Journal of Hydroscience and Hydraulic Engineering
Vol. 9, No. 1, May, 1991, 27-42

ONE-DIMENSIONAL NUMERICAL MODELING OF DAM-BREAK WAVE PROPAGATION IN MILD SLOPING NATURAL STREAMS

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SYNOPSIS

Wave propagation resulting from multiple dam failures in a mild sloping natural stream is numerically investigated by a 1 -D dynamic model. The model is based on a divergent form of Saint Venant equations and relies upon three difference schemes (Vasiliev, Godunov and Lax-Wendroff) to simulate dam-breaking. The schemes are first tested in rectangular channel through standard procedures and compared for their shock-capturing capabilities. In accordance with the test results Lax-Wendroff scheme is modified to damp the parasitic oscillations behind the wave front and the contributions respectively of Lax-Wendroff's and Godunov's fractional steps in the final solution are evaluated. Finally, a real-life application of the model is attempted with the most attractive features of all three schemes used in conjunction to provide the numerical process with appropriate initial conditions and obviate computational shortcomings. All numerical results are discussed in detail.

INTRODUCTION

In 1892, almost one century ago, Ritter gave the first solution of the dam-break problem, recognized nowadays as one of the most important and challenging subjects in rapidly varied unsteady flow.

During the last three decades considerable attention and efforts were devoted to reasonably simulate the complex flow conditions resulting from dam-breaking. Proportionally to the amount of effort focused on the field and under an increasing demand for speed, economy and precision the number of numerical models has rapidly grown. Hence, in present days investigators dispose of a huge arsenal of sophisticated numerical algorithms and promising modeling techniques.

The dam-break problem is of major concern for engineers in applied hydraulics although most of them are not expert in computational hydraulics. Accordingly when facing the dam-break problem, the use of one of the several available commercial dam-break packages (Wurbs (25)) should be considered as the most appropriate solution. However experienced hydraulicians fear the "black-box syndrome" and seldom accept a role confined to this of ordinary users consulting user's manuals. Such a conduct is founded also on the conviction that there is no single best method for numerical simulation of dam-break wave propagation. To build a new model therefore is often conceived as a valuable alternative.

Indeed building a numerical model has a lot of advantages for every engineer. The major advantage is certainly the possibility of formalizing his own concepts of the basic physical processes that are to be modeled. In addition, extending the field of application of the model by adding new features, understanding factors affecting the flow variables, conducting numerical experiments in varying the flow parameters and experimenting different numerical schemes and boundary conditions are significant advantages, as well. However if adopted, this alternative implies a lot of difficulties to overcome. For instance numerical modeling includes not only the development of an algorithm simulating the phenomena under investigation but also the acquisition of a solid theoretical background providing the most appropriate set of governing equations and pair of dependent variables. Gaining a deep understanding of numerical solution methods as well as of the pertinent criteria for numerical method assessment, stability analysis and numerical accuracy is required too. It means going far beyond the primary purpose of obtaining rational results but neglecting this vast area of numerical knowledge has always disastrous consequences.

Since no universal ready solutions to the dam-break problem are given in the current technical literature usually rough guide lines have to be followed. Extending the implementation of numerical techniques and difference schemes to natural streams is suggested or advised, but in some cases conclusions are drawn from numerical experiments in straight rectangular channels. It invites great criticism and one must bear constantly in mind that whatever the recipes provided no satisfaction from the obtained results is guaranteed.

Basic Concepts and Guide Lines

The basic concepts and guide lines underlying the present work stem from the following considerations.

/1/ Dam-breaking in a natural stream is a large scale problem. Thus a macro-hydraulic approach is, in general, considered worth even if there exist applications containing terms accounting for the diffusion of momentum due to turbulence (Garcia and Kahawita(8)).

/2/ As most flows in nature the flow resulting from the gradual or sudden dam collapse in a natural stream is 3 -D no matter how far from the dam site the flow is referenced. Nevertheless the unsteady flow generated by the dam-breaking is traditionally simulated as 1 -D across its entire length since usually long reaches are investigated. It should be noted that the use of hybrid numerical models obtained by mixing the 1 -D and 2 -D concepts is extending but a lot of numerical experience is needed to carefully select how, when and where to switch from one concept to the other. Still few in number pure 2-D applications seem to become more attractive with the advent of powerful digital computers. First experiences in numerical modeling are mostly 1 -D.

/3/ In present day computational hydraulics the use of complete dynamic models for dam-break simulation is justified by yielding more superior results and comprehensive study of the complex flow conditions.

/4/ In 1 -D modeling of dam-breaking based on complete dynamic models Saint Venant equations may be selected as a governing set of equations. They are assumed valid elsewhere except near the propagating discontinuities. The numerical methods used to solve Saint Venant equations are classified in general in two groups: shock fitting methods and through methods. In the former the discontinuities are detected and followed along their paths. In the latter discontinuities are assumed narrower than the distance between two computational points and are implicitly taken into account when the basic set of equations is written in divergent form. Since discontinuities are smeared out, flows including shocks may be computed without setting up special logic. The legitimacy of this approach relies on the theory of weak solutions. The actual trend seems to favor through methods implementing the "shock-capturing" technique.

/5/ As far as dependent variables are concerned a pair subject to slow changes over distance is preferred. For natural streams usually recommended are z and Q = water level and water discharge respectively.

/6/ To solve numerically the governing system of non-linear hyperbolic partial differential equations there are three fundamental techniques:

-finite element method; The advantage of the method in 1 -D flow simulation is lacking sufficient argumentation. Meanwhile its application to time dependent problems appears not always clear to some researchers whereas others are promoting its widespread use.

-method of characteristics; There is no doubt about the physically and theoretically appealing features of the method but its application to wave propagation in natural streams is a tremendous work because tracking down of several reflected and superposed wave fronts is in general involved.

-method of finite differences; The method is currently used in the great majority of industrial applications. When discussing the two basic categories of finite difference schemes the following conclusions are established as "rules" in present days.

Explicit methods are relatively simple to program for numerical computations. However because of the restriction imposed on the time step by Courant stability condition their use is not appreciated for real-life applications.

For 1 -D numerical simulation of rapidly varied unsteady flow, especially when long stream reaches are considered, the unconditionally stable implicit methods have become more popular.

These "rules" are less stringent in numerical modeling of dam-break wave propagation. In general the time steps are taken small enough in order to accurately simulate the physical phenomena investigated. Therefore for both explicit and implicit methods nearly equal time steps are used.

After the breaking of the dam, in the downstream channel, flow may become super -critical while it generally remains sub -critical upstream of the dam site. It must be noted that not all numerical schemes are capable of simulating mixed sub- and super-critical flows. Usually schemes based on centered space differences cannot handle super -critical flows. If super -critical flow is expected to occur one has to make sure that the appropriate numerical scheme has been selected.

In addition it has to be taken under consideration that second-order accurate numerical schemes are preferred for the simulation of dam-break wave propagation because superior results as compared to first-order accurate schemes are produced. Furthermore, the diffusive properties of the schemes are connected to the order of accuracy too. First-order accurate schemes cause dissipation of the numerical solution, that is smearing the solution over several grid intervals, whereas dispersion is caused by second-order accurate schemes, thus making parasitic oscillations an inherent quality of the numerical solution.

