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STRUCTURE OF FLOW AND DISPERSION PROCESS OF SUSPENDED PARTICLE OVER TWO-DIMENSIONAL DUNES

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ABSTRACT

The characteristics of turbulent flows over the two-dimensional dunes are investigated by the numerical simulation by k-ε turbulence model. The computed results of k-ε model agree well with the measured data of mean velocity and turbulent intensities profiles.

In the calculated flow field, the time series of the velocity field is simulated by Monte-Carlo simulation in which one-dimensional Markov-chain principle is adopted. The trajectory of suspended particle is traced in the simulated flow field.

Finally the non-equilibrium dispersion process of suspended particle over the dunes is predicted based on the proposed simulation model. The profile of the suspended particle concentration over dunes agree well with the results of experiment.

INTRODUCTION

On a discussion of the dune formation process caused by non-equilibrium bed load transport, the relation between bed shear stress and bed configuration plays a very important role. From this point of view, many of previous studies (Raudkivi (12), Kikkawa & Ishikawa (4), Fredsφe (2), Nakagawa et al. (8)) performed measurement and analytical estimation of bed shear stress on upstream surface of dune. In early stage of dune formation, suspended load is often negligible, because tractive force is not large enough to generate suspended load. Whereas, in dune evolution process, suspended load is not negligible. Since suspended load distributes whole the flow region in evolution process, it is required to investigate not only the bed shear stress but the total flow structures. Furthermore, in evolution process, the flow structure is complicated; dunes have asymmetrical configuration and separation occurs at crest. Because of these complicated conditions, the analytical solution of flow field can not be obtained.

In this study, to obtain quantitative information of velocity profile and turbulent intensity profile in the whole region of the flow over two-dimensional dunes, the numerical simulation with the aid of k-ε turbulent model is executed. Subsequently, based on the flow properties investigated by k-ε model, the time-dependent flow field is simulated by Monte-Carlo simulation of Markov process, and suspended particle movement in turbulent flow is traced based on the Tchen's equation of motion. Finally, by superposition of the individual particle trajectories, the probability of existence height of a suspended sediment, which is often identified with the concentration distribution, is estimated.

SIMULATION OF THE FLOW OVER TWO-DIMENSIONAL DUNES

A numerical simulation of the flow structure is executed with the aid of k-ε turbulent model. In order to confirm the applicability of this simulation, the calculation is carried out in the same condition as the experiment by authors (7). Flow field can be simulated by solving the following five basic equations. The equation of continuity is:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \qquad (1)$$

in which U, V=longitudinal and vertical velocities, respectively. The Reynolds equation in the longitudinal and the vertical components are:

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = g\left(\sin\theta - \frac{\partial h}{\partial x}\cos\theta\right) - \frac{\partial}{\partial x}\left(\frac{P'}{\rho}\right) + \frac{\partial}{\partial x}(-u^2) + \frac{\partial}{\partial y}(-uv) + \nu\nabla^2 U \qquad (2)$$

$$U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left(\frac{P'}{\rho}\right) + \frac{\partial}{\partial x}(-uv) + \frac{\partial}{\partial y}(-v^2) + \nu\nabla^2 V \qquad (3)$$

in which g=gravitational acceleration; θ=averaged bed slope; h=flow depth; P'=deviation from static pressure; ρ=mass density of fluid; ν=kinematic viscosity; and ∇=Laplacian operator. The k-equation is:

$$U\frac{\partial k}{\partial x} + V\frac{\partial k}{\partial y} = \frac{\partial}{\partial x}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial y}\right) + G - \varepsilon \qquad (4)$$

in which ν_t=kinematic eddy viscosity; σ_k=empirical constant in Table 1; and G=generation term which is given by

$$G = \nu_t\left[2\left\{\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2\right\} + \left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right)^2\right] \qquad (5)$$

The ε-equation is:

$$U\frac{\partial \varepsilon}{\partial x} + V\frac{\partial \varepsilon}{\partial y} = \frac{\partial}{\partial x}\left(\frac{\nu t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\nu t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial y}\right) + \frac{\varepsilon}{k}(C_{1\varepsilon}G - C_{2\varepsilon}\varepsilon) \qquad (6)$$

in which σ_ε, $C_{1\varepsilon}$ and $C_{2\varepsilon}$=empirical constants in Table 1. The values of empirical constants in Table 1 are determined by Launder & Spalding (3). The coordinate system and the bed geometry are shown in Fig. 1.

Table 1 Empirical constants of k-ε turbulent model

$C\mu$	$C_{1\varepsilon}$	$C_{2\varepsilon}$	σ_k	σ_ε
0.09	1.44	1.92	1.00	1.30

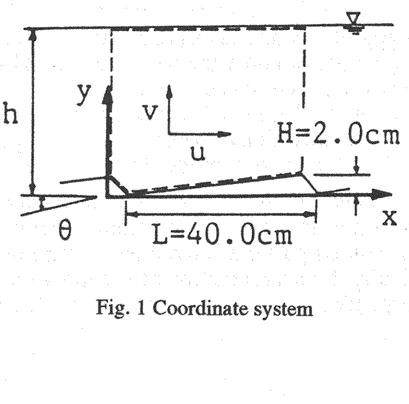

Fig. 1 Coordinate system

By using a Boussinesq approximation, the Reynolds stress term are proportional to the mean-velocity gradients in Einstein's notation, as follows;

$$-u_iv_j = \nu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) + \frac{2}{3}k\delta_{ij} \qquad (7)$$

in which δ_{ij}=Kronecker delta. Here the kinematic eddy viscosity is dictated by a dimensional analysis:

$$\nu_t = C_\mu \frac{k_2}{\varepsilon} \qquad (8)$$

in which C_μ=empirical constant in Table 1. By employing eddy viscosity concept in Eqs.7 and 8, Eqs. 1, 2, 3, 4 and 6 are closed.

The boundary conditions at the bottom boundary are given by well-known wall function (14). The velocity parallel to the wall at the first grid point U_{fir} is given by log law. With the local equilibrium assumption (G=ε) and the log law, there follows for k_{fir} and ε_{fir}.

$$k_{fir} = \frac{u_*^2}{\sqrt{C_\mu}} \quad ; \quad \varepsilon_{fir} = \frac{u_*^3}{\kappa y} \qquad (9)$$

At the free surface, symmetry condition is modified by Nezu & Nakagawa's method (11). In order to calculate the turbulent energy at the free surface k_{sur}, turbulent energy calculated by symmetry condition k_{sym} is modified with the damping coefficient D_w.

$$k_{sur} = D_w k_{sym} \qquad (10)$$

They recommend the D_w=0.8 from the experiment. The velocity at the free surface satisfy the relation, as follows;

$$\left(\frac{\partial U}{\partial y}\right)_{sur} = \frac{1}{\nu_t}(-uv)_{sym} \qquad (11)$$

Energy dissipation ε at the free surface is calculated by a symmetry condition. The periodic boundary condition is satisfied between the outflow- and inflow-boundaries.

Table 2 Experimental condition

Mean velocity	:	17.138*cm/sec*
Flow depth	:	6.7*cm*
Energy gradiant	:	0.00122
Froud number	:	0.21
Reynolds number	:	4.59×10^4

The condition of the calculation shown in Table 2 is set to be the same as the condition of the experiment executed by the authors (7). The Reynolds number in Table 2 is defined as Re≡ $4hU_m/\nu$ in which U_m is bulk mean velocity at crest. A comparison between the calculated flow properties and the experimental data are shown in Fig.2. An agreement in the calculated mean velocity field with the measured one is excellent. From the downstream of the crest to the reattachment point, the distribution of the mean velocity U coincides with Gaussian distribution which is often adopted in the theory of a mixing layer. Although an internal boundary layer develops from the downstream of the reattachment point, the growth of the boundary layer is suppressed by the effect of flow acceleration caused by bed configuration. The increasing rate of the internal boundary layer thickness over a dune is smaller than that in the backward-facing step flow, in which the effect of acceleration does not exist. Therefore, a uniformity of the mean velocity distribution at the crest of dune is stronger than that at the step in backward-facing step flow. Nakagawa et al. (9) reported this property in mean velocity distribution through the measurement by Laser Doppler Anemometers.

The distribution of V shows that a downward flow occurs from the crest to the reattachment point and that an upward flow occurs form the downstream of the reattachment point. This tendency is expressed well by calculation, if one takes the difference in the order between U and V into consideration. An agreement between the measured data and the calculated data is fairy good.

The turbulent intensities are estimated in two ways: (i) by using Rodi's algebraic stress relation (13) in Einstein's notation; they are written as

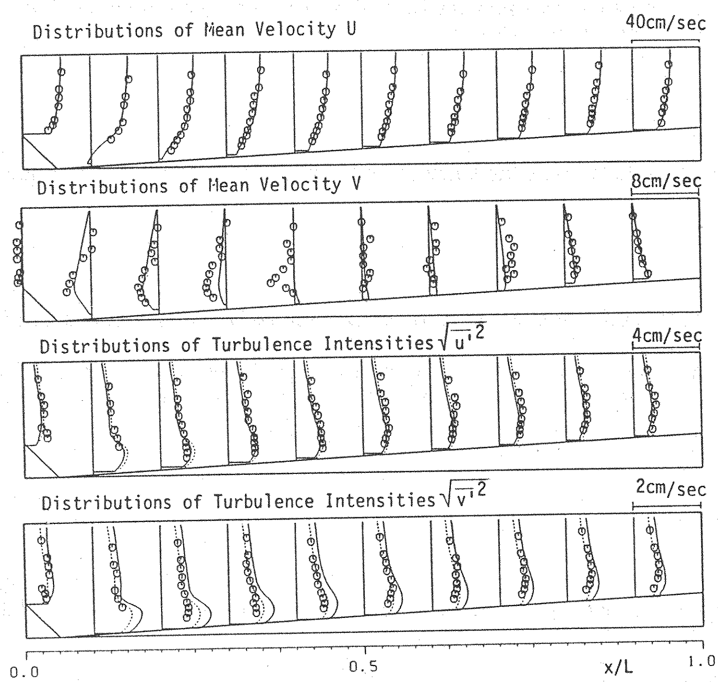

Fig. 2 Comparison between calculated field and experimental data

$$-u_iu_j = k\left[\frac{2}{3}\delta_{ij} + \frac{(1-\alpha)\left(\frac{P_{ij}}{\varepsilon} - \frac{2}{3}\delta_{ij}\frac{P}{\varepsilon}\right)}{C_1 + \frac{P}{\varepsilon} - 1}\right] \tag{12}$$

in which P_{ij}=stress production of u_iu_j; P=stress production of k; α=0.764; and C_1=1.5. (ii) By using ratio of the coefficient in the universal function of turbulent intensities proposed by Nezu (10), the turbulence intensities u_{rms}, v_{rms} are written as

$$u_{rms} = 1.10k \quad ; \quad v_{rms} = 0.34k \tag{13}$$

The calculated result shows an increase of the turbulent intensities along the shear layer and it suggests the turbulent diffusion in the vertical direction over the upstream surface of dune. The method (ii) is better for redistributing the turbulent energy into each component, therefore the ratio of each turbulent-energy component to the total turbulent energy in uniform flow can be applied to the flow over dunes in which the acceleration effect exists.

NUMERICAL SIMULATION OF SUSPENDED PARTICLE'S MOVEMENT

Most of the studies on the concentration of suspended sediment by diffusion equation supposed the two assumptions: (i) The distribution of diffusion coefficient of suspended sediment ν_s is proportional to a kinematic eddy viscosity ν_t. (ii) The distribution of ν_s is independent of the longitudinal location.

Figure 3 shows the distribution of the kinematic eddy viscosity calculated by a k-ε turbulent model. In Fig.3, it is evident that the longitudinal variation of the kinematic eddy viscosity is not negligible, then the longitudinal variation of ν_s is not negligible too, even if the proportional relation between ν_s and ν_t is available. On the other hand, there is not universal relation between ν_s and ν_t.

In this study, these two points are considered: (i) The calculated result of k-ε model which conclude the properties of turbulent flow field is used so as to simulate the velocity fluctuation instead of the consideration of ν_t distribution. (ii) The movement of suspended particle is traced based on the equation of motion in a simulated flow field, instead of the consideration of ν_s and ν_t relation.

The equation of motion of a suspended particle is known as Tchen's equation. It is written as

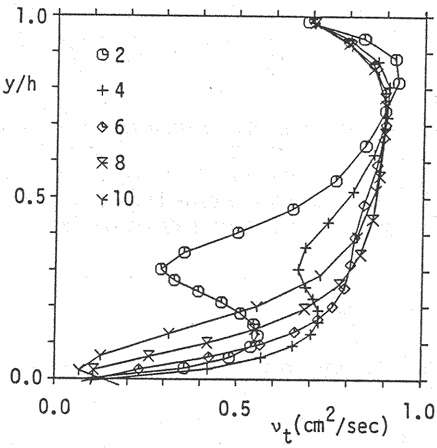

Fig. 3 Distribution of kinematic eddy viscosity calculated by k-ε turbulent model

$$\rho\left(\frac{\sigma}{\rho}+C_M\right)A_3d^3\frac{du_p}{dt}=\frac{1}{2}C_D\rho A_2d^2\sqrt{(u_f-u_p)^2+(v_f-v_p)^2}\,(u_f-u_p)+\rho(1+C_M)A_3d^3\frac{du_f}{dt} \quad (14)$$

$$\rho\left(\frac{\sigma}{\rho}+C_M\right)A_3d^3\frac{dv_p}{dt}=\frac{1}{2}C_D\rho A_2d^2\sqrt{(u_f-u_p)^2+(v_f-v_p)^2}\,(v_f-v_p)$$

$$+\rho(1+C_M)A_3d^3\frac{dv_f}{dt}-\rho\left(\frac{\sigma}{\rho}-1\right)gA_3d^3 \quad (15)$$

in which, u_f, v_f=instantaneous velocity in the longitudinal and the vertical directions, respectively; u_p, v_p=instantaneous velocities of a particle in the longitudinal and the vertical directions, respectively; σ= particle's mass density; C_M=added mass coefficient (=0.5); C_D=drag coefficient; d=diameter of particle; and A_2, A_3 =geometric coefficients of suspended particle. In Eqs.14 and 15, Basset term and the effect of the particle's rotation are neglected. The drag coefficient C_D is estimated as

$$C_D=2.0+\frac{24}{Re} \quad (16)$$

Here, Re=Reynolds number ($\equiv\sqrt{(u_f-u_p)^2+(v_f-v_p)^2}d/\nu$).

In order to simulate the instantaneous velocity field, which is required for tracing the particle movement from Lagrangian viewpoint, the flow field calculated by k-ε model is combined with Monte-Carlo simulation. First, the mean velocity (U, V), turbulent intensity (u_{rms}, v_{rms}) at the particle's position are estimated by the linear interpolation of the calculated flow field. Second, with the aid of Monte-Carlo simulation, the instantaneous velocities (u(t),v(t)) are estimated from the turbulent intensity. For this purpose, two methods are attempted. One is : (i) simple Monte-Carlo simulation, where

$$u(t)=u_{rms}\cdot r \quad ; \quad v(t)=v_{rms}\cdot r \quad (17)$$

in which r=random variables following the one-dimensional Gaussian distribution. The other is (ii) one-dimensional Markov process connected with Monte-Calro simulation, where

$$u(t+\Delta t)=\alpha_u u(t)+r_u \quad ; \quad v(t+\Delta t)=\alpha_v v(t)+r_v \quad (18)$$

in which, α_u, α_v=Lagrangian auto-correlation coefficients; and r_u, r_v=random variables following the two-dimensional Gaussian distribution. The random variables (r_u, r_v) shows the two-dimensional Gaussian distribution, of which probability density function is written as

$$f(r_u, r_v) = \frac{1}{\sqrt{2\pi}\sigma_{ru}} \exp\left(-\frac{r_u^2}{2\sigma_{ru}^2}\right) \frac{1}{\sqrt{2\pi}\sigma_{ru}\sqrt{1-\gamma^2}} \exp\left\{-\frac{1}{2(1-\gamma^2)\sigma_{rv}^2}\left(r_v - \gamma\frac{\sigma_{rv}}{\sigma_{ru}} r_u\right)^2\right\} \quad (19)$$

in which σ_{ru}, σ_{rv}=standard deviation of the random variables in the longitudinal and the vertical directions ($\sigma_{ru}=u_{rms}\sqrt{1-\alpha_u^2}$; $\sigma_{rv}=v_{rms}\sqrt{1-\alpha_v^2}$); and γ=cross-correlation coefficient. The validity of Gaussian distribution to the turbulence has been demonstrated Nakagawa et al. (7).

The Lagrangian autocorrelation coefficient is written as

$$\alpha_u = \alpha_v = \exp\left(-\frac{\Delta t}{T_L}\right) \quad (20)$$

in which T_L=Lagrangian time scale. The ratio of Lagrangian time scale T_L and Eulerian time scale T_E is a constant:

$$\frac{T_L}{T_E} = \beta \quad (21)$$

which has a probable number between 3 and 5, after Hay & Pasqill (2). From the hypothesis of frozen turbulence, the Eulerian length scale L_E is written as

$$L_E = u_m \cdot T_E \quad (22)$$

in which u_m= mean velocity in longitudinal direction. Substituting Eqn.21 into Eqn.22 and supposing that the Eulerian time scale L_E is proportional to the height above the bottom, the Lagrangian time scale T_L is

$$T_L = c_{TL}\frac{y}{u_m} \quad (23)$$

where c_{TL}=constant about 2.4 (Hall (1)).

Generally, method (i) simulates the broad-scale characteristics of turbulence using a large time interval. On the other hand, method (ii) can treat a correlation between velocities at successive steps using a small time interval. Method (i) is regarded as the simplified case of method (ii). In other word, supposing $\alpha_u=\alpha_v=0$ in method (ii) is equivalent to method (i).

The time interval Δt should be decided considering the characteristics of the particle's response to turbulence. The time interval Δt is decided by the analysis of a particle's response to turbulence. The responding amplitude of displacement of the particle to the turbulence which is given as a sinusoidal function with maximum amplitude u_{rms} is calculated, and the period, in which responding amplitude is about 0.2d, is adopted for the time interval of this simulation. If the time interval is not small enough, the accuracy of calculation is sensitively affected by the time interval. The present adoption of the time step makes it possible to trace the particle's trajectory in the accuracy of one-fifth of particle's diameter. The mass density of a particle is set 1.03 and the diameter 0.128cm in this simulation, in order to compare the theory with the experiment in the next section.

Most of the previous studies (Yalin & Krishnappan (15)) employed the method (i) to estimate the instantaneous velocity field, and thus, the hysteresis of particle movement is not considered. On the other hand, in the method (ii), Markovian process model is adopted, so that the fluctuating components at (t+ Δt) are somewhat correlated to that at t.

Figure 4 shows a comparison between the trajectory calculated by the method (i) and that by the method (ii). Although the great discrepancies observed between the result of two method can be reduced by adoption a large time interval to method (i). But, method (i) with a large time interval fails to account for the rapidly change of turbulent properties, because the variation of the turbulent properties' distribution is great in this flow field. In the result of the method (ii), the suspended particle which moves downstream extending over three of four dunes can be simulated. Because such particles are observed in the experiment in the following section, the method (ii) is considered to be better to express the real particle movement than the method (i).

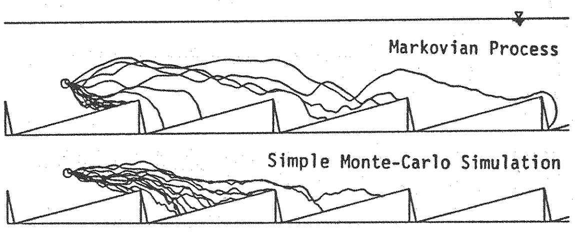

Fig. 4 Trajectory of particle simulated by Monte-Calro simulation

Table 3 Experimental condition

Mean velocity	:	12.067 cm/sec
Flow depth	:	7.5 cm
Energy gradiant	:	0.001
Froud number	:	0.14
Reynolds number	:	3.60×10^3

NON-EQUILIBRIUM SUSPENSION OVER DUNE BED

The concentration-profile of suspended sediment over dunes is estimated in the following manner. First, individual particle's trajectory is traced in simulated turbulent flow; secondly, the probability density function of the existence height of a suspended particle is evaluated by superposing respective individual particle trajectories. The probability density function of the existence height is identified with the concentration profile.

Table 3 shows the experimental conditions. Polystyrene particle which has 0.128cm diameter and 1.03 specific gravity is used. Movement of particles was analyzed by a video system in the following manner. First, the test section was divided into many cells; secondly, the number of particles in each cell was counted, and finally, the time averaged value of the particle number in each cell was calculated.

In this simulation, the initial location of a suspended particle at the upstream crest was given so as to be coincident with the experimented results. And then the trajectory of each suspended particle was traced. During tracing the particle's trajectory, the instantaneous location of the suspended particle was recorded in the computer's memory. This tracing particle's trajectory procedure was continued until the particle deposited on the bed.

When the concentration profile at the upstream crest is equal to that of the downstream crest, the amount of the particle which turns from bed-load motion to suspension is equal to that of depositing particle in each dune segment. In this simulation ,the transition from bed-load motion to suspension was treated as mentioned below. After tracing each particle's trajectory, the number of deposit particle was counted. And then, the transition rate from bed load motion to suspension was calculated using by the following equation proposed by Nakagawa et al. (4).

$$p_{T*} \equiv p_T \sqrt{d/(\frac{\sigma}{\rho}-1)g} = F_{0T}\left(\frac{u_*}{w_0}\right)^n \left[1-\frac{(u_*/w_0)_c}{(u_*/w_0)}\right]^m \qquad (24)$$

in which w_0=terminal velocity of particle. The constants in Eq.24 were determined as follows: F_{0T}= 0.0175; n=0.4; m=1.10. $(u_*/w_0)_c$ is the value of (u_*/w_0) corresponding to p_{T*}=0.

The deposit particles were distributed to each location in the upstream bed surface of the dune, and then the trajectory was traced again. The existence probability density of the suspended particle which turns from bed load is estimated. Finally, the existence probability density of suspended particle is calculated by summing up that of convection-particle from the upstream crest and that of particle which turns from bed load.

Figure 5 shows the result of the present simulation being compared with the experimental result.

The existence probability density obtained in the experiment is averaged in each segment which has 1 cm height. Although the present simulation can estimate more detailed profile, the result of simulation is averaged in the same manner which is adopted in the experimental data treatment, so as to compare the result of the simulation with the experiment result directly.

The following characteristics are recognized in Fig.5: The gravity center of the concentration gradually descends down in the region from the upstream crest to the reattachment point. In the downstream of the reattachment point, the gravity center of the concentration gradually rise up again, because of the effect of the upward flow and the transition from bed-load motion to suspension, and then, at the sixth cross-section the concentration profile has already fully developed. In the downstream section from sixth cross-section, almost the same profiles have been maintained.

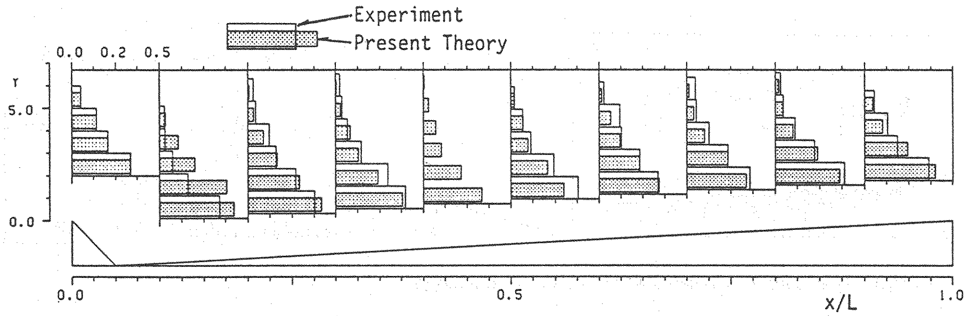

Fig. 5 Comparison between simulated result and experimental data

The result of the simulation shows an excellent agreement to the experimental result except the second cross-section. Such disagreement might be caused by the difficulty in observing each particle, because the second cross-section is included in the separating region in which a number of particles are rotating. Although the specific gravity of the suspended particle which is adopted in this study is smaller than that of a natural sand, this simulation model can be applied to the suspension of a natural sand considering the response of a sand particle to the turbulence.

CONCLUSIONS

In this paper, the flow structure over a dune bed has been investigated by turbulence measurement and k-ε model simulation. Furthermore, numerical simulation of suspended sediment has been conducted to estimate the concentration profile of suspended sediment over dune bed. The results obtained in this study are summarized below:

(1) The mean velocity and turbulent intensity over a dune bed has been simulated by k-ε turbulent model. Calculated field and measured data show good agreement, then the applicability of k-ε turbulent model to the flow over the dune bed configuration has been confirmed.

(2) To simulate the trajectory of a suspended particle, Monte-Calro simulation, in which random variables following Markovian process are generated to simulate the instantaneous turbulent field, has been carried out. At least qualitatively, the present method simulates well the characteristics of a suspended particle.

(3) In order to check whether the present simulation of suspended particle movement is adequate, the existence probability density of suspended particle has been obtained by the simulation and compared with the experimental results. The agreement between the simulation and experiment is fairy good, and thus, the applicability of the present simulation has been confirmed not only qualitatively but quantitatively.

Most of the contents of this study was already published in Japanese (7), though some parts has been modified herein.

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APPENDIX-NOTATION

The following symbols are used in this paper:

A_2, A_3	= two-and three-dimensional geometrical coefficients of sand;
C_D, C_M	= drag coefficient and added mass coefficient;
C_1	= constant in Rodi's algebraic stress relation;
C_m	= empirical constant of standard k-ε model;
$C_{1\varepsilon}$, $C_{2\varepsilon}$	= empirical constants of standard k-ε model;
d	= diameter of sand;
F_{T0}	= empirical constant in transition-probability formula;
g	= gravitational acceleration;
G	= generation term;
k	= turbulent energy;
k_{fir}	= turbulent energy at first grid point;
k_{sur}	= turbulent energy at water surface;
h	= flow depth;
m, n	= empirical constant in transition-probability formula;
p_T, p_{T*}	= probability density of transition per unit time from bed-load motion to suspension and its dimensionless form;
P	= stress production of k;
P'	= deviation from static pressure;
P_{ij}	= stress production of $u_i v_j$;
Re	= Reynolds number:

r	= random variable following to uniform distribution;
r_u, r_v	= random components following to two-dimensional normal distribution;
u, v	= instantaneous turbulent velocity in longitudinal and vertical direction;
u_f, v_f	= instantaneous velocity in longitudinal and vertical direction ($u_f \equiv U+u$, $v_f \equiv V+v$);
u_i	= fluctuating velocity component in x_i direction;
u_p, v_p	= instantaneous velocity of particle in longitudinal and vertical direction;
u_*	= shear velocity;
u_{rms}, v_{rms}	= turbulent intensities in longitudinal and vertical direction;
U, V	= mean velocity in longitudinal and vertical direction;
U_{fir}	= mean velocity at first grid point;
U_i	= mean velocity component in x_i direction;
U_m	= bulk mean velocity at crest;
U_{sur}	= mean velocity at water surface;
x, y	= longitudinal and vertical coordinates;
α	= constant in Rodi's algebraic stress relation;
α_u, α_v	= Lagrangian auto correlation coefficients in longitudinal and vertical direction;
g	= correlation coefficient in random variables;
δ_{ij}	= Kronecker delta;
ε	= energy dissipation;
ε_{fir}	= energy dissipation at first grid point;
ε_{sur}	= energy dissipation at water surface;
θ	= averaged bed slope;
ν	= kinematic viscosity;
ν_s	= diffusion coefficient of suspended particle;
ν_t	= kinematic eddy viscosity;
ρ	= mass density of fluid;
σ	= mass density of particle;
σ_ε	= empirical constant in k-ε turbulent model; *and*
σ_{ru}, σ_{rv}	= variances of random component in longitudinal and vertical direction.

Subscripts

f	= fluid;
p	= particle;
fir	= first grid point;
sym	= symmetry condition; *and*
sur	= water surface.

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