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EXPERIMENT AND MACROSCOPIC MODELLING OF FLOW IN HIGHLY PERMEABLE POROUS MEDIUM UNDER FREE-SURFACE FLOW

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ABSTRACT

When the seepage flow in a highly permeable porous medium is accompanied by a turbulent free surface flow above it, an appreciable interaction between seepage flow and surface flow takes place. This interaction means mass and momentum exchanges between the surface flow and the seepage flow. The induced velocity fluctuation inside the porous medium contributes the momentum exchange and it plays an important role on determining the structure of the seepage flow.

In this study, experiments are conducted to measure the velocity profile and the vertical mass transport in the porous medium beneath the free surface flow, and subsequently a macroscopic model is proposed to describe a seepage-flow velocity profile based on an eddy-viscosity assumption.

INTRODUCTION

For the accurate estimation of the flow discharge from a relatively small mountain drainage basin, or the evaluation of the sediment discharge in a debris region, the physical mechanism of the interaction between the seepage flow and the free surface flow is of great importance. Because the stream bed or the drainage basin in mountain region is often covered by highly permeable debris materials, the interaction between the free surface flow and the seepage flow in the porous layer affects not only the flow discharge but also the sediment discharge.

The effects of the interaction between the seepage flow and the surface flow can be summarized as the following two: The first is the presence of a non-zero velocity at the permeable boundary ("slip velocity"); and the other is the existence of a fluctuating transpiration (injection and suction) velocity at the boundary between the surface-flow region and the porous medium. The latter is just an elementary events of the interaction between the two regions; while the former is rather a result of the momentum exchange between the seepage flow and the surface flow. The fluctuating transpiration causes the somehow organized fluctuation on the seepage-flow and it behaves just like an appearance of the "Reynolds stress". The reason why such an organized fluctuation appears is a non-linear effect due to non-Darcy law as a resistance law for the flow in a highly permeable porous medium. This mechanism was skillfully described by Chu & Gelhar (1), and their analysis was applied to open channel flows by Nakagawa et al. (4, 5). As a result of the presence of the slip velocity and the apparent "Reynolds stress", the so-called turbulent Couette flow takes place inside the porous medium. On the other hand, the seepage flow also affects the free surface flow as an increase of the friction factor (Lovera & Kennedy (2); Chu & Gelhar (1); Nakagawa & Nezu (4); Yamada & Kawabata (8); Nakagawa et al. (5,7)).

In this study, the influence of a free surface flow on the structure of the seepage flow in a highly permeable medium is focussed. A model porous layer was made of glass beads in a laboratory flume. The time-averaged velocity profile was measured by a tracer method. In order to estimate the magnitude of the vertical momentum exchange based on the Reynolds analogy, the vertical mass dispersion test is conducted. Furthermore, a macroscopