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DIFFUSION BEHAVIOR OF VERTICAL FORCED PLUMES IN LINEARLY STRATIFIED CROSS FLOWS

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SYNOPSIS

The characteristics of a vertical forced plume in a uniform cross flow of stably linear-stratified environment are studied both numerically and experimentally. The empirical formulas for the diffusion characteristics of a vertical forced plume (trajectory, dilution, etc.) are presented experimentally. A three-dimensional numerical model is used to simulate the behavior of the vertical forced plume in these ambient conditions. The adaptability of the three-dimensional model is evaluated by comparison of numerical results with experiments. The numerical model is applied to discuss the production and disappearance process of vortex pairs induced both in the plume and its surrounding environment.

INTRODUCTION

It is important in hydraulic engineering to estimate the dispersion process of forced plumes in natural environment accurately. The behavior of the forced plume in a stratified cross flow is complicated, and significantly affected by its discharge conditions and ambient conditions such as currents and density stratification in environment. For example, the diffusion process of heated water discharge or effluent in coastal zones depends on both sea currents and vertical density stratification, especially in summer. Very few experimental and theoretical studies , however, have been carried out to reveal the compound effects of ambient current and density stratification on the diffusion process of a forced plume.

The purpose of this study is to investigate the characteristics of a vertical forced plume in a uniform cross flow of stably linear stratified environment. Firstly, the mean trajectories of forced plume and dilution along trajectories in stratified cross flows are investigated experimentally. They are compared with the experiments which were carried out in non-stratified cross flows to estimate the effects of ambient current and stratification. The several empirical formulas for characteristics of forced plumes in linearly stratified cross flows are proposed.

In numerical computations of plumes or jets, the simple model have been used to estimate the diffusion characteristics. This model is based on the entrainment hypothesis and expressed by using ordinary differential equations (E. Hirst (1) and G. A. L. Delvigue (3) etc.). M. Mizutori and N. Katano et al. (6) simulated a vertical forced plume by using a three-dimensional numerical model which is derived from the conservation laws of mass, momentum, temperature and salinity without entrainment hypothesis and boundary layer approximation. The model could calculate the diffusion process of plumes more accurately than such simple differential models in complicate environmental conditions. Secondly, this model is applied to investigate the diffusion process of a vertical forced plume in a linearly stratified cross flow. The adaptability of the model is evaluated by the comparison of numerical results with experimental measurements. The numerical model is applied to discuss the production and disappearance process of vortex pairs which grow as the plume flows downstream.

EXPERIMENTAL STUDY

Equipment and Methods

The series of experiments were carried out by using a flume illustrated in Fig. 1. The flume was 1.0m wide, 0.8m deep and 10.0m long, the horizontal bottom and side walls being made of stainless steel and plexiglass plates, respectively. The working fluids were vertically separated into five layers and were circulated independently. Salt water was used as working fluid of environment, and fresh water as one of forced plume. The fresh water was discharged upward into the flume through the outlet (diameter D=1.6mm) attached at the height of 30D from the flume bottom (H/D=470, H:Depth of the outlet). A heat-exchange system and mixing slits at the upper stream part of the flume were used to control the temperature of circulating salt water in order to make the density stratification in ambient fluid. Mean trajectories and dilution of forced plumes were observed by using fluorescence analysis. The fluorescent dyes contained in a plume were measured with a fluorometer and analyzed, which enabled us to study the plume behavior in sufficiently diluted zone. Measured points were located on the half side of a cross section owing to plane symmetry of the phenomenon.

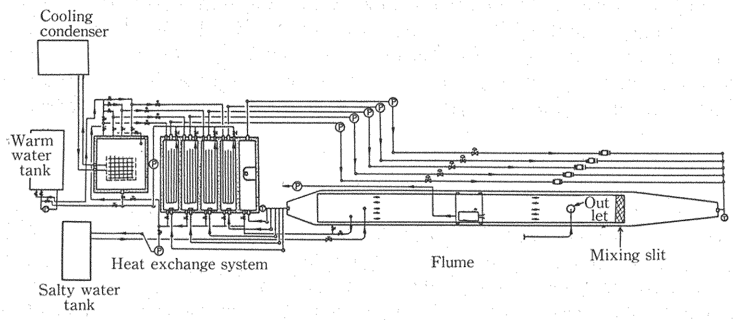


Fig. 1 Schematic experimental equipment

Experimental Conditions

In the case of stratified cross flow, the characteristics of a forced plume are depending on the initial momentum flux Mo, the initial buoyancy flux Bo, the ambient cross flow u_e and the strength of ambient density stratification $\partial\rho/\partial Z$. These four parameters can be written as follows.

$$M_o = \rho_0 w_0{}^2 \pi D^2/4$$

$$B_o = g\Delta\rho w_0 \pi D^2/4$$

$$u_e \qquad (1)$$

$$\frac{\partial\rho}{\partial z}$$

where ρ_0 is the density of forced plume at the outlet and $\Delta\rho$(= 0.025g/cm^3) is the density difference between forced plumes and ambient fluid at the outlet. The dimensional analysis gives four non-dimensional parameters which have major effect on the diffusion of plumes, and the range of experiment are shown as follows.

$$Fro = \frac{w_o}{(\Delta\rho_0/\rho_0 gD)^{1/2}} = 40,60 \qquad (2)$$

$$Fru = \frac{u_e}{(\Delta\rho_0/\rho_0 gD)^{1/2}} = 0.191\text{-}0.638$$

$$k = w_o/u_e = 21\text{-}94 \quad (2)$$

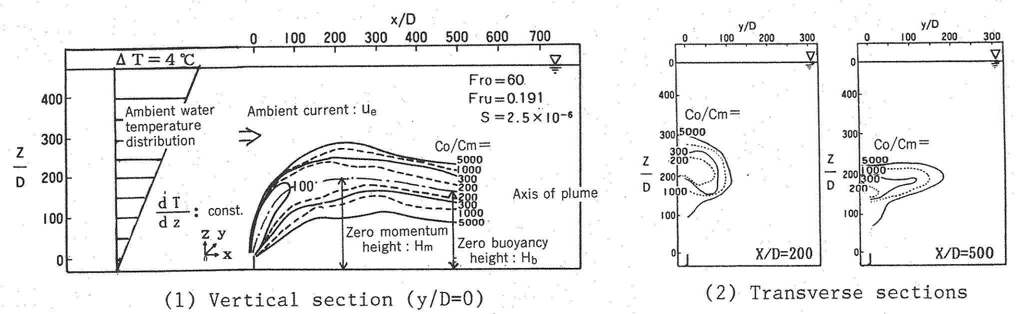
$$S = \frac{D}{\rho_0}\frac{\partial\rho}{\partial z} = 1.1 \times 10^{-6} - 3.9 \times 10^{-6}$$

Experimental Results

Figure 2 shows typical contours of dilution of a forced plume in vertical and transverse sections. In stratified cross flows, the diffusion of a plume which primarily depends upon a cross flow and density stratification is classified into three patterns shown in Fig. 3. A diffusion pattern is able to be distinguished by the comparison of H with zero momentum height H_m and zero buoyancy height H_b as shown in Fig. 3. All results of this experiment were divided into C pattern.

Experimental results as shown in Fig.2 show a tendency that a density stratification in environment makes vertical diffusion of a plume decreased, trajectories of a plume low, and dilution contours in a transverse section flat. The separation of plume is recognized as well as in non-stratified cross flows, but is not clear as plume flows downstream.

