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AN ANALYSIS OF THE SPREAD OF OIL SPILLED ON THE SURFACE OF WATER FLOW

By

Hideaki NOGUCHI
Marine Resources Division, Government Industrial Research Institute, CHUGOKU, Hiro, Kure, Japan

Kesayoshi HADANO
Associate Prof., Department of Civil Engineering, Yamaguchi University, Ube, Japan

and

Muneo HIRANO
Professor, Department of Civil Engineering Hydraulics and Soil Mechanics Kyushu University, Hakozaki, Fukuoka, Japan

SYNOPSIS

In this paper, the spread of continuously spilled oil in a uniform flow are presented both experimentally and theoretically. The laboratory experiments were performed in a circulating water basin. The oil was continuously poured on the surface of uniform water flow from the small outlet in order to observe the spreading oil slick. A mathematical model of oil spreading on the surface of water flow was developed by using the kinematic wave theory. The governing equations of the location and velocity of oil front are derived. Calculations of the slick propagation show a similar tendency to experimental data.

INTRODUCTION

Oil spills result from accidents in the oil production and transportation, and there is little hope to eliminate them entirely. When such a problem arises, there is an urgent need to estimate how rapidly the oil will spread and how far it will cover the water surface at a definite time. This forms a basis for engineering prevention measures to find out the best method dealing with such problems. The spread of an oil slick may be approximately viewed as composed of two part, the first consisting of convection by winds and currents, and the second of the increase in area of the oil due to the tendency of the oil to spread in calm water. These phenomena have been studied by a number of researchers, and much valuable information have been obtained. In previous studies, attention was mainly focused on the behavior of oil slick formed on the surface of stationary water. These studies have shown that the phenonenon is governed by the forces of inertia, gravity, viscous and surface tension, and that the dominating force changes in this order with time. In most studies the phenomenon is treated by dividing into three regions such as inertia-gravity, gravity-viscous and viscous-surface tension forces, according to the dominant forces. The laws of spread associated with the above regions were proposed using dimensional analysis. Theoretical treatments based on the governing equations of fluid mechanics were also proposed; e.g. Hoult(3) and Fannelop & Waldman(2) based on similarity law, and Abott(1) and Noguchi & Hirano(7) based on the kinematic wave method. Noguchi et al.(7) predicted an axisymmetrical oil spread on calm water on a theoretical basis, in which the equation of conservation of volume and the equation of motion were used to express the characteristics of oil slick. By using the kinematic wave theory, those equations were transformed into the characteristic equations along the characteristic curve. These simultaneous equations were solved to give the spreading rate of the oil front. This model describes the motion of the oil slick ranging from the gravity-inertial to the gravity-viscous spread.

There, however, exist only a few studies treating the oil slick on the surface of flowing water. Using the data from the Chevron spill, Murray(4) studied the characteristics of oil slicks generated from a continuous spillage. Field observations of oil slick geometries and current speeds of the Chevron spill of March 1970 at the Gulf of Mexico showed that the rates of slick expansion agreed with the Taylor's turbulent diffusion theory quite well. The approximate size and shape of a given oil slick could be predicted from the Fickian diffusion theory when the current speed, horizontal eddy diffusivity, and the oil discharge rate were given. His treatment, however, was not based on the fluid mechanics, i.e. does not take into account the forces such as buoyant force which are usually attributed to as the dominant ones. The reason for success of Murray's treatment seems to be that there exerted other forces comparable or dominant to the above mentioned forces.

This paper deals with the unsteady spread of continuously spilled oil on a uniform flow, experimentally and theoretically. The laboratory experiments were performed in a circulating water basin. Layer averaged equations were employed to describe the flow of oil slick. Based on the kinematic wave theory, a mathematical model of oil spread in water flow was developed. The front velocity of oil slick was also obtained as a function of the location at the maximum width section of oil slick. The theory was examined by comparing with the experimental results.

EXPERIMENT

A large circulating water basin, 3.8m wide, 40m long and 4m deep at the working section, was employed for laboratory experiments on the spread of continuously spilled oil on a uniform flow. The oil used in the experiments were machine oil No.1 (kinematic viscosity of about 135cSt at 15°C), machine oil No.10 (kinematic viscosity of about 23cP at 15°C) and light oil(kinematic viscosity of about 6cP at 15°C). The oil was continuously poured onto the water surface through the small outlet and allowed to spread over the surface of the uniform flow. The pattern of the spreading oil slicks were recorded with a 35mm still camera.

