Journal of Hydroscience and Hydraulic Engineering
Vol. 9, No. 1, May, 1991, 49-61

NON-EQUILIBRIUM PROFILE OF SUSPENDED SEDIMENT CONCENTRATION

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ABSTRACT

The concentration distribution of suspended sediment under non-equilibrium conditions due to the change of bed shear stress is investigated by focussing an analogy between the transition process of the Reynolds-stress distribution and that of the turbulent flux of suspended sediment. The impulse response of the Reynolds-stress distribution empirically determined and confirmed by the recent turbulence measurements in open channel flows is applied to the transition process of the turbulent flux of suspended sediment under non-equilibrium conditions. By using the boundary condition at the water surface, the non-equilibrium concentration profile of suspended sediment is deduced. Subsequently, the non-equilibrium process of the bottom concentration of the suspended sediment is discussed. As a result, the non-equilibrium process of suspended sediment can be reasonably described by a simple method.

INTRODUCTION

Recently sediment transport under unsteady, non-uniform conditions is one of the important topics in fluvial hydraulics and several papers or lectures have emphasized in several international symposia (Jain (10), Di Silvio (7), Klaassen (15), Tsujimoto (21), Yen (29) and others). The report of the research group encouraged by the Hydraulic Committee of JSCE (Tsujimoto et al. (23)) also pointed out its importance, and it reviewed the recent works.

Bed-material load is in general treated by dividing it into bed load and suspended load. The latter has longer excursion length than the step length of the former, and thus the non-equilibrium process of suspended load is more important than that of bed load from the view point of engineering. Since the experiment of Yalin & Finlayson (28) where they measured the profiles of suspended sediment concentration on the movable bed at the downstream region of a rigid bed, lots of researchers were interested in the non-equilibrium profile of suspended sediment.

As for the bed-load motion, Nakagawa & Tsujimoto (18) developed a model characterized by the pick-up rate and the step length, and succeeded in explaining non-equilibrium bed-load transport and the subsequent sand bed instability as a cause of small scale sand waves reasonably. In their model, the distribution function of the step length plays a role of the impulse response of the bed-load transport rate to the spatial change of the pick-up rate which responds to the change of the bed shear stress immediately, and in this sense, their model might be called a "relaxation model."

As for the suspended sediment, Nakagawa et at. (19) proposed a similar model to their non-equilibrium bed-load model, where the transition probability density per unit time for a bed-load particle turns into suspension and the excursion length of a suspended particle were used instead of the pick-up rate and the step length for bed load, respectively. Although it can explain the non-equilibrium process of the transport rate, it cannot explain the change of the concentration profile which is also appreciable and practically important for suspended sediment.

The aforementioned models belong to the Lagrangian modelling, and its more typical one is stochastic simulation. Ashida & Fujita (2), Bechteler & Fäber (5) and others tried stochastic simulation of suspended sediment transport under non-equilibrium and obtained interesting results. On stochastic simulation of suspended particles, however, there still remain problems such as that it cannot determine the profile analytically and how to choose the time step in the calculation reasonably.

Another approach is one based on a diffusion equation but it cannot be accomplished without numerical computation. Some numerical computations were tried by Kerssens et al. (14), Michiue et al. (17), Çelik & Rodi (6), and they succeeded in explaining several experiments of non-equilibrium suspended sediment transport (Yalin & Filayson (28), van Rijn (27), Ashida & Okabe (4)). Moreover, Kerssens et al. (14) and Michiue et al. (17) investigated the adaptation length of suspended sediment concentration profile to the change of supplied sediment concentration and the change of bed shear stress, respectively. The previous works on this line, however, could not deduce relatively general characteristics of non-equilibrium suspended sediment transport systematically.

Recently, Kuroki et al. (16) proposed a mathematical expression of non-equilibrium suspended sediment concentration profile based on a two-dimensional diffusion equation by adaptating it to satisfy the boundary conditions but without directly solving it. According their method, the non-equilibrium profile can be deduced if the bottom concentration is evaluated. They estimated non-equilibrium bottom concentration based on the depth-averaged one dimensional diffusion equation by applying the proposed relative concentration profile. However, despite the longitudinal variation of bottom concentration is affected by non-equilibrium concentration profile and the non-equilibrium profile is obviously a function of the longitudinal distance, but these facts are not taken into account in their model.

This study to describe the non-equilibrium process of suspended sediment is based on the following two keys: (i) an analogy between the momentum exchange by turbulence (the Reynolds stress) and the exchange of suspended sediment by turbulence; and (ii) the relaxation model of the Reynolds-stress distribution of which relaxation distance increases with the relative height from the bed. Based on the above ideas, the unit impulse response of the turbulence flux of suspended sediment can be identified with that of the Reynolds-stress. The relaxation model with the impulse response will describe the non-equilibrium process of suspended sediment for arbitrary pattern of the variation of bed shear stress.

RELAXATION MODEL OF REYNOLDS-STRESS DISTRIBUTION

When the impulse response is known, the transition process of the Reynolds-stress distribution can be described as follows (Tsujimoto et al. (22)):

$$\tau(\eta\,|\xi) = \int_0^\infty \tau_e(\eta\,|\xi-\delta)\cdot g_R(\delta\,|\eta)d\delta \tag{1}$$

in which $\tau(\eta\,|\xi)$=Reynolds-stress distribution at ξ; $\xi \equiv x/h$; x=longitudinal distance; $\eta \equiv y/h$; y=distance from the bed; h=flow depth in the case of open channel flow; $\tau_e(\eta\,|\xi)$=equilibrium Reynolds-stress distribution; and $g_R(\delta\,|\eta)$=impulse response of Reynolds stress at the relative height η. The impulse response is often approximated by an exponential function as follows:

$$g_R(\xi\,|\eta) = \frac{1}{\Lambda(\eta)}\exp\left[-\frac{\xi}{\Lambda(\eta)}\right] \tag{2}$$

in which $\Lambda(\eta)$=relaxation scale non-dimensionalized by h at the relative height η. $\Lambda(\eta)$ was determined as follows according to the wind-tunnel data by Jacobs (11):

$$\Lambda(\eta) = 20\eta(1+1.5\eta^3) \tag{3}$$

Under equilibrium, the Reynolds-stress distribution is triangular, and thus Eq.1 is rewritten as

$$\tau(\eta\,|\,\xi) = (1-\eta)\int_0^{\infty}\tau_b(\xi-\delta)\cdot g_R(\delta\,|\,\eta)\mathrm{d}\delta \equiv (1-\eta)\cdot\tau_b(\xi)\cdot\Gamma(\eta\,|\,\xi) \tag{4}$$

$$\Gamma(\eta\,|\,\xi) \equiv \frac{\int_0^{\infty}\tau_b(\xi-\delta)\cdot g_R(\delta\,|\,\eta)\mathrm{d}\delta}{\tau_b(\xi)} = \int_0^{\infty}\psi_b(\xi-\delta)\cdot g_R(\delta\,|\,\eta)\mathrm{d}\delta \tag{5}$$

in which $\tau_b(\xi)$=longitudinal change of bed shear stress; and $\psi_b(\xi-\delta)\equiv\tau_b(\xi-\delta)/\tau_b(\xi)$. $\Gamma(\eta\,|\,\xi)$ represents the degeneration of the Reynolds-stress profile from the equilibrium profile $(1-\eta)$. When the mixing length theory is applied to the degenerated Reynolds-stress distribution, the velocity distribution under non-equilibrium is obtained by integrating the following equation:

$$\frac{\mathrm{d}u^+}{\mathrm{d}\eta} = \frac{\sqrt{\Gamma(\eta\,|\,\xi)}}{\kappa\eta} \tag{6}$$

in which $u^+\equiv u/u_*$; κ=Kármán constant; u=local velocity; u_*=shear velocity and the mixing length is assumed to be $\kappa y\sqrt{1-\eta}$. The above model was applied to the recently refined turbulence measurements for open channel flow with an abrupt change of bed roughness by Nezu et al. (20), and it was clarified that the calculation based on the present model could well describe the data without any modification of impulse response determined by wind-tunnel data (The detail was reported in another paper of the authors' (22)).

TURBULENT FLUX OF SUSPEDED SEDIMENT UNDER NON-EQUILIBRIUM

The turbulent momentum flux (the Reynolds stress) and the turbulent flux of suspended sediment are written as follows, respectively:

$$\tau = -\rho\overline{u'v'} = \rho\nu_T\frac{\mathrm{d}u}{\mathrm{d}y}\ ; \qquad \Psi \equiv \overline{c'v'} = -\varepsilon_s\frac{\mathrm{d}C}{\mathrm{d}y} \tag{7}$$

in which u' and v'=turbulence components in the longitudinal and the vertical directions, respectively; ρ=mass density of fluid; ν_T=eddy kinematic viscosity (turbulent diffusion coefficient of momentum); c'=concentration fluctuation of suspended sediment; C=concentration of suspended sediment; and ε_s=turbulent diffusion coefficient of suspended sediment.

Considering the analogy between the mass exchange and the momentum exchange due to turbulence, one can write the following equation for the relaxation process of the turbulent flux of sediment with the common impulse response:

$$\Psi(\eta\,|\,\xi) = \int_0^{\infty}\Psi_e(\eta\,|\,\xi-\delta)\cdot g_R(\delta\,|\,\eta)\mathrm{d}\delta \tag{8}$$

in which $g_R(\delta\,|\,\eta)$ is given by Eq.3; and the subscript e represents a value under equilibrium.

Under equilibrium, the upward flux of sediment by turbulence ($\overline{c'v'}$) must equal the falling flux by the terminal velocity (Cw_0), and thus,

$$\Psi_e(\eta\,|\,\xi) = w_0\cdot C(\eta\,|\,\xi) \tag{9}$$

in which w_0=terminal velocity of sediment. By applying Eq.7 to it, the equilibrium concentration profile of suspended sediment is obtained. When the diffusion coefficient of suspended sediment is identified with the depth-averaged value of the eddy kinematic viscosity ($\varepsilon_s = \kappa u_* h/6$), the equilibrium concentration profile is obtained as follows:

$$\frac{C_e(\eta)}{C_a} = \exp(-E\eta) \qquad (10)$$

in which C_a=bottom concentration (C at η=0); and $E \equiv w_0 h/\varepsilon_s = (6/\kappa)(w_0/u_*)$. Then,

$$\Psi_e(\eta\,|\xi) = C_{ae} w_0 \exp(-E\eta) \qquad (11)$$

As for the bottom concentration under equilibrium, several works were carried out, but still it is difficult what gives the best evaluation among them. Thus, the simple method is applied here based on Einstein's idea (8), where it is assumed that the thickness of the bed-load layer is twice the sand diameter, the bed-load particle's speed is $11.6u_*$ and the bed-load discharge is estimated by the so-called Swiss formula. Then,

$$C_{ae} = 0.345\tau_*\left(1 - \frac{\tau_{*c}}{\tau_*}\right)^{1.5} \qquad (12)$$

in which $\tau_* \equiv u_*^2/[(\sigma/\rho - 1)gd]$; τ_{*c}=dimensionless critical tractive force; σ=mass density of sediment; g=gravitational acceleration; and d=sediment diameter. When the properties of sediment are given, τ_* can be replaced by (w_0/u_*), as follows:

$$\tau_* = \left(\frac{u_*}{w_0}\right)^2\left[\frac{w_0}{\sqrt{(\sigma/\rho - 1)gd}}\right]^2 \;; \qquad \frac{w_0}{\sqrt{(\sigma/\rho - 1)gd}} = \sqrt{\frac{2}{3} + \frac{36}{d^*}} - \sqrt{\frac{36}{d^*}} \qquad (13)$$

in which $d^* \equiv (\sigma/\rho - 1)gd^3/\nu^2$. The second equation of Eq.13 is Rubey's equation. As for the critical tractive force, Iwagaki's formula (9) is familiar, and it can be approximated by the following equation.

$$\frac{u_{*c}}{w_0} = (0.0455d^*)^{-0.65} + 0.25 \qquad (14)$$

in which u_{*c}=shear velocity corresponding to the critical tractive force. As shown in Fig.1, Eq.14 well approximates the Iwagaki's formula.

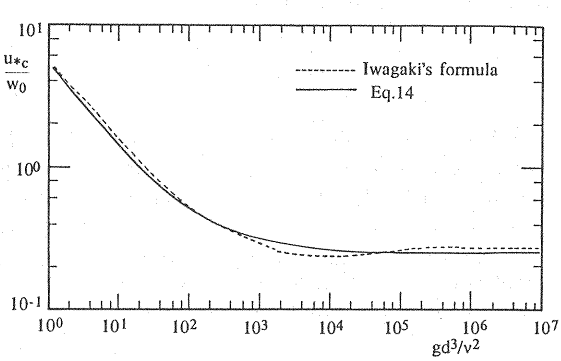

Fig.1 Critical shear velocity

When the shear velocity changes abruptly at ξ=0 from u_{*0} to u_{*1}, the transition process of the vertical distribution of the turbulent flux of suspended sediment is expressed as follows:

$$\Psi(\eta\,|\xi) = \Psi_{0e}(\eta)\cdot\exp\left(-\frac{\xi}{\Lambda(\eta)}\right) + \Psi_{1e}(\eta)\cdot\left[1 - \exp\left(-\frac{\xi}{\Lambda(\eta)}\right)\right] \qquad (15)$$

in which

$$\Psi_{e0}(\eta) = C_{ae0} w_0 \cdot \exp(-E_0\eta) \;; \qquad \Psi_{e1}(\eta) = C_{ae1} w_0 \cdot \exp(-E_1\eta) \qquad (16)$$

The subscripts 0 and 1 represent the values before and after the abrupt change of the bed shear stress, respectively. Eq.15 is normalized by the bottom value of the turbulent flux of sediment before the change of the bed shear stress ($\Psi_{00} \equiv \Psi_{e0}(0) = C_{ae0}w_0$) as follows:

$$\frac{\Psi(\eta\,|\xi)}{\Psi_{00}} = \exp(-E_0\eta)\bullet\exp\left[-\frac{\xi}{\Lambda(\eta)}\right] + \gamma_c\exp(-\gamma_E E_0\eta)\left\{1-\exp\left[-\frac{\xi}{\Lambda(\eta)}\right]\right\} \tag{17}$$

in which $\gamma_c \equiv C_{ae1}/C_{ae0}$; and $\gamma_E \equiv E_1/E_0 = u_{*0}/u_{*1}$. The relation between γ_c and (u_{*0}/u_{*1}), which is obtained by Eqs.12 and 13, is shown in Fig.2. In this figure, the relation between γ_E and (u_{*0}/u_{*1}) is also depicted.

The calculated examples of non-equilibrium distribution of the turbulent flux of suspended sediment are shown in Fig.3. The flux responds to the new equilibrium increasingly from the bed to the region far from the bed. The data of the turbulent flux of the suspended sediment in the transition process recently obtained by Kanda et al. (13), which shows appreciable scattering, suggest that the present model can describe the behavior of the turbulent flux of suspended sediment in the non-equilibrium process properly.

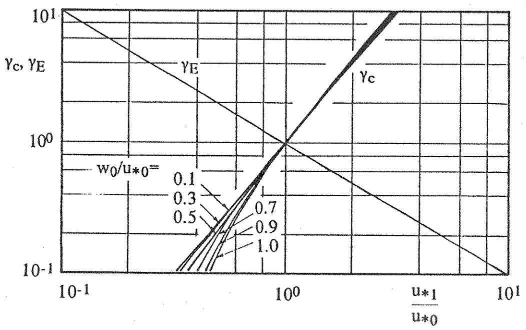

Fig.2 Variation of γ_c and γ_E against u_{*1}/u_{*2}

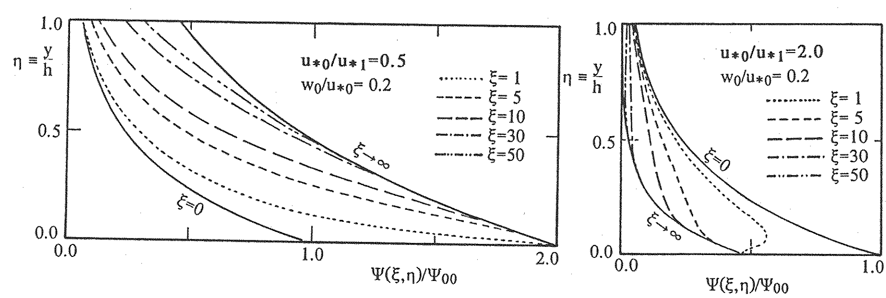

Fig.3 Transient process of turbulent flux of suspended sediment due to abrupt change of shear velocity

NON-EQUILIBRIUM CONCENTRATION PROFILE OF SUSPENDED SEDIMENT

When the turbulence flux of suspended sediment is estimated, its integration through the flow depth deduces a concentration profile as follows:

$$C(\eta\,|\xi) = -\frac{h}{\varepsilon_s}\int_0^{\eta}\Psi(\zeta\,|\xi)\mathrm{d}\zeta + C_a(\xi) \equiv C_{ae0}\Omega(\eta\,|\,\xi) + C_a(\xi) \tag{18}$$

in which ε_s is the depth averaged value and thus it is independent of η. $\Omega(\eta\,|\,\xi)$ implies that

$$\Omega(\eta\,|\xi) \equiv \frac{C(\eta\,|\xi)-C_{a}(\xi)}{C_{ae0}} \tag{19}$$

The relation between Ω and η is depicted in Fig.4 with ξ as a parameter. Although $C_a(\xi)$ has not been determined yet, $\Omega(\eta\,|\xi)$ expresses that the concentration profile reaches the equilibrium one increasingly from the bottom.

When the adaptation length L_{as} is defined as the distance where the difference between Ω and its new equilibrium Ω_e reaches ±5% of Ω_e at the water surface (η=1), the relationship between the adaptation length and (u_{*0}/u_{*1}) is obtained as shown in Fig.5. Such an adaptation length increases with the difference between u_{*1} and u_{*2}, and it takes a longer distance to adapt the profile to the new equilibrium one in the case of deposition ($u_{*1}<u_{*0}$) than in the case of erosion ($u_{*1}>u_{*0}$). Moreover, the adaptation length for the larger value of (w_0/u_{*1}) is longer.