Figures 4 and 5 show the typical trajectories of forced plumes in stratified cross flows and dilution along them, respectively. The solid lines in figures denote the empirical formulas by M. Mizutori and N. Katano et al. (5) in non-stratified cross flows. In the region close to the outlet, the initial momentum of forced plume plays an important role in its diffusion process, and trajectories of forced plume and dilution along them in stratified cross flows are agreement with those in non-stratified cross flows. However, as forced plumes rise and dilute, trajectories and dilution are significantly affected by density stratification in environment, which results the difference from the empirical formula in non-stratified cross flows. Trajectories in stratified close flows and dilution along them, therefore, can be given by



(1) Vertical section (y/D=0)

(2) Transverse sections

Fig. 2 Experimental results of dilution contours

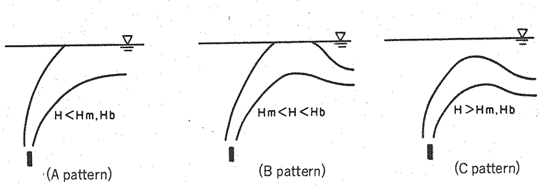


Fig. 3 Diffusion patterns of vertical forced plume in stratified cross flows

(trajectories) $\frac{z}{\ell} = 0.60S^{-1/3}F_{ru}^{-5/2}F_{ro}^{-2/3}$ (3)

(dilution) $\frac{Cm}{Co}kF_{ru}^{-4} = 4.06S^{1/3}F_{ru}^{-5/2}F_{ro}^{3/4}[(\frac{X}{\ell})^2 + 2F_{ru}^{4}(\frac{X}{\ell})]^{-1/3}$ (4)

where $\ell(=k\ D\ F_{ru}^{-2})$ denotes characteristic length of buoyancy.

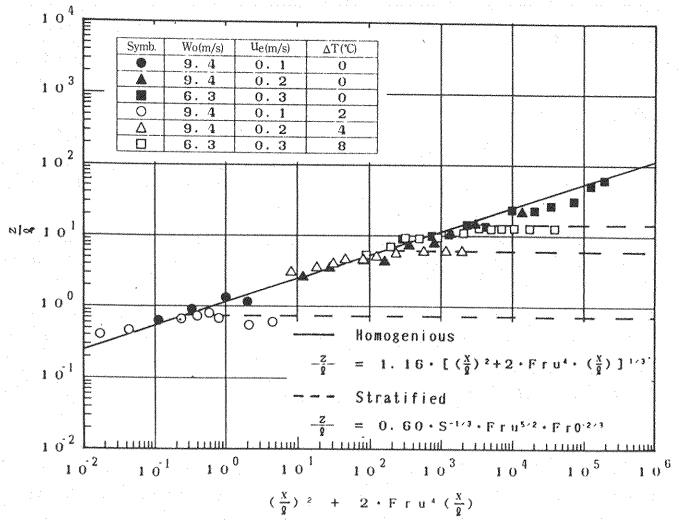


Fig. 4 Trajectories of the forced plume in stratified cross flow

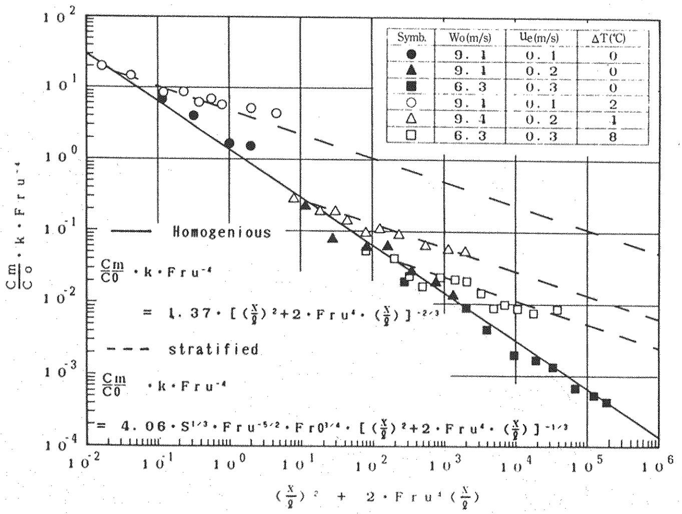


Fig. 5 Dilution along the plume trajectories in stratified cross flow

Figures 6 and 7 show zero momentum height H_m where the vertical momentum of a forced plume is equal to zero, and dilution at H_m, respectively. Zero momentum height and dilution at that height are presented by using S, F_{ro} and F_{ru} defined in equation (2).

