Journal of Hydroscience and Hydraulic Engineering
Vol. 10, No. 2, November, 1992, 27-37

LONGITUDINAL DISPERSION OF BUOYANT MATTER IN TURBULENT OPEN CHANNEL FLOW

By

Kazuhiro FUJISAKI, Noriyuki YOSHITAKE, Hideki HAYASHI

Department of Civil Engineering, Kyushu Institute of Technology
Kitakyushu, Fukuoka, Japan

and

Yoichi AWAYA

Department of Water and Soil Development,
Kyushu Kyoritsu University, Kitakyushu, Fukuoka, Japan

SYNOPSIS

The effect of density gradient on longitudinal dispersion is studied in a two-dimensional turbulent open channel flow. Using the k-ε turbulence model, the longitudinal dispersion coefficient is evaluated as a function of the non-dimensional density gradient parameter σ . It is confirmed that the dispersion coefficient D_L increases with σ when $\sigma > 0$, and D_L is weakly sensitive to the value of σ in the range of $-0.2 < \sigma < 0$. The relationship between D_L and σ is almost same as the author's previous results in which a mixing length model and the Monin-Obukhov theory have been employed. The damping trend of eddy viscosity with Richardson number calculated by the k-ε model is found to be similar to those reported so far. The computations indicate reasonable agreement with the experiments.

INTRODUCTION

This paper deals with the effect of density gradient on longitudinal dispersion in a turbulent open channel flow. When solute is injected into turbulent flow, it spreads under the combined action of longitudinal convection and the turbulent mixing. In case of buoyant solute, the vertical distribution of the solute causes vertical density gradient, so that, the flow field becomes stable. On the other hand, if the solute is heavier than the ambient fluid, the flow field becomes unstable. In both cases, the longitudinal density gradient affects the velocity profile through the static pressure difference.

The turbulent mixing of different density fluid is often observed such as the flow of fresh water from rivers into estuaries and the discharge of heated water from industrial plants into rivers. Therefore there are many studies on the mixing of different density fluid. There, however, seems not so much work which treats the effect of density gradient on the dispersion phenomenon.

Empirical models representing the dispersion dependency on density gradient have been presented by several researchers (Holley et al(18), Harleman and Thatcher(17) and Brocard and Harleman(3)). The dispersion of buoyant matter was discussed in laminer flow through a tube(8) and in estuaries(6) by Erdogan and Chatwin. To take account of the effect of density gradient, they introduced additive terms to the dispersion coefficient derived at first by Taylor(31). Smith(29,30) showed that the secondary flow reduces the value of longitudinal dispersion coefficient in a shallow channel.

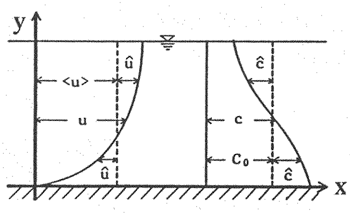

Fig.1 Coordinate system

Using the mixing length model and the Monin-Obukhov similarity theory, Fujisaki et al.(9,10) studied the effect of density on longitudinal dispersion as an extension of Elder's work(7). Furumoto et al(12) also studied the similar problem by employing the k-l turbulence model.

In this paper, we discuss the dispersion phenomena by the k-ε turbulence model, and propose the longitudinal dispersion coefficient as a function of a nondimensional density gradient parameter.

BASIC EQUATIONS AND NUMERICAL COMPUTATION

Characteristics of flow with density gradient

The flow field is assumed to be two-dimensional. Turbulent mixing is represented by eddy diffusivity. Furthermore, it is assumed that the Boussinesq approximation holds. Taking x axis in flow direction and y axis vertical(Fig.1), the equations of motion are

$$0 = \rho g i - \frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\rho \nu_t \frac{\partial u}{\partial y}\right) \tag{1}$$

$$0 = -\rho g - \frac{\partial p}{\partial y} \tag{2}$$

where u= flow velocity in the x direction, ν_t = eddy viscosity, ρ = density of fluid, p= pressure, i= the slope of the channel bottom and g=acceleration due to gravity The concentration of solute is assumed so dilute that the excess density of fluid is proportional to its concentration c, thus

$$\rho = (1-c)\rho_0 + c\rho_c = \rho_0 + c\,(\rho_c - \rho_0) \tag{3}$$

where ρ_0 and ρ_c = the density of fluid and matter respectively, and the subscript 0 refers to values of the case c=0.

Substituting Eq.3 to Eq.2 and integrating, we have vertical distribution of static pressure p. So that Eq.1 can be rewritten

$$0 = \rho g i - \rho g \frac{dh}{dx} - (\rho_c - \rho_0) g \int_y^h \frac{\partial c}{\partial x} dy + \frac{\partial}{\partial y}\left(\rho \nu_t \frac{\partial u}{\partial y}\right) \tag{4}$$

In the case that the dispersion theory can be applied, the distribution of concentration must be fully developed. Therefore the concentration can be expressed by

$$c = c_0\left(1 + \left[\frac{\partial c}{\partial x}\right] x_1\right) + \hat{c}(y) \tag{5}$$

$$x_1 = x - \langle u \rangle t \tag{6}$$

in which t= time, c_0= the reference concentration, $\left[\frac{\partial c}{\partial x}\right]$= a the longitudinal gradient of c, and $\langle\ \rangle$ means cross-sectional mean over flow depth. The assumption given by Eq.5 is generally employed in discussing the longitudinal dispersion(31,7). Substituting Eq.5 in Eq.4, we obtain the equation of motion in a nondimensional form

$$0 = 1 - \frac{1}{i}\frac{d\bar{h}}{d\bar{x}} - 2\sigma(1-\bar{y}) + \frac{\partial}{\partial \bar{y}}\left(\bar{\nu}_t \frac{\partial \bar{u}}{\partial \bar{y}}\right) \tag{7}$$

in which

$$\sigma = \frac{1}{2}\ \frac{\rho_c - \rho_o}{\rho}\ c_o \cdot \frac{gh_o}{u_*^2}\left[\frac{\partial c}{\partial x}\right] h \qquad (8)$$

$$\left.\begin{array}{l} \bar{h} = h / h_o \quad , \quad \bar{x} = x / h_o \quad , \quad \bar{y} = y / h_o \\ \bar{\nu_t} = \nu_t / (hu_*) \quad , \quad \bar{u} = u / u_* \end{array}\right\} \qquad (9)$$

$$u_*^2 = gh_o i \qquad (10)$$

where h_o and h= flow depth in case of $\sigma=0$ and $\sigma \neq 0$ respectively, u_* = shear velocity. The parameter σ means the ratio of excesses hydrostatic pressure gradient to the bottom shear of the bulk flow.

The purpose of this study is to investigate the value of longitudinal dispersion coefficient D_L in turbulent open channel flow with longitudinal density gradient. In the following, we will discuss the effect of σ on the longitudinal dispersion.

Another basic equation is the equation of conservation of solute, given by

$$\bar{u}\frac{\partial c}{\partial \bar{x}} = \frac{\partial}{\partial \bar{y}}\left(\frac{\bar{\nu_t}}{\sigma_t}\ \frac{\partial c}{\partial \bar{y}}\right) \qquad (11)$$

where σ_t =turbulent Schmidt number 1.0 in the present work.

To the turbulent diffusivity, we employ the standard k-ε model. In the model, the turbulent energy k and the viscous dissipation rate of turbulence ε, are controled by the following transport equations

$$\bar{\nu_t}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 + \frac{\partial}{\partial \bar{y}}\left(\frac{\bar{\nu_t}}{\sigma_k}\ \frac{\partial \bar{k}}{\partial \bar{y}}\right) + 2\sigma\frac{\bar{\nu_t}}{\sigma_t}\ \frac{\partial c}{\partial \bar{y}} - \bar{\varepsilon} = 0 \qquad (12)$$

$$C_{1\varepsilon}\frac{\bar{\varepsilon}}{\bar{k}}\left\{\bar{\nu_t}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 + 2\sigma(1 - C_{3\varepsilon})\frac{\bar{\nu_t}}{\sigma_t}\ \frac{\partial \hat{c}}{\partial \bar{y}} + \frac{\partial}{\partial \bar{y}}\left(\frac{\bar{\nu_t}}{\sigma_\varepsilon}\ \frac{\partial \bar{\varepsilon}}{\partial \bar{y}}\right) - C_{2\varepsilon}\frac{\bar{\varepsilon}^2}{\bar{k}}\right\} = 0 \qquad (13)$$

in which

$$\bar{k} = k / u_*^2 \quad , \quad \bar{\varepsilon} = \varepsilon h_o / u_*^3 \qquad (14)$$

and the eddy viscosity is represented by

$$\bar{\nu_t} = C_\mu \bar{k}^2 / \bar{\varepsilon} \qquad (15)$$

where C_μ is an empirical constant.

To the boundary condition at the channel bottom, we assume the logarithmic flow velocity distribution and the local equilibrium of turbulence at near bottom point. Thus we have the following normally used wall boundary conditions

$$\bar{u} = \frac{1}{\kappa}\ln\frac{u_* y}{\nu} + E \qquad (16) \qquad\qquad \bar{k} = 1 / \sqrt{c_\mu} \qquad (17)$$

$$\bar{\varepsilon} = 1 / \kappa\bar{y} \qquad (18) \qquad\qquad \bar{\nu_t} / \sigma_t \cdot \partial c / \partial \bar{y} = 0 \qquad (19)$$

where y= Δy is the height of the first grid cell from channel bottom, κ is the Karman constant, 0.4 and E is a numerical constant, 5.5.

A condition of symmetry is adapted as boundary condition at the water surface for u,c and ε, thus

$$\partial \bar{u} / \partial \bar{y} = 0 \quad , \quad \partial \bar{c} / \partial \bar{y} = 0 \quad , \quad \partial \bar{\varepsilon} / \partial \bar{y} = 0 \qquad (20)$$

It is not appropriate to use the symmetry condition in k equation at water surface, because the turbulence damps vertically at the water surface(4,15). In the present study, a simple method proposed by Nezu and Nakagawa(24) is used. The outline of the method is as follows. At first we seek the solution with the symmetry condition(Eq.21.1), then the new boundary condition is given by Eq.21.2,

$$\partial \bar{k} / \partial \bar{y} = 0 \qquad (21.1), \qquad \bar{k} = \alpha \overline{k_s} \qquad (21.2)$$

where $\overline{k_s}$ is the value of $\bar{k}$ at the water surface obtained with Eq 21.1, and α is a numerical constant. Using this boundary condition(Eq.21.2), the total flow field is computed again. The value of α is taken 0.8, because it was confirmed that this gives a good agreement with the measurement in turbulent open channel uniform flow(24).

In the computation, the value of u_* is assumed to be constant independent of σ. The value of dh/dx in Eq.1 is determined by an iteration to satisfy this condition. Numerical solutions are obtained for $Re=hU_m/\nu$ =10000, where U_m is the mean velocity. The cartesian, non-eqidistance mesh is employed in the calculations. The flow depth is divided by 100 node in the lower part($0<y<0.05$) and 200 nodes in the upper part($0.05=<y<1.0$). For the computation of the k-ε model so-called TEACH(16) code is referred. The values of numerical constant in the k-ε model are given in Table 1 and these are widely used values except $C_{3\varepsilon}$ =2.5. The effect of the values of $C_{3\varepsilon}$ will be discussed later.

σ_k	σ_t	σ_ε	$C_{1\varepsilon}$	$C_{2\varepsilon}$	$C_{3\varepsilon}$	C_μ
1.0	1.0	1.3	1.44	1.92	2.5	0.09

Table 1 Values of the constants in the k-ε model

Dispersion coefficient

Substituting Eq.5, in Eq.11, we have

$$\frac{\hat{c}}{c_0} = \int_0^{\bar{y}} \frac{\sigma_t}{\bar{\nu}_t} \int_0^{\bar{y}} \hat{\bar{u}}\, d\bar{y}^2 + A \qquad (22)$$

$$\hat{\bar{u}} = \bar{u} - \langle \bar{u} \rangle \qquad (23)$$

where A is a numerical constant.

The dispersion coefficient D_L in this case is given by

$$\overline{D_L} = D_L / (h_0 u_*) = -\langle \hat{\bar{u}}\, \hat{c} \rangle \qquad (24)$$

EXPERIMENTS

Dispersion experiments were carried out in the 10m length, 0.4m width strait tilting flume as shown in Fig.2. A wire roughness is set at the bottom of the upper end, to get the logarithmic velocity profile. The injection point of matter is at 2m downstream from the upper end of the flume. Five probes of the conductivity meter are set as shown in Fig.2.

A flow velocity is varied 0.25 to 0.4m/s and water depth 2.5 to 4cm. Salt water is used as solute, and the variation of salt concentration along the flow direction are measured by using the conductivity meters.