As shown in Fig.1, two cases of oil spread are considered, the velocity ratio $K_s \leqq 1$ and $K_s > 1$. K_s is defined as the ratio of the velocity of spilled oil at the outlet U_o to the velocity of water flow U_a, $K_s = U_o/U_a$. In the first case($K_s \leqq 1$), the oil poured onto the surface soon spreads in the similar pattern as the oil were spilled from an outlet which moves at the same speed of the ambient flow. In the latter case($K_s > 1$), at the region near the outlet, the oil spreads as a jet

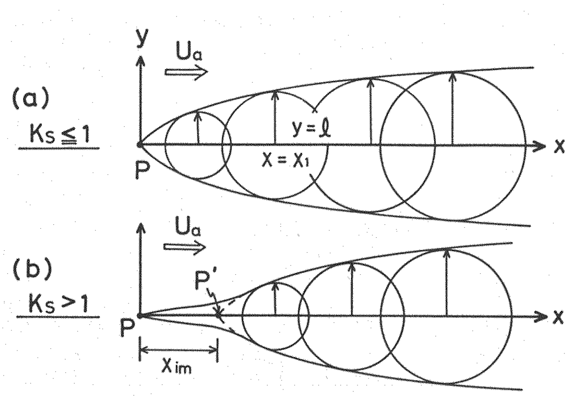

Fig.1 Two cases of spreading pattern of the oil slick ($K_s \leq 1$ and $K_s > 1$).

like flow because the oil injecting from the outlet has the relative velocity(U_o-U_a) to that of water flow. But the velocity of oil slick gradually approaches that of the ambient flow as the oil is far away from the outlet. At this stage the pattern of spread of oil slick is similar to that of the case of $K_s \leqq 1$.

Figure 2 shows the experimental results in the case of $K_s \leqq 1$. In this figure, the dimensionless slick width l/l_{Tc} is plotted against the dimensionless distance $x/(U_a t_{Tc})$, where x is the distance taken downstream direction from the outlet as

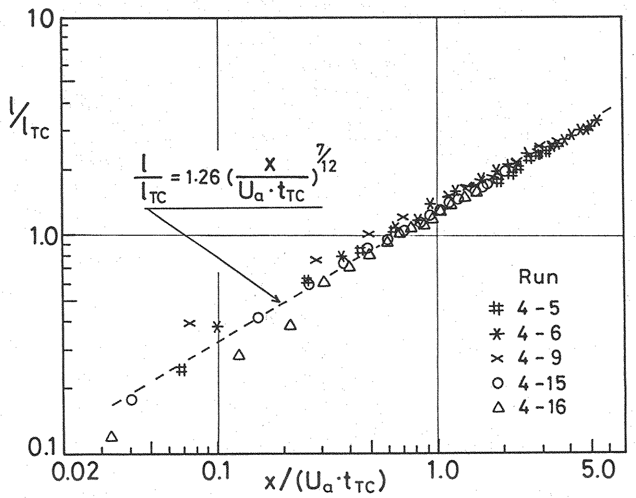

Fig.2 Experimental relationship between l/l_{Tc} and $x/(U_a t_{Tc})$. (for $K_s \leqq 1$)

shown in Fig.1 ; l is the half width of the slick at distance x ; and l_{Tc} and t_{Tc} are the length and time scales associated with the released volume respectively. The functional form of l_{Tc} and t_{Tc} were shown by Noguchi(5) as

$$l_{Tc} = (1 - \Delta)^{3/4} (\Delta g \nu_\omega^{\ 3})^{-1/8} Q^{5/8} \quad (1)$$

$$t_{Tc} = (1 - \Delta) (\Delta g \nu_\omega)^{-1/2} Q^{1/2} \quad (2)$$

where g is the gravitational acceleration; ρ_o and ρ_w are the density of oil and ambient fluid respectively ; $\Delta = 1 - (\rho_o / \rho_w)$; ν_w is the kinematic viscosity of water ; and Q is the flow rate of oil pouring through the outlet. From this figure, the relationship between l/l_{Tc} and $x/(U_a t_{Tc})$ is given as

$$\frac{l}{l_{Tc}} = 1.26 \left(\frac{x}{U_a t_{Tc}} \right)^{7/12} \quad (3)$$

which is similar to that of axisymmetrical oil spread on calm water.