/7/ The foregoing considerations may be combined in distinct sets and further on implemented. Generally this leads to different approaches and therefore different solutions with far reaching practical consequences.

This paper reports on the results of an investigation carried out by a 1 -D complete dynamic model in a through approach. The model is build on Saint Venant equations written in divergent form. Three difference schemes, two implicit and one explicit are used in the numerical simulation. The implicit schemes are Vasiliev and Godunov schemes while Lax-Wendroff is the explicit one. The shock-capturing capabilities of these schemes in a long rectangular channel are compared. The analogy existing in Godunov's and Lax-Wendroff's schemes is fully exploited to evaluate the contributions of their respective fractional steps in the final solution. The behavior of Lax-Wendroff scheme is modified to better fit an existent analytical solution and damp parasitic oscillations behind the wave front. Later on, under initial conditions provided by either Godunov or Lax-Wendroff schemes in rectangular channel, both Godunov and Vasiliev schemes are applied within their range of validity to numerically simulate multiple dam failures in mild sloping natural stream.

GOVERNING EQUATIONS

Different forms of the governing set of equations are used to describe the unsteady 1 -D flow produced by dam failure in a channel with arbitrary cross-section and small bottom slope. All forms are derived from the original Saint Venant equations and assumed valid elsewhere except near the wave front.

In non-divergent form the equations are as follows.

Conservation of mass equation:

$$B\frac{\partial z}{\partial t}+\frac{\partial Q}{\partial x}=q \tag{1}$$

Conservation of momentum equation:

$$(1-Fr)\frac{\partial z}{\partial x}+\frac{1}{g\omega}\left(\frac{\partial Q}{\partial t}+2V\frac{\partial Q}{\partial x}\right)=\left[i_0+\frac{1}{B}\left(\frac{\partial \omega}{\partial x}\right)_{h=const}\right]Fr-\frac{Q\mid Q\mid}{K^2} \tag{2}$$

The divergent form of Eqs. 1 and 2 may be written

Mass:

$$\frac{\partial \omega}{\partial t}+\frac{\partial Q}{\partial x}=q \tag{3}$$

Momentum:

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left[\left(\frac{P}{\rho}\right)+\left(\frac{Q^2}{\omega}\right)\right]=g\omega\left(i_0-\frac{Q\mid Q\mid}{K^2}\right);\quad \frac{P}{\rho}=g\int_0^h (h-\xi)B(x,\xi)\,d\xi \tag{4}$$

where $B(x,z)$ = cross sectional width at free surface elevation; $z(x,t)$ = water surface elevation; $Q(x,t)$ = water discharge; $q(x,t)$ = continuous lateral inflow or outflow per unit length; $Fr=(V/c)^2$ = Froude number; $\omega(x,z)$ = cross-sectional flow area perpendicular to the flow direction; $V(x,t)$ = mean velocity; $i_0(x)$ = bed slope; $K(x,t)$ = channel conveyance factor; g = acceleration due to gravity; $h(x,t)$ = water depth; c = wave celerity; and ξ = variable for integration representing the distance measured from the fixed bottom to the centroid of a water layer of uniform thickness $d\xi$.

The set of Eqs. 1 and 2, respectively 3 and 4 are hyperbolic partial differential equations based on conservation principles. The derivatives in these sets of differential equations are approximated by finite-differences to form algebraic finite difference relationships.

A modified form of Eq. 2 written as

$$\frac{\partial Q}{\partial t} + 2V\frac{\partial Q}{\partial x} + B(c^2 - V^2)\frac{\partial z}{\partial x} = \Phi Q^2; \quad \Phi = \left[Bi_0 + \frac{\partial \omega}{\partial x}\right]\frac{1}{\omega^2} - sign(Q)\frac{g\omega}{K^2} \tag{5}$$

is also used through the study related in this paper.

FINITE-DIFFERENCE SCHEMES

In the last decades Godunov, Vasiliev and Lax-Wendroff difference schemes have been extensively used in unsteady open-channel flow simulations.

In the middle of the sixties Vasiliev and Gladyshev (24) recommended the use of Godunov scheme for numerical simulation of dam-break wave propagation in natural streams. Relying on the mathematical legitimacy of weak solutions the scheme has been later applied with confidence to dam-break flow analysis by Gladyshev (9), Alalykin, Godunov, Kireeva and Pliner (2) and Sudobitcher (20). These works as the detailed description of the scheme by Richtmyer and Morton (17), Roache (18) and Holt (10) may be recommended as references for the Godunov scheme.

Several applications of Lax-Wendroff scheme in numerical modeling of dam-break wave propagation have been reported and the studies of Takeshi, Hirosho and Ohira (21), Price (15), Liggett and Cunge (11), Rajar (16) are considered relevant to the subject matter. A detailed development of the scheme can be found in Roache (18).

Vasiliev scheme is reserved mainly to gradually varied unsteady flows. A necessary prerequisite for its implementation are the works by Vasiliev and Godunov (22), Vasiliev, Temnoyeva and Shugrin (23), Liggett and Cunge (11), Cunge, Holly and Verwey (6).

Detailed descriptions of the schemes together with the appropriate boundary and initial conditions are beyond the intended scope of this paper. For the sake of continuity the general structure of the schemes and their most salient features are briefly reviewed.

Godunov Scheme

Godunov second order accurate difference scheme is a fractional two-step procedure which can be applied on both fixed or movable computational grid. In the first half time step, the numerical solution is advanced to an intermediate layer $t^{k+\vartheta} = t^k + \vartheta\Delta t$ for the $z(x,t)$ and $Q(x,t)$ pair of dependent variables. For this first step, usually called predictor, Eqs. 1 and 5 are written in finite difference form using the following approximations of the derivatives

$$\frac{\partial f}{\partial t} = (\frac{f_i^{k+\vartheta} - f_i^k}{\vartheta\Delta t}); \qquad \frac{\partial f}{\partial x} = (\frac{f_{i+1}^{k+\vartheta} - f_{i-1}^{k+\vartheta}}{2\Delta x}) \tag{6}$$

where f_i^k represents the value of the grid function at point x_i and t_k . Applying the same discretization to the entire computational domain yields a system of equations with a banded three diagonal matrix. After the appropriate boundary conditions have been supplied, the double sweep method is used to simultaneously solve the system of equations at the intermediate time layer.

The values of the dependent variables thus obtained at integral computational sections are first-order accurate. They are further submitted to smearing procedures where values at half integral computational sections are determined. The smearing procedures are interpolations formulas of the following form:

$$f_{i+\frac{1}{2}}^{k+\vartheta} = \begin{cases} \frac{3}{8}(f_i^{k+\vartheta} + f_{i+1}^{k+\vartheta}) + \frac{1}{8}(f_{i-1}^{k+\vartheta} + f_{i+2}^{k+\vartheta}) & (i = 2, \ldots, n-2) \\ \frac{1}{2}(f_i^{k+\vartheta} + f_{i+1}^{k+\vartheta}) & (i = 1, n-1) \end{cases} \tag{7}$$

In the second half time step, called corrector, the crest explicit scheme is applied to Eqs. 3 and 4. The new dependent variables, $\omega(x,t)$ and $Q(x,t)$, computed at the final layer, are second-order accurate in contrast with the first-order accurate results obtained after the implementation of the predictor step. In order to obey through methods requirements a restriction is imposed on the corrector step namely, the crest scheme must be applied to a basic set of equations in divergent form.

