Journal of Hydroscience and Hydraulic Engineering
Vol. 8, No. 2, December, 1990, 53-63

# INFLUENCE OF HYDRAULIC FACTORS ON RIVER BED SCOUR

by

Yasuharu Watanabe
Civil Engineering Research Institute, Sapporo, Japan,

Kazuyoshi Hasegawa
Hokkaido University, Sapporo, Japan

and

Kouji Houjyou
Hokkaido Development Agency, Tokyo, Japan

## SYNOPSIS

River bed scour prediction is important in river improvement works. There have been few theoretical studies and quantitative measurements of river bed scour. This paper uses field data and focuses on the relationship between hydraulic factors and bed scour. It is found that the tendency of bed scour can be expressed by two factors, which are the hight of bars and the curvature of bends. With this expression, bed scour can be simply predicted even in the field.

## INTRODUCTION

In recent years, flood inundation due to bank destruction has decreasesd, dne to advances in river improvement works. However, the confined flood flow still causes large damage to revetments and other facilities in river courses. For this reason the cost of maintaining river courses has tended to increase. In addition to flood prevention, considerations of prevention of disasters in river courses is then of practical importance. Disasters related to river courses are classified into a variety of causes, and bed scour is the most important phenomenon and one of the main causes. When bed scour occurs near the river bank, it influences bank erosion, high water level, and the direction of flow, and also accelerates damage to structures and revetments. To prevent river bed scour, several countermeasures have been implemented on the basis of past experience. However, considerable time is required till completion of final plans, and at this time it is necessary to carry out essential and urgent countermeasures to prevent bed scour. There are only few studies (10) on the mechanism of river bed scour and prevention works based on field observations, and most river planning have relied largely on experience. However, the stepwise construction plan for facilities in river courses demands the development of methods to estimate bed scour depth accurately and simply. The present study aims to investigate the relationship between flow or basic hydraulic conditions and bed scour, and to make it possible to determine countermeasures against disasters in river courses.

## DERIVATIONS

### Studied Rivers

Hydraulic data of the rivers investigated in the present study are shown in Table 1. They are all A class rivers in Hokkaido, and the geometry of the river courses is considered to be determined by the snowmelt flow which corresponds to the largest mean flow when considering duration. Considering this, the investigation of bed scour, which is the major cause of river course disasters, was performed with data from the snowmelt flood period. Fig. 1 shows the regime criterion for meso-scale bed forms by Kuroki and Kishi(8), the data covers the region from no bars to braids, for all river course geometries.

Table. 1 Hydraulic data of the rivers investigated in the present study

| Name of river | Reach km | Discharge $m^3/s$ | River width m | Water depth m | Gradient of water surface | Bed material diameter mm | Symbol |
|---|---|---|---|---|---|---|---|
| Shiribetsu | 16.0 - 16.4 | 517 | 103 | 2.98 | 1/825 | 16.9 | ○ |
| | 16.6 - 16.9 | 517 | 89 | 3.01 | 1/714 | 16.9 | ○ |
| | 17.0 - 17.6 | 517 | 110 | 2.96 | 1/903 | 17.0 | ○ |
| | 17.8 - 18.6 | 517 | 114 | 2.93 | 1/907 | 18.9 | ○ |
| | 18.8 - 19.8 | 517 | 128 | 2.44 | 1/478 | 20.8 | ○ |
| | 20.0 - 21.0 | 517 | 124 | 2.51 | 1/661 | 19.6 | ○ |
| Koetoi | 16.0 - 21.6 | 385 | 20 | 2.70 | 1/1116 | 20.0 | ● |
| Teshio | 65.0 - 74.0 | 2971 | 215 | 3.85 | 1/1813 | 14.1 | ⦿ |
| Chyubetsu | 9.0 - 20.0 | 397 | 175 | 2.07 | 1/515 | 122.0 | ⊙ |
| Mu | 9.0 - 17.8 | 836 | 225 | 2.47 | 1/880 | 9.7 | □ |
| Tokachi | 72.0 - 79.0 | 603 | 218 | 1.80 | 1/305 | 30.0 | ■ |
| Uryu | 7.0 - 25.0 | 360 | 101 | 3.21 | 1/1200 | 17.8 | ⊡ |
| Shiribetsu | 3.2 - 8.2 | 894 | 113 | 2.95 | 1/2310 | 1.1 | ◎ |
| Yuubetsu | 9.0 - 13.0 | 280 | 165 | 1.11 | 1/300 | 36.4 | ◬ |
| Toyohira | 1.8 - 10.8 | 530 | 84 | 3.56 | 1/1321 | 0.2 | △ |
| | 11.0 - 21.0 | 500 | 100 | 2.40 | 1/143 | 73.4 | △ |
| Kushiro | 60.6 - 62.4 | 130 | 54 | 1.18 | 1/327 | 16.2 | ▲ |

