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A PROBABILITY MODEL FOR EVALUATING DESIGN STORM PATTERNS WITH MULTI-LOCAL PEAKS

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SYNOPSIS

This paper describes a probability model of heavy storm patterns with multi-local peak intensities. The definitions of a storm cluster and a storm part lead that these occurrences follow Poisson and logarithmic series probability distributions, respectively. Using combined Freund's bivariate probability density functions we have the joint probability distribution of duration, depth, and local peak for a storm part. From those of multi-storm parts, the joint probability distribution function of total duration, local depths, and local peak intensities for a single storm cluster with multi-storm parts is derived. Examples of 2-peak design storm patterns are demonstrated.

INTRODUCTION

According to the technical manual for river engineering and erosion control (a revised draft), Ministry of Construction, Japan, published in 1977, a design storm for flood-control and multi-purpose projects in river drainage areas in Japan was defined to have three characteristics, the total rainfall, temporal, and spatial distributions. Considering the magnitude of drainage area, the rainfall, and regional characteristics, we usually set the design storm duration at 1 to 3 days. In essence, the magnitude of project has been evaluated by the return period of the design total rainfall. Following the determination of the design total rainfall depth, the time distribution-shape (storm pattern) is determined to be almost the same as the storm pattern of a historical heavy storm that had caused a large flood. This method is called the enlargement method of historical storms, and is easy and simple in application. However, this method often leads to overestimation of the peak rainfall intensity, so that some empirical modification of the storm pattern should be required. Thus, the current method for determination of a design storm pattern in Japan has not been strongly supported by the theory of probability. Especially, the joint probability of the design total rainfall and hourly rainfall intensities around the peak intensity governing the maximum discharge of flood has not been clearly clarified, although it is very important for the determination of the design storm pattern.

Hashino(4) proposed a new method for evaluation of the joint occurrence probability of total rainfall and peak intensity for a single storm part using Freund's bivariate probability density function, and stochastic criteria for determination of design storm patterns were demonstrated. This method takes into account the autocorrelation coefficient of heavy hourly rainfall intensities around the peak, as well as the crosscorrelation coefficient of total rainfall and peak intensity.

Although the theory of this method is based on a single storm part with a single peak, actual heavy storms with long durations often have multi-local peaks (local maximum rainfall intensities). Therefore, in this paper we propose a probability model for evaluating multi-local peaked design storms using combined Freund's bivariate probability density functions that were developed by Hashino(2).

DEFINITION OF A STORM CLUSTER AND A STORM PART

As shown in Fig. 1 a storm cluster is defined as a sequence of rainy 1-hr periods bounded by two dry periods, where there is no rain or no local maximum rainfall intensity exceeding a threshold value of x_c, at the beginning and at the end. The duration D of the storm cluster is assumed to be longer than a threshold value d_c, as well as the local maximum(peak) rainfall intensity $y_i (i=1,2,\cdots)$ larger than x_c. Otherwise, it is considered to be a drizzle, and these drizzles are not modeled because the aim of this paper is to formulate heavy storm patterns with multi-local peaks for design floods.

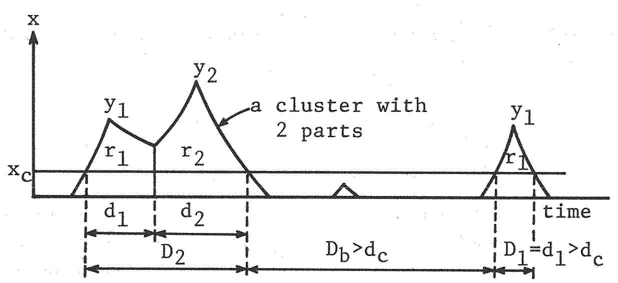

Fig. 1 Schematic depiction of a storm cluster and a storm part

A storm cluster often has multiple local maximum intensities. If a storm cluster has n(≧2) local peaks, then we subdivide the storm cluster into n parts that are refferred to as storm parts.

In practise, selection of storm clusters and subdivision of them into storm parts are based on the smoothed series obtained by a 3rd-order simple moving average scheme. The reason is that mesoscale precipitation areas are observed to have average life spans of 2 - 4 hours in Japan.

PROBABILITY MODEL FOR A STORM CLUSTER WITH MULTI-LOCAL PEAKS

Probability Distributions of Numbers of Clusters and Parts

Consider a certain period of time, such as the rainy season and the typhoon season. The occurrence number N_c of storm clusters in the period is assumed to follow a Poisson process:

$$P[N_c = n] = \Lambda^n \cdot \exp(-\Lambda)/n! \qquad (n = 0,1,2,\cdots) \qquad (1)$$

where Λ = occurrence rate of storm clusters in the given period.

We assume that the number N_p of storm parts given a storm cluster follows a logarithmic series probability distribution with the parameter θ $(0<\theta<1)$:

$$P[N_p = n|N_c = 1] = \gamma \cdot \theta^n/n \qquad (n = 0,1,2,\cdots) \qquad (2)$$

where

$$\gamma = -\{\log(1-\theta)\}^{-1} \qquad (3)$$

A negative binomial distribution for the total number of storm parts in a certain period can arise as the distribution of the sum of N_c independent variables each having the same logarithmic series distribution, when N_c has a Poisson distribution.

Joint Probability Distribution Function for a Storm Part

A single storm part is considered to consist mainly of three components of the duration d, the depth r, and the local peak intensity y as shown in Fig. 1. The trivariate joint probability distribution function H(d,r,y) is defined by

$$\begin{aligned} H(d,r,y) &= P[D_p<d,R_p<r,Y_p<y] \\ &= P[D_p<d,R_p<r]\cdot P[Y_p<y|D_p<d,R_p<r] \end{aligned} \tag{4}$$

If the crosscorrelation coefficient ρ_{ry} between r and y is larger than ρ_{dy} between d and y, the following approximation can be permitted.

$$P[Y_p<y|D_p<d,R_p<r] = P[Y_p<y|R_p<r] = P[R_p<r,Y_p<y]/P[R_p<r] \tag{5}$$

Therefore, the trivariate joint probability for a storm part can be approximately represented by a couple of bivariate joint probability distribution functions; that is,

$$H(d,r,y) = H(d,r)\cdot H(r,y)/H(r) \tag{6}$$

In essentials we make use of Freund's exponential distribution as the bivariate distribution. If a combination of two or three Freund's distributions yields a better fit to observations, then the combined Freund's distribution may be employed. A combined density function $f(\eta,\xi)$ composed of two Freund's distributions is defined as

$$f(\eta,\xi) = \begin{cases} a_1b_2\exp[-b_2\eta-(a_1+b_1-b_2)\xi] & (0<\xi<\eta,\xi<u_1) \quad (7a) \\ b_1a_2\exp[-a_2\xi-(a_1+b_1-a_2)\eta] & (0<\eta<\xi,\eta<u_1) \quad (7b) \\ \alpha_1\beta_2\exp[-\beta_2(\eta-v_1)-(\alpha_1+\beta_1-\beta_2)(\xi-v_1)] & (0<\xi<\eta) \quad (7c) \\ \beta_1\alpha_2\exp[-\alpha_2(\xi-v_1)-(\alpha_1+\beta_1-\alpha_2)(\eta-v_1)] & (0<\eta<\xi) \quad (7d) \end{cases}$$

where a_1, b_1, a_2, b_2=parameters for the lower class density functions of η and ξ: Eqs. 7a and 7b; $\alpha_1,\beta_1,\alpha_2,\beta_2$=parameters for the upper class density functions: Eqs. 7c and 7d; v_1=constant; and u_1=critical(connective) value of η and ξ that classifies the combined Freund's distribution into the two regions shown in Eqs. 7a; 7b and 7c; 7d. In the case of $u_1=\infty$, Eqs. 7a and 7b become the original Freund's equations(Freund(1)).

From continuity conditions of the marginal density function $f_d(\eta)$, the conditional distribution function $F(\xi|\eta)$, and the marginal distribution functions $F_d(\eta)$ and $F_r(\xi)$ for $\eta=\xi=u_1$, the following equations are derived.

$$b_1 = \beta_1; \quad a_2 = \alpha_2 \tag{8}$$

$$v_1 = [1-\{(a_1+b_1)/(\alpha_1+\beta_1)\}]\cdot u_1 \tag{9}$$

Under the continuity conditions shown in Eqs. 8 and 9, independent variables of the combined Freund density function in Eq. 7 are the parameters: a_1, b_1, a_2, b_2, α_1, and β_2. These six parameters can be estimated by the maximum likelihood method as

$$\left.\begin{aligned}
1/a_1 &= \{ \sum_{j=1}^{N_{11}} \xi_j^{<11>} + \sum_{j=1}^{N_{12}} \eta_j^{<12>} + N_2 u_1 \}/N_{11} \\
1/b_1 &= N_{11}/(N_{12} a_1) \\
1/a_2 &= \sum_{j=1}^{N_{12}} (\xi_j^{<12>} - \eta_j^{<12>})/N_{12} \\
1/b_2 &= \sum_{j=1}^{N_{11}} (\eta_j^{<11>} - \xi_j^{<11>})/N_{11}
\end{aligned}\right\} \quad (10)$$

$$\left.\begin{aligned}
1/\alpha_1 &= \{ \sum_{j=1}^{N_{21}} \xi_j^{<21>} + \sum_{j=1}^{N_{22}} \eta_j^{<22>} - N_2 u_1 \}/N_{21} \\
1/\beta_2 &= \sum_{j=1}^{N_{21}} (\eta_j^{<21>} - \xi_j^{<21>})/N_{21}
\end{aligned}\right\} \quad (11)$$

where $N_2=N_{21}+N_{22}$; $(\eta_j^{<11>}, \xi_j^{<11>})$, $(\eta_j^{<12>}, \xi_j^{<12>})$, $(\eta_j^{<21>}, \xi_j^{<21>})$, and $(\eta_j^{<22>}, \xi_j^{<22>})$=samples of the bivariate (η, ξ) satisfying the regions: $(0<\xi<\eta, \xi<u_1)$, $(0<\eta<\xi, \eta<u_1)$, $(u_1<\xi<\eta)$, and $(u_1<\eta<\xi)$, respectively; and N_{11}, N_{12}, N_{21}, N_{22}=sample sizes for these four regions. If $N_{21}=0$, then the continuity conditions of $f_d(\eta)$ and $F(\xi|\eta)$ should be replaced by $f_r(\xi)$ and $F(\eta|\xi)$. Consequently, when $N_{21}=0$, the parameters α_1, β_1, α_2, and β_2 of the upper class distribution are re-estimated by the following equations.

$$\alpha_1 = a_1; \qquad \beta_2 = b_2 \quad (12)$$

$$\left.\begin{aligned}
1/\beta_1 &= \{ \sum_{j=1}^{N_{22}} \xi_j^{<22>} + \sum_{j=1}^{N_{22}} \eta_j^{<22>} - N_2 u_1 \}/N_{22} \\
1/\alpha_2 &= \sum_{j=1}^{N_{22}} (\xi_j^{<22>} - \eta_j^{<22>})/N_{22}
\end{aligned}\right\} \quad (13)$$

This parameter estimation method can be easily extended for a combined density function composed of three Freund's distributions. The details are omitted from here on.

The crosscorrelation coefficient $\rho_{\eta\xi}$ of η and ξ for the upper class density function shown in Eqs. 7c and 7d can be expressed in terms of the corresponding parameters as

$$\rho_{\eta\xi} = (\alpha_2\beta_2-\alpha_1\beta_1)/\sqrt{(\alpha_2^2+2\alpha_1\beta_1+\beta_1^2)(\beta_2^2+2\alpha_1\beta_1+\alpha_1^2)} \quad (14)$$

It is easily found from Eq. 14 that $\rho_{\eta\xi}$ varies within the domain: $-1/3<\rho_{\eta\xi}<1$.

In practise, the original bivariate, say (d, r), should be transformed to the following dimensionless bivariate (η, ξ):

$$\eta = (d-u_{dc})/\{\varepsilon_d\sigma_d\}; \qquad \xi = (d-u_{rc})/\{\varepsilon_r\sigma_r\} \quad (15)$$

where σ_d, σ_r=standard deviations of exceedances $(d-u_{dc})$ and $(r-u_{rc})$, respectively; u_{dc}, u_{rc}=specified base levels; ε_d, ε_r=coefficients of modification. For a given value set of u_1, u_{dc}, u_{rc}, ε_d, and ε_r the maximum likelihood method provides the parameters of the combined Freund distribution. We have to search for a set of u_1, u_{dc}, u_{rc}, ε_d, and ε_r that leads to a satisfactory fitting of combined Freund marginal distributions $F_d(\eta)$ and $F_r(\xi)$ to the corresponding empirical distributions plotted by Gringorten's formula.

Formulation of the Storm Pattern Given the Peak Intensity

Consider a single storm part with a peak intensity y as shown in Fig. 2. Let x_i be the rainfall intensity at discrete time i before or after the peak, measured to the left or right of the peak, respectively, and x_{i-1} be the rainfall intensity at the time (i-1) with time interval Δt(see Fig. 2). By the definition of a single storm part with a peak, the rainfall intensity decreases monotonously away from the peak; that is, $x_i<x_{i-1}$. Therefore, we can obtain the conditional probability distribution function $G(x_i|x_{i-1})$ from a Freund distribution with identical marginal distributions where $u_1=\infty$, $a_1=b_1=\alpha$, and $a_2=b_2=\beta$ in Eqs. 7a and 7b. The rainfall intensity x_i is transformed into a reduced variate z_i as

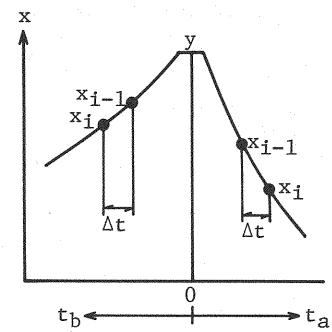

Fig. 2 A single storm part with a peak

$$Z_i = (x_i - u_z) / \sigma_z \qquad (0<u_z<x_i) \tag{16}$$

where u_z=base intensity level and σ_z=standard deviation of the exceedance (x_i-u_z). We can obtain the same conditional probability distribution function $G(z_i|z_{i-1})$ as that of $G(x_i|x_{i-1})$. Considering that the variance of the reduced variate z_i equals to unity, and rewriting the equation of $G(z_i|z_{i-1})$ with respect to z_i, we have the following equation(Hashino, (3)).

$$\left.\begin{aligned} &\exp(-\lambda z_i) = 2G\cdot(1-k)\exp(-\lambda z_{i-1})+(1-G) && (k\neq 1/2,\ 0<k\leqq 1) \\ &z_i = G\cdot z_{i-1}+(2\sqrt{7}/7)\cdot G && (k=1/2) \end{aligned}\right\} \tag{17}$$

where

$$k \equiv \alpha/\beta \tag{18}$$

$$\lambda \equiv (2k-1)\sqrt{3k^2+1}\,/2k \tag{19}$$

with G denoting a given value of $G(z_i|z_{i-1})$. The ratio $k\equiv\alpha/\beta$ governs the autocorrelation coefficient of z_i :

$$\rho_z = (1-k^2)/(1+3k^2) \tag{20}$$

It is clear from Eq. 20 that $\rho_z=0$ for k=1, $\rho_z\to 1$ for $k\to 0$, and $\rho_z\to -1/3$ for $k\to\infty$. In practise, an appropriate value of ρ_z in the range of $0\leqq\rho_z<1$ will be adopted, so that the ratio k may be in the range of $0<k\leqq 1$.

For the purpose of stochastic formulation of design storm patterns, the conditional probability G may be assumed to be time-invariant before and after the peak. Hence, the values of G before and after the peak are denoted as G_b and G_a, respectively.

In summary if k, G, and y are given, Eqs. 16 and 17 provide a sequence of x_i or a discrete hyetograph in time interval Δt. Therefore, a design one-sided hyetograph before or after the peak with the sub-duration d_o, the sub-depth r_o and the peak intensity y can be obtained by searching for the optimal couple of parameters: k and G which minimize the following sum-of-squares function.

$$S(k, G|d_o, r_o, y) = (r_o - \sum_{i=1}^{d_o} x_i)^2 + (x_L - x_{ba})^2 \tag{21}$$

where x_i=hourly rainfall intensity calculated by Eqs. 16 and 17; x_L=calculated intensity at the last hour of the sub-storm part after the local peak; and x_{ba}=dummy parameter defined as

$$x_{ba} = \begin{cases} 0 & \text{for the beginning and the last sub-storm parts} \\ x_L & \text{for sub-storm parts after the local peaks} \\ x_L' & \text{otherwise} \end{cases} \tag{22}$$

with x_L'=calculated intensity at the last hour of the preceding storm part.

Joint Distributions of Depths, Durations, and Multi-Local Peaks for a Storm Cluster

Let a single storm cluster consist of two storm parts with d_i, r_i, and y_i (i=1,2). Assuming that d_1, r_1, and y_1 of the first part are independent of d_2, r_2, and y_2 of the second part, the joint probability distribution given two parts can be represented as

$$\begin{aligned} &H(d_1, r_1, y_1; d_2, r_2, y_2|N_p = 2, N_c = 1) \\ &= P[D_{p1}<d_1, R_{p1}<r_1, Y_{p1}<y_1; D_{p2}<d_2, R_{p2}<r_2, Y_{p2}<y_2] \\ &= P[D_{p1}<d_1, R_{p1}<r_1, Y_{p1}<y_1] \cdot P[D_{p2}<d_2, R_{p2}<r_2, Y_{p2}<y_2] \end{aligned} \tag{23}$$

Using Eqs. 5 and 6 to Eq. 23 gives

$$\begin{aligned} H(d_1,r_1,y_1;d_2,r_2,y_2|N_p = 2,N_c = 1) &= \prod_{i=1}^{2} H(d_i,r_i) \cdot H(r_i,y_i)/H(r_i) \\ &= \prod_{i=1}^{2} F_{dr}(\eta_i,\xi_i) \cdot F_{ry}(\xi_i,\zeta_i)/F_r(\xi_i) \end{aligned} \tag{24}$$

where $F_{dr}(\eta_i,\xi_i)$, $F_{ry}(\xi_i,\zeta_i)$=combined Freund bivariate distributions for dimensionless variates η_i and ξ_i of d_i and r_i of the i-th storm part and dimensionless variates ξ_i and ζ_i of r_i and y_i, respectively. Thus, Eq. 24 leads to the conditinal joint probability distribution for a storm cluster with n storm parts as

$$\begin{aligned} &H(d_1,r_1,y_1;d_2,r_2,y_2;\cdots;d_n,r_n,y_n|N_p = n,N_c = 1) \\ &= \prod_{i=1}^{n} F_{dr}(\eta_i,\xi_i) \cdot F_{ry}(\xi_i,\zeta_i)/F_r(\xi_i) \qquad (n=1,2,\cdots) \end{aligned} \tag{25}$$

The n-fold convolution of the marginal distribution $F_d(\eta)$ of d per storm part gives the probability distribution of the total duration $D_n(=d_1+\cdots+d_n)$ for a single storm cluster with n parts under the assumption that $d_i(i=1,2,\cdots,n)$ are mutually independent. Therefore, the distribution of the total duration D per storm cluster is given as

$$H_D(D|N_c = 1) = \sum_{n=1}^{\infty} P[D_n<D|N_p = n, N_c = 1]\cdot P[N_p = n|N_c = 1]$$

$$= \sum_{n=1}^{\infty}\{\gamma\theta^n/n\}\ F_r^{n*}(\eta) \quad (26)$$

where $F_d^{n*}(\eta)$=distribution of dimensionless variate η of D obtained from the n-fold convolution of the marginal distribution $F_d(\eta)$. Similarly, we can derive the marginal distribution of the total depth R per storm cluster as

$$H_R(R|N_c = 1) = \sum_{n=1}^{\infty} P[R_n<R|N_p = n, N_c = 1]\cdot P[N_p = n|N_c = 1]$$

$$= \sum_{n=1}^{\infty}\{\gamma\theta^n/n\}\ F_r^{n*}(\xi) \quad (27)$$

where $F_r^{n*}(\xi)$=distribution of dimensionless variate ξ of R obtained from the n-fold convolution of the marginal distribution $F_r(\xi)$.

If χ_{Dn} and χ_{Rn} are the maxima of total durations D_n and total depths R_n with n-peak storm clusters, the probability distributions $H_D^{(n)}(\chi_{Dn})$ and $H_R^{(n)}(\chi_{Rn})$ can be derived as

$$H_D^{(n)}(\chi_{Dn}) = \exp[-\Lambda\{\gamma\theta^n/n\}\{1-F_d^{n*}(\eta^{(n)})\}] \quad (28)$$

$$H_R^{(n)}(\chi_{Rn}) = \exp[-\Lambda\{\gamma\theta^n/n\}\{1-F_r^{n*}(\xi^{(n)})\}] \quad (29)$$

where $\eta^{(n)}$, $\xi^{(n)}$=dimensionless variates of χ_{Dn} and χ_{Rn}, respectively.

Similarly, if χ_D and χ_R are the maxima of total durations D and total depths R, respectively, of storm clusters occurred within the season, the non-exceedance distributions $H_D(\chi_D)$ and $H_R(\chi_R)$ are derived as

$$H_D(\chi_D) = \exp[-\Lambda\{1-H_D(\chi_D|N_c = 1)\}]$$

$$= \exp[-\Lambda\{1-\sum_{n=1}^{\infty}\{\gamma\theta^n/n\}F_d^{n*}(\eta)\}] \quad (30)$$

$$H_R(\chi_R) = \exp[-\Lambda\{1-\sum_{n=1}^{\infty}\{\gamma\theta^n/n\}F_r^{n*}(\xi)\}] \quad (31)$$

Thus, the return periods T_{Dn}, T_{Rn}, T_D, and T_R of χ_{Dn}, χ_{Rn}, χ_D, and χ_R, respectively, are defined as

$$T_{Dn} = 1/[1 - H_d^{(n)}(\chi_{Dn})]; \qquad T_{Rn} = 1/[1 - H_R^{(n)}(\chi_{Rn})];$$

$$T_D = 1/[1 - H_D(\chi_D)]; \qquad T_R = 1/[1 - H_R(\chi_R)] \quad (32)$$

Now, consider a design storm cluster which consists of n storm parts with d_i, r_i, and y_i $(i=1,2,\cdots,n)$. If we adopt a simple relation between d_i and d_{i-1} as

$$d_i = \phi_d\cdot d_{i-1} = \phi_d^i\cdot d_1 \qquad (i=2,\cdots,n) \quad (33)$$

with the coefficient $\phi_d(>0)$, then the total duration $D_n=d_1+d_2+\cdots+d_n$ can be represented by

$$D_n = d_1(\phi_d^n - 1)/(\phi_d - 1) \quad (34)$$

The joint probability distribution of D_n; $r_1,\cdots,r_n$; and $y_1,\cdots,y_n$ is given as

$$H(D_n;r_1,\cdots,r_n;y_1,\cdots,y_n|N_p = n,N_c = 1)$$
$$= F_d^{n*}(\eta^{(n)}) \prod_{i=1}^{n}\{F_r(\xi_i|\eta<\eta_i)\cdot F_y(\zeta_i|\xi<\xi_i)\} \quad (35)$$

where $F_r(\xi_i|\eta<\eta_i)$, $F_y(\zeta_i|\xi<\xi_i)$=interval-conditional probability distribution functions. Let F_{ri} and F_{yi} denote the values of $F_r(\zeta_i|\xi<\xi_i)$ and $F_y(\zeta_i|\xi_i)$, respectively. If we adopt the following simple relations with respect to F_{ri} and F_{yi} as

$$F_{ri} = \phi_r \cdot F_{ri-1}, \quad F_{yi} = \phi_y \cdot F_{yi-1} \quad (i=2,3,\cdots,n) \quad (36)$$

with the coefficients ϕ_r and $\phi_y(>0)$, then Eq. 35 is rewritten as

$$H(D_n;r_1,\cdots,r_n;y_1,\cdots,y_n|N_p = n,N_c = 1) = F_d^{n*}(\eta^{(n)})\cdot\prod_{i=1}^{n}(F_{ri}\cdot F_{yi})$$
$$= F_d^{n*}(\eta^{(n)})\cdot(F_{r1}\cdot F_{y1})^n\cdot(\phi_r\cdot\phi_y)^{n(n-1)/2} \quad (37)$$

Now, we propose a new procedure for making design storm patterns against a design return period T_R^* and a design maximum total duration D* as follows.

1) Determine the value of the design total depth R* from Eqs. 31 and 32.
2) Determine the values of the number n of design storm parts and the design total duration D_n^* satisfying the conditions: $T_{Rn}<T_R^*$ and $0<P[D_n<D^*]=F_d^{n*}(\eta^{(n)})<(T_R^*-1)/T_R^*$. A storm cluster with n peaks unsatisfying these conditions is excluded from design storms.
3) Determine d_1 using Eq. 34 for D_n^* and a given value of ϕ_d, and calculate $d_i(i=2,\cdots,n)$ by Eq. 33.
4) Calculate $r_i(i=1,\cdots,n)$ for d_i using F_{ri} and ϕ_r.
5) Calculate $y_i(i=1,\cdots,n)$ for r_i using F_{yi} and ϕ_y.
6) Determine the sub-duration d_{oi} and the sub-depth r_{oi}, which are the duration and depth before and after, respectively, the peak for each storm part, by the following equations.

$$d_{oi} = \psi_d\, d_i\,; \quad d_{oi} = d_{bi} \text{ or } d_{ai} \quad (38)$$

$$r_{oi} = \psi_r\, r_i\,; \quad r_{oi} = r_{bi} \text{ or } r_{ai} \quad (39)$$

where ψ_d, ψ_r=coefficients($0<\psi_d,\psi_r<1$); and d_{bi}, d_{ai}; r_{bi}, r_{ai}=sub-durations and sub-depths before and after, respectively, the peak of the i-th storm part.

7) Search for a set of parameters: k and G which minimizes the sum-of-squares function(Eq. 21) for given values of the sub-duration d_{oi}, sub-depth r_{oi}, and peak intensity y_i for each one-side before and after the peak of a storm part. Calculate the temporal rainfall intensities by Eqs. 16 and 17 for each one-side of each storm part.

APPLICATION

Distributions of Numbers of Storm Clusters and Parts

Hourly rainfall observations for Kitoh in Tokushima prefecture, which belongs to a high-rain area with an annual mean precipitation of 3500mm. Since this area has the rainy and the typhoon seasons, observations for 30 years were divided into two seasons, the rainy season from May to July and the typhoon season from August to October. Using threshold values of x_c=3mm/hr and d_c=6hr and using 3rd-order simple moving average series, we counted the number N_c of storm clusters per season and the number N_p of storm parts per storm cluster, whose relative frequencies are shown in Figs. 3 and 4, respectively. The Kolmogorov-Smirnov test shows that the hypotheses of a Poisson distribution for N_c and a logarithmic series distribution for N_p can be accepted at a significance level of 90%.

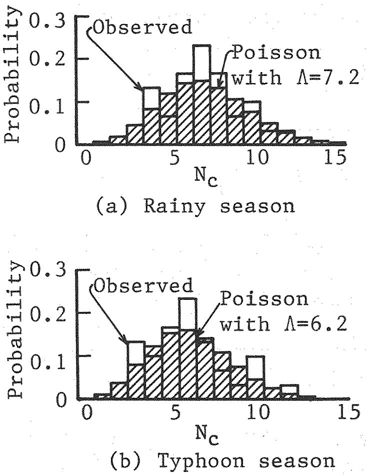

Fig. 3 Number N_c of storm clusters per season

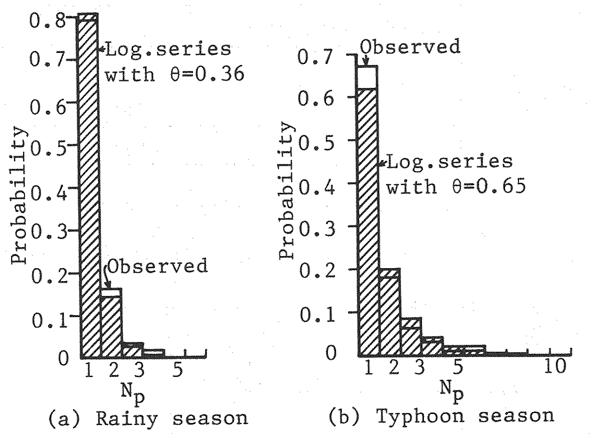

Fig. 4 Number N_p of storm parts per storm cluster

Cross and Auto Correlations of Three Components of Storm Parts

For three couples(d and r; d and y; and r and y) of components for a storm part, crosscorrelation coefficients ρ_{dr}, ρ_{dy}, and ρ_{ry}, were computed. Auto(part to part) correlation coefficients ρ_{dd}, ρ_{rr}, and ρ_{yy} of d, r, and y, respectively, were also computed, and shown in Table 1. From the inequality relation: $\rho_{ry}>\rho_{dr}>\rho_{dy}$ seen in Table 1 we can accept the approximation of Eq. 4. Since $\rho_{dd}=0$, the independence assumption of duration used in Eq. 26 can be accepted. From the fact of $\rho_{dr}>\rho_{rr}$ and $\rho_{ry}>\rho_{dy}$, ρ_{yy} the interval conditional probability distributions $F_r(\xi_i|\eta<\eta_i)$ and $F_y(\zeta_i|\xi<\xi_i)$ more effective than $F_y(\zeta_i|\eta<\eta_i)$ are used in modeling of design storm parts as seen in Eq. 35.

Table 1 Correlation coefficients for components(d,r,y) of storm parts

Correlation coefficient	Rainy season (May-July)	Typhoon season (Aug.-Oct.)
ρ_{ry}	0.872	0.834
ρ_{dr}	0.433	0.587
ρ_{dy}	0.150	0.272
ρ_{dd}	-0.026	-0.031
ρ_{rr}	0.148	0.133
ρ_{yy}	0.189	0.214

Results of data analyses for Tokushima and Osaka beside Kitoh in the rainy and typhoon seasons showed that ρ_{ry}(0.704-0.932) is the highest and ρ_{dr} (0.433-0.733) is higher than ρ_{dy}(0.049-0.557).

Fitting of Combined Freund Distributions

Table 2 shows means of three components d, r, and y for three groups: (i)single peak clusters, (ii) 2-5 peak clusters and (iii) 6 or more peak clusters in the typhoon season. We can clearly find from Table 2 that means of duration of storm parts are almost constant regardless of the number n of storm parts, while means of depth and local peak intensity are larger with increasing n.

Table 2 Means of components(d,r,y) of storm parts in the typhoon season

No. of storm parts	1	2 - 5	6 or more
d_i (hr)	8.7	8.8	9.0
r_i (mm)	57.9	86.1	122.0
y_i(mm/hr)	16.1	21.5	31.1

Figures 5 and 7 show the fitting of marginal distributions of combined Freund distributions of d and r to observations of storm parts for each group of storm clusters((i), (ii), and (iii)) in the typhoon season. Using the identified

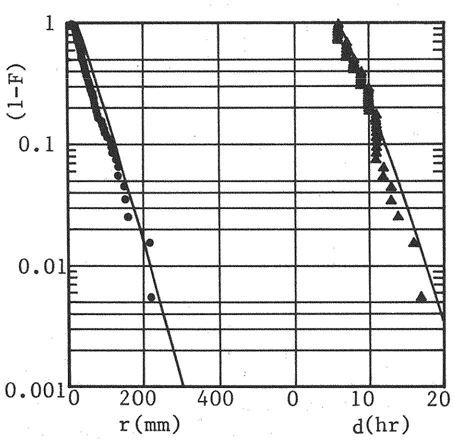

Fig. 5 Marginal distribution of d and r for single peak clusters

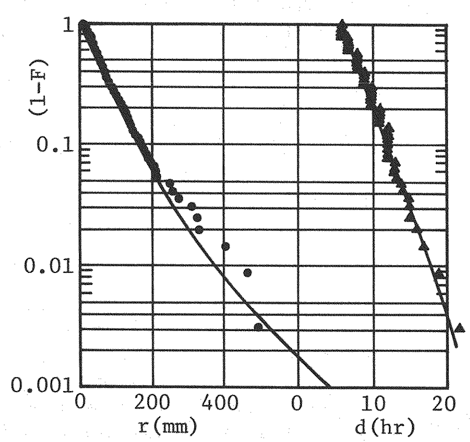

Fig. 6 Marginal distributions of d and r for 2-5 peak clusters

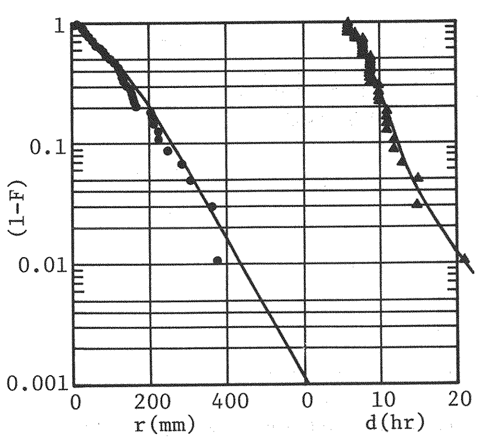

Fig. 7 Marginal distributions of d and r for 6 or more peak cluster

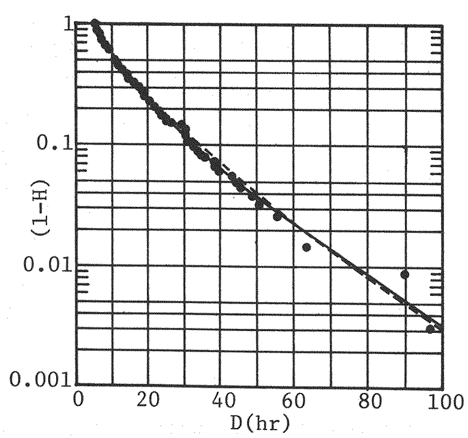

Fig. 8 Marginal distribution of D per storm cluster

parameters for three groups of storm clusters to Eqs. 26 and 27, we have the theoretical marginal distributions of the total duration D and the total depth R per storm cluster, which are shown in Figs. 8 and 9, respectively, where empirical distributions are plotted by Gringorten's formula.

Dotted lines in Figs. 8 and 9 represent the theoretical distributions for the case of no-grouping, while solid lines represent those for three groups of storm clusters. Figure 9 shows the solid line to be much better fitted to the empirical distribution than the dotted line. Therefore storm clusters should be divided into 2 or 3 groups based on the three properties of d, r, and y.

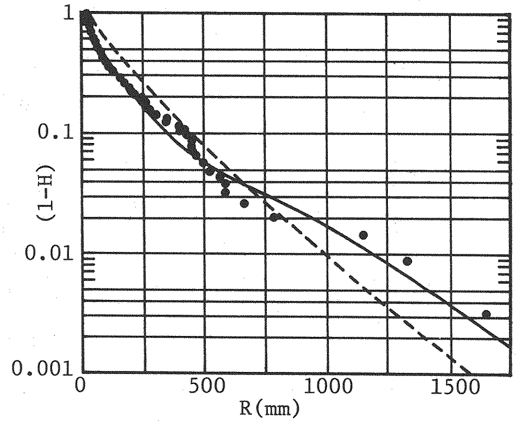

Fig. 9 Marginal distribution of R per storm cluster

Extremely Heavy Storms and Their Hyetographs

Historical extremely heavy storms for the period 1958-1987 at Kitoh are listed in Table 3, where d_{max}, r_{max}, and y_{max} denote the maxima of d_i, r_i, and $y_i(i=1,2,\cdots,n)$, respectively, and the underlines show the maxima of all observations for each item. The return periods T_{Dn} and T_d of total durations Dn and D calculated by Eqs. 28, 30, and 32 are also shown in Table 3. It was found

Table 3 Properties of extremely heavy storms and return periods of D_n and D

Date	N_p	d_{max} (hr)	r_{max} (mm)	y_{max} (mm/hr)	D (hr)	R (mm)	T_{Dn} (yr)	T_D (yr)
Aug.,1960	2	22	470	52	32	502	34	2
Aug.,1971	2	17	496	67	31	591	27	2
Sep.,1961	6	11	337	100	49	1113	16	5
Sep.,1976	11	15	252	63	97	1650	298	62

from Table 3 that the maximum storm parts of N_p=11 occurred in September, 1976, that the maxima of d_{max}, r_{max}, and y_{max} per storm part were 22hr in August, 1960, 496mm in August, 1971, and 100mm/hr in September, 1961, and that the maxima of total durations D and total depths R per storm cluster were 97hr and 1650mm in September, 1976.

The hyetographs of the 2-peak cluster in August, 1971 and the 6-peak cluster in 1961 are shown in Fig. 10, where symbol • denotes observed intensities, while solid lines denote fits obtained by using Eqs. 16 and 17 with parameters k and G

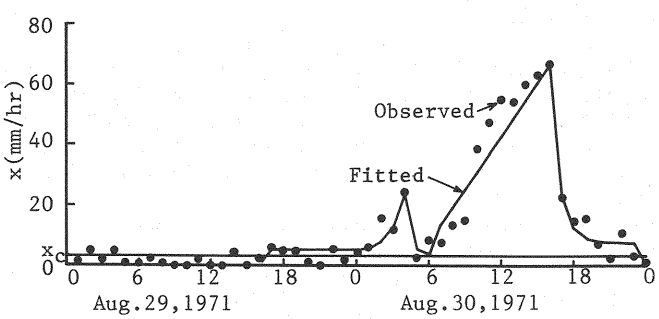

Fig. 10(a) Hyetograph of the 2-peak storm cluster in August, 1971

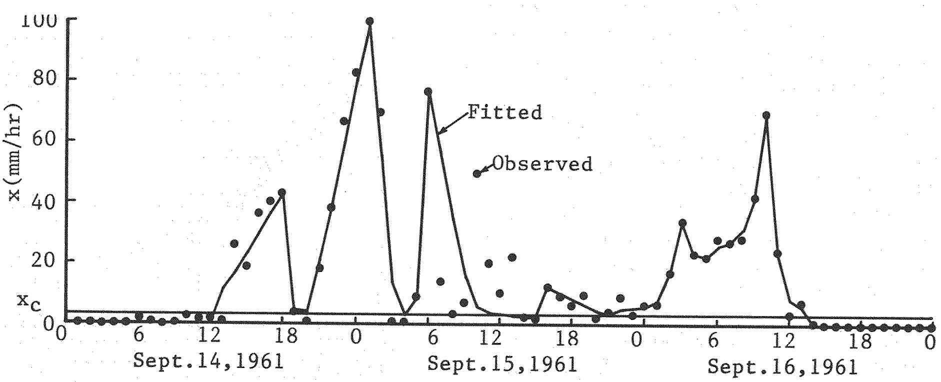

Fig. 10(b) Hyetograph of the 6-peak storm cluster in September, 1961

to each one-side hyetograph before and after the peaks of storm parts given observed hourly intensities. It is suggested from fairly good fit seen in Fig. 10 that if appropriate design values of d, r, and y of a design storm part are given taking account of properties of historical heavy storms, the optimal set of parameters k and G can provide a reasonable hourly hyetograph of the design storm part. The unsatisfactory fit after the peak of the third storm part seen in Fig. 10b is not due to a failure in searching for the optimal set of k and G, but due to that since the next of the third local peak has a duration less than d_c=6hr, the peak embeds in the third storm part.

Properties of Parameters for Design Storm

Regarding five parameters: $\phi_d \equiv d_i/d_{i-1}$ (where i=rank of a value in a data list ordered by descending magnitude of d), $\phi_r \equiv F_{ri}/F_{ri-1}$, $\phi_y \equiv F_{yi}/F_{yi-1}$, $\psi_d \equiv d_{bi}/d_i$, and $\psi_r \equiv r_{bi}/r_i$, which are defined by Eqs. 33, 36, 38, and 39, means and standard deviations are calculated for observations in the typhoon season, and shown in Table 4. Means of ϕ_d and ψ_r seemed to be almost constant regardless of the number n of storm parts, while means of ϕ_r, ϕ_y, and ψ_d tended to be slightly larger with increasing n. Since there was no remarkable tendency in occurrence sequence of duration d_i, as a design storm pattern we could adopt so called a last peaked type which has the longest duration d_1 in the last storm part of the design cluster.

Table 4 Means and standard deviations of parameters

Parameter	m	σ
ϕ_d	0.759	0.151
ϕ_r	0.492	0.328
ϕ_y	0.714	0.243
ψ_d	0.705	0.163
ψ_r	0.662	0.161

Examples of 2-Peak Storm Clusters

According to the procedure for making design storm patterns mentioned before, we can make storm patterns. Herein, let the design total depth R* with a design return period T_R^* and the total duration D_n^* of n storm parts be given by steps 1) and 2) in the procedure. Consider storm patterns with n=2 local peaks. With reference to the heaviest 2-peak cluster in August, 1971 in Fig. 10a, let R* and D_2^* be 600mm and 31hr, respectively.

Figure 11 shows three examples of 2-peak patterns made according to steps 3) to 7), for which we used values of parameters ϕ_d, F_{r1}, ϕ_r, F_{y1}, ϕ_y, ψ_d, and ψ_r shown in Table 5. In order of the procedure step the following explanation is added. In step 3) we used the mean of ϕ_d=0.759 in Table 4.

In step 4) the conditional probability F_{r1} of r_1 for the longest duration d_1

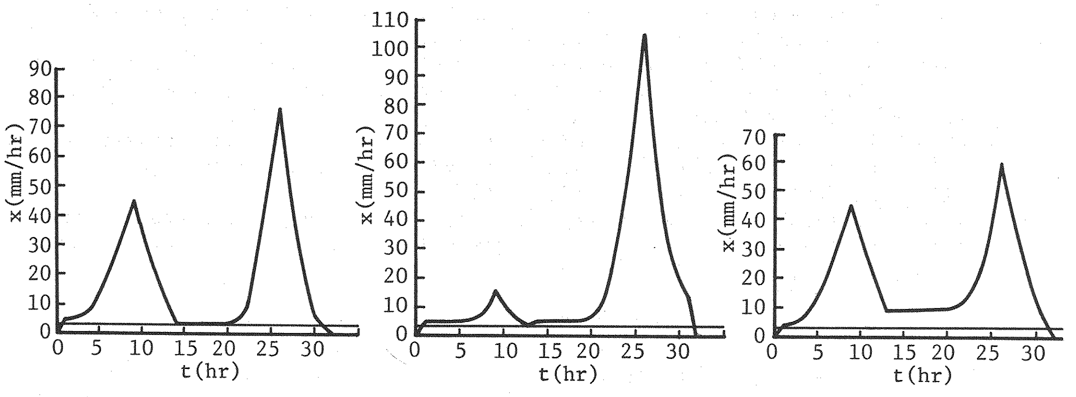

Fig. 11 Examples of 2-peak storm clusters

Table 5 Parameters for examples of 2-peak storm cluster

Case	ϕ_d	F_{r1}	ϕ_r	F_{y1}	ϕ_y	ψ_d	ψ_r
a	0.759	0.640	0.988	0.730	0.292	0.750	0.662
b	0.759	0.839	0.114	0.990	0.252	0.750	0.662
c	0.759	0.640	0.988	0.200	0.990	0.750	0.662

and the ratio ϕ_r are important especially in deciding storm patterns. Giving a large $F_{r1}(<1)$, we have a large depth r_1. Since the second conditional probability F_{r2} for r_2 is given by the product of F_{r1} and ϕ_r, the smaller ϕ_r is, the more we have different values of r_i as seen in Figs. 11a and 11b. Since the sum of r_i should be equal to R*, the allowable range of ϕ_r is restricted by the value of F_{r1}, and the residual of the sum of r_i to R* is modified by allotting it to r_i in proportion to the magnitudes of r_i.

In step 5) values of the conditional probability F_{y1} of y_1 for d_1 and the ratio ϕ_y should give a value of y_i less than $(r_i \cdot t_i/2)$, since the shape of a storm part is almost triangular. Giving a large F_{y1}, we have a large peak y_1. As seen in Figs. 11a and 11c, the smaller ϕ_y is, the more different the peak intensities y_i become each other. In step 6) combinations of ψ_d and ψ_r can give various shapes of storm parts, although we used means of ψ_d and ψ_r in Table 4 for these examples.

CONCLUDING REMARKS

It is clear from observations at Kitoh in Tokushima prefecture that the number of storm clusters per season follows a Poisson process, and that the number of storm parts per storm cluster follows a logarithmic series probability distribution. By using combined Freund bivariate probability distribution functions the trivariate joint distribution of duration, depth, and local peak for a storm part is approximately defined. For a single storm cluster with multi-storm parts the joint distribution function of total duration, local depths, and local peak intensities is theoretically derived from those of storm parts. With reference to the heaviest 2-peak cluster in August, 1971, three examples of 2-peak design storm clusters with R*=600mm and D_2^*=31hr are demonstrated.

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APPENDIX - NOTATION

The following symbols are used in this paper:

a_1, b_1, a_2, b_2 = parameters for the lower class density functions of the combined Freund bivariate probability distribution;

d, r, y = duration, depth and peak intensity, respectively, of a single storm part;

d_c = threshold value of storm duration and time between storms;

d_i, r_i, y_i = duration, depth and peak intensity, respectively, of the i-th storm part;

d_o, r_o = sub-duration and sub-depth before or after a local peak, respectively;

$d_{max}, r_{max}, y_{max}$ = maxima of d_i, r_i, and y_i, respectively;

d_{ai}, a_{bi} = sub-durations after and before, respectively, the peak of the i-th storm part;

d_{oi} = sub-duration equal to d_{ai} or d_{bi};

D, R = total duration and total depth, respectively, of a storm cluster;

D_b = time between storms;

D_n, R_n = total duration and total depth of a storm cluster with n storm parts;

D_p, R_p, Y_p = probability variables of duration, depth, and local intensity, respectively, for a single storm part;

$f_d(\eta), f_r(\xi)$ = marginal probability density functions of dimensionless variates η and ξ, respectively;

$f(\eta, \xi)$ = joint probability density function of η and ξ;

$F(A|B), G(A|B)$ = conditional probability distribution functions of argument A given argument B;

$F_d(\eta), F_r(\xi)$ = marginal probability distribution functions of dimensionless variates η and ξ, respectively;

$F_d^{n*}(\eta), F_r^{n*}(\xi)$ = probability distribution functions of dimensionless variates η and ξ obtained from the n-fold convolution of the marginal distributions $F_d(\eta)$ and $F_r(\xi)$, respectively;

$F_{dr}(\eta,\xi)$, $F_{ry}(\xi,\zeta)$ = combined Freund probability distribution functions of (η,ξ) and (ξ,ζ);

$F_r(\xi_i|\eta<\eta_i)$, $F_y(\zeta_i|\xi<\xi_i)$, $F_y(\zeta_i|\eta<\eta_i)$ = interval-conditional probability distribution functions of ξ_i, ζ_i, and ζ_i given $\eta<\eta_i$, $\xi<\xi_i$, and $\eta<\eta_i$, respectively;

F_{ri}, F_{yi} = given values of $F_r(\xi_i|\eta<\eta_i)$ and $F_y(\zeta_i|\xi<\xi_i)$, respectively;

G = given value of the conditional probability distribution function $G(z_i|z_{i-1})$;

G_a, G_b = conditional no-exceedance probabilities for rainfall intensities after and before the peak;

H(r) = univariate probability distribution function of r;

$H_D(D|N_c=1)$, $H_R(R|Nc=1)$ = probability distribution functions of total duration D and total depth R, respectively, of a storm cluster;

H(d,r), H(r,y) = bivariate joint probability distribution function of (d,r) and (r,y), respectively;

H(d,r,y) = trivariate joint probability distribution function of d, r and y;

$H_D(\chi_D)$, $H_R(\chi_R)$ = non-exceedance probability distribution functions of the annual maxima χ_D and χ_R, respectively;

$H_D^{(n)}(\chi_{Dn})$, $H_R^{(n)}(\chi_{Rn})$ = non-exceedance probability distribution functions of the annual maxima χ_D and χ_R, respectively, of a n-peak storm cluster;

i, j, n = counting variates;

$k = \alpha/\beta$ = parameter related to the autocorrelation coefficient ρ_z of rainfall intensity;

N_{11}, N_{12}, N_{21}, N_{22} = sample sizes of bivarate (η,ξ) satisfying regions: $(0<\xi<\eta,\ \xi<u_1)$, $(0<\eta<\xi,\ \eta<u_1)$, $(u_1<\xi<\eta)$, and $(u_1<\eta<\xi)$, respectively;

N_c = number of storm clusters in a season;

N_p = number of storm parts per storm cluster;

$P[\cdot]$ = probability of argument;

$P[A|B]$ = conditional probability of argument A given argument B;

r_{ai}, r_{bi} = sub-depths after and before, respectively, the peak of the i-th storm part;

r_{oi} = sub-depth equal to r_{ai} or r_{bi};

$S(k,G|d_o,r_o,y)$ = sum-of-squares function defined by Eq. 21;

t = time;

t_a, t_b = time after and before the peak;

T_D, T_{Dn} = return periods of χ_D and χ_{Dn};

T_R, T_{Rn} = return periods of χ_R and χ_{Rn};

u_1 = critical value of η and ξ that classifies the combined Freund distribution into the two regions shown in Eqs. 7a, 7b and 7c, 7d;

u_{dc}, u_{rc}, u_z = specified base levels of duration, depth, and rainfall intensity, respectively;

v_1 = constant;

x = rainfall intensity;

x_{ba} = dummy parameter defined by Eq. 22;

x_c = threshold value of rainfall intensity;

x_i = rainfall intensity at discrete time i;

x_L = calculated rainfall intensity at the last hour of the sub-storm part after the local peak considered;

x_L' = calculated rainfall intensity at the last hour of the preceding storm part;

z_i = reduced variate of x_i as defined by Eq. 16;

α, β = parameters equal to $a_1=b_1$ and $a_2=b_2$, respectively;

α_1, β_1, α_2, β_2 = parameters for the upper class density functions of the combined Freund bivariate probability distribution;

$\gamma = -\{\ln(1-\theta)\}^{-1}$ = parameter defined by θ;

θ = parameter of the logarithmic series probability distribution;

λ = parameter defined by Eq. 19;

Λ = occurrence rate of storm clusters per season;

η, ξ, ζ = dimensionless variates of d, r, and y, respectively;

η_i, ξ_i, ζ_i = dimensionless variates for d_i, r_i, and y_i of the i-th storm part, respectively;

$(\eta_j^{\langle 11\rangle}, \xi_j^{\langle 11\rangle})$, $(\eta_j^{\langle 12\rangle}, \xi_j^{\langle 12\rangle})$, $(\eta_j^{\langle 21\rangle}, \xi_j^{\langle 21\rangle})$, $(\eta_j^{\langle 22\rangle}, \xi_j^{\langle 22\rangle})$ = satisfying the regions: $(0<\xi<\eta,\ \xi<u_1)$, $(0<\eta<\xi,\ \eta<u_1)$, $(u_1<\xi<\eta)$, and $(u_1<\eta<\xi)$, respectively;

$\eta^{(n)}$, $\xi^{(n)}$ = dimensionless variates of χ_D and χ_R;

ρ_z = autocorrelation coefficient of rainfall intensity defined by Eq. 20;

ρ_{dr}, ρ_{dy}, ρ_{ry}; ρ_{dd}, ρ_{rr}, ρ_{yy} = cross and auto correlation coefficients for components (d,r,y) of storm parts;

$\rho_{\eta\xi}$ = crosscorrelation coefficient between η and ξ;

σ_d, σ_r, σ_z = standard deviations of variables: $(d-u_{dc})$, $(r-u_{rc})$, and (x_i-u_z), respectively;

$\phi_d = d_i/d_{i-1}$ = parameters representing the ratio of the i-th duration for the (i-1)-th duration;

$\phi_r = F_{ri}/F_{ri-1}$, $\phi_y = F_{yi}/F_{yi-1}$ = parameters representing ratios of non-exceedance probabilities for depth and local peak intensity, respectively;

χ_D, χ_R = annual maxima of total durations D and total depths R of storm clusters occurred within the season;

χ_{Dn}, χ_{Rn} = annual maxima of total durations D_n and total depths R_n of storm clusters with n storm parts occurred within the season; and

$\psi_d = d_{oi}/d_i$, $\psi_r = r_{oi}/r_i$ = parameters representing the ratios of sub-durations and sub-depths for durations and depths of the i-th storm

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