(zero momentum height)

$$\frac{H_m}{D} = 0.17S^{-1/3}F_{ru}^{-2/3}F_{ro}^{2/3} \tag{5}$$

(dilution at H_m)

$$\frac{C_m}{C_0} = 2.00S^{2/3}F_{ru}^{-2/3}F_{ro}^{2/3} \tag{6}$$

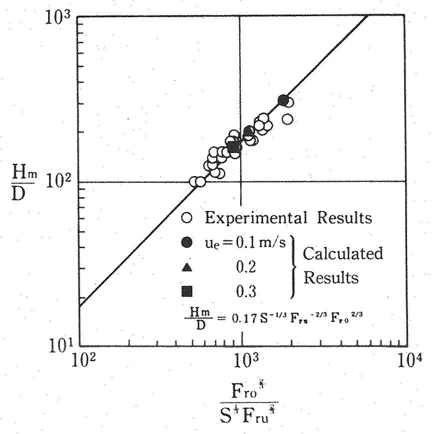


Fig. 6 Zero momentum height Hm

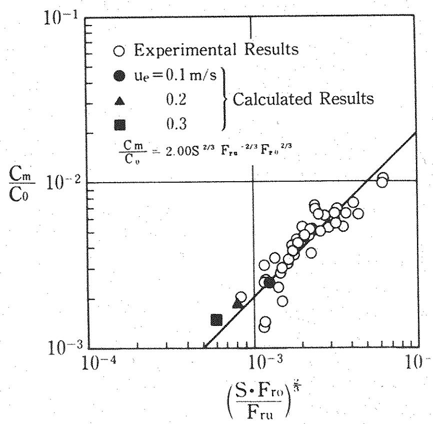


Fig. 7 Dilution at Hm

NUMERICAL SIMULATION

Numerical Model and Boundary Conditions

The investigation with the three-dimensional numerical model were carried out to study the characteristics of a forced plume in stratified cross flows. It treats conservation laws of mass, momentum, temperature and salinity simultaneously without entrainment hypothesis and boundary layer approximations. The density of water is computed by using Knudsen's formula, and density stratification in environment is expressed as a function of only water temperature. Assuming the Boussinesq approximation that density variation is account for only in gravitational term, the govering equations for the velocity, temperature and salinity are

(conservation of mass)

$$\frac{\partial ui}{\partial xi} = 0$$

(conservation of momentum)

$$\frac{\partial ui}{\partial t} + \frac{\partial}{\partial xj}(ui\ uj) = -\frac{1}{\rho}\frac{\partial p}{\partial xi} - g\frac{\Delta\rho}{\rho}\delta 3i + \frac{\partial}{\partial xj}\left(\varepsilon mj\frac{\partial ui}{\partial xj}\right) \tag{7}$$

(conservation of temperature)

$$\frac{\partial T}{\partial t} + ui \frac{\partial T}{\partial xi} = \frac{\partial}{\partial xi} (\varepsilon_{Ti} \frac{\partial T}{\partial xi})$$

(conservation of salinity)

$$\frac{\partial s}{\partial t} + ui \frac{\partial s}{\partial xi} = \frac{\partial}{\partial xi} (\varepsilon_{si} \frac{\partial s}{\partial xi})$$

where ui is the velocity vector, ρ is the dynamic pressure and s is the salinity. The viscosity, conductivity and molecular diffusion are neglected because they are small compared with turbulent stress and turbulent fluxes. The eddy viscosity is denoted by ε_m, and the eddy diffusivities of temperature and salinity are denoted by ε_T and ε_s, respectively. On the basis of the mixing length theory, the eddy viscosity ε_m in plume is modelled by using the characteristic width of a plume.