In the case of $K_s>1$ as shown in Fig.3, the dimensionless slick width $l/(DF_o^{\ 2})$ is plotted against the dimensionless distance $x/(DF_o^{\ 2})$, where D is the diameter of the outlet; and F_o is the densimetric Froude number $U_o/(\Delta g D)^{1/2}$. At the first stage correspondent to the jet like flow, the slick width of oil in each experiment varies in a manner as indicated by the solid line in Fig.3.

$$\frac{l}{DF_o{}^2} = 0.403 \left\{ \frac{x}{DF_o{}^2} \right\}^{1.42} \qquad (4)$$

After that, it shifts to the each dotted lines according to the values of K_s. In the downstream region from an intersection of the solid and dotted lines, the spread of oil would be similar to that in case of $K_s \leqq 1$.

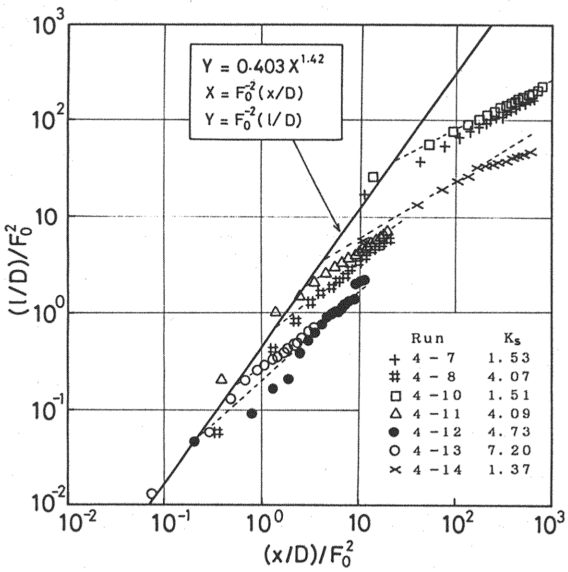

Fig.3 Relationship between $(l/D)/F_o{}^2$ and $(x/D)/F_o{}^2$ (for $K_s>1$).

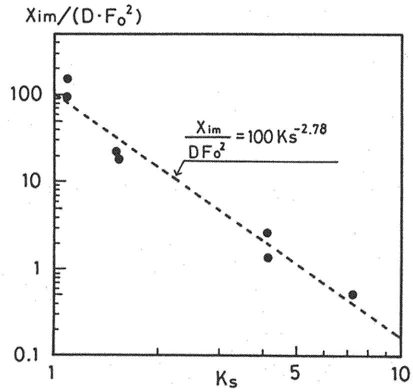

Fig.4 Distance $x_{im.}$ of the jet like flow region of the slick. (for $K_s>1$)

In Fig.4, position of the intersection of the two lines $x_{im.}$ are plotted against K_s. The dimensionless slick width l/l_{Tc} is plotted in Fig.5 against the dimensionless distance ($x-x_{im.}$) / $(U_a t_{Tc})$. As a result, an equation similar to Eq.(3) is obtained ;

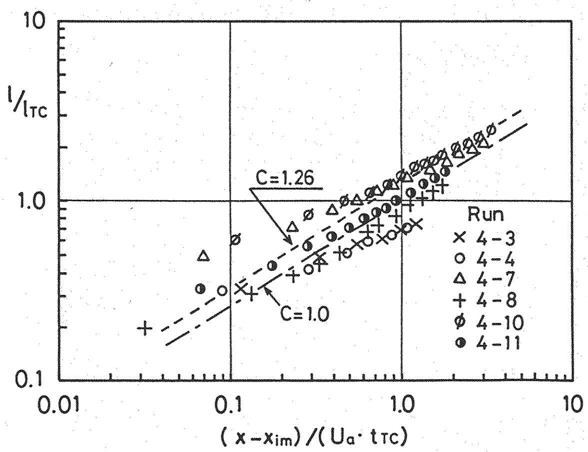

Fig.5 Experimental relationship between l/l_{Tc} and $(x-x_{min.})/(U_a t_{Tc})$ (for $K_s>1$) .

$$\frac{l}{l_{Tc}} = C\left(\frac{x - x_{im.}}{U_a t_{Tc}}\right)^{7/12} \tag{5}$$

where, C is the constant which is estimated approximately between 1 and 1.26 from Fig.5.