In addition, Godunov scheme incorporates a parameter ϑ which is used to smooth the irregular parasitic oscillations characterizing second-order accurate difference schemes. As stated in (2) for $\vartheta = 1.0$ the final solution is first order accurate and no oscillations are detected, whereas a second order solution is obtained for $\vartheta = 0.5$

Vasiliev Scheme

Vasiliev scheme is a one step unconditionally stable implicit difference scheme. The scheme is of first-order of accuracy with respect to time and space as long as it is applied on an non-uniform computational grid. At each time step the solution is advanced for both dependent variables $z(x,t)$ and $Q(x,t)$, which are computed at all grid points, that means, on a fully dense grid. To obtain the difference form of the governing differential equations, namely Eqs. 1 and 5, the partial derivatives are replaced by the following difference operators:

$$\frac{\partial f}{\partial t} = (\frac{f_i^{k+1} - f_i^k}{\Delta t}); \qquad \frac{\partial f}{\partial x} = (\frac{f_{i+1}^{k+1} - f_{i-1}^{k+1}}{2\Delta x}) \tag{8}$$

The scheme links together three consecutive computational sections or two less than Godunov scheme. Vasiliev scheme and the first half step in Godunov scheme for $\vartheta = 1.0$ in Eq. 6 have recourse to the same finite-difference discretization. Likewise, they are applied on non-divergent form of equations.

Lax-Wendroff Scheme

Characterized by a strong shock-capturing capability this explicit scheme has been successfully implemented in numerical simulation of rapidly varying flows where its advantages and drawbacks have been fully demonstrated. The scheme belongs to the class of fractional step methods and is a standard reference for second-order accurate explicit schemes. In the present study the original Lax-Wendroff scheme of finite differences to Eqs. 3 and 4 and a scheme referenced hereafter as a modified Lax-Wendroff scheme are used. To obtain the modified Lax-Wendroff scheme the original discretization applied over the time derivatives in the corrector step $\partial f/\partial t = (f_i^{k+1} - f_i^k)/\Delta t$ is replaced by

$$\frac{\partial f}{\partial t} = \frac{f_i^{k+1} - \left[\alpha f_i^k + \frac{1-\alpha}{2}\left(f_{i+1}^k + f_{i-1}^k\right)\right]}{\Delta t} \tag{9}$$

The later approximation is borrowed from Lax diffusive scheme (see Liggett and Cunge (11)). It is evident that for $\alpha = 1.0$ in Eq. 9 the modified Lax-Wendroff scheme reduces to the original one. As noted in the same reference Lax-Wendroff scheme is analogous to Godunov scheme except that in Godunov scheme an implicit method is used to obtain the solution for the predictor step. The smearing procedure included in Godunov scheme is another significant difference between these two schemes and should not be omitted when drawing the above analogy.

IMPLEMENTED APPROACH

Within the context of computational hydraulics the schemes presented above are not the only suitable schemes to solve the dam-break problem and neither are they the best. Representing both explicit and implicit schemes, their selection is conform with the "rules" previously commented. The schemes are submitted to a series of standard tests and the results obtained are examined in the next section. Moreover it is necessary to approve the numerical approach in general. To do this let us remind that any numerical solution may be contested from two different standing points, namely field measurements and experiments.

According to Miller and Chaudhry (13) and Dammuller, Bhallamudi and Chaudhry (7) field measurements obtained from historic dam failures in natural channels are questionable for the verification of mathematical models. The scarcity of collected data and/or the lack of similitude between observed and simulated flows strengthen this statement.

On the other hand experiments conducted in laboratory flume, although a suitable tool for testing and calibrating numerical schemes usually fail to predict even within a one-dimensional approach more complex flow conditions occurring in long natural streams.

A more efficient and certainly a more flexible approach is employed hereafter. For this purpose three difference schemes, approved for their reliability, are run in parallel. The schemes selected in the present investigation include analogous procedures and common computational

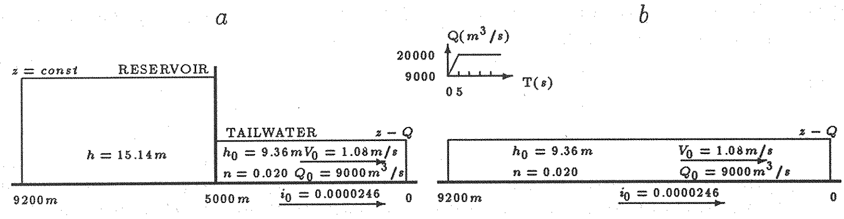

Figure 1: Definition sketch

steps. Besides the similarity already stressed between Godunov's predictor step and Vasiliev scheme, Godunov and Lax-Wendroff schemes share the same second half time step. This simple analogy is fundamental to assess the contribution of each of the computational steps in the final results and offers a natural basis for their comparison. Moreover by comparative studies any incidence of random errors on the numerical solution can be easily discovered and eliminated.

The above approach is adopted in order to analyze the performances of the schemes after a numerical simulation of the dam-break problem in mild sloping natural stream with arbitrary cross section and sudden changes in geometry was unsuccessfully attempted by Godunov scheme on a fixed computational grid. Results from preliminary studies (Botev (3)) led to the conclusion that parasitic oscillations behind the front of the shock destroy the numerical solution when superimposed on sudden changes in river bed geometry. On the other hand the scheme is reported to be well suited for real-life applications and the problem is classified as solved by Atavin, Vasiliev, Voevodin and Shugrin (1). Mainly for this reason additional studies prove useful as they put the validity of the scheme in a proper perspective.

NUMERICAL EXPERIMENTS IN RECTANGULAR CHANNEL

Sample Problems

The performances of the schemes in modeling two sample problems given in Fig. 1. are initially investigated. All numerical experiments presented in this subsection are conducted in a rectangular channel with topographic and hydraulic features corresponding to a $9200m$ long reach that is a part of the natural stream later investigated. The channel width is $857m$, the bottom slope $i_0 = 0.0000246$ and Manning's $n = 0.020$. Dam-breaking is analyzed through the sample problem, given in Fig. 1a. The channel conveys initially $Q = 9000m^3/s$ with downstream velocity $V_0 = 1.08m/s$ and normal depth $h_0 = 9.36m$, while $V_0 = 0.67m/s$ and $h_0 = 15.14m$ in the reservoir. At $T = 0s$ the dam is instantaneously removed across its entire width and the resulting flow pattern is computed up to time $T = 100s$. All computations are carried out with time step $\Delta t = 5s$, except for Godunov scheme where additional computations with $\Delta t = 10s$ are performed. The left boundary condition is $z = const$, whereas for the right boundary a stage/discharge rating curve corresponding to a gauging section in the natural stream is prescribed.

In the second sample problem (Fig. 1b) there is no dam in the channel. The initial flow conditions being uniform with $Q = 9000m^3/s$ and $h_0 = 9.36m$ the inlet water discharge at the left boundary is raised from 9000 to 20000 m^3/s in $5s$, thus generating a traveling wave in downstream direction. The right boundary condition and the geometric features of the channel are unchanged.

For all experiments the channel is subdivided into 92 equal-length computational reaches. The length of the channel is specified long enough such that waves propagating upstream and downstream do not reach the boundaries for the duration of the tests. Thus the boundaries do not affect the computed results.