(eddy viscosity)

$$\varepsilon_m = C\, b_{1/2} w_m \quad (0 \leqq r \leqq b_{1/2}) \qquad (8)$$

where w_m denotes the mean centerline velocity of a plume, $b_{1/2}$ is the half-width of $0.5w_m$, r is the distance from the centerline, and the experimental constant C is 0.0384 in this calculation. The eddy diffusivities of temperature and salinity are estimated by using turbulent Schmit (or Plandtl) number Sc(or Pr), which is assumed to be 0.7.

$$(\varepsilon_T,\ \varepsilon_s) \quad \varepsilon_T(=\varepsilon_s) = \varepsilon_m/Pr(=\varepsilon_m/Sc) \qquad (9)$$

In environment where $r>b_{1/2}$, ε_m, ε_t and ε_s are given as constants.

(ε_m, ε_T and ε_s)

$$\varepsilon_m = 90\ cm^2/s,\ \varepsilon_T(=\varepsilon_s) = 1\ cm^2/s\ (horizontal), 0.1\ cm^2/s\ (vertical) \qquad (10)$$

The dynamic pressure p is calculated by using HSMAC scheme. The equations are discretized onto a grid using third-order up-wind differencing for advection term and central differencing for diffusion term on a non-uniform mesh. In the case of stratified cross flows, the boundary conditions of the model were schematically shown in Fig. 8.

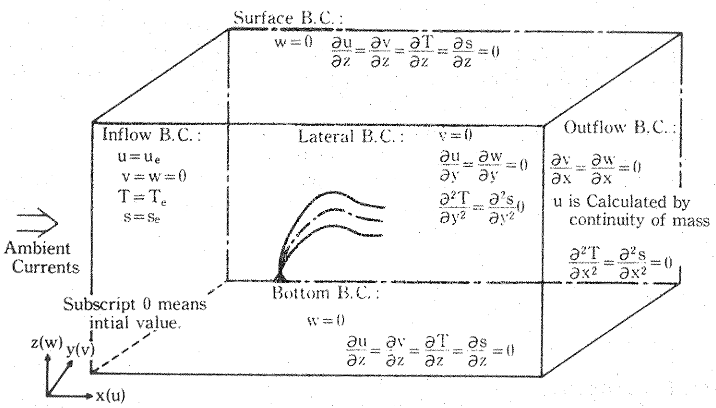


Fig. 8 Boundary conditions

Table 1 Numerical conditions

W_0 : Discharged velocity at outlet	6.3 m/s
D : Diameter of outlet	7.5 cm
$\Delta\rho$: $\rho_{e,0} - \rho_0$ ($\rho_{e,0}$, ρ_0 are ambient and discharged density at outlet)	0.026 g/cm³
H : Depth at outlet	44 m
ue : Ambient flow velocity	0.1, 0.2, 0.3 m/s
ΔT : Ts − To (Ts and To are ambient temperature at surface and outlet)	2 ℃
F_{ro} : $\dfrac{W_0}{(\Delta\rho_0/\rho_0 g D)^{1/2}}$	46
F_{ru} : $\dfrac{u_e}{(\Delta\rho_0/\rho_0 g D)^{1/2}}$	0.72, 1.45, 2.17
S : $\dfrac{D}{\rho_0}\dfrac{\partial\rho_e}{\partial z}$	7.15×10^{-7}

Numerical Results

The above-mentioned three-dimensional model was applied to simulate the behavior of forced plume in stratified cross flows. Calculating conditions are shown in Table 1. In this study, the ambient current was changed in the three cases, ue = 0.1, 0.2, 0.3 m/s. The computational domain is schematically shown in Fig. 8, which denotes a half region of plume owing to the plane symmetry of it, and the size of computational mesh is 20*38*33 in x, y and z directions.

Figures 9 and 10 show the comparison between the calculated results and the empirical formulas proposed in chapter 2 as to the trajectory and the dilution along them. The three-dimensional model can be found to predict accurately the behavior of plume in not only the near region which is close to the outlet in

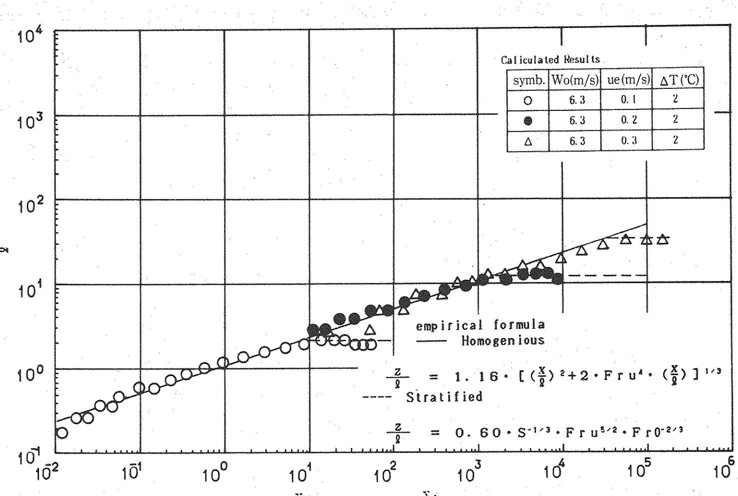


Fig. 9 Comparison of trajectories of forced plume between calculated results and empirical formulas

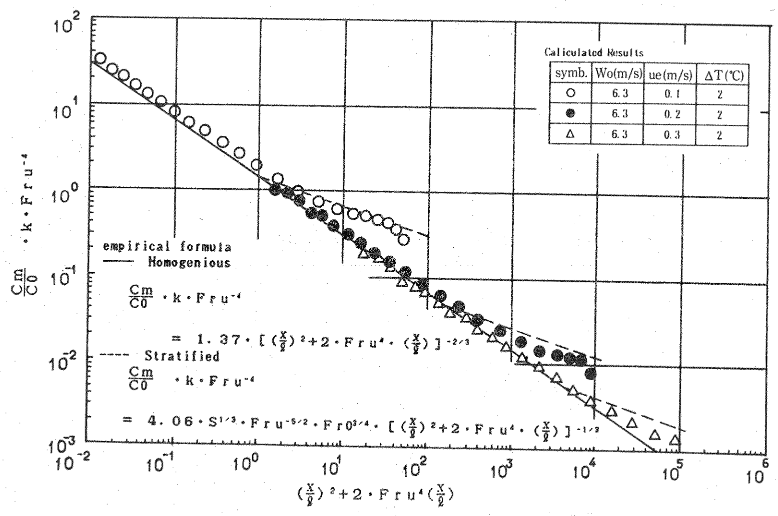


Fig. 10 Comparison of dilution along the plume trajectories between calculated results and empirical formulas

which plume momentum predominates but also the far region in which the effect of momentum decrease and the density stratification in environment is dominant beyond the momentum.

Presented in Figs. 6 and 7 in chapter 2, calculated results of zero momentum height H_m and dilution at H_m are compared with experimental results and empirical formula. There is a good agreement between computations and experiments.

Figure 11 shows dilution contours in the vertical section at y/D=0 in the case of u_e=0.1m/s. The forced plume ascends to zero momentum height at x/D=400, and descends slowly to zero buoyancy height on account of negative buoyancy.

Figure 12 shows dilution contours in transverse sections at x/D=133, 400 and 533. Dilution contours tend to be flat as plume flow downstream by the effect of density stratification in environment. The separation of plume may be not clear as well as the experimental results.

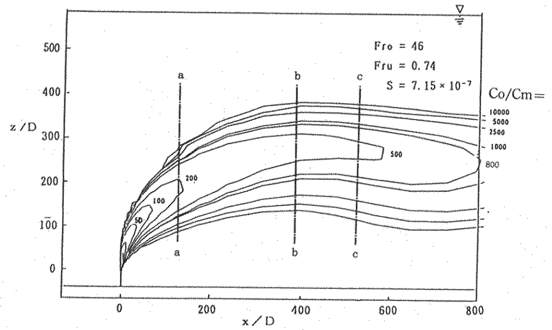


Fig. 11 Dilution contours in the verical section at y/D=0 (Calculated results)

Figure 13 shows secondary flow vectors in the same transverse sections as Fig. 12. At x/D=133 where the forced plume is rising, one vortex pair symmetrical about the plane y/D=0 appears clearly visible. Further downstream at x/D=400, it is remarkable that secondary vortex pairs are induced above and below the primary vortex pair. The primary vortex pairs tend to disappear gradually as plume flows downstream (x/D=533). Such production and disappearance process of vortex pairs are not recognized in the non-stratified environment.