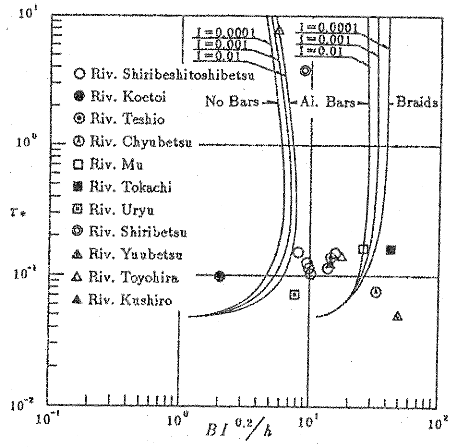


Fig. 1 Regime criterion for meso-scale bed forms

## Relationship Between Flow and Bed Scour

The shape and characteristics of river courses are regulated by the river flow, and conversely the river flow is regulated by the shapes of the river course. Hence it is necessary to determine the relationship between flow conditions and river bed scour when considering the bed scour. Fig. 2 shows the surface flow velocity vector and the contour of the river bed obtained from aerial photographs of the Shiribetsu River during the snowmelt period in 1987. The outer bend forms a pool and the inner bend is accreting, and the surface velocities are maximum or minimum values in these areas. It shows that the value of the scour depth $\eta$ is determined by the surface flow velocity or mean flow velocity near the bank. However, it is also determined by the secondary flow ( spiral flow ) which is created by the curvature of the river course. The relationship between $\eta/h$ and $1/r$ in Fig. 4 shows this phenomenon clearly.

Change in scour near the river bank before and after a flood is the most serious factor in river bank destruction. The changes are defined by the difference between the amounts of sediment which flow into and out of a certain area. In other words, it is necessary to consider the spatial variation in river bed shear stress in the downstream direction of the river bed. The bed shear stress is related to the flow velocity, as shown in Eq. 1:

$$\begin{aligned} \tau &= \rho g h i_e \\ &= \rho g N^2 h^{-1/3} U^2 \end{aligned} \qquad (1)$$