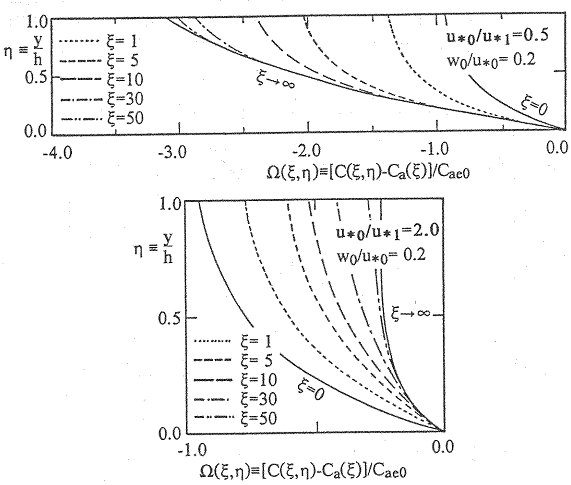

Fig.4 Longitudinal variation of $\Omega(\eta)$ in the transient process

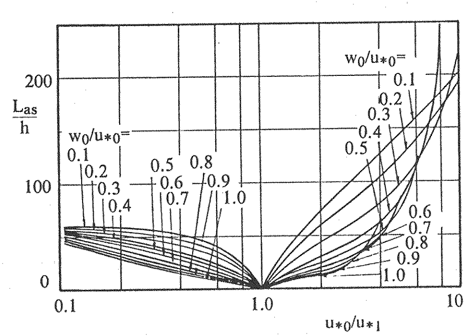

Fig.5 Adaptation length of non-equilibrium suspended sediment concentration profile

The bottom boundary condition is difficult to determine under non-equilibrium. On the other hand, the boundary condition at $y=h$ (η=1) is easily obtained by the fact that a suspended particle cannot pass through the free surface, and that is,

$$w_0 C(1\,|\xi) = \Psi(1\,|\xi) \tag{20}$$

The right hand side has been already known by applying the relaxation model, and thus the boundary condition at the free surface is determined as follows:

$$\frac{C(1\,|\xi)}{C_{\mathrm{ae0}}} = \exp(-E_0)\bullet\exp\Big(-\frac{\xi}{\Lambda_s}\Big) + \gamma_c\exp(-\gamma_E E_0)\bullet\Big[1 - \exp\Big(-\frac{\xi}{\Lambda_s}\Big)\Big] \tag{21}$$

in which $\Lambda_s \equiv \Lambda(1)$.

Finally, the non-equilibrium suspended sediment concentration distribution is obtained as follows:

$$\frac{C(\eta\,|\xi)}{C_{\mathrm{ae0}}} = \Omega(\eta\,|\xi) - \Omega(1\,|\xi) + \frac{C(1\,|\xi)}{C_{\mathrm{ae0}}} \tag{22}$$

The calculated non-equilibrium suspended sediment concentration distributions are shown in Fig.6. The gradient of the concentration profile adapts the new condition increasingly from the bed. When the concentration is normalized by the local bottom concentration, the calculated results are shown in Fig.7, where the change of the profile is not monotonous.

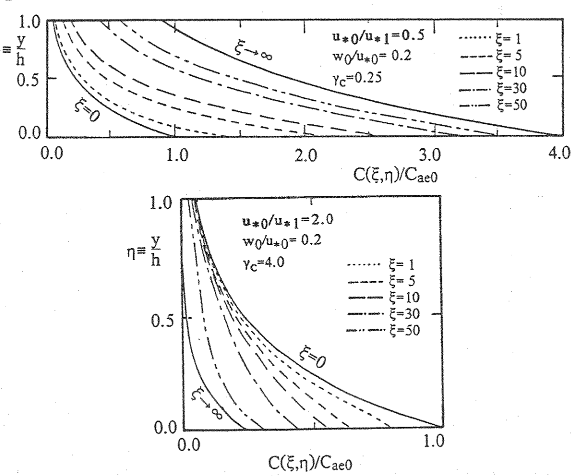

Fig.6 Variation of suspended sediment concentration distribution under non-equilibrium conditions

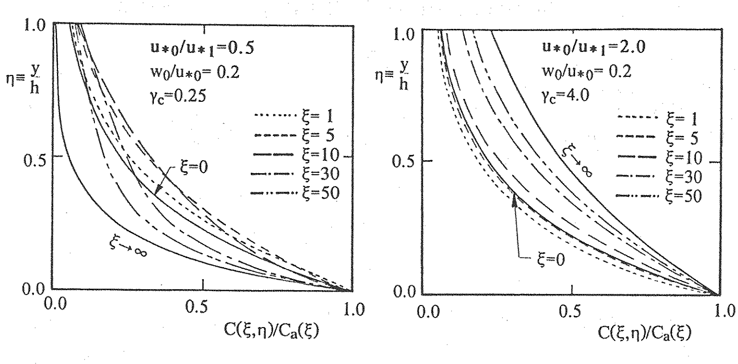

Fig.7 Variation of normalized profile of sediment concentration under non-equilibrium conditions

Now that the absolute concentration is deduced as Eq.22, the bottom concentration is easily obtained by setting η=0, as follows:

$$\frac{C_a(\xi)}{C_{\mathrm{ae0}}} = \frac{C(1\,|\xi)}{C_{\mathrm{ae0}}} - \Omega(1\,|\xi) \tag{23}$$

The calculated non-equilibrium process of the bottom concentration is demonstrated in Fig.8. As for the bottom concentration, the adaptation length is also defined as a distance at which the difference between the bottom concentration and its equilibrium value reaches ±5% of the equilibrium bottom concentration. The adaptation length of the bottom concentration, L_{ac}, is plotted against (u_{*0}/u_{*1}) with (w_0/u_{*0}) as a parameter, in Fig.9. L_{ac} is the same order of L_{as}, but L_{ac} is relatively longer than L_{as} in the case of $u_{*1}>u_{*0}$.