Results and Discussion

Standard numerical tests are conducted to determine if the schemes behave properly for a wide range of initial and boundary conditions. The flow pattern resulting from the first sample problem is modeled by all schemes. The second sample problem is used to test the performance

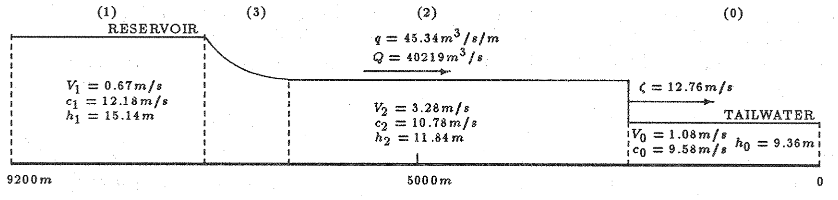

Figure 2: Stoker's solution

Difference schemes	$\delta z(\%)$	$\delta Q(\%)$	$\delta L(\%)$
Vasiliev	≈ 1.0	< 8.5	≈ 5.0
Godunov($\vartheta = 1.0$)	< 1.4	≈ 10.0	≈ 5.0
Original L-W.($\alpha = 1.0$)	< 0.4	< 9.0	≈ 3.0
Modified L-W.($\alpha = 0.9$)	< 0.4	< 3.0	< 3.0

Table 1: Comparison between numerical and analytical solutions

of the modified Lax-Wendroff scheme and to calibrate the parameter α in Eq. 9. For both sample problems analytical solutions are given.

For the first sample problem besides comparison between analytical and numerical solutions mass conservation through the site of the collapsed dam ($5000m$) is tested. Dampening of high-frequency oscillations appearing behind the wave front while producing the correct wave speed and keeping the front width in reasonable limits is also attempted. The analytical solution for the dam-break problem is obtained by using Stoker's method (Stoker (19)). Some of the results calculated by using this method are shown in Fig. 2. The downstream front propagates into the uniform flow (zone 0) at speed $\zeta = 12.76m/s$. Because $V_2 < c_2$ only sub -critical flow is considered in the present work. In the comparison between analytical and numerical solutions which is further conducted it must be borne in mind that friction and bottom slope are accounted in the numerical solutions. Water level (z_{DAM}) and water discharge (Q_{DAM}) at dam location, as well as the distance L which the front travels downstream for $100s$ are compared. The first two quantities refer to the time immediately after dam collapse. Table 1 contains the differences between numerical and analytical results obtained by Stoker's method. Clearly, with respect to the above cited parameters - (z_{DAM}, Q_{DAM} and L) - the implicit schemes yield results with almost the same accuracy. Both explicit schemes simulate more accurately z_{DAM} and L, whereas for Q_{DAM} the modified Lax-Wendroff scheme fits best the analytical solution (the performances of the modified Lax-Wendroff scheme are discussed below). Water level profiles computed with $\Delta t = 5s$ for all schemes are plotted in figures 3 to 6. L for the analytical front is depicted by a vertical dashed line.

Apparently the front speed is correctly simulated although non-significantly delayed, most probably due to friction included in the numerical solution. These generally successful results are attributed to the divergent form of equations used. Tests for mass conservation during the computations show that at $T = 100s$ the errors introduced in all numerical solutions (except the one obtained by Vasiliev scheme) are less than 0.5%. For Vasiliev scheme, build on a non-divergent form of equations, the error however does not exceed 1.0%.

Unfortunately, wiggles behind the wave front are present in all profiles computed by Godunov (for $\vartheta = 1.0$) and the original Lax-Wendroff schemes. Lagging oscillations with significant amplitude are the most serious drawback of both schemes. It is true that, as far as computations proceed in a straight rectangular channel these parasitic oscillations do not damage in a large extent the numerical solution. Likewise, their maximum amplitude is clearly seen to decrease as they travel downstream. As already mentioned, in natural streams however, interaction between oscillations and sudden changes in river bed geometry destroy the numerical solution. Therefore, the presence of parasitic oscillations behind the front should not be neglected and a remedy to eliminate them must be conceived at this stage of study. A classical remedy is to use the close relationship existing between the numerical damping of parasitic oscillations and the Courant number. To achieve the strongest damping Courant number Cr should be kept close to unity. Tests run with Godunov scheme with $\Delta t = 10s$ that means $Cr \approx 1.0$ for the present case explore this possibility for damping the oscillations. The effectiveness of the method can be estimated after careful examination of the results given in Fig. 7. A reduction of parasitic oscillations is observed as the front curvature decreases, thus increasing the front width (evaluated in mesh increments). As noticed by Roache (18) a method of handling shocks is generally considered successful if the shock width is spread over 3 to $5\Delta x$. In the present case it is worth noting the following: at $T = 100s$ while a moderate dampening of oscillations is distinguished, the width of the wave front modeled by Godunov scheme extends from $13\Delta x$ to $16\Delta x$ whereas Cr increases from ≈ 0.5 to ≈ 1.0. Meanwhile for $Cr \approx 0.5$ Vasiliev and the original Lax-Wendroff schemes produce a front width of $9\Delta x$ and $4\Delta x$ respectively. As expected Vasiliev first-order solution is oscillation-free.

In the results given so far Lax-Wendroff scheme demonstrates an important advantage, namely the one of simulating the sharpest wave front (i.e. smallest front width). However the solution obtained is not exempted of oscillations (see Fig. 5) with approximately the same order of magnitude as those generated by Godunov scheme. The second sample problem suggests a tool for their elimination.

To do this a commonly used practice of introducing some additional viscosity in the second step of Lax-Wendroff methods is followed. The diffusive-like approximation of the time derivative (Eq. 9) that replaces the original one is manipulated to reach a reasonable compromise between the above discussed properties of the numerical solutions, namely minimum front width and minimum amplitude of the oscillations behind the wave front. A group of numerical experiments is conducted to calibrate the parameter α. The calibration process consists to vary the value of α in the range between 0.5 and 1.0 so as to fit satisfactory the analytical solution for the second sample problem, that is a traveling wave in downstream direction with a height $\Delta h' = \frac{\Delta Q'}{c'B} = \frac{11000}{11.52887.0} = 1.07m$. Here $\Delta Q'$ = the increase in water discharge produced at the left boundary; and c' = the wave celerity computed as $c' = V_0 + \sqrt{g(\omega_0/B + 1.5\Delta h')} = 1.08 + \sqrt{9.81(9.36 + 1.51.07)} = 11.45m/s$

Fig. 8 shows the newly established uniform flow level after the analytical wave has filled the whole channel. The analytical wave front is again represented by a vertical dashed line. For convenience only the part of the channel containing the region of interest is included in the plots. To conserve space all but four numerical solutions computed by the modified Lax-Wendroff scheme for $\alpha = 0.50, 0.75, 0.90$ and 1.00 are presented herein.

As previously noted, for $\alpha = 1.0$ the modified Lax-Wendroff scheme is equivalent to the original one. Oscillations with large amplitude, similar to those observed in the first sample problem are present behind the wave front. Judged in terms of minimum front width and minimum oscillation amplitude the most satisfactory results are obtained for $\alpha = 0.90$. One can state in good conscience that the introduced Lax approximation acts in the present case as "explicit" artificial viscosity. The resulting damping is effective only in the front region. Away of this region, the effect of the modified approximation may be considered negligible. However, damping oscillations with $\alpha = 0.90$ increases the front width by one mesh increment, thus spreading the front over 6 mesh increments instead of 5 for the original Lax-Wendroff scheme (Fig. 8).

Unlike $\alpha = 0.90$, for $\alpha = 0.75$ and 0.50 over-smeared solutions with a front height far below the theoretical one are obtained. For low values of α the relative merits of the present approach are eliminated. Obviously calibration of α is necessary before future implementation of the modified Lax-Wendroff scheme. In addition, attention is called on two important facts: /1/oscillations develop merely in the second half step of the scheme and /2/noticeable improvements of the solution are accomplished due to non-significant deviation from the original Lax-Wendroff scheme (i.e.$\alpha = 0.90$ and even $\alpha = 0.95$).

All above cited conclusions concerning the performance of the modified Lax-Wendroff scheme offer the possibility to reconsider the solution of the first sample problem. Attempts to simulate the dam-break problem by the modified Lax-Wendroff scheme with $\alpha = 0.90$ achieve the plots in Fig. 6. The solution is significantly improved as far as the front width and oscillation amplitude are considered. It i s concluded that a smooth solution is obtained without increasing the front width or altering the front speed. Likewise, the results are concordant with the introductory remarks concerning the implementation of high-order schemes. They are sufficiently convincing and computations may be stopped right here whenever a short rectangular channel has to be investigated and as far as a small computational step imposed by Courant stability criteria is not a major inconvenient. This phase in the present study is however an intermediate step, half a way from the assigned goal of simulating dam-breaking in a natural stream.

Justifying the generally satisfactory results computed by the modified Lax-Wendroff scheme is far beyond the scope of this work. One reasonable explanation however may be found in considering that during computations of rapidly varied flows in prismatic channels the time derivatives undergo the most important changes. It seems to be the main reason for the increased "sensitivity" of the solution to the alterations introduced in the original Lax-Wendroff scheme, through variations of the parameter α.

Finally we note that it is straightforward to expand the validity of the above conclusions and statements to the analysis of Godunov scheme. Obviously the high-frequency oscillations generated by Godunov scheme (see Fig. 4) are contributed by its second half step too. Such a conclusion is confirmed hereafter in a direct comparison with results computed by Vasiliev scheme.

We start reminding that Vasiliev scheme is analogous to Godunov's first step for $\vartheta = 1.0$. Accordingly, in evaluating the behavior of Vasiliev scheme actually the isolated Godunov's first half step is evaluated. Furthermore from the comparison of Vasiliev and Godunov schemes it becomes clear (see Fig. 3) that Vasiliev scheme reproduces in only one step a first-order accurate solution exempted from oscillations (a quality intrinsic to all first-order solutions). Nevertheless, Godunov scheme applied in its integrity, presents oscillations in the solution when the final layer is reached. It comes next the firm conviction that these oscillations result from the second step of the scheme and their damping, if necessary, must follow rather than precede this step. The present author suggests here to incorporate the damping in the second step, as demonstrated earlier on Lax-Wendroff scheme. It can be easily realized by simply exchanging the second step of the original Godunov scheme with the second step of the modified Lax-Wendroff scheme.

NUMERICAL EXPERIMENTS IN NATURAL STREAM

The finite difference schemes discussed so far are included in the present model designed to numerically simulate dam-break wave propagation in mild sloping natural stream. Variable bed geometry, flow with convective and storage cross-sectional areas and lateral outflow over breaking levees are among the most significant features of the model. In the present section however attention is drawn mainly on the results achieved by the newly constructed model. A detailed description of the model and its initial calibration are given elsewhere (Botev (4,5)).

An instantaneous and complete collapse of two consecutive dams is modeled. The dams are $80km$ apart and their respective failures are conditioned by the maximum disastrous effect produced. A $400 - km$ long natural channel down from the upstream dam is included in the computations. This distance is divided into 24 reaches, each one subdivided in maximum 50 sub-reaches. The time step adopted for the final computations is set to $15min$ and the problem assigned in the current application required wave propagation to be simulated for inasmuch as 72 hours. Sub-critical flow develops through the entire length of the stream after dam-breaking and centered space difference schemes, as those so far tested are capable of handling such a flow.

As reported earlier superimposition of high-frequency oscillations over sudden changes in river bed geometry caused all initial executions with Godunov scheme to be interrupted. Obviously local disturbances behind the wave front tend to expand and increase their amplitude. Hence computations abort after few time steps while results with no physical meaning are produced. It was concluded that even such a strong and complex smearing of the numerical solution at the intermediate time layer as the one achieved by Eq. 7 fails to prevent the destruction of the solution. Accordingly, computations with Godunov scheme for values of $\vartheta < 1.0$ were abandoned because of the evident increase in oscillation amplitude for high-order solutions. Thereupon the concept of using all three discussed above schemes in conjunction was retained after tests run in rectangular channel demonstrated solutions with some attractive qualities.

In brief Godunov scheme provides the frame of the current solution. Shortcomings in the numerical process, seemingly inherent to Godunov scheme are overcome by implementing Godunov's or Lax-Wendroff's solutions for prismatic channel, whereas Vasiliev scheme is introduced for reasons of computational simplicity and reduction of computational time. The procedure is briefly described below.

The model resorts to Godunov's solution for rectangular channel to avoid the destruction of the numerical process in natural stream. Actually, Godunov's solution for rectangular channel is transferred and adjusted over the disturbed flow simulated by the same scheme in natural stream. The coupling of solutions affects only the computational reach in close proximity of the collapsing dam. It is accomplished after oscillations in rectangular channel are sufficiently reduced as not to deteriorate the following numerical process. The effective computational step to realize the coupling is selected in agreement with the overall time step governing the simulation. The time period necessary for Godunov scheme to smear oscillations in the solution impose however a severe limitation on the selection of the appropriate computational time step for coupling. Tests run with Godunov scheme in rectangular channel show that smearing of oscillations takes inasmuch as $5min$ of the simulated process.

It becomes soon evident that superimposing a solution for rectangular channel obtained by the modified Lax-Wendroff scheme over Godunov's solution for natural stream reaches by far more satisfactory results. The later approach has the virtue of providing with a possibility for coupling solutions practically right after dam-break, because the modified Lax-Wendroff scheme gives a smooth solution much earlier than Godunov scheme. In this case the problem of solution coupling is reduced to the one of defining appropriate initial conditions for dam-breaking although somewhat delayed in time (about $40-50s$ of the simulated process). No other obstacles are encountered further in simulating dam-breaking in natural stream by Godunov scheme.

As the dam-break wave front travels downstream and the flow becomes friction dominated sudden changes in bed geometry no more alter the solution. The front steepness gradually decreases and the dam-break wave itself transforms into a classical flood wave. Henceforth comparative studies involving both Godunov and Vasiliev schemes show an amazing resemblance between the produced flow patterns. Fig. 9 presents plots of water level and water discharge variations with respect to time at a site located approximately $100km$ downstream from the upper dam. The later is instantaneously removed at $T = 0$. All results in Fig. 9 are computed by Vasiliev scheme. Godunov scheme performs identically.