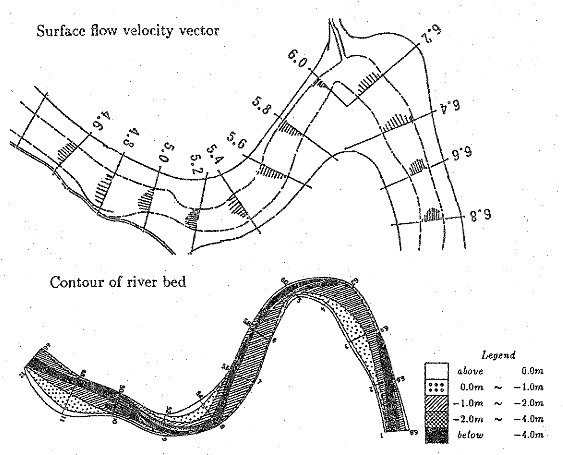


Fig. 2 Surface flow velocity vector and contour of river bed

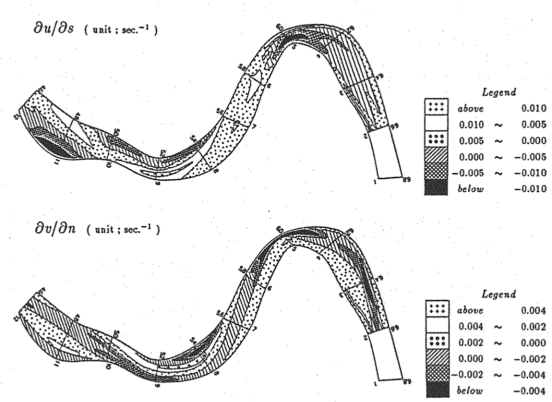


Fig. 3 Contour figures of variation of flow velocity component

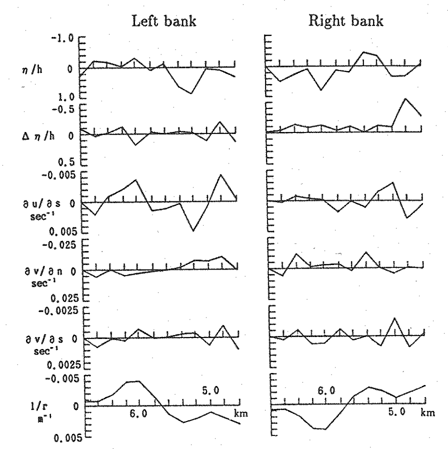


Fig. 4 Bed scour depth and flow velocity component in neighborhood of river bank

where $\tau$ = the shear stress of the river bed; $\rho$ = the density of water; $g$ = gravitational acceleration; $h$ = water depth; $i_e$ = energy gradient; $N$ = Manning's coefficient of roughness; and $U$ = the flow velocity.

Hence, changes in the river bed in the direction of the stream is highly correlated with variations in flow velocity $\partial u/\partial s$. Similarly, cross sectional changes in the river bed are also related to variations in flow velocity $\partial v/\partial n$.

Fig. 4 demonstrates the relationship between $\Delta\eta/h$ and the variations in the flow velocity components in the neighborhood of river banks. The rate of bed scour (or deposition) tends to become large at positions where there are accretions (pools) and scour (deposition) seems to occur, from the variation of the flow velocity, such as at K.P. (Kilometer Points) 6.6 and 4.8 on the left bank and K.P. 6.4 $\sim$ 6.2 on the right bank. This makes it easy to understand where the bed is eroded or deposition takes place, due to the variation in the flow velocity components. One of the authors has obtained similar results of the relationship between flow velocity components and volume changes in river beds, using the solution to flow velocities by a two dimensional shallow water flow model (11).

Fig. 3 shows contour maps for $\partial u/\partial s$ and $\partial v/\partial n$. These values are influenced by the bars as clearly observed in Fig.2. It follows from the results of Fig. 3 that both $\partial u/\partial s$ and $\partial v/\partial n$, which regulate the shape of the river bed, are closely related to the relative positions of bars as clearly shown in Fig. 2.

The above results suggest that it is necessary to analyze the influences of bars and curvature on the flow velocity, to predict the amount and position of bed scour.

## Simplified Equation to Estimate Scour Depth

Estimation of scour depth is important to determine the depth of foot protection of revetments. This paper develops a simple method for calculating scour depth under the assumption that the flow velocity components are determined by two factors only, i.e., the presence of bars and the curvature of the river course.

Field data which are measured from the cross sectional view of a river are plotted on a graph with vertical axis $\eta/h$ and horizontal axis $B/r$. Some examples are shown in Fig. 5, where $B$ is the river width (water surface width). It is possible to draw a regression line by the least-squares method for the data of the respective rivers. This shows that Eq. 2 can be used to estimate the scour depth for the rivers, here $\alpha$ is the intersection of the vertical axis, and $\beta$ the inclination of the regression line.

$$\frac{\eta}{h} = \alpha + \beta(\frac{B}{r}) \tag{2}$$

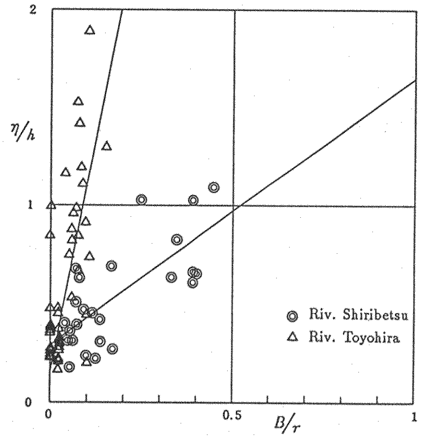


Fig. 5 Relationship between bed scour depth and curvature of river course

It is necessary to provide a physical interpretation of $\alpha$ and $\beta$. As the cross section of the river is surveyed at intervals of 200 $\sim$ 500$m$, the maximum values of the scour depth are treated as avaraged values for the interval of the cross sectional survey in this paper.