Two types of injection are carried out ; (A) pulse response, in which the solute is hand injected instantaneously as a slug and (B) step response, in which the injection is continued for a certain time. This continuous injection is done

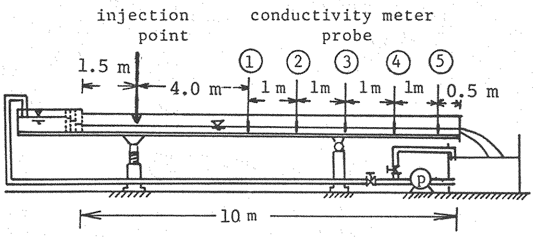

Fig.2 Laboratory flume

	type of injection	
	(A) pulse	(B) step
salt concentration (wt. %)	1 ∿ 30	0.5 ∿ 10
injected volume	20 cc	180 cc
injection time	——	12 sec.

Table 2 Condition of injection

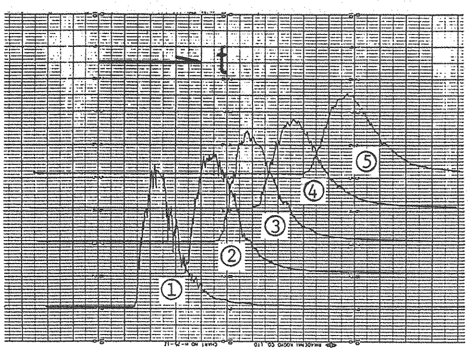

Fig.3 Example of recorded chart pulse response

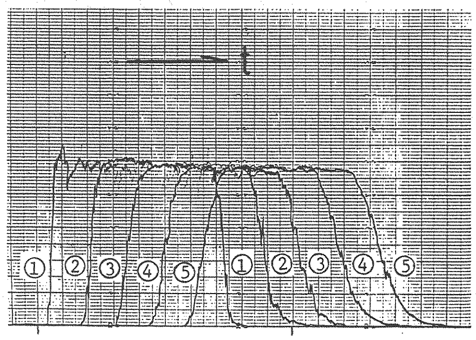

Fig.4 Example of recorded chart step response

by the discharge trough set laterally in the channel. In both cases, the solute is injected from the position 1 cm above the water surface. Another conditions for injection are given in Table 2.

Examples of output signal of conductivity meter are shown in Figs.3 and 4. To evaluate dispersion coefficients from these output data shown in Fig.3, we employed the method used by Taylor(31).

$$D_L = \frac{1}{2}\frac{ds^2}{dt} \tag{25}$$

where s is the standard deviation of measured concentration curve and t is time. In the forward part of longitudinal concentration distribution shown in Fig.3, the concentration decreases with distance x , thus $dc/dx<0$, so that $\sigma<0$ and in the backward part $\sigma>0$. In the present study, therefore, the value of s is obtained from each half part of the cloud of solute, since they are influenced by density gradient.

The step response type injection is done to get rather large value of σ. In case of step injection, the concentration distribution of solute can be divided into three parts as shown in Fig. 4. They are increasing region from C=0 to C=C0, constant concentration region C=C0 and decreasing region from C=C0 to C=0. These three part respectively correspond to $\sigma<0$, $\sigma=0$ and $\sigma>0$.

The longitudinal dispersion coefficient can also be obtained from the variation of the concentration distribution of both these two regions $\sigma>0$ and $\sigma<0$, because this phenomena corresponds to the integration of the pulse response with respect to time(5). The value of σ between two proves in the flow is approximated to be constant in all experiments, thus we can obtain the relation between D_L and σ, as shown in Fig.10.