Now, we can draw the configuration of the leading edge of the oil slick. Figure 6 shows a schematic view of the coordinate system to represent the coordinates (x,y) of the configuration of the slick front. We put the origin of coordinates on the section of maximum width b_m of the slick. In figure 7, the observed configurations are represented in dimensionless form. In the figure, a dotted line shows the half circle with radius equal to unity. It can be seen that the plane view of the leading edge makes approximately a half circle of radius $b_m/2$.

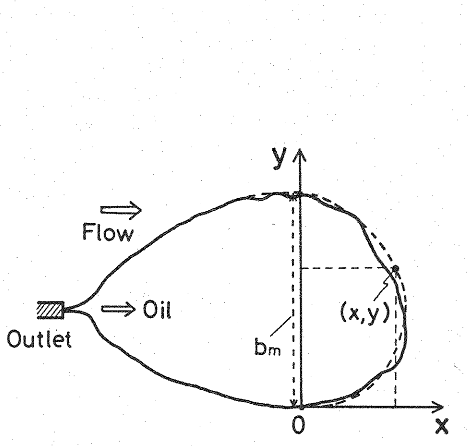

Fig.6 Schematic view of the leading edge of the slick.

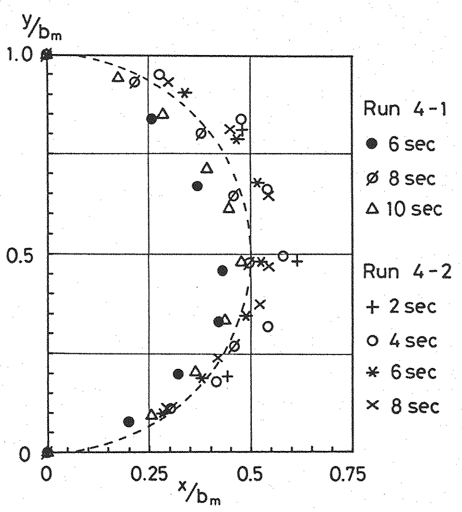

Fig.7 Experimental results of the plane view of the leading edge.

THEORY

Governing equations of the oil slick

If oil is released on a flowing water, the spreading of oil on the water surface is caused by the combined effect of the water flow and oil-water interaction forces. As shown in Fig.8, we employ the two layers model. Since the direction of the injection of oil is the same as that of the water stream, it is expected that the streamwize motion dominates also for the oil flowing downstream.

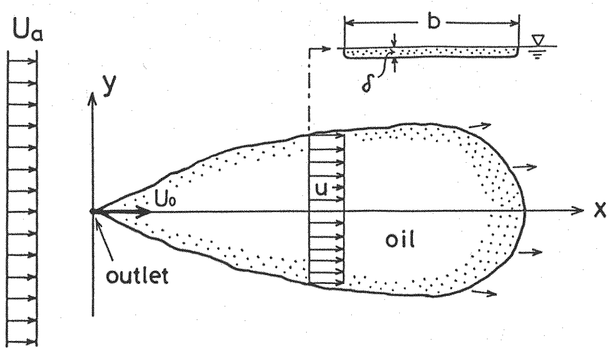

Fig.8 Schematic model of oil slick spreading over uniform flow.

Thus, the one-dimensional model is employed. Further, the cross-sectional geometry of the oil slick is assumed to be rectangular for convenience. Basic equations used are the conservation equation of volume and the x-component of momentum equation. They are written in the following forms;

$$\frac{\partial A}{\partial t} + \frac{\partial (uA)}{\partial x} = 0 \tag{6}$$

$$\frac{\partial (Au)}{\partial t} + \frac{\partial (Au^2)}{\partial x} = - \frac{\partial (\alpha \delta^2 b)}{\partial x} + \frac{\tau}{\rho_o} b \tag{7}$$

where t is the time ; x is the horizontal dimension of length ; δ is the thickness of slick ; b is the width of slick ; and A is the vertical cross-sectional area of oil slick taken perpendicular to the x-axis. It is assumed that the cross section A is rectangular with an area of $A=b\delta$; u is the averaged horizontal velocity of oil over a cross section A ; τ is the frictional drag force acting on an oil-water interface ; $\Delta=1-(\rho_o/\rho_w)$ and $\alpha=(\Delta g)/2$. The frictional drag force τ is written as

$$\frac{\tau}{\rho_o} = \frac{f_i}{2} \mid U_a - u \mid (U_a - u) \tag{8}$$

where f_i is the drag coefficient, and U_a is the velocity of the ambient uniform flow. Equations (6) and (7) are rewritten into following forms, when one uses the relation of $A=b\delta$ and Eq.(8);