The parameter, $\alpha$ shows the amount of scour, when the curvature of a river course is infinity (i.e. straight). Then $\alpha$ is approximately equal to the wave hight of bars developed in a straight channel. A theoretical equation which expresses the bed scour depth in a uniformly curved channel course is given by Eq. 3.

$$\frac{\eta}{h} = N_*(\frac{\mu_s\mu_k\tau_{*0}}{\tau_{*c}})^{0.5}(\frac{B/2}{r}) \qquad (3)$$

$$N_* = \begin{cases} 11 \sim 11.5 & \text{Rozovskii} \\ 7 & \text{Engelund} \\ \frac{4.52+8\sqrt{2C_f}}{3.83\sqrt{2C_f}+27.12C_f} & \text{Zimmermann} \\ 1.226(\sqrt{\frac{1}{C_f}} - 1.584) & \text{Ikeda} \end{cases}$$

where $\mu_s$ = the static friction factor of sediment particles; $\mu_k$ = the dynamic friction factor of sediment particles; $\tau_{*0}$ = the shield stress; and $\tau_{*c}$ = the critical shield stress. Different values have been proposed for $N_*$ (3). Hence, $\beta$ has the same physical significance as $N_*$ which is a parameter explaining the strength of the secondary flow. So Eq. 2 which simply expresses the bed scour depth, takes the form of a linear weighted sum of amounts caused by bars and the curvature of the river course.

When examining the properties, $\alpha$ and $\beta$, $\alpha$ is related to the parameter for the regime criterion of meso-scale bed forms, $BI^{0.2}/h$ (Kuroki and Kishi parameter), while $\beta$ is related to the resistance coefficient of flow, $C_f$, because the strength of the secondary flow can be expressed by $C_f$ (3, 6). In this paper, $C_f$ is calculated from the basic hydraulic conditions which are avaraged for the whole distance. In addition, $\beta$ is arranged by the value $2\beta/\sqrt{\mu_s\mu_k\tau_*/\tau_{*c}}$ to express the value of $N_*$. The resulting plots of $\alpha$ and $\beta$ are shown in Figs. 6 and 7.

The value of $\alpha$ becomes larger as $BI^{0.2}/h$ increases. It is nearly zero in no-bar regions and increases in regions that change from alternating bars to braids. Eq. 4 shows the result of curve fitting of $\alpha$ in the respective rivers.

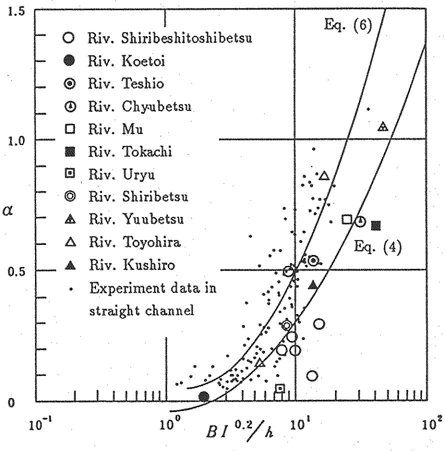


Fig. 6 Relationship between $\alpha$ and parameter of regime criterion for meso-scale bed forms

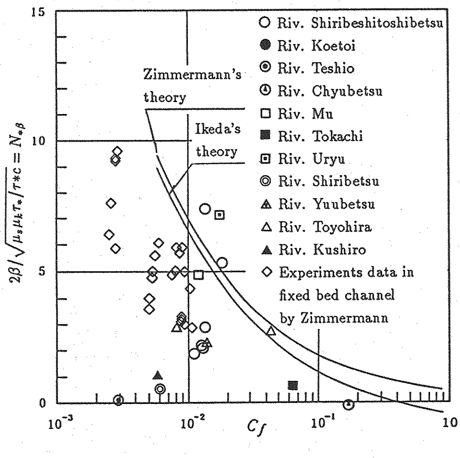


Fig. 7 Relationship between $\beta$ and resistance coefficient of flow

$$\alpha = 0.361X^2 - 0.0224X - 0.0394 \tag{4}$$

$$X = \log_{10}(\frac{BI^{0.2}}{h})$$

As defined in Eq. 2, $\alpha$ is estimated by the nondimensional displacement of the bed surface from an averaged bed surface in each cross section. Therefore, the following adjustment is required to give $\alpha$ the same physical meaning as the wave height of the bars, $Z_b$. When the wave form of bars is approximated by a cosine curve, Eq. 5 is obtained as the mean wave hight of bars over one wave length $L$.