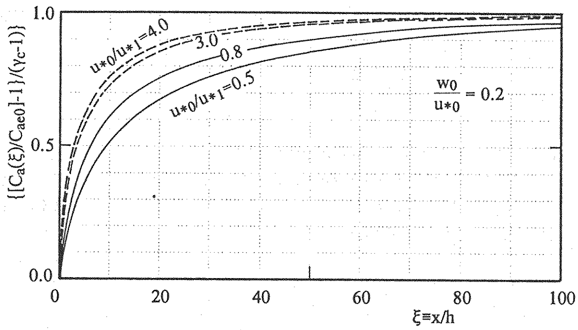

Fig.8 Variation of bottom concentration of suspended sediment under non-equilibrium conditions

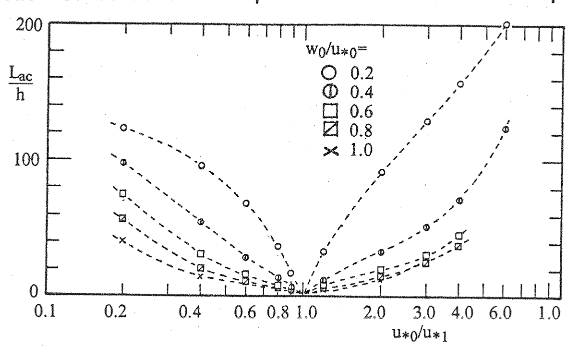

Fig.9 Adaptation length of bottom concentration of non-equilibrium suspended sediment

COMPARISON WITH EXPREIMENTAL DATA

Several experiments were conducted for non-equilibrium suspended load transport process at the downstream of a rigid bed (Yalin & Finlayson (28), Van Rijn (27)). In this case, the shear velocity does not change (u_*). The equilibrium bottom concentration is easily estimated and it is represented by C_{ae} here. The transition process of the turbulent flux of suspended sediment is described as follows:

$$\frac{\Psi(\eta\,|\,\xi)}{\Psi_e} = \exp(-E\eta)\cdot\left[1 - \exp\left(-\frac{\xi}{\Lambda(\eta)}\right)\right] \qquad (24)$$

in which $\xi \equiv x/h$; x=distance from the upstream end of the mobile bed; and Ψ_{e0}=equilibrium turbulent flux of suspended sediment. The boundary condition at the free surface is written as

$$\frac{C(1\,|\,\xi)}{C_{ae}} = \exp(-E)\cdot\left[1 - \exp\left(-\frac{\xi}{\Lambda_s}\right)\right] \qquad (25)$$

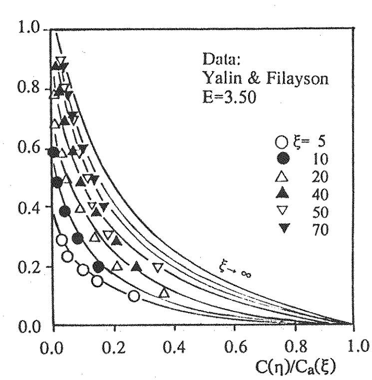

Fig.10 Example of non-equilibrium concentration distribution downstream of rigid bed (data by Yalin & Finlayson)

Integrating Eq.24 with the boundary condition Eq.25, one can obtain the non-equilibrium concentration distribution of suspended sediment. Fig.10 demonstrates the comparison between the experimental data of Yalin & Finlayson (26) and the calculated results. Though there were no description on the detail experimental condition in their paper, E was estimated from the equilibrium profile they presented in the paper.

As for more general non-equilibrium suspended sediment transport, there were few data in the previous researches because of the difficulties of the adjustments of the conditions. Nevertheless, Ashida & Okabe (1) and Kanda et al. (13) conducted interesting experiments. They controled the sediment supply at the upstream end of the flume, and changed the bed roughness. Under such conditions, sediment transport was unsaturated (it was rather transported as wash load) but the profile could reach equilibrium for the given bed shear stress. The change of the bed roughness implies the change of the bed shear stress. Thus, for example, the growth of the concentration profile is observed in the rough bed at the downstream of a smooth bed; while the decay of the profile in the smooth bed at downstream of the rough bed.

The non-equilibrium concentration distribution of suspended sediment in such cases were calculated and compared with the experimental results (Kanda et al. (13)) in Fig.11. It demonstrates a good agreement between the experimental data and the calculated results. In such cases, the value of γ_c cannot be determined because the sediment transport rate even with the equilibrium profile has no unique relation with the hydraulic parameter (because it is wash load or unsaturated). In calculation, γ_c was estimated from the data of the concentrations with equilibrium profiles. The value of γ_E was also estimated from the concentration profiles under equilibrium before and after the change of the bed roughness. The estimated values of them are shown in the figures.