$$\frac{\partial \delta}{\partial t} + u \frac{\partial \delta}{\partial x} = - \frac{\delta}{b} V_b - \delta \frac{\partial u}{\partial x} \tag{9}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = - 2\alpha \left(\frac{\partial \delta}{\partial x} + \frac{\delta}{2b} \frac{\partial b}{\partial x} \right) + \frac{f_i}{2\delta} |U_a - u| (U_a - u) \tag{10}$$

where, $$V_b = \frac{\partial b}{\partial t} + u\frac{\partial b}{\partial x} = \frac{db}{dt} \quad . \tag{11}$$

By using the kinematic wave theory, Eqs.(9) and (10) are transformed into the following forms.

$$\frac{dt}{1} = \frac{dx}{u} = \frac{d\delta}{-\frac{\delta}{b}V_b - \delta\frac{\partial u}{\partial x}} \tag{12}$$

$$\frac{dt}{1} = \frac{dx}{u} = \frac{du}{-2\alpha\left(\frac{\partial\delta}{\partial x} + \frac{\delta}{2b}\frac{\partial b}{\partial x}\right) + \frac{f_i}{2\delta}|U_a - u|(U_a - u)} \tag{13}$$

Then, δ and u are expressed along the same characteristic curve dx/dt=u as

$$\frac{d\delta}{dx} = -\frac{1}{u}\left(\frac{\delta}{b}V_b + \delta\frac{\partial u}{\partial x}\right) \tag{14}$$

$$\frac{du}{dx} = -\frac{2\alpha}{u}\left(\frac{\partial\delta}{\partial x} + \frac{\delta}{2b}\frac{\partial b}{\partial x}\right) + \frac{f_i}{2\delta u}|U_a - u|(U_a - u) \tag{15}$$

Propagation of the section of maximum width of the slick

It is difficult to obtain the velocity of oil slick from Eq.(14) and (15), because the right hand sides of these equations contain the partial differential terms whose correct estimates are almost impossible. To make further development of these equations possible, the cross section of maximum width b_m of the oil slick is traced. Clearly the term $\partial b/\partial x$ is zero in such a section,i.e.

$$\left.\frac{\partial b}{\partial x}\right|_{x=x_m} = 0 \tag{16}$$

where suffix "m" denotes the value at the section of maximum width b_m.

Substituting Eq.(16) into Eq.(11) yields

$$V_{b=b_m} = \frac{db_m}{dt} \tag{17}$$

Now, near the section of maximum width b_m, we introduce assumptions of similarity for the velocity u and the thickness δ, they can be expressed as below ;

$$\frac{u}{u_m} = \xi\left(\frac{x}{x_m}\right) , \qquad \frac{\delta}{\delta_m} = \psi\left(\frac{x}{x_m}\right) \tag{18}$$

where $\xi(x/x_m)$ and $\psi(x/x_m)$ denote the functions of x/x_m. Differentiating Eq.(18) with respect to x yields ;

$$\left.\frac{\partial u}{\partial x}\right|_{x=x_m} = K_1\frac{u_m}{x_m} \tag{19}$$

$$\left.\frac{\partial \delta}{\partial x}\right|_{x=x_m} = K_2\frac{\delta_m}{x_m} \tag{20}$$

where parameters $K_1 = \xi'(1)$ and $K_2 = \psi'(1)$.

Substituting Eqs.(16),(17),(19) and (20) into Eqs.(14) and (15), we obtain

$$\frac{d\delta_m}{dx_m} = -\frac{\delta_m}{u_m}\left(\frac{1}{b_m}\frac{db_m}{dt} + K_1\frac{u_m}{x_m}\right) \tag{21}$$

and

$$\frac{du_m}{dx_m} = -\frac{2\alpha}{u_m}K_2\frac{\delta_m}{x_m} + \frac{f_i}{2\delta_m u_m}|U_a - u_m|(U_a - u_m) \tag{22}$$

Thus the partial differential terms have been eliminated and the simultaneous ordinary differential equations to predict the variations of δ_m and u_m are obtained as a function of x_m. In order to obtain the solution of the simultaneous equations (21) and (22), another relation between b_m and x_m is needed. Considering the experimental results indicated in Figs.2 and 5, the maximum width b_m can be determined as bellow, similarly to Eqs.(3) and (5).