At this phase of the study some important features of the schemes have to be mentioned. In regions where the flow is no more dominated by inertial forces computations realized by Godunov scheme take almost twice as long as Vasiliev scheme reproducing identical results. It would seem that the additional complexity of the smearing procedure (Eq. 7) and Godunov's corrector have no more beneficial effect on the solution. In other words, only the first Godunov's step builds the solution, an argument that is easily proven by comparison with Vasiliev solution. Regardless of the fact that the flow has been generated by dam-breaking the author is inclined to conclude that Vasiliev scheme becomes soon more than adequate to capture the physics of the events. Logically the present model switches the computations from Godunov to Vasiliev scheme right after equivalence of solutions obtained by both schemes is established. Because the range of validity of Vasiliev scheme cannot be defined beforehand comparative studies with Vasiliev and Godunov schemes precede final executions.

It would be correct to note that in the present work the use of Godunov scheme has been limited to the case of fixed grid following the early approach of Vasiliev and Gladyshev (24). This is a one-sided treatment of the dam-break problem that obviates lot of subtle difficulties in natural stream applications together with the most desirable features of Godunov scheme. For instance, the great facility in adjusting the precision of the solution by varying the parameter ϑ although decisive for the initial selection of the scheme was unfortunately not exploited because of the appearance of parasitic oscillations. In this aspect it must be confessed that from the analysis of the excellent results for Favre wave in prismatic channel obtained by the above authors it is quite difficult to anticipate the computational problems encountered in the current investigation.

More promising alternatives stem from the idea of using adaptive grids. In this case the finite difference grid is allowed to adapt in time and more grid points are placed in regions where the solution undergo rapid spatial changes. A successful application of Godunov scheme on moving grid is demonstrated in (2) but a rather simplified bed geometry is used. A smoothed solution was introduced as initial condition in the dam site and the natural channel was approximated by series of prismatic channels.

Since solving the dam-break problem on fixed grid in natural stream still presents interest further research is needed to guarantee the applicability of Godunov scheme when arbitrary cross sections and sudden changes in geometry in the vicinity of the collapsing dam are encountered. While superimposing solutions from rectangular channel to natural stream, as it is done in the present case, reach satisfactory results, one may find the coupling procedure somewhat tiresome. Therefore, exchanging the second step of the original Godunov scheme with the second step of the modified Lax-Wendroff scheme emerges also as a valuable solution of the problem.

SUMMARY AND CONCLUSIONS

Following some basic guidelines given in the current technical literature multiple dam failures in mild sloping natural stream has been numerically investigated by one-dimensional complete dynamic model based on a divergent form of Saint Venant equations. A set of three difference schemes, approved for their reliability and including analogous procedures was conceived in order to obviate some frustrating results obtained by Godunov scheme in natural stream. Before applying the model for a real case study however the shock-capturing capabilities of all three schemes were investigated in rectangular channel. To complete the first part of the investigation the contribution in the final solution of each of Godunov's and Lax-Wendroff's fractional steps was determined. Moreover, Lax-Wendroff scheme has been modified to damp parasitic oscillations behind the wave front. Although the modification of Lax-Wendroff scheme is drawn upon intuition confidence in the approach is bolstered by satisfactory results achieved in rectangular channel. Hence, the case study carried out in the second part of the investigation is provided by solution coupling with appropriate initial conditions in the proximity of the collapsing dam.

Finally, the interaction between the schemes is demonstrated in a real-life application and their range of validity for dam-break wave propagation is defined. Likewise, to eliminate oscillations in Godunov solution for natural stream without additional procedures the idea to replace Godunov's second step by the corrector step of the modified Lax-Wendroff scheme is suggested.

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APPENDIX-NOTATION

The following symbols are used in this paper:

$B(x,z)$	= cross sectional width at free surface elevation;
c	= wave celerity;
Cr	= Courant number;
f_i^k	= value of the grid function at point x_i and t_k;
Fr	= Froude number;
g	= acceleration due to gravity;
$h(x,t)$	= water depth;
$i_0(x)$	= bed slope;
$K(x,t)$	= channel conveyance factor;
L	= distance traveled downstream by the wave front;
n	= Manning's roughness coefficient;
$q(x,t)$	= continuous lateral inflow or outflow per unit length;
$Q(x,t)$	= water discharge;
Q_{DAM}, z_{DAM}	= water discharge and water level at dam location;
t	= time;
$\triangle t$	= computational time step;
$V(x,t)$	= mean velocity;
x	= distance;
$\triangle x$	= distance increment;
$z(x,t)$	= water surface elevation;
α, ϑ	= parameters;
ζ	= front speed.
ξ	= distance measured from the fixed bottom to the centroid of a water layer of uniform thickness $d\xi$; and
$\omega(x,z)$	= cross-sectional flow area perpendicular to the flow direction.

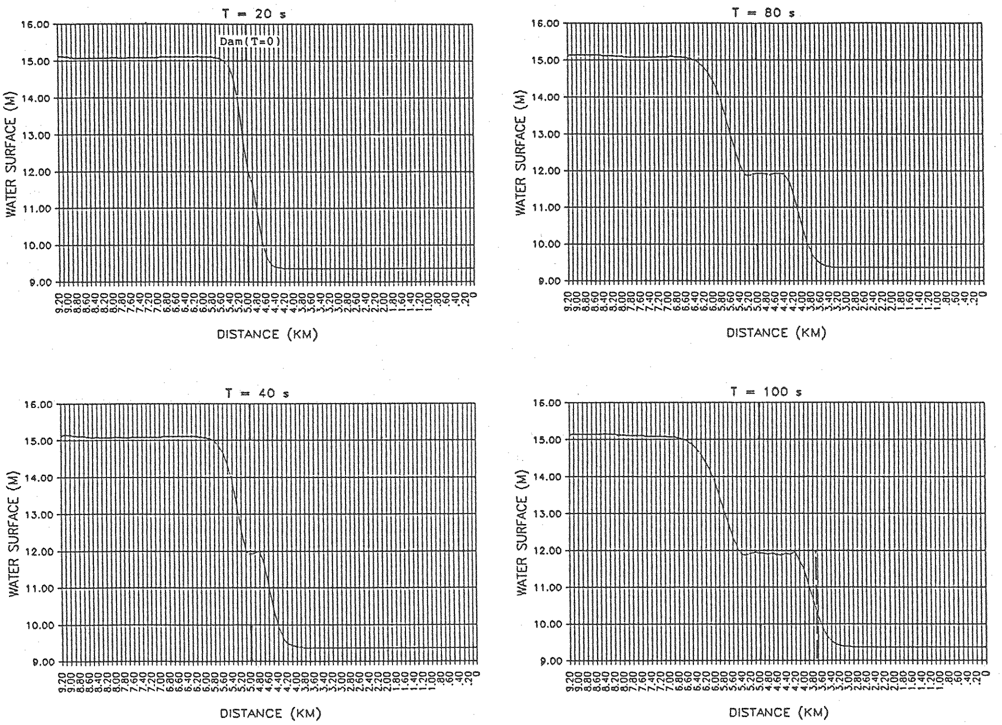


Figure 3: Dam-break in rectangular channel: Vasiliev scheme, $\Delta t = 5s$.