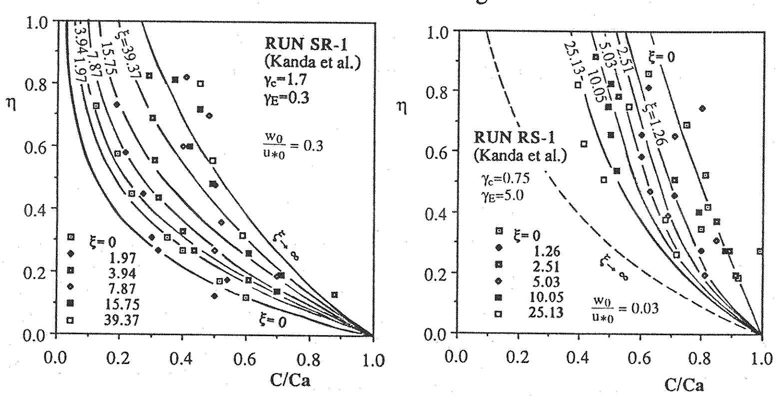

Fig.11 Examples of development and decaying processes of suspended sediment concentration distributions (Experimental data were obtained by kanda et al.)

DISCUSSION ON BOTTOM CONCENTRATION

In the above treatment, the concentration distribution is determined by applying the boundary condition at the water surface. This is available when the diffusion coefficient at the water surface is not zero as assumed to be constant in the present analysis. If the parabolic distribution of the diffusion coefficient, which deduces Rouse's equation, is applied, the zero diffusion coefficient at η=1 leads C=0 at η=1 and thus the boundary condition at the free surface becomes meaningless.

Though the boundary condition at the bottom is reversely determined in the preceding analysis in this study, it is not impossible to determine it directly. The analyses were already conducted by Ashida (1) and Kalinske (12). A similar approach is performed hereinafter.

For simplicity, it is assumed that the flow is quasi-uniform, and thus h and the depth-averaged velocity U are constant along x (see Fig12a). When the depth-averaged suspended sediment concentrations at the two sections (the distance is Δx; Δx is infinitesimally small), I and II, are represented as $\overline{C}_{\mathrm{I}}$ and $\overline{C}_{\mathrm{II}}$, respectively, the continuity equation of sediment transport is written as

$$(\overline{C}_{\mathrm{II}}-\overline{C}_{\mathrm{I}})Uh = (C_{ae}-C_a)w_0\Delta x \qquad (26)$$

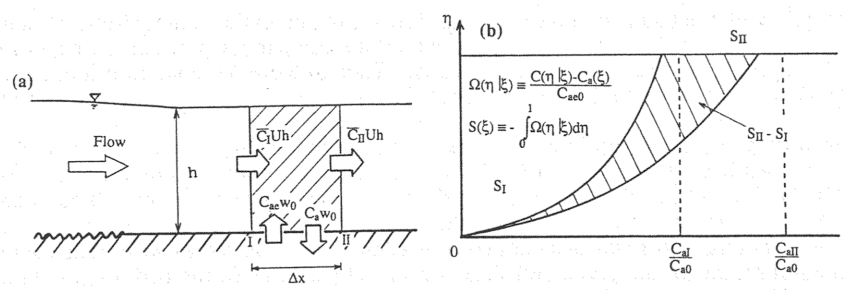

Fig.12 Schematic figures

While, referring Fig.12b, the following relation is valid.

$$\overline{C_{II}}-\overline{C_I} = C_{aII}-C_{a0}S_{II} - (C_{aI}-C_{a0}S_I) \tag{27}$$

in which

$$S(\xi) \equiv - \int_0^1 \Omega(\eta\,|\xi)d\eta \tag{28}$$

in which the subscripts I and II indicate the values at the section I and II, respectively. From Eqs.26 and 27, the differential equation with respect to C_a is obtained as follows ($\Delta x \rightarrow \infty$):

$$\frac{dC_a}{dx} = (C_{ae}-C_a)A + C_{a0}\frac{dS}{dx} \tag{29}$$

in which $A \equiv (w_0/u_*)/\phi$; and $\phi \equiv U/u_*$. Eq.29 emphasizes that the variation of the bottom concentration is affected by the transition process of the concentration profile (dS/dx). Fig.13 depicts the computed solution of Eq.29, and the transition of the bottom-concentration is promoted by the effect of the transition of the concentration profile in the both stages of growth and decay of sediment concentration. The non-equilibrium concentration distributions determined by the bottom concentration estimated by this model is not always quantitatively consistent with those determined by the boundary condition at the free surface. The reason of such a difference might be caused by depth-averaged approximations to deduce Eqs.26 and 27. The detailed reason will be further investigated.