$$\frac{b_m}{l_{Tc}} = C_1\left(\frac{x_m}{U_a t_{Tc}}\right)^n \tag{23}$$

where C_1 and exponent n are constant. Differentiating Eq.(23) with respect to t yields

$$\frac{db_m}{dt} = \frac{n C_1 l_{Tc}}{(U_a t_{Tc})^n} x_m^{n-1} u_m \tag{24}$$

Substituting Eqs.(23) and (24) into Eq.(21) yields

$$\frac{d\delta_m}{dx_m} = -(n + K_1)\frac{\delta_m}{x_m} \tag{25}$$

Since $(n+K_1)$ is assumed to be constant, then Eq.(25) is easily integrated as below

$$\frac{\delta_m}{\delta_{mo}} = \left(\frac{x_m}{x_{mo}}\right)^{-(n+K_1)} \tag{26}$$

where, x_{mo} is the position of the maximum width of oil slick at a time $t=t_o$, and δ_{mo} is the corresponding value of δ at that time and position. Since $\partial\delta/\partial x$ near the section of maximum slick width b_m is supposed to be small, we may put $K_2=0$ in Eq.(20). Substituting Eq.(26) into Eq.(22), and putting $K_2=0$,

$$\frac{du_m}{dx_m} = \frac{f_i}{2\delta_{mo}}\left(\frac{x_m}{x_{mo}}\right)^{n+K_1}\frac{|U_a - u_m|(U_a - u_m)}{u_m} \tag{27}$$

Now, the Eq.(27) can be rewritten in dimensionless form;

$$\frac{du_{m*}}{dx_{m*}} = \frac{f_i}{2}E_o x_{m*}^{n+K_1}\frac{|U_{a*} - u_{m*}|(U_{a*} - u_{m*})}{u_{m*}} \tag{28}$$

where $E_o = x_{mo}/\delta_{mo}$; u_{m*} is the dimensionless velocity of the oil slick, $u_{m*} = u_m / u_{mo}$; $x_{m*} = x_m / x_{mo}$; $U_{a*} = U_a / u_{mo}$; u_{mo} is the value of u_m at $x_m = x_{mo}$.

When f_i is assumed to be constant, Eq.(28) can be integrated to give the dimensionless velocity at the section of maximum slick width as follows;

$$\log\left\{\frac{u_{m*} - U_{a*}}{1 - U_{a*}}\right\} + \frac{U_{a*}\,(\,u_{m*} - 1\,)}{(\,1 - U_{a*}\,)\,(\,u_{m*} - U_{a*}\,)} = \Phi \tag{29}$$

$$\text{For } (\,U_{a*} - u_{m*}\,) < 0 : \quad \Phi = \frac{f_i\,E_0}{2\,\lambda}(1 - x_{m*}{}^{\lambda})$$

$$\text{For } (\,U_{a*} - u_{m*}\,) > 0 : \quad \Phi = \frac{f_i\,E_0}{2\,\lambda}(x_{m*}{}^{\lambda} - 1)$$

and $\lambda = K_1 + n + 1$.

Propagation of the leading edge of the slick

As shown in Fig.6 and Fig.7, we assume that the leading edge of oil slick makes a half circle of radius $b_m/2$. Then, the location of the leading edge $x_f(t)$ at any moment is written as

$$x_f(t) = x_m(t) + \frac{1}{2}\,b_m(t) \tag{30}$$

Substituting Eq.(23) into Eq.(30), the dimensionless location of oil front x_{f*} is obtained as

$$x_{f*} = x_{m*} + \frac{\varepsilon}{n}\,x_{m*}{}^{n} \quad , \tag{31}$$

where

$$\varepsilon = \{\ (nC_1 l_{Tc})\ x_{mo}{}^{n-1}\ \}\ /\ \{\ 2\ (U_a t_{Tc})^n\ \} \quad . \tag{32}$$

Differentiating Eq.(30) with respect to t, the dimensionless velocity of oil front u_{f*} is determined as

$$u_{f*} = \{\ 1 + \varepsilon\,x_{m*}{}^{n-1}\ \}\ u_{m*} \tag{33}$$

COMPARISON OF THE THEORY WITH EXPERIMENT

In order to examine the applicability of the theory, the front velocity u_{f*} and the location x_{f*} are used to compare with those determined experimental data. In the following, we use the same value of f_i(=0.006) proposed by Noguchi et al.(6). Also, we use the values of experimental result of Eq.(5) for the values of n and C_1 in Eq.(22), that is n=7/12 and C_1=2 (because C=1 and $L=b_m/2$ in Eq.(5)). The initial condition of b_m, δ_m, x_m and u_f are determined by the use of experimental data of the plane view of the slick at 2 second after the initiation of injection. Then, the initial condition of u_m is calculated from Eq.(33).

