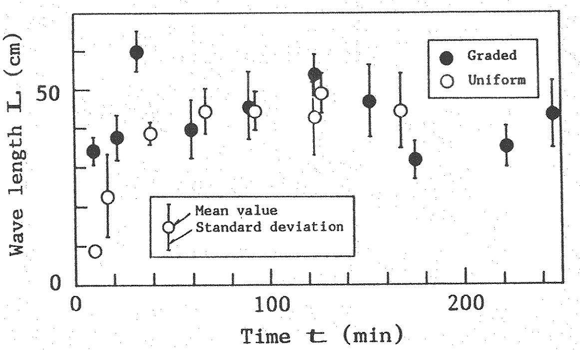

Fig.2 Temporal variation in wave length.

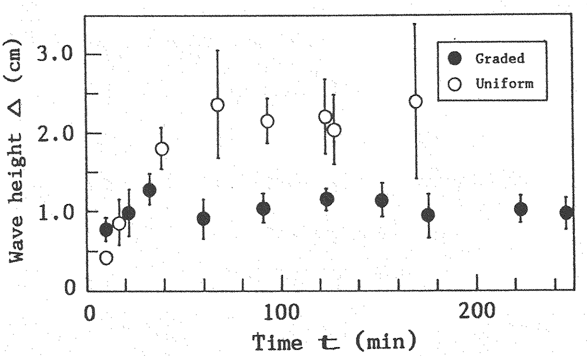

Fig.3 Temporal variation in wave height.

that for uniform sediment in the initial formation stage of sand waves; but, such a difference tends to be small in the fully developed stage. Wave lengths of fully developed sand waves were four to six times the mean flow depth in all experiments in this study. Temporal variations in the wave heights of sand waves for both cases are shown in Fig.3. The wave height for the graded sediment reached an equilibrium in a short time, about 30 minutes in this case, after the experiment began. The wave heights of the fully developed sand waves for the graded sediment were approximately 50% less than those for the uniform sediment.

The relation between the migration velocity, V_B, of the fully developed sand waves and the dimensionless shear stress, τ_{*m}, for the graded and uniform sediments is shown in Fig.4. The migration velocity is normalized by the mean shear velocity of each experiment, and the dimensionless shear stress is defined by the mean diameter of the sediment used. The migration velocities of the sand waves of graded sediment are larger than those of the uniform sediment at the same dimensionless shear stress.

These results show that grain size composition, such as the graded and uniform sediment, affects the development of sand waves.

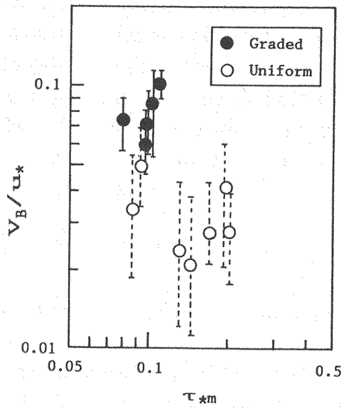

Fig.4 Migration velocity of sand waves versus dimensionless shear stress.

Sediment Sorting in Bed with Sand Waves

Grain size distributions of the surface layer at several locations in a single sand wave are shown in Fig.5. The sampling sites are indicated in Fig.6. The thickness of the

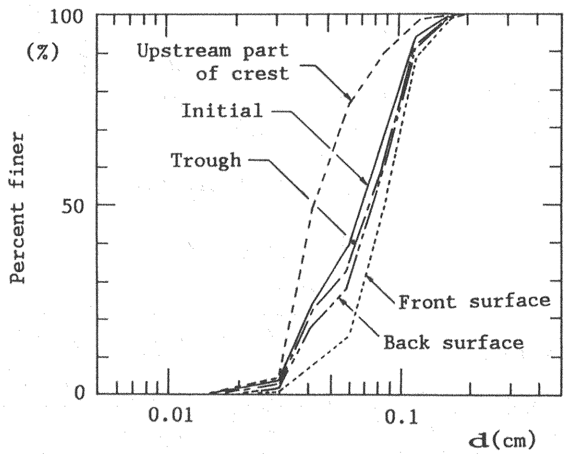

Fig.5 Grain size distribution in the bed surface of a fully developed sand wave.