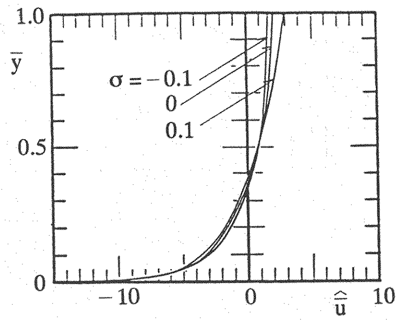

Fig.5 Effect of density gradient on flow velocity u

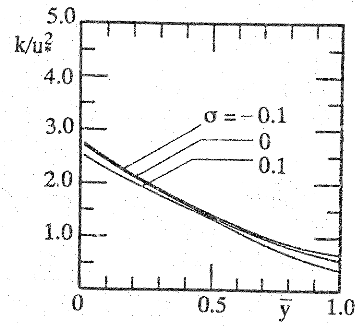

Fig.6 Effect of density gradient on turbulent energy

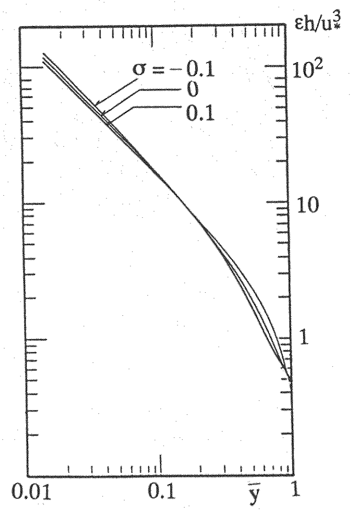

Fig.7 Effect of density gradient on turbulent dissipation

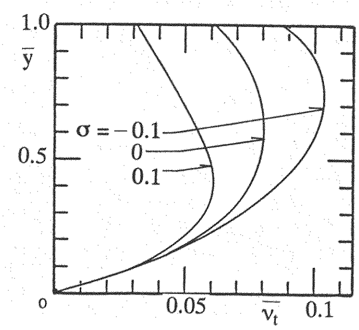

Fig.8 Effect of density gradient on eddy viscosity

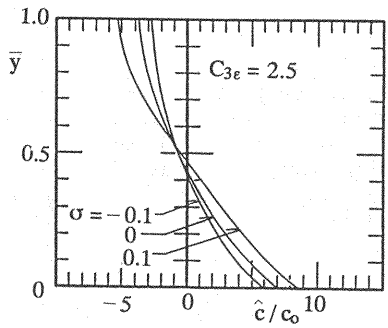

Fig.9 Effect of density gradient on additional concentration

RESULTS AND DISCUSSIONS

Effect of density gradient on flow field

Figures 5-9 show the variation of flow characteristics for several values of σ. As given in Eq.8, σ means the ratio of potential energy due to excess density to the bottom shear of the bulk flow and σ is also proportional to the longitudinal density gradient. Therefore the flow stability is determined by the sign of σ, that is $\sigma>0$; stable, $\sigma=0$; neutral, $\sigma<0$; unstable.

The velocity profiles are shown in Fig.5. In the case of $\sigma>0$, the flow velocity near the water surface becomes large due to depressed turbulent viscosity. Turbulent energy k shown in Fig.6 takes larger values with decrease in the value of σ . Figure 7 shows that the turbulent dissipation ε also increases with decrease in σ , in the lower part of the flow depth.

The values of turbulent viscosity calculated by Eq.15 is given in Fig.8. It should be mentioned that there is clear effect of σ on ν_t, while the effect of σ on k and ε seemed not so large. In addition, at the lower part of flow depth, ν_t shows apparent insensitivity to the value of σ . This may be caused by the boundary condition Eqs.16 - 19, that is the flow velocity profile is assumed to be logarithmic in the near bottom region.

The additive term of concentration of solute is shown in Fig.9, which is obtained from Eq.22 and the numerical constant A in Eq.22 is taken so as to $\langle c\rangle=0$.

Effect of longitudinal density gradient on dispersion coefficient

The longitudinal dispersion coefficient calculated by Eq.25 is shown in Fig.10. This figure shows that the value of D_L increases rapidly with increase of σ if $\sigma>0$. On the other hand, when $\sigma<0$, the effect of σ on D_L is not so sensitive. Open circles in Fig.10 indicate the experimental results. It can be concluded that the theoretical solution of the present work well predicts the experimental results.

In Fig.10, the results of previous study(10) are also compared, in which the Monin-Obukhov theory is employed in addition to the mixing length model. Using the perturbation method with respect to σ, the dispersion coefficient has been given by

$$D_L = 6.25 + 45.7\sigma + 196\sigma^2 \qquad (26)$$

Furumoto's numerical solution which is obtained by using the one-equation turbulence model(12) is also presented in Fig.10.

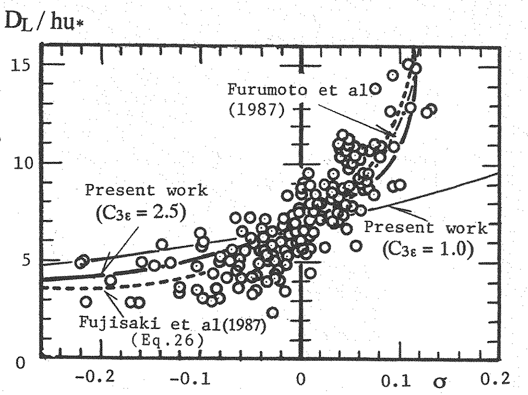

Fig.10 Effect of density gradient on longitudinal dispersion coefficient

Fig.10 shows that the three theoretical predictions takes close values with each other, although they are derived by making use of different turbulence models.

Numerical values of the parameter $C_{3\varepsilon}$

The k-ε model has nine numerical parameters to be given, which values are given in Table 1 and they are widely used values except $C_{3\varepsilon}$. There seems no definite value of $C_{3\varepsilon}$ so far. Plumb and Kennedy(25), To and Humphery(32) and Iwasa et al(20) pointed out that the buoyancy term in the ε-equation is quite sensitive for the thermal stratification of water. On the other hand, Gibson and Launder(14), Murota et al(22,23) and some others(11,28) report that the effect of buoyancy term in ε-equation on flow field is not so important, therefore they recommend to take $C_{3\varepsilon}=1$ in Eq.13.

To determine the numerical value of $C_{3\varepsilon}$, numerical solutions are obtained for various values of $C_{3\varepsilon}$ and compared with measurements. As the result, the value of 2.5 is used throughout this paper, because this value gives the best fit to experimental data. In addition, we couldn't predict well experimental results in case of $C_{3\varepsilon}=1$, as shown in Fig.10.

Effect of density gradient on turbulent viscosity

The relationships between turbulent viscosity ν_t and flux Richardson number R_f, defined by Eq.27, are shown in Fig.11.

$$R_f = -g\frac{\partial\rho}{\partial y}\Big/\left\{\rho\left(\frac{\partial u}{\partial y}\right)^2\right\} \qquad (27)$$

Open circles plotted in Figs.11 and 12 indicate numerical solutions of various flow condition.

To the relationship between ν_t and Rf, the following equation is generally used (21,25,27).

$$\nu_t = \nu_{t0}(1 + \beta R_f)^{\,n} \qquad (28)$$

the numerical values of n and β are taken within a range of $1 \leq n < 1.5$, $2.5 < \beta < 13$ (2,19). As can be seen in Fig.11, the following values of these parameter fit the plotted values well.

$$\begin{aligned} n &= -1 \quad \beta = -4.5 \quad (\sigma>0) \\ n &= -1 \quad \beta = -2.0 \quad (\sigma<0) \end{aligned} \qquad (29)$$

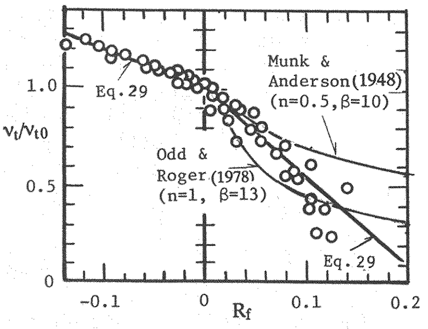

Fig.11 Variation of eddy viscosity with Richardson number

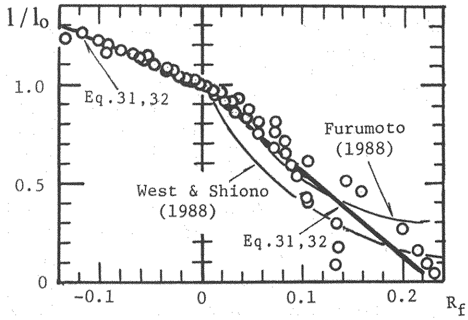

Fig.12 Variation of mixing length with Richardson number

In our previous studies(9,10), the values of n=-1 and β=-5 were obtained.

On the relationship between turbulent intensity and density gradient, the mixing length l is also examined. The mixing length is defined in Eq.30

$$\nu_t = l^2 \left|\frac{du}{dy}\right| \quad , \quad l = c_\mu \frac{k^{3/2}}{\varepsilon} \qquad (30)$$

The dependency of l on R_f is given in Fig.12, where l_o is the value of uniform fresh water. Using the mixing length model, the damping trend of eddy viscosity due to the density gradient has been reported(1,27,33,34). Some of them are also presented in Fig.11. The following relationship between l and Rf is often used.

$$l / l_o = (1 - \beta' R_f) \qquad (31)$$

In this study, the numerical values of parameter may be taken as

$$\beta' = 4.4 \quad (R_f>0) \quad , \quad \beta' = 2.3 \quad (R_f<0) \qquad (32)$$

Furumoto(13) also investigated the effect of R_f on l and obtained semi-empirical equation as shown in Fig.12. It can be concluded that the k-ε model and k-l model give similar numerical results to the effect of the Richardson number on the mixing lengh.

CONCLUDING REMARK

The paper has described the effect of density gradient on longitudinal dispersion in two-dimensional turbulent open channel flow. By employing the k-ε turbulence model, the longitudinal dispersion coefficient is given as a function of nondimensional density gradient parameter σ. The results obtained in this work gives very close value to our previous work in which a mixing length model and Monin-Obukhov theory has been employed.