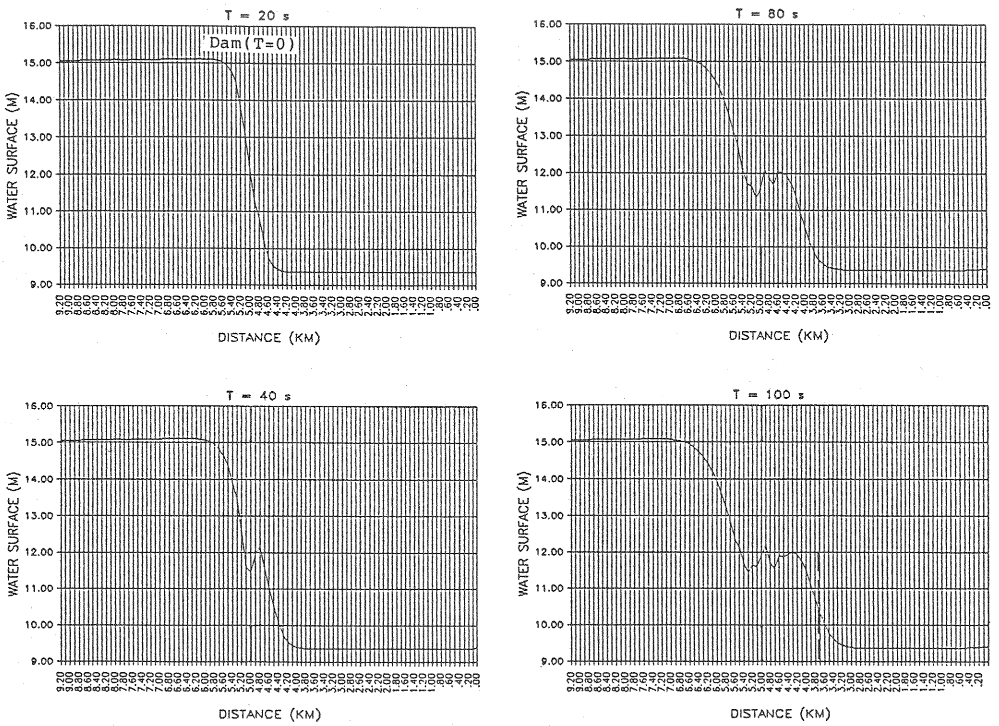


Figure 4: Dam-break in rectangular channel: Godunov scheme, $\vartheta = 1.0, \Delta t = 5s$.

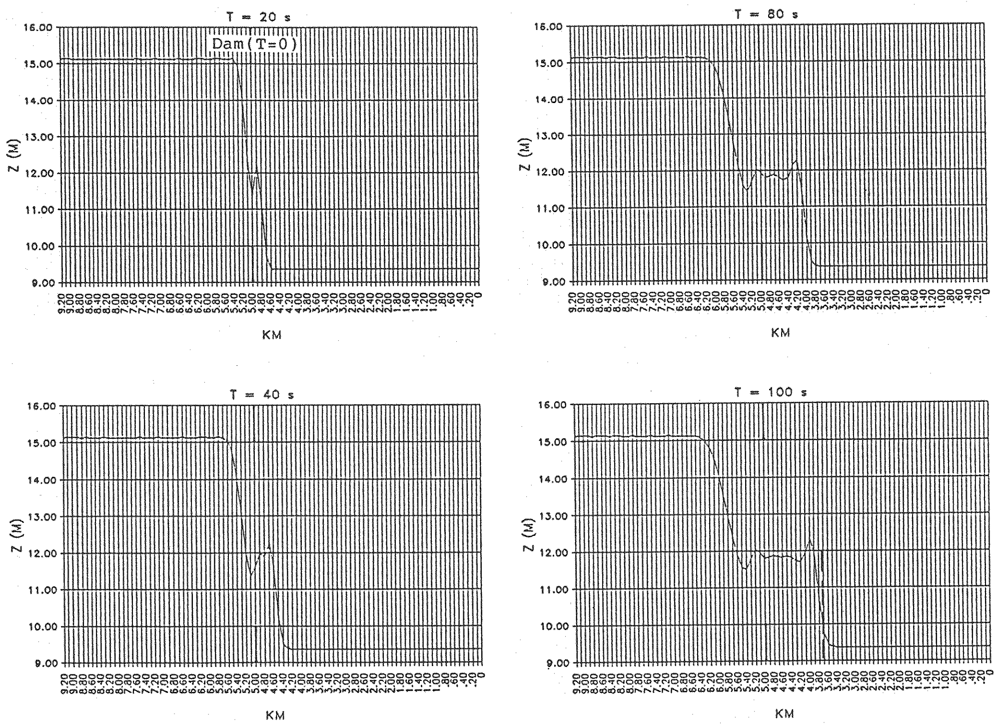


Figure 5: Dam-break in rectangular channel: Original Lax-Wendroff scheme, $\Delta t = 5s$.

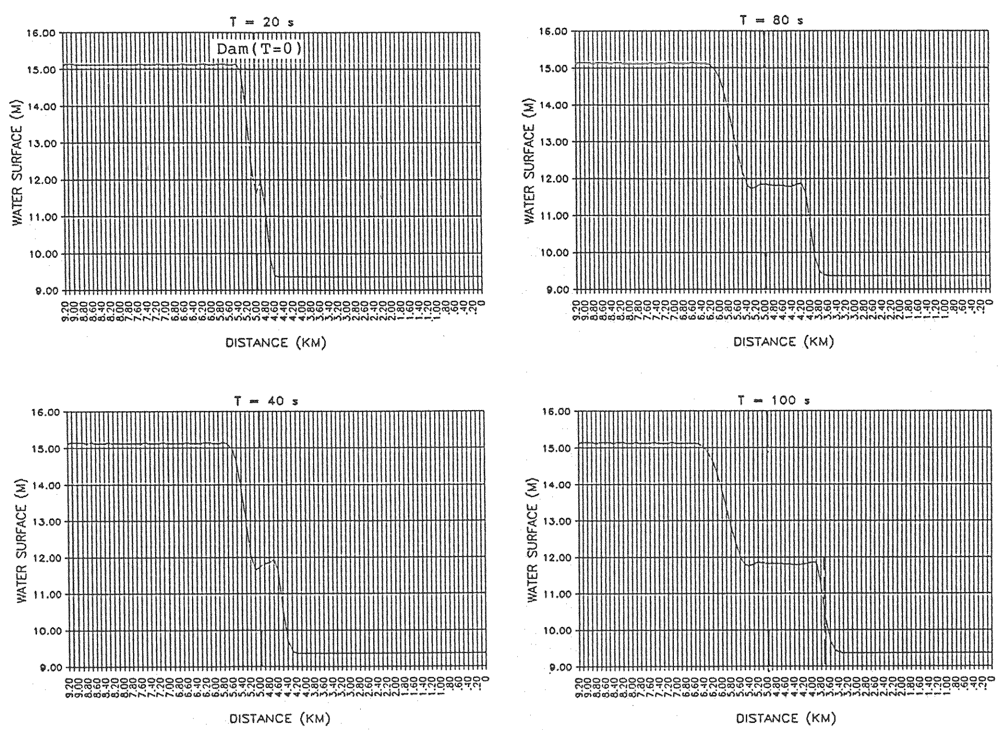


Figure 6: Dam-break in rectangular channel: Modified Lax-Wendroff scheme, $\alpha = 0.90, \Delta t = 5s$.