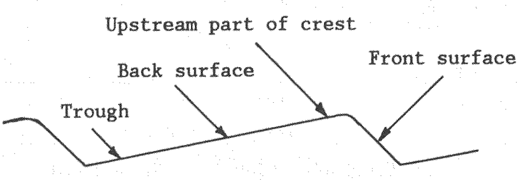

Fig.6 Sampling sites.

sampled layer was as deep as the maximum grain size of the sediment used. The results shown in Fig.5 indicate that in comparison with the initial sediment size, the sediment of the front surface is coarser, the sediment of the upstream part of the crest is finer, and the sediment of the back surface and trough is somewhat coarser. These results confirm that longitudinal alternation of sediment sorting takes place on the bed with sand waves. This variability in sediment sorting is caused by the cyclic distribution of bed shear stress along the bed surface and the difference in the mobility of the grains. The bed shear stress on one sand wave is smallest at the reattachment point of flow and largest at the crest. Mobility of each particle depends greatly on particle size and tractive force. Just behind the reattachment point, coarse grains are transported less actively than fine grains; whereas, these are transported as actively as fine grains from the area of the back surface to the crest. The fine grains then are deposited where coarse grains have been picked up, but not vice versa. As a result, the degree of exposure to the flow of the coarse grains is greater than that of fine grains. The coarse grains located in the high velocity range of flow always move faster than the fine grains, avalanching down the front surface of the sand wave. The sediment of the front surface, therefore, tends to become coarse, and that of the upstream part of the crest to become fine. This phenomenon propagates downstream with the migration of the sand waves. The migration velocity of the sand waves, therefore, corresponds to the propagation velocity of the longitudinal alternation of sediment sorting.

The development and migration of sand waves are produced by the erosion of sediment from their upstream surfaces and its deposition on their front surfaces. The sediment deposited on the front surfaces is buried into subsurface layers as sand waves develop and migrate. Sediment sorting, thus, takes place even in subsurface layers.

Grain size distributions in subsurface layers under the upstream part of the crest are shown in Fig.7. These layers are classified into I, II, and III on the basis of the differences in grain

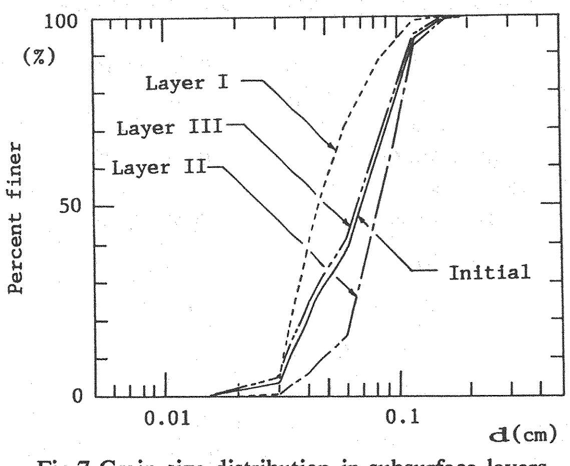

Fig.7 Grain size distribution in subsurface layers.

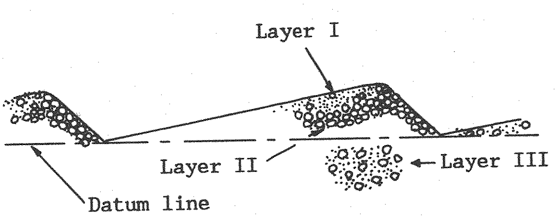

Fig.8 Layers sampled for subsurface sediment.

size (see Fig.8). The grains of layer I are fine; those of layer II are coarse due to deposition of coarse grains on the front surface of the sand waves; and those of layer III are similar to the initial material. Ikeda and Iseya (4) reported similar phenomena occurred when the sand fraction exceeded the gravel fraction in experiments where sand and gravel mixtures were used.