$$\begin{aligned}(\frac{Z_b}{2h})_{mean} &= \frac{1}{L/4}\int_0^{L/4} \frac{Z_b}{2h}\cos\left(\frac{2\pi}{L}s\right) ds \\ &= 0.64(\frac{Z_b}{2h})\end{aligned} \tag{5}$$

The wave height of bars, $Z_b$ measured from experiments in straight channels (1) are also shown in Fig. 6, $[Z_b/(2h)]_{mean}$ can be expressed as a function of $BI^{0.2}/h$, which is given by Eq. 6 as:

$$(\frac{Z_b}{2h})_{mean} = 0.601X^2 - 0.173X + 0.0615 \tag{6}$$

Although $\alpha$ changes similarly to $[Z_b/(2h)]_{mean}$, the inclination of the curve is rather smaller. The value of $BI^{0.2}/h$ is larger than 5 in the region where the bars are generated. The value of $[Z_b/(2h)]_{mean}/\alpha$ is a constant around 1.6 in this region, as shown in Fig. 8.

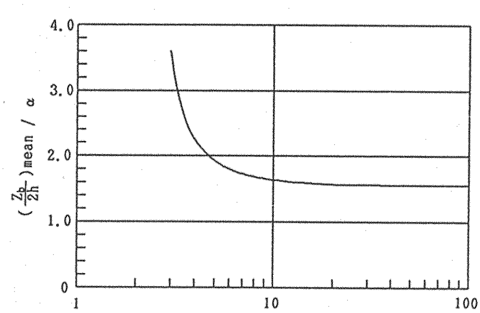


Fig. 8 Relationship between $\alpha$ and wave hight of bar

Kuroki *et al.* (7) have obtained the wave hight of bars analytically, although the influence of the river bed gradient is not clear. According to them, $Z_b/h$ is $1.0 \sim 3.0$ in the region of alternating bars, judged from the value of $BI^{0.2}/h$, while the value of $Z_b/h$ becomes larger toward the region of braids. This value corresponds to a $[Z_b/(2h)]_{mean} = 0.3 \sim 0.9$ which is almost the same as the value by Eq. 5. It is of practical interest that the value of $\alpha$ is expressed by $BI^{0.2}/h$ which defines the regime criterion for meso-scale bed forms. Fujita *et al.* (2) have related the wave height of bars to another parameter of the regime criterion for meso-scale bed forms.

Ikeda *et al.* (4) use dimensional analysis to obtain the wave hight of bars by Eq. 7.

$$\frac{Z_b}{h} = 9.34(\frac{B}{d})^{-0.45} \exp[2.53erf(\frac{\log_{10}\frac{B}{h} - 1.22}{0.594})] \tag{7}$$

where $d =$ the grain diameter of river bed materials. The mean scour depth over one wave of bars is obtained by Eq. 8, using Eqs. 5 and 7.

$$(\frac{Z_b}{h})_{mean} = 2.97(\frac{B}{d})^{-0.45} \exp[2.53erf(\frac{\log_{10}\frac{B}{h} - 1.22}{0.594})] \tag{8}$$

Fig. 9 compares the observed values of $\alpha$ in each river with Eq. 8, which shows fairly good correspondence between data and theory, even though there is some scattering.

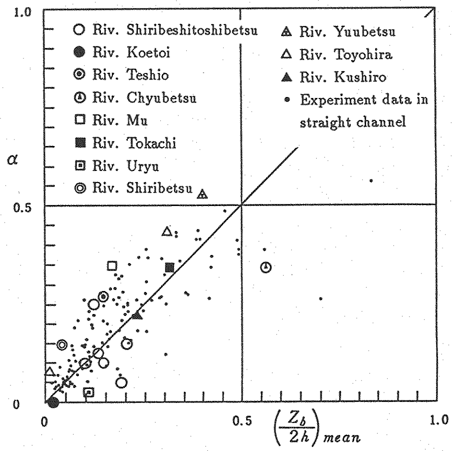


Fig. 9 Relationship between $\alpha$ and wave hight of bar by Ikeda's theory

The above investigations show that $\alpha$ is expressed by $BI^{0.2}/h$, and has a slightly smaller value than the wave hight of bars. That $\alpha$ is smaller than the wave hight of bars is attributed to the fact that the grain diameter of river bed materials is not taken into consideration. However, accurate values for rough estimates of river bed scour can be obtained by using Eq. 4. Furthermore, when accurate calculations of the wave hight of bars become available, $\alpha$ will improve. Half the wave hight of bars is recommended as the value of $\alpha$ to adequately estimate the scour.