Figures 14 and 15 show temperature and buoyancy flux contours in same transverse sections. The production and the disappearance of vortex pairs can be explained owing to the buoyancy flux distribution in transverse sections. In the most part of the cross section at x/D=133 buoyancy flux is positive by the effect of initial buoyancy of the plume. However, negative buoyancy flux is induced above and below the plume at x/D=400, because cold water in environment rises by the effect of primary vortex pairs as plume assends. However, the construction of vortex pairs is transformed as a plume flows downstream, and it is expected that all vortex pairs are finally decreasing.

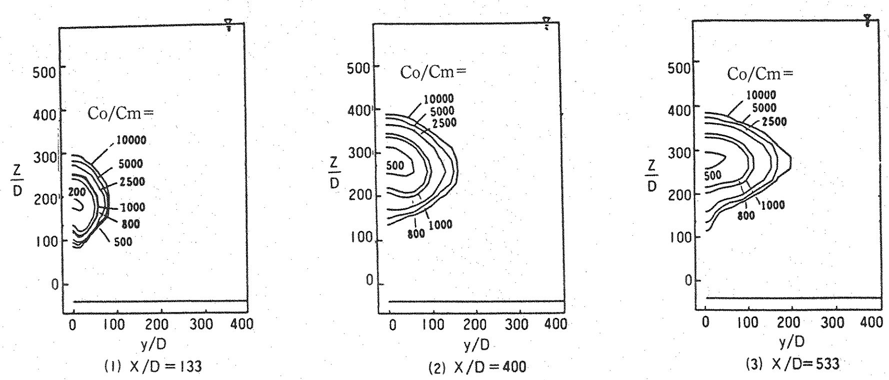


Fig. 12 Dilution contours in transverse sections
(Calculated results)

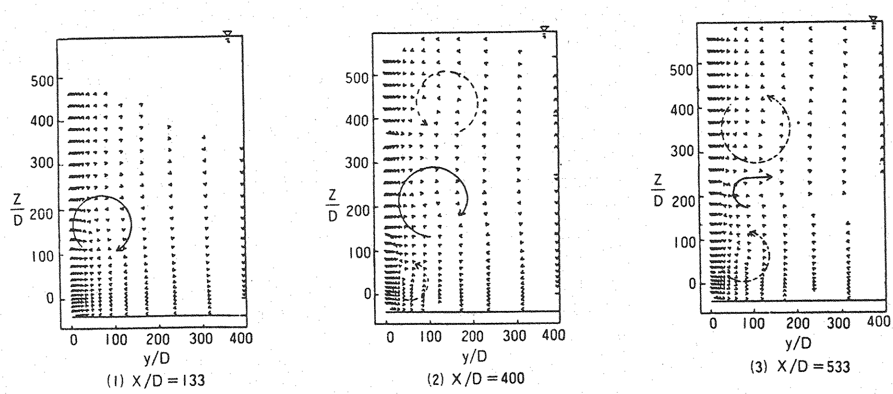


Fig. 13 Secondary flow vectors
(Calculated results)

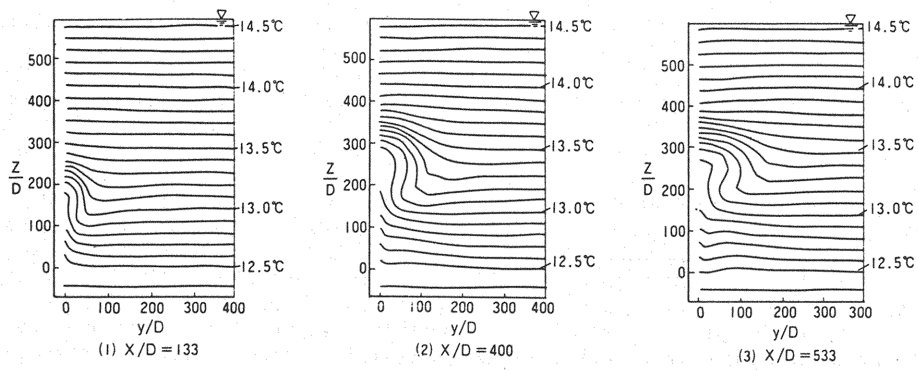


Fig. 14 Water temperature contours in transverse sections
(Calculated results)