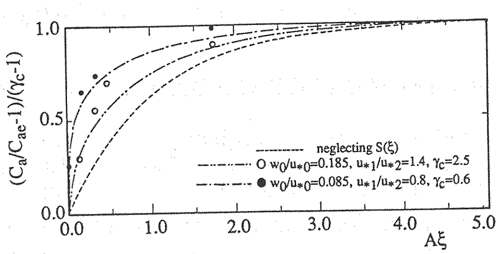

Fig.13 Effect of non-equilibrium profile on variation of bottom concentration of suspended sediment

CONCLUSIONS

In this study, the relaxation model proposed for the transition process of the Reynolds-stress distribution is applied to description of non-equilibrium sediment concentration distribution based on an analogy between the turbulent exchange of momentum and that of suspended sediment. The results obtained in this study are summarized below:

(1) The relaxation model composed as a convolution integral with an impulse response has been proposed for the transition process of the turbulent flux distribution of suspended sediment. The impulse response of turbulent flux of suspended sediment is regarded to be equivalent to that of the Reynolds-stress, the mathematical expression of which was already established previously.

(2) By applying the relaxation model, the characteristics of the spatial change of the turbulent flux distribution of suspended sediment in the transition process due to an abrupt change of the bed shear stress have been discussed. For simplicity, the diffusion coefficient is assumed to be constant along the depth. The turbulent flux distribution of sediment adapts the new condition of the bed shear stress increasingly from the bed.

(3) The concentration distribution is obtained by integrating the turbulent flux distribution through the flow depth. Even without referring the boundary condition, the geometrical degeneration of the concentration profile can be argued, and the adaptation length of the concentration profile has been obtained. It is longer for the finer sediment, and it has a tendency to become longer for deposition cases.

(4) The boundary condition at the free surface is given directly from the turbulent flux of sediment at the free surface which is already predicted, and by using it the concentration distribution is determined.

(5) The spatial change of the bottom concentration in the non-equilibrium transport process of suspended sediment has been calculated. The so-called adaptation length of the bottom concentration is almost the same order of that of the concentration profile, but there is little difference between erosion and deposition cases.

(6) The bottom concentration under non-equilibrium might be determined by using the continuity equation of sediment transport. By using the depth-averaged relations to compose the continuity equation, the differential equation with respect to the bottom concentration has been deduced. The solution suggests that the relaxation of the bottom concentration is promoted by that of the concentration profile obviously. However, the bottom concentrations obtained by this approximated approach might have some error compared with those obtained from the concentration distribution determined by using the boundary condition at the free surface.

The most part of this paper is translated from the authors' paper published in Japanese (25), but some are added and/or revised herein.

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APPENDIX - NOTATION

The following symbols are used in this paper:

A	$\equiv (w_0/u_*)/\phi$;
C, c'	= concentration distribution of suspended sediment and its fluctuation;
$\overline{C}$	= depth-averaged concentration of sediment;
C_a	= bottom concentration of suspended sediment;
d	= sediment diameter;
d^*	$\equiv (\sigma/\rho-1)gd^3/\nu^2$;
E	$\equiv w_0 h/\varepsilon_s$;
g	= gravitational acceleration;
$g_R(\delta \mid \eta)$	= impulse response of turbulent flux at the relative height η;
h	= depth of flow;
L_{ac}	= adaptation length of bottom concentration of sediment;

L_{as}	= adaptation length of non-equilibrium profile of suspended sediment;
S	= defined by Eq.27;
u, u^+	= time-average velocity and its dimensionless expression (u/u_*);
U	= depth-averaged velocity;
u_*, u_{*c}	= shear velocity and critical shear velocity;
u', v'	= turbulence in longitudinal and vertical directions;
w_0	= terminal velocity of sand;
x	= longitudinal distance;
y	= height from the bed;
γ_c	$\equiv C_{ae1}/C_{ae0}$;
γ_E	$\equiv E_1/E_0$;
$\Gamma(\eta\mid\xi)$	= deviation of turbulent flux under non equilibrium from equilibrium one and defined in Eq.4;
ε_s	= diffusion coefficient of suspended sediment;
η	$\equiv y/h$ = relative height;
κ	= Kármán constant;
$\Lambda(\eta)$, Λ_s	= relaxation scale non-dimensionalized by h at η and its value at the surface;
ν, ν_T	= kinematic viscosity and eddy kinematic viscosity;
ξ	$\equiv x/h$;
ρ, σ	=mass density of water and mass density of sand;
τ	= Reynolds stress;
τ_b	= bed shear stress;
τ_*, τ_{*b}	= dimensionless bed shear stress and dimensionless critical tractive force;
ϕ	$= U/u_*$;
$\psi_b(\xi-\delta)$	$\equiv \tau_b(\xi-\delta)/\tau_b(\xi)$;
Ψ	= turbulent flux of suspended sediment; *and*
$\Omega(\eta)$	= defined by Eq.18.

Subscripts:

e	= values under equilibrium conditions;
0	= values before an abrupt change; *and*
1	= values after the abrupt change

(Received December 25, 1990 ; revised March 13, 1991)