Numerical value of parameter $C_{3\varepsilon}$ is taken as 2.5, since this value gives the best fit to the experimental results. The numerical solution of this work also predicts well the Richardson Number dependency of turbulent viscosity reported so far.

The theoretical solution of this work is compared with laboratory experimental data and found to be in reasonable agreement.

ACKNOWLEDGMENT

The authors wish to express their thanks to graduate student Hiroyuki Oki for his help in performing experiments and to Associate Professor Keiji Nakatsuji of Osaka University for his help in obtaining some published papers.

REFERENCES

1. ASCE Task Committee: Turbulence models in hydraulic computations, Journal of Hydraulic Division,ASCE, Vol.114, No.9,pp.970-1173,1988
2. Bloss,S., Lehfeld,R. and Pattersons,J.C: Modeling turbulent transport in stratified estuary, Journal of the Hydraulics Division,ASCE,Vol.114,No.HY9, pp.1115-1133,1988
3. Brocard,D.N and Harlerman,R.F:One-dimensional temperature predictions in unsteady flows, Journal of the Hydraulics Division,ASCE, Vol.102,No.Hy3,pp.227-240,1975
4. Celik,I. and Rodi,W: Simulation of free surface effects in turbulent channel flows, Physicochemical Hydrodynamics, Vol.5,No.3/4,pp.217-227,1984
5. Chatwin,P.C.:On the interpretation of some longitudinal dispersion experiments, Journal of Fluid Mechanics,Vol.48,part 4,pp.689-702,1971
6. Chatwin,P.C.:Some Remarks on the Maintenance of the Salinity Distribution in Estuaries,Estuarine and Coastal Marine Science, Vol.4, pp.555-566,1976
7. Elder,J.W: The dispersion of marked fluid in turbulent shear flow, Journal of Fluid Mechanics,Vol.5,part 4,pp.544-560,1959
8. Erdogan,M.E. and Chatwin,P.C: The effect of curvature and buoyancy on the laminer dispersion of solute in a horizontal tube, Journal of Fluid Mechanics, Vol.29, part 3, pp.465-484,1967
9. Fujisaki,K,Furumoto,K,Oura,Y and Awaya,Y: Effect of density on longitudinal dispersion of sediment particles, Proc. of the 29th Japanese Conference on Hydraulics, pp.443-448,1985 (In Japanese)
10. Fujisaki,K., Minami,Y. and Awaya,Y.:Effect of density on longitudinal dispersion, Proc.of IAWPRC specialized Conference on Coastal and Estuarine Pollution, Fukuoka, Japan, pp.237-244, 1987
11. Fukusima,Y:Numerical simulation of density underflow by the k-ε turbulence model, Journal of Hydroscience and Hydraulic Engineering, Vol.8,No.1,pp.31-40,1990
12. Furumoto,K,Tobita,H,Ichinose,K.and Tubaki,T: Experiment on stratified shearing flow with longitudinal density gradient,Proc. of the 31st Japanese Conference on Hydraulics, pp.461-466,1987 (In Japanese)
13. Furumoto, Studies on the density current in esturaries, Dissertation,1988(In Japanese)
14. Gibson,M.M. and Launder,B.E: On the calculation of horizontal, turbulent, free shear flows under gravitaional influence, Journal of Heat Transfer, Trans. ASME,Vol.98, pp.81-87,1976
15. Gibson,M.M. and Rodi,W: Simulation of free surface effects on turbulence with a Reynolds stress model,Journal of Hydraulic Research Vol.27,No.2, pp.233-244,1989
16. Gosmann,A.D.,Launder,B.E. and Reece,G.J.:Computer Aided Engineering,Heat Transfer and Fluid flow,John Wiley and Sons,1985
17. Harlemann,D.R.F and Thatcher M.L.: Longitudinal dispersion and unsteady salinity intrusion in estuaries, La Houille Blanche, Vol.29,No.1/2,pp.25-33,1974
18. Holley,A.M.,Harleman,R.F. and Fishcer,H.B.: Dispersion in homogeneous estuary flow, Journal of the Hydraulics Division,ASCE,Vol.96,No.HY8,pp.1691-1709,1970
19. Henderson-Sellers,B: A simple formula for vertical eddy diffusion of nonneutral stability, Journal of Geophysical Research,Vol.87,No.C8,pp.5860-5864,1982
20. Iwasa,Y,Hosoda,T and Ito,K: Numerical analysis of buoyant surface jets by means of turbulence model, Annuals, Disas.Prev.Res.Inst.,Kyoto Univ.,No30B-2,pp.1-13,1987 (In Japanese)
21. Munku,W.H. and Anderson,E.R.:Notes on a theory of the thermocline,Journal of Marine Research,Vol.7,pp.276-295,1948