The term $2\beta/\sqrt{\mu_s\mu_k\tau_*/\tau_{*c}}$ can be represented by the resistance coefficient of the flow and can be obtained by extending the value of $N_*$ measured in Zimmermann's experiments on fixed beds. Ikeda *et al.* (5) estimate the value of $N_*$ analytically in a meandering river course by Eq. 9.

$$N_* = 1.226(\frac{1}{\sqrt{C_f}} - 1.584)\chi \cos(\frac{2\pi}{L}s - \sigma) \tag{9}$$

$$\chi = \frac{1}{1.5C_f(\frac{1.11}{\sqrt{C_f}} - 1.42)\sin\sigma + \cos\sigma}$$

$$\sigma = \tan^{-1}[1.5C_f(\frac{1.11}{\sqrt{C_f}} - 1.42)]$$

where $L$ = the meandering length of the river course. $\beta$ is determined similarly to $\alpha$ from the averaged bed surface in each cross section. Hence $\beta$ is expressed by Eq. 10 which is an averaged value of Eq. 9 over one meander.

$$N_{*mean} = \frac{2}{\pi}1.226(\frac{1}{\sqrt{C_f}} - 1.584)\chi \tag{10}$$

The values defined by Eq. 10 are plotted in Fig. 7 with the theoretical curve by Zimmermann.

Both curves appear to express the tendency of $N_*$ obtained from $\beta$. However, there are cases where the $N_*$ obtained from $\beta$ shows a considerably smaller value than the $N_*$ calculated from the theoretical value. The reasons for this discrepancy can be interpreted as follows: The secondary flow in a meandering river is not developed as strongly as that in a uniformly curved channel. On the contrary, when the length of a meander is sufficiently long, the secondary flow is weakened by the fact that the vertical distribution of flow velocity is made uniform (9). Therefore, the scale of the secondary flow and the meander length are related. The relationship between the ratio $N_{*\beta}/N_{*I}$ and the value of meander lengths of a river course is considered. $N_{*\beta}$ is the value of $N_*$ estimated by using $\beta$, while $N_{*I}$ is the value of $N_*$ obtained from Eq. 10. The value of $N_{*\beta}/N_{*I}$ is a maximum when $L/B$ is about 20. Further investigation is needed, however, because the relationship between the development of secondary flow and the meander length is not clear.

Some examples are provided to test the validity of the simplified equation for estimating the bed scour depth by field data. Fig. 10 compares the bed scour depth at K.P. 21 $\sim$ 7 of Toyohira River with the calculated values, where the bed scour depth is shown on the vertical axis, while the downsteam distance on the horizontal axis. The values were calculated from three sets of parameters, i.e., (a) $\alpha$ by Eq. 4 and $\beta$ by Eq. 10, (b) $\alpha$ by Eq. 8 and $\beta$ by Eq. 10, and (c) $\alpha$ by (Eq. 7)/2 and $\beta$ by Eq. 10. The hydraulic conditions used are estimated from the discharge during the snowmelt in 1988 by a one dimensional nonuniform flow model. Because Eq. 2 does not consider differences in phase between the meander of a river course and the position where the secondary flow occurs, the reproduced values are far from the observed ones at K.P. ( Kilometer Points) 9 $\sim$ 7. However, the estimated values reproduce actual measurements of bed scour with a high accuracy except for this section. The values by methods (a) and (b) are similar the (c) values are supposed to be on the safe side, because $\alpha$ is half of the wave hight of bars and this fact is clear in the analysis of Toyohira River.

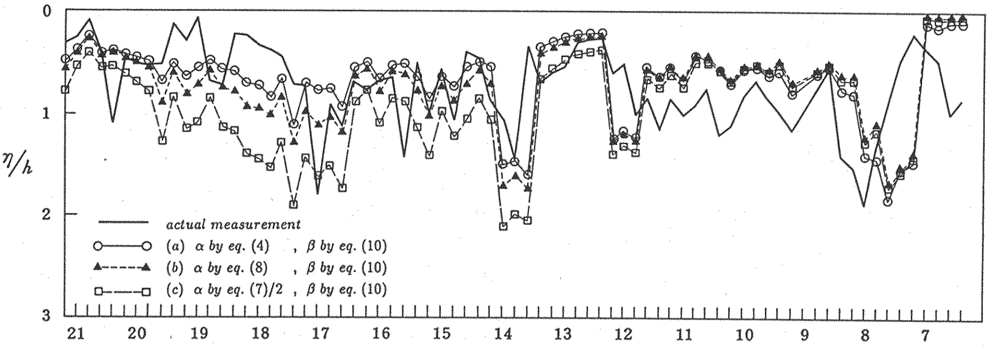


Fig. 10 Comparison of bed scour depth calculated by Eq. 2 with the value of actual measurement

The reproducibility of the simplified equation for estimating the river bed scour depth was tested in other rivers. Mean squares errors of the simplified equations are shown in Table 2.