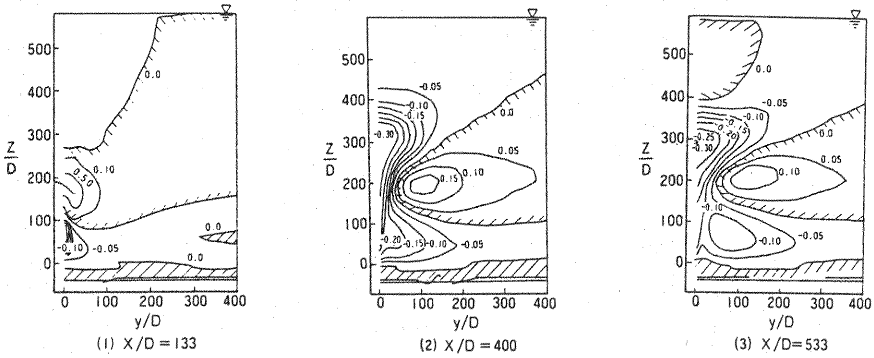


Fig. 15 Buoyancy flux contours in transverse sections
(Calculated results)

CONCLUSION

The characteristics of vertical forced plumes in a uniform cross flows of stably linear stratified environment were studied both numerically and experimentally. We propose four empirical formulas for diffusion characteristics of forced plume: trajectory, dilution along them, zero momentum height H_m and dilution at H_m. The three-dimensional numerical model based on the mixing length theory is applied to simulate the plume behavior in stratified cross flows. The model predicts the characteristics of plume in such a environment accurately. The secondary vortex pairs in transverse sections are induced by the primary one changes as a plume flows downstream.

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APPENDIX-NOTATION

The following symbols are used in this paper:

$b_{1/2}$	= distance from centerline to the point where velocity is half of w_m;
B_o	= initial buoyancy flux;
C_o	= concentration of plume at outlet;
C_m	= concentration of plume at the center of plume;
C	= experimental constant in the formula of eddy viscosity;
D	= diameter of outlet;
F_{ro}	= internal Froude number at outlet;
F_{ru}	= Froude number using ambient velocity;
g	= gravitational acceleration;
H	= depth of outlet;
H_b	= zero buoyancy height;
Hm	= zero momentum height;
k	= ratio of u_o and u_e;
ℓ	= characteristic length of buoyancy;
M_o	= initial momentum flux;
p	= dynamic pressure;
Pr	= turbulent Prandtl number;
r	= distance from centerline;
s	= salinity;
s_e	= ambient salinity;
S	= dimensionless parameter concerned in ambient density structure;
Sc	= turbulent Schmit number;
t	= time;

T_o	= water temperature at outlet;
T_e	= ambient water temperature;
T_s	= water temperature at surface;
ΔT	= temperature variation between T_o and T_s;
u,v,w	= flow velocity components in x,y,z directions;
u_e	= ambient flow velocity;
w_o	= discharged velocity at outlet;
w_m	= flow velocity at the center of plume;
x,y,z	= coordinate axis in cartesian coordinate system;
ρ	= density of plume;
ρ_o	= density of plume at outlet;
ρ_e	= ambient density;
$\rho_{e,o}$	= ambient density at outlet;
$\Delta\rho$	= density variation between ρ_e and ρ;
$\Delta\rho_o$	= density variation between $\rho_{e,o}$ and ρ_o;
ε_m	= eddy viscosity;
ε_T	= eddy diffusivity of temperature; and
ε_s	= eddy diffusivity of salinity.

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