22. Murota,A,Nakatsuji,K and Fujisaki,Y: Application of turbulence model to stratified shear flow,Proc. of the 33rd Japanese Conference on Hydraulics,pp.583-588,1989 (In Japanese)
23. Murota,A,Nakatsuji,K and Fujisaki,Y: Calculation of vertical mixing in two-dimensional turbulent buoyant surface jet with turbulence models,Proc. of the Japan Society of Civil Engineers,No.411,pp.35-44, 1989 (In Japanese)
24. Nezu,I and Nakagawa H: Numerical calculation of turbulent open-channel flows in consideration of free-surface effects,Annuals,Disas.Prev.Res.Inst.,Kyoto Univ.,No29B-2,pp.647-673,1986 (In Japanese)
25. Odd,N.V.M.and Roger J.G.:Vertical mixing in stratified tidal flows,Journal of the Hydraulics Division,ASCE,Vol.104,No.HY3,pp.337-351,1978
26. Plumb,O.A. and L.A.Kennedy:Application of the k-ε model to natural convection from a vertical isothermal surface, Journal of Heat Transfer,ASME, Vol.99,C1, pp.79-85,1977
27. Rodi,W: Turbulence models and their application in hydraulics, state of the art paper,IAHR,1984
28. Simonin,O.,Uittenbogaard,R.E.,Baron.F.and Viollet,P.L.:Possibilities and Limitations of the k-ε model to simulate turbulent fluxes of mass and momentum, measured in a steady, stratified mixing layer, Proceedings of the International Association of Hydraulic Research, pp.A55-A63, 1989
29. Smith,R: Longitudinal dispersion of a buoyant contaminant in a shallow channel ,Journal of Fluid Mechanics,Vol.87,part 4,pp.677-688,1976
30. Smith,R:Buoyancy effects upon lateral dispersion in open channel flow,Journal of Fluid Mechanics,Vol.90,part 4,pp.761-779,1979
31. Taylor,G.I: The dispersion of matter in turbulent flow through a pipe, Proceedings of the Royal Society, A,Vol.223,pp.466-488,1954
32. To,W.M. and Humphrey,J.A.C.: Numerical simulation of buoyant,turbulent flow-I:International Journal of Heat and Mass Transfer, Vol.29,pp.573-592,1986
33. West,J.R,Knight,D.W and Shiono,K: A note on the determination of vertical turbulent transport coefficients in a partially mixed estuary, Proceedings of Institution of Civil Engineers,Vol.79,Part 2, pp.235-246,1985
34. West,J.R. and Shiono,K.: Vertical turbulent mixing processes on ebb tides in partially mixed estuaries, Estuarine, Coastal and Shelf Science, Vol.26,pp.51-66,1988

APPENDIX-NOTATION

The following symbols are used in this paper

C = volume concentration of solute;

$\hat{C}$ = additive concentration given by Eq.22;

$C_{1\varepsilon}$ = coefficient in ε-equation(Eq.13);

$C_{2\varepsilon}$ = coefficient in ε-equation(Eq.13);

$C_{3\varepsilon}$ = coefficient in ε-equation(Eq.13);

C_{μ} = coefficient in Eq.15;

D_L = longitudinal dispersion coefficient;

g = acceleration due to gravity;

h = flow depth;

i = slope of the cannel bottom;

$\left[\frac{\partial c}{\partial x}\right]$ = longitudinal concentration gradient as defined by Eq.5;

k = kinetic energy of turbulence;

l = mixing length;

u = flow velocity in the x direction;

u_* = shear velocity;

R_f = Richardson number (Eq.27);

x = coordinate in the flow direction;

y = coordinate normal to the x direction;

ε = viscous dissipation rate of turbulence;

κ = von Karman's universal constant;

ν_t = eddy viscosity;

σ = non-dimensional density gradient(Eq.8);

σ_t = turbulent Scmidt number;

ρ = density of water with solute;

ρ_c = density of the solute and

ρ_o = density of the fresh water.

notation

$\langle * \rangle$: mean value of * over flow depth

$\bar{*}$: non-dimensional expression of *

(Received October 9, 1991; revised may 15, 1992)