As $\beta$ cannot be estimated exactly in the Teshio and Shiribetsu Rivers, the reproducibility of the equations is not good. However, the errors are less than 20 % of the water depth in other rivers. The result clearly shows that the simplified equation is of practical use to compute the bed scour depth. But further study is needed to estimate accurate values of $\beta$.

There is no difference in the errors in the values of $\alpha$ by three methods of estimation. Therefore, it seems reasonable to calculate the bed scour depth by using case (c) which yields safer values for the river bed scour depths.

Table. 2 Mean squares errors of the simplified equations for bed sour depth

| | | Case a | Case b | Case c |
|---|---|---|---|---|
| Riv. Shiribetsu | $\alpha$ | 0.258 | 0.063 | 0.099 |
| | $\beta$ | 26.619 | 26.619 | 26.619 |
| | Error | 1.118 | 1.088 | 1.093 |
| Riv. Shiribeshitoshibetsu | $\alpha$ | * | * | * |
| | $\beta$ | * | * | * |
| | Error | 0.115 | 0.090 | 0.092 |
| Riv. Teshio | $\alpha$ | 0.394 | 0.394 | 0.062 |
| | $\beta$ | 9.254 | 9.254 | 9.254 |
| | Error | 0.568 | 0.568 | 0.549 |
| Riv. Chyubetsu | $\alpha$ | 7.715 | 0.112 | 0.175 |
| | $\beta$ | 0.672 | 0.672 | 0.672 |
| | Error | 0.953 | 0.073 | 0.066 |
| Riv. Koetoi | $\alpha$ | 0.010 | 0.140 | 0.210 |
| | $\beta$ | 0.010 | 0.010 | 0.010 |
| | Error | 0.123 | 0.100 | 0.089 |
| Riv. Yuubetsu | $\alpha$ | 0.941 | 0.069 | 0.109 |
| | $\beta$ | 2.164 | 2.164 | 2.164 |
| | Error | 0.101 | 0.185 | 0.180 |
| Riv. Tokachi | $\alpha$ | 0.854 | 0.606 | 0.951 |
| | $\beta$ | 1.340 | 1.340 | 1.340 |
| | Error | 0.279 | 0.264 | 0.286 |
| Riv. Kushiro | $\alpha$ | 0.650 | 0.077 | 0.122 |
| | $\beta$ | 4.769 | 4.769 | 4.769 |
| | Error | 0.181 | 0.125 | 0.126 |
| Riv. Toyohira | $\alpha$ | * | * | * |
| | $\beta$ | * | * | * |
| | Error | 0.160 | 0.177 | 0.169 |
| Riv. Uryuu | $\alpha$ | * | * | * |
| | $\beta$ | * | * | * |
| | Error | 0.181 | 0.175 | 0.174 |

* ; $\alpha$ and $\beta$ are calculated in each meander of river course.

## Conclusion

As a result of analyses of flow velocity components by field observation of rivers, river bed scour was found to depend mainly on variations in flow velocities in both the flow and cross sectional directions, as well as it depends on the curvature of the river course. In particular, the bed scour depth tends to increase where the three factors interact. A simplified estimation equation for river bed scour is

$$\frac{\eta}{h} = \alpha + \beta(\frac{B}{r})$$

The relationship between the value of $N_*$ and the length of meanders is still not finally established, however the parameter $\alpha$ can be determined by $BI^{0.2}/h$ which is used for meso-scale bed forms, and the parameter $\beta$ can be determined by the resistance coefficient of flow, $C_f$ which determines the strength $N_*$.

Several studies of river bed scour were conducted, with data from rivers. These studies have failed to separate the influence of hydraulic factors on bed scour such as bars and the curvature of the river course. Therefore, it is extremely difficult to apply these studies to rivers which have curved river courses and bars in the river bed. The present study, allows a better estimate of river bed scour as the causes of the scour are separated into the separate factors, bars and the curvature of the river course.