Substituting Eqs.(19), (20) and (24) into Eq.(9), we obtain

$$\frac{\partial \delta_m}{\partial t} = -\ (n + K_1 + K_2)\ \frac{u_m\,\delta_m}{x_m} \tag{34}$$

Since $\partial\delta/\partial t$ is negative in Eq.(34), then $(n + K_1 + K_2) \geqq 0$. Moreover, since K_2 is considered to be zero in Eq.(20), we then obtain $K_1 \geqq -n$. Calculation were performed for various values of K_1 from -0.5 to 4. Results of the calculations of

u_{f*} and x_{f*} using Eqs.(29), (31) and (33) are given in Fig.9 together with experimental results. The figure shows that the result of calculation using $K_1 = 4$ shows a fairly similar tendency to the experimental data. As the results, K_1 is estimated to be about 4. In these calculations, the spreading time t_* is given by

$$t_* = t_{*o} + \Sigma\, \Delta t_*(k) \quad , \; k=0,1,2,3 \cdots\cdots \qquad (35)$$

where, $\Delta t_* = \{x_{m*}(k+1) - x_{m*}(k)\} \,/\, [\, \{u_{m*}(k+1) + u_{m*}(k)\} \,/\, 2\,]$; $t_* = t/(\delta_{mo}/\, u_{mo})$; $\Delta t_* = \Delta t/(\delta_{mo}/\, u_{mo})$; and $t_{*o} = t_0/(\delta_{mo}/\, u_{mo})$.

The calculated curves of x_{f*} using K_1=4 for the another experimental results are shown in Fig.10. The predicted curves agree well with the experimental results.

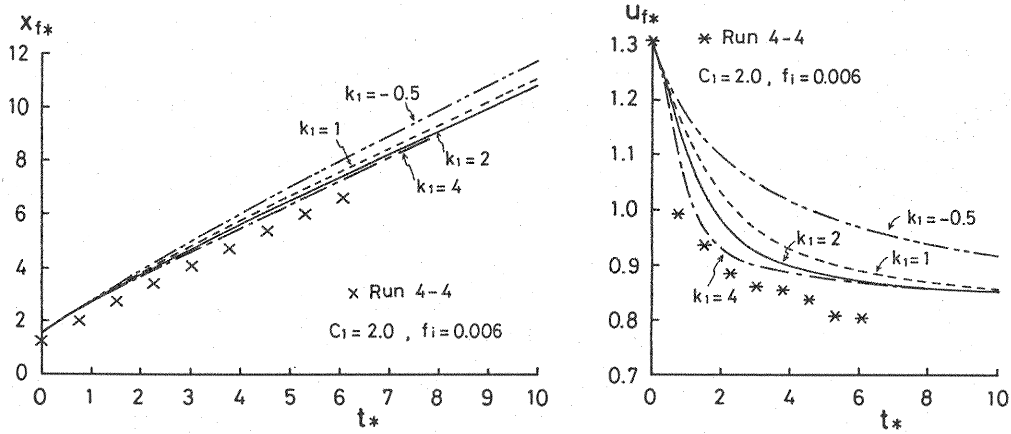

Fig.9 Comparison of the calculated and measured values of x_{f*} and U_{f*}.

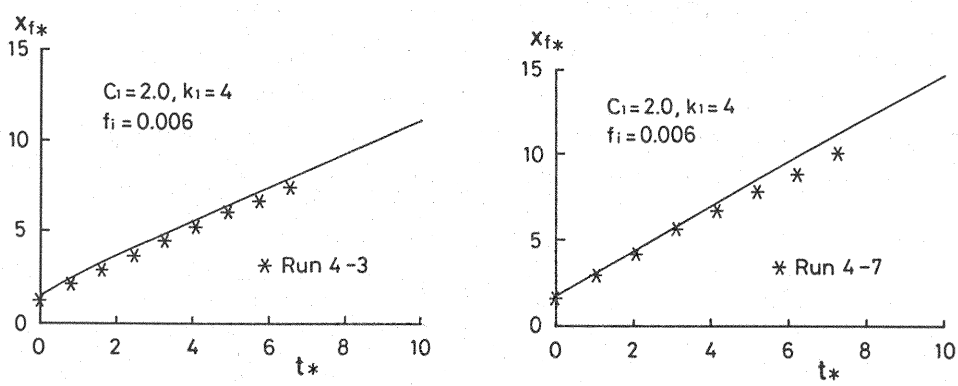

Fig.10 Comparison of the calculated and measured values of x_{f*}.