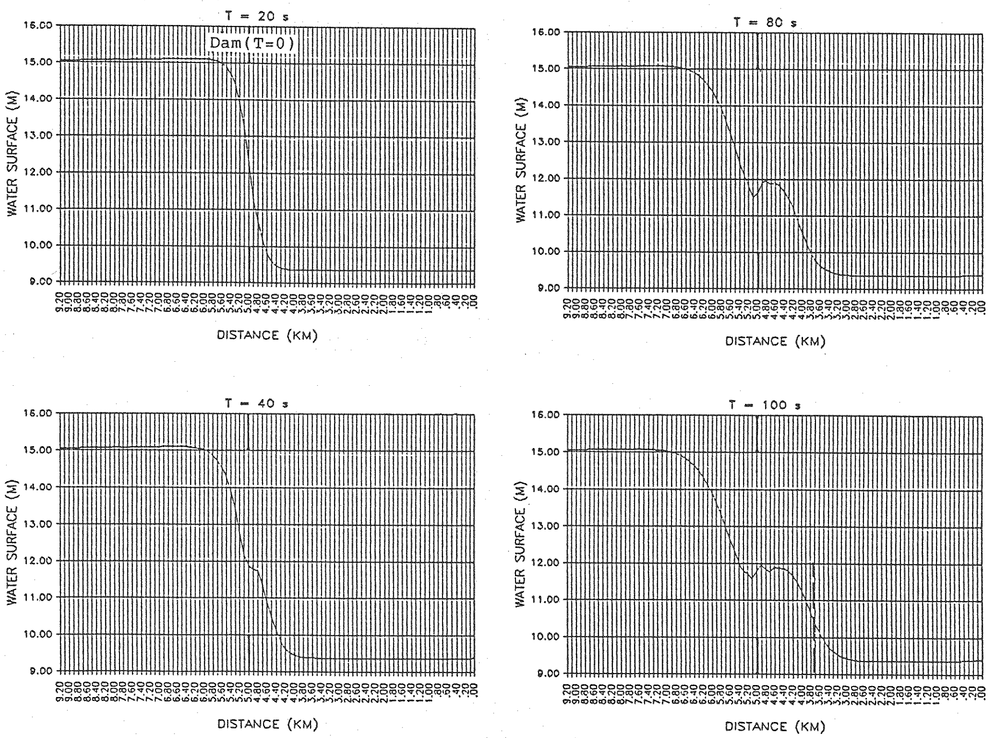


Figure 7: Dam-break in rectangular channel: Godunov scheme, $\vartheta = 1.0, \Delta t = 10s$.

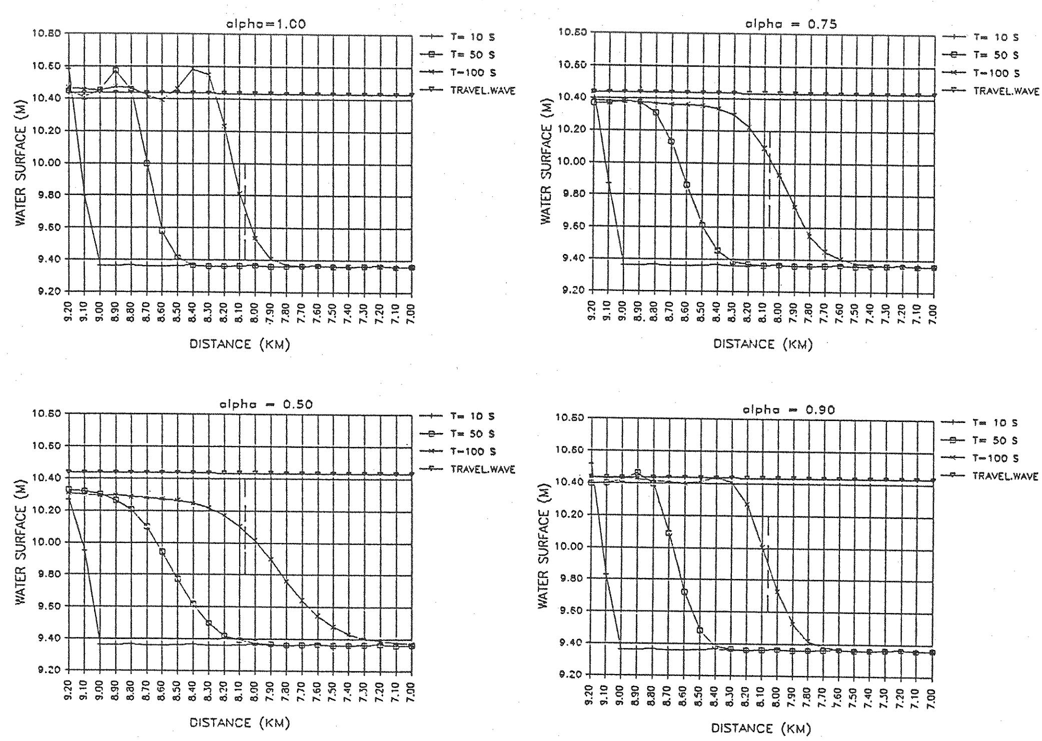
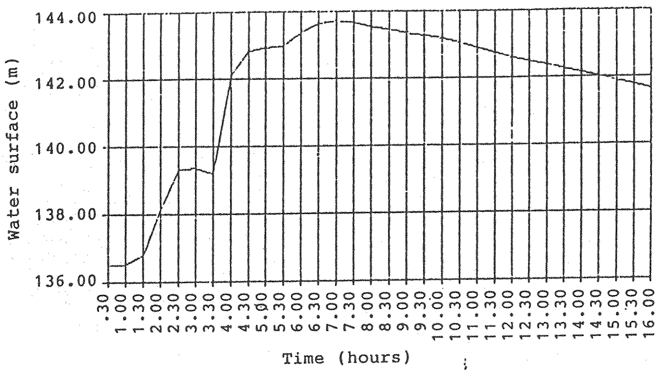


Figure 8: Traveling wave: Modified Lax-Wendroff scheme, $\Delta t = 5s$ for $\alpha = 1.00, \alpha = 0.90, \alpha = 0.75, \alpha = 0.50$.



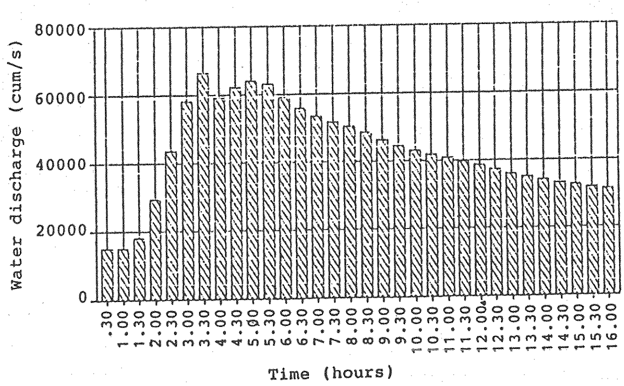


Figure 9: Water level and water discharge variations resulting from multiple dam-failures and partial destruction of protecting levees in natural stream.

(Received May 23, 1990 ; revised March 13, 1991)