In recent years, calculation methods of river bed variations have been developed by extensive use of computers, and river bed scour can easily be estimated. However, much time and effort are required for the calculations of changes in river beds over the whole length of a river to determine the foot protection depth of revetments. When using the present method as a first approximation, it is possible to save considerable time and labor in computations of the river bed changes over an arbitrary stretch of river. In addition, the calculations of river bed scour can be conducted also for other rivers, because of simplicity of the method.

## REFERENCES

1. Committee Report, Task Committee on three-dimensional structures of flood and channel process, Committee on Hydraulics and Hydralic Engineering, Japan Society Civil Engrs, 1982 (in Japanese).

2. Fujita, Y. , Y. Muramoto, S. Horiike and T. Koike : On the mechanism of alternating bar development, Proceedings of the 26th Japanese Conference on Hydraulics, pp.25-30, 1982 (in Japanese).

3. Hasegawa, K. : A study on flows, bed topographies and plane forms of alluvial meanders, Thesis, Dr. Engrg., Hokkaido Univ., 1983 (in Japanese).

4. Ikeda, S. : Wavelength and hight of single row alternate bars, Proceedings of the 27th Japanese Conference on Hydraulics, pp.689-695, 1983 (in Japanese).

5. Ikeda, S. and T. Nishimura : Flow and bed profile in meandering sand-silt rivers, J. Hydraul. Eng., Am. Soc. Civ. Eng., Vol.112, No.7, pp.562-579, 1986.

6. Kikkawa, H. , S. Ikeda and A. Kitagawa : Flow and bed topography in curved open channels, J. Hydraul. Div., Am. Soc. Civ. Eng., Vol.102, HY9, pp.1342-1342, 1976.

7. Kishi, T. , T. Itakura, M. Kuroki and A. Mori : Characteristics of bed configurations and flows in alluvial rivers, Research Report, Hokkaido Development Bureau, 1989 (in Japanese).

8. Kuroki, M. and T. Kishi : Regime criteria on bars and braids in alluvial straight channels, Proceedings, Japan Soc. Civ. Eng. No.342, pp.87-96, 1984 (in Japanese).

9. Mori, A. and T. Kishi : A study on the secondary flow in open channel bend with transverse sloping bed, Proceedings of the 28th Japanese Conference on Hydraulics, pp.751-755, 1984 (in Japanese).

10. Suga, K. , S. Tsuchiya and T. Asano : On river morphology and local scouring - from the viewpoint of lateral bed profiles - , Proceedings of the 28th Japanese Conference on Hydraulics, pp.789-794, 1984 (in Japanese).

11. Takahashi, K. , Y. Watanabe and T. Hata : Investigation of hydraulic factors on bed scour, Proceedings of the 43rd Annual Congress of Civil Engineering, Min. of Construction, River section, pp.25-32, 1989 (in Japanese).

## APPENDIX - NOTATION

The following symbols are used in this paper:

$B$ = river width;
$C_f$ = resistance coefficient of flow;
$d$ = mean diameter of bed sand grain;
$g$ = gravitational acceleration;
$h$ = water depth;
$i_e$ = energy gradient;
$I$ = water surface gradient;
$L$ = meander length;
$n$ = transverse horizontal coordinate;
$N$ = Manning's coefficient of roughness;
$N_*$ = parameter for representing the strength of secondary flow;
$N_{*I}$ = $N_*$ by Ikeda's theory;
$N_{*\beta}$ = $N_*$ by using field data;
$r$ = radius of curvature along channel centerline;
$s$ = streamwise coordinate along channel center line;
$u$ = streamwise component of flow velocity;
$U$ = flow velocity;
$v$ = $n-$direction component of flow velocity;
$Z_b$ = wavehight of bar;
$\alpha$ = amount of bed scour when the river course is straight;
$\beta$ = parameter for representing the scale of secondary flow;
$\eta$ = displacement of bed surface measured downward from average bed surface for cross section;
$\mu_s, \mu_k$ = static and dynamic friction factor of sediment particles;
$\rho$ = density of water;
$\tau$ = bed shear stress;
$\tau_{*0}$ = Shields stress; and
$\tau_{*c}$ = critical Shields stress.

(Received August 1, 1990; revised November 26,1990)