CONCLUSIONS

The spread of continuously spilled oil in the surface of a uniform flow are discussed experimentally and theoretically. A mathematical model starting from the layer averaged equations were developed by the use of the kinematic wave theory. Equations(31) and (33) were proposed to determine the location and velocity of the front of spreading oil slick. In the development of the theory, the parameter K_2 was taken to be 0 and the parameter K_1 in Eq.(19) was estimated to be 4 from comparison between the calculated and measured value of x_{f*}. The calculated curves are in good agreement with the experimental data.

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APPENDIX - NOTATION

The following symbols are used in this paper:

A = vertical cross-sectional area of oil slick taken perpendicular to the x-axis;

C = coefficient in the equation of slick width given by Eq.(5);

C_1 = constant in the equation of slick width defined by Eq.(23);

D = diameter of the outlet of oil;

F_o = densimetric Froude number, defined by $F_o=U_o/(\Delta gD)^{1/2}$;

K_1 = parameter denoted by $K_1=\xi'(1)$;

K_2 = parameter denoted by $K_2=\psi'(1)$;

K_s = velocity ratio of the velocity of spilled oil at the outlet U_o to the velocity of water flow U_a ;

Q = flow rate of oil pouring through the outlet;

U_a = velocity of ambient uniform water flow;

U_{a*} = dimensionless velocity of ambient uniform water flow , defined by $U_{a*}=U_a/u_{mo}$;

U_o = velocity of spilled oil at the outlet;

V_b = the rate of change of slick width defined by Eq.(11);

b = width of the oil slick

b_m = maximum width of the slick;

f_i = drag coefficient;

g = gravitational acceleration;

l = half width of the slick at distance x;
l_{Tc} = length scale associated with the released volume of oil;
n = exponent in the equation of slick width defined by Eq.(23);
t = time;
t_* = dimensionless time, defined by $t_*=t/t_o$;
t_{*o} = dimensionless time at $t=t_o$, defined by $t_{*o}=t_o/(\delta_{mo}/u_{mo})$;
t_{Tc} = time scale associated with the released volume of oil;
u = averaged horizontal velocity of oil over a cross section A;
u_f = velocity of oil front;
u_{f*} = dimensionless velocity of oil front, defined by $u_{f*}=u_f/u_{mo}$;
u_m = averaged horizontal velocity of oil over a cross section A at the section of maximum width b_m;
u_{mo} = averaged horizontal velocity of oil over a cross section A at the section of maximum width b_m at a time $t=t_o$;
u_{m*} = dimensionless velocity of the oil at the section of maximum width b_m, defined by $u_{m*}= u_m/u_{mo}$;
x = downstream distance from the outlet;
$x_f(t)$ = location of the leading edge at any moment;
x_{f*} = dimensionless location of oil front, defined by Eq.(31);
$x_{im.}$ = downstream distance of jet like flow region from the outlet (see Fig. 1);
x_m = downstream distance from the outlet to the section of maximum width b_m;
x_{m*} = downstream distance from the outlet to the section of maximum width, defined by $x_{m*}=x_m/x_{mo}$;
x_{mo} = downstream distance from the outlet to the section of the maximum width of oil slick at a time $t=t_o$;
α = constant, defined by $\alpha=(\Delta g)/2$;
ε = parameter defined by Eq.(32);
δ = thickness of the oil slick;
δ_m = thickness of the oil slick at the section of maximum width b_m;
δ_{mo} = thickness of the oil slick at the section of maximum width b_m at a time $t=t_o$;
λ = constant, defined by $\lambda=K_1+n+1$;
ν_w = kinematic viscosity of water;
ξ, ψ = functions of x/x_m;
ρ_o = density of oil;
ρ_w = density of water;
Δ = $1 - (\rho_o/\rho_w)$;
τ = frictional drag force acting on an oil-water interface;

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