Temporal variations in the thickness of each layer are shown in Fig.9. The vertical axis denotes the dimensionless distance in terms of wave height from the datum line chosen at the level of trough of sand waves, Δ' denotes the wave height at the time when the thickness was measured. This figure shows that the thicknesses of layers I and II vary during the formation stage of sand waves but are constant in the fully developed stage. Moreover, layer III is always under the datum line. The results in Fig.9 indicate that the exchange of grains due to the development and migration of sand waves occurs only above the datum line. Therefore, the thickness of the exchange layer of grains in beds with sand waves is equivalent to the wave height.

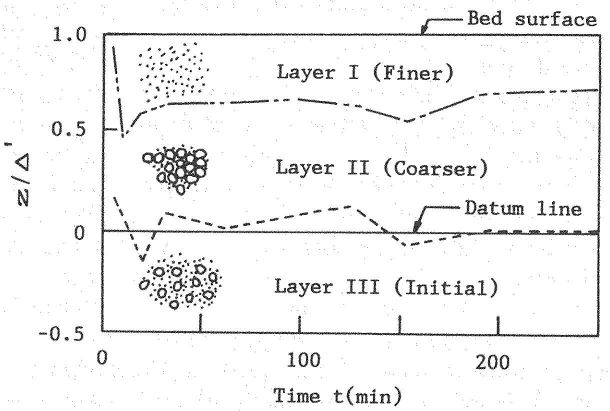

Fig.9 Temporal variation in thickness of each layer.

The profile of shear stress distribution along the bed surface varies as sand waves develop, and does not vary after the sand waves reach equilibrium. Therefore, the degree of sediment sorting, such as coarsening and fining, progresses during the formation of sand waves, after which the variation in sorting becomes stable in the fully developed stage. This indicates that the progressive coarsening of the front surfaces of sand waves is closely related to the development of sand waves. Taking into account the variation in wave height in the development of sand waves, the relation between sediment sorting and the development of sand waves can be found from the temporal variations in the grain size distribution of the front surface and wave height. Figure 10 shows temporal variations in the grain size distribution of layer II. The sediment in this layer corresponds to that in the front surface. The Coarsening process

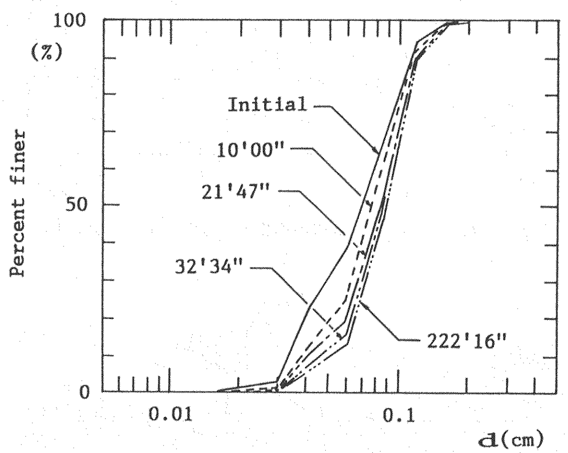

Fig.10 Temporal variation in the grain size distribution of layer II.

reaches equilibrium roughly within 30 minutes, after which the grain size distribution hardly varies. The wave heights reach equilibrium at the same time (see Fig.3). These results show that wave height increases during the progressive coarsening of the front surfaces of sand waves and the development of wave height stops when coarsening reaches equilibrium.

REFERENCE GRAIN SIZE FOR DETERMINING WAVE HEIGHT

As shown in the previous section, coarse grains of the non–uniform sediment play an important role in the development of sand waves. The coarse grain is regarded as a grain roughness of upstream surface of sand waves because these are transported over finer grains by the sorting effect. Thus, taking into account wave height as geometric scale of sand waves, the coarse grain size is defined as a reference grain size that determines the condition for the wave heights of fully developed sand waves This coarse grain size is regarded as the reference grain size for determining wave height.

Ishikawa (3) suggested that this condition is given by the dimensionless shear stress at the tops of fully developed sand waves, and proposed the following criterion based on the study of Garde and Raju (1):

$$\tau_{*r} = 0.0147 \left(\frac{h_m - \Delta/2}{d_m} \right)^{0.532} \quad (1)$$

in which τ_{*r} = dimensionless shear stress at the tops of fully developed sand waves.

The distribution of the bed shear stress along the upstream surface of a sand wave can be calculated from

$$\frac{\tau(x)}{\rho u_m^2} = \Pi \left(\frac{k_s}{h_m} \right) \left(\frac{x}{k_s} \right)^{\frac{3}{4}} \quad (2)$$

in which $\Pi = (B/A)^2 C^{1/4}$, A=8.94, $B=\{2/(2-L/h_m \cdot \lambda)\}\{L/h_m \cdot (1-5.1\lambda)\}^{-1/2}$, and $C=A^2\{0.26+\lambda/(F_r \cdot B)^2\}$. Herein, ρ = mass density of water; u_m = mean flow velocity; k_s = equivalent roughness ($k_s = 2d_m$ is adopted here); λ = wave steepness; F_r = Froude number; and x = distance from the reattachment point of flow. The details of derivation of these formulas are shown in Appendix.

Introduction of the reference grain size into the equivalent roughness yields the following equation from Eq.2:

$$\tau_{*r} = \Pi' \left(\frac{d_r}{d_m} \right)^{\frac{1}{4}} F_{*m} \quad (3)$$

in which $\Pi' = (B'/A)^2 C^{1/4}$, $B' = \{2/(2-L/h_m \cdot \lambda)\}\{L/2d_m \cdot (1-5.1\lambda)\}^{-1/8}$, and $F_{*m} = u_m / \sqrt{(\sigma/\rho - 1) g d_m}$. Herein, d_r = reference grain size for the determination of the wave height; σ = mass density of sand; and g = gravitational acceleration.

The reference grain size is obtained from Eqs.1 and 3. The relation between the reference grain size, d_r, and the dimensionless shear stress, τ_{*m}, is shown in Fig.11. The reference grain size is given as the ratio to the mean diameter of sediment used. The reference grain size for the uniform sediment is almost constant and equal to the mean diameter, d_m; whereas, that for the graded sediment is approximately 2 times larger than d_m.

Geometric scale of sand waves depends on the ratio of the flow depth to the grain size as well as the tractive force (Yalin & Karahan (13)). Thus, the difference in wave height between the graded and uniform sediments may be due to the difference in their reference grain size. Figure 12 shows the variation of wave steepness with the relative tractive force, τ_0/τ_c, for the relative depth, h_m/d_m. Since the wave length of fully developed sand waves is nearly same for the uniform and graded sediments, deference in wave steepness for both sediments corresponds to it in wave height. In Fig.12, the solid lines show the following equation proposed by Yalin and Karahan (13);

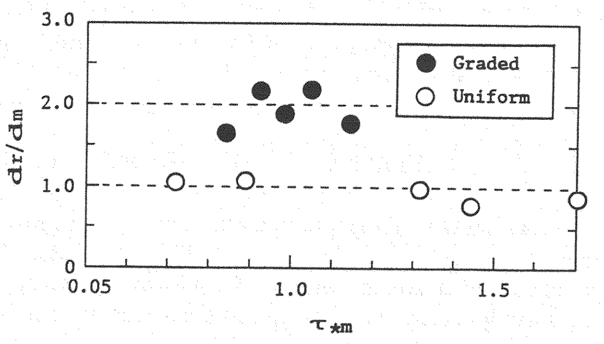

Fig.11 Reference grain size for determination of wave height.

$$\lambda = \Delta/L = 0.0127\,(\eta-1)\,\exp\left(\frac{1-\eta}{\eta_m-1}\right) \tag{4}$$

in which $\eta = \tau_0/\tau_c$. Herein, τ_0 = total shear stress; τ_c = critical shear stress; and η_m = relative tractive force which gives the maximum value of wave steepness for each relative depth. Eq.4 indicates that wave steepness decreases with decrease of the relative depth in case of the same relative tractive force.

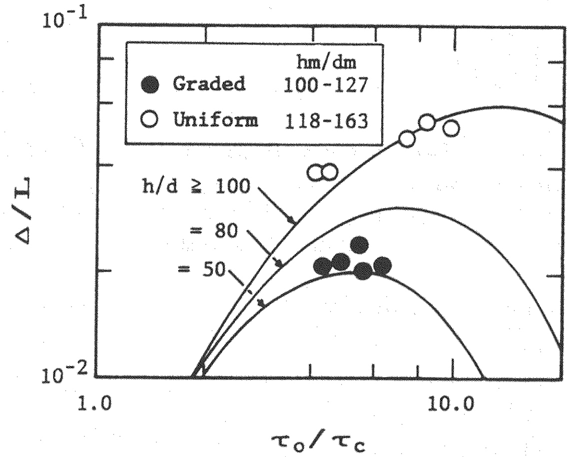

Fig.12 Variation of wave steepness with relative tractive force for relative depth.

The experimental results for the uniform sediment are suitably expressed by Eq.4. While, those for the graded sediment are smaller than those for uniform sediment in spite that h_m/d_m >100, but the experimental data for graded sediment are plotted around the line of h_m/d_m =50. Such a result indicates that the reference grain size must be used instead of the mean diameter for graded sediment. Then, the relative depth, h_m/d_r, for the reference grain size is given by about a half of that, h_m/d_m, for the mean diameter for graded sediment. Since coarse grains affect the development of sand waves in beds composed of graded sediment, wave heights of sand waves are determined by the coarse grains, which are defined as the reference grain here. In other words, the wave heights of sand waves for graded sediment are smaller than those for uniform sediment under the condition of the same mean diameter and the same tractive force.

CONCLUSIONS

The results of this study are as follows:

(1) The wave heights of sand waves in beds composed of graded sediment are smaller than those in beds composed of uniform sediment, and the migration velocities of sand waves in the former are greater than those in the latter under the same initial hydraulic conditions.

(2) Longitudinal alternation of sediment sorting takes place in stream beds with sand waves. This causes the sediment of the front surfaces of the sand waves to become coarse and the sediment of the bed surfaces of the upstream part of the crests to become fine. Such a phenomenon propagates downstream as the sand waves migrate.

(3) Sand waves develop with sediment sorting. The moving coarse grains have a larger exposure to the flow and velocity than the fine grains because of sediment sorting on the upstream surfaces of sand waves. This causes coarsening of their front surfaces, and development of the sand waves stops when coarsening process reaches equilibrium.

(4) Coarse grains affect the development of sand waves in beds composed of graded sediment, and wave heights are determined by the coarse grain which is regarded as the reference grain. The coarse grain size is approximately 2 times the mean diameter of the sediment mixture used.

(5) The reference grain size which determines the wave heights of sand waves is different between the uniform and graded sediments, that is, the reference size is the mean diameter for uniform sediment, but about 2 times the mean diameter for graded sediment. This causes that wave height for graded sediment is smaller than that for uniform sediment under the same mean diameter and the same flow condition.

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REFERENCES

1. Garde, R.J. and K.G.R. Raju : Resistance relationship for alluvial channel flow, Journal of the Hydraulics Division, ASCE, Vol.92, No.HY4, pp.77–100, 1966.
2. Guy, H.P., D.B. Simons and E.V. Richardson : Summary of alluvial channel data from flume experiments, 1956–61, Geological Survey Professional Paper, 462–J, 1966.
3. Ishikawa, T. : Fundamental study on sediment transport in alluvial river, Technical Report,Department of Civil Engineering, Tokyo Institute of Technology, No.25, Chapter 5, 1979 (in Japanese).
4. Ikeda, H. and F. Iseya : Experimental study of heterogeneous sediment transport, Environmental Research Center Papers, The University of Tsukuba, No.12, Chapter VI, 1988.
5. Kuhnle, R.A. and J.B. Southard : Bed load fluctuations in a gravel bed laboratory channel, Water Resources Research, Vol.24, No.2, pp.247–260, 1988.
6. Nakagawa, H., T. Tsujimoto, S. Murakami and Y. Mizuhashi : Spatial distribution of bed load transport rate over two–dimensional dunes, Proc. 28th Japanese Conference on Hydraulics, JSCE, pp.735–741, 1984 (in Japanese).

7. Miwa, H. and A. Daido : Development of sand waves and sorting mechanisms of sand mixtures in alluvial channel, Memoirs of Maizuru College of Technology, No.26, pp.69–81, 1991 (in Japanese).
8. Miwa, H. and A. Daido : Flow resistance and bed load transport rate for sand waves, Proc. 36th Japanese Conference on Hydraulics, JSCE, pp.123–130, 1992 (in Japanese).
9. Miwa, H. and A. Daido : Study on flow over sand waves and flow resistance, Memoirs of Maizuru College of Technology, No.28, in press (in Japanese).
10. Snishchenko, B.F., A.M. Muhamedov and T.Sh. Mazhidov : Bed load composition effect on dune shape parameters and on flow characteristics, Proc. XXIII Congress of IAHR, Ottawa, Canada, pp.B105–B112, 1989.
11. Suzuki, K. and M. Michiue : On the dune height and the influence of sand mixture on its characteristics, Proc. 23rd Japanese Conference on Hydraulics, JSCE, pp.151–156, 1979 (in Japanese).
12. For example, Suzuki, K. and M. Michiue : Study on the influence of channel width on the bed form, Proc. 25th Japanese Conference on Hydraulics, JSCE, pp.35–40, 1981 (in Japanese).
13. Yalin, M.S. and Karahan, E. : Steepness of sedimentary Dunes, Journal of the Hydraulic division, ASCE, Vol.105, No.HY4, pp.381–392, 1979.
14. Yamamoto, K. : Research note on mechanisms of sediment transport in an alluvial river, Data of Public Work Research Institute, Ministry of Construction, No.1416, Chapter I, 1978 (in Japanese).

APPENDIX – LONGITUDINAL DISTRIBUTION OF BED SHEAR STRESS

Eq.2 is derived as follows (8,9): On the upstream surface of a sand wave, a boundary layer develops along the bed surface from the reattachment point of flow (Fig.13). The bed shear stress is calculated from the momentum equation for the boundary layer (6);

$$\frac{\partial (U\delta^*)}{\partial t} + U^2\frac{\partial \theta}{\partial X} + (2\theta+\delta^*)U\frac{\partial U}{\partial x} - \frac{\delta(x)}{\rho}\frac{\partial p}{\partial x} = \frac{\tau_0(x,t)}{\rho} \qquad (5)$$

in which U = velocity outside the boundary layer; δ^* = displacement thickness; θ = momentum thickness; δ = boundary-layer thickness; and p = hydrostatic pressure.

By assuming a steady flow, neglecting variation in the flow surface, and considering spatial variation in the flow depth due to the shape of the sand wave, Eq.5 is rewritten as follows:

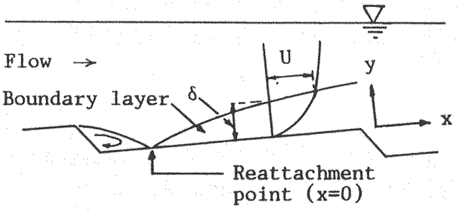

Fig.13 Coordinate definition.

$$\frac{\tau_0(x)}{\rho} = \frac{\partial}{\partial x}(U^2\theta) + U\delta^*\frac{\partial U}{\partial x} + g\,\delta(x)\,\lambda \qquad (6)$$

in which λ = wave steepness.

The velocity distribution in the boundary layer can be expressed in form of a power law:

$$\frac{u}{U} = \left(\frac{y}{\delta(x)}\right)^{\frac{1}{6}} \qquad (7)$$

The following equation proposed by Nakagawa et al. (6) is used to obtain the velocity outside the boundary layer;

$$\frac{U}{u_m} = B\left(\frac{x}{h_m}\right)^{\frac{1}{2}} \tag{8}$$

in which $B=\{2/(2-L/h_m\cdot\lambda)\}\{L/h_m\cdot(1-m\cdot\lambda)\}^{-1/2}$, $m=L_s/\Delta$. Herein, L_s = separation length of flow; Δ = wave height; and m = parameter for the mean separation length, m=5.1 having been obtained by Miwa et al. (7).

The resistance law is written in terms of the Manning–Strickler formula;

$$\frac{U}{u_*} = A\left(\frac{\delta(x)}{k_s}\right)^{\frac{1}{6}} \tag{9}$$

in which A=8.94; and k_s = equivalent roughness.

Substituting Eqs.7–9 into Eq.6, the bed shear stress distribution is

$$\frac{\tau(x)}{\rho u_m^2} = \frac{u_*(x)^2}{u_m^2} = \Pi\left(\frac{k_s}{h_m}\right)\left(\frac{x}{k_s}\right)^{\frac{3}{4}} \tag{10}$$

in which $\Pi=(B/A)^2C^{1/4}$, $C=A^2\{0.26+\lambda/(F_r\cdot B)^2\}$.

The bed shear velocity distribution calculated from Eq.10 is given in Fig.14, in which the shear velocity is normalized by the shear velocity at the crest, $\hat{u}_*$. Therein, the experimental data were obtained by assuming the log–law to the velocity distributions which were measured with the propeller anemometer on the sand wave model.

Figure 14 indicates that Eq.10 gives a good estimation for the shear stress distribution on sand waves.

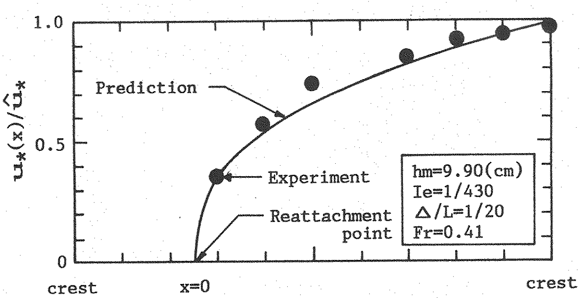

Fig.14 Shear stress distribution on a sand wave.

APPENDIX–NOTATION

The following symbols are used in this paper:

- d = grain diameter;
- d_r = reference grain size for determination of wave height;
- d_m = mean sediment diameter;
- F_r = Froude number;
- g = gravitational acceleration;
- h = flow depth;
- h_m = mean flow depth;
- I_e = energy slope;
- k_s = equivalent roughness;

L = wave length;
L_s = separation length;
p = hydrostatic pressure;
q_w = water discharge per unit width of channel;
u = velocity in the boundary layer;
u_m = mean flow velocity;
u_* = shear velocity;
$\hat{u}_*$ = shear velocity at crest;
U = velocity outside the boundary layer;
δ = boundary–layer thickness;
δ^* = displacement thickness;
Δ = wave height;
Δ' = wave height during development of sand waves;
η = relative tractive force $(=\tau_0/\tau_c)$;
η_m = relative tractive force to maximum value of wave steepness;
θ = momentum thickness;
λ = wave steepness;
ρ = mass density of water;
σ = mass density of sand;
τ = bed shear stress;
τ_0 = total shear stress $(=\rho g h_m I_e)$;
τ_c = critical shear stress;
τ_{*m} = dimensionless shear stress for mean diameter; and
τ_{*r} = dimensionless shear stress at crest for fully developed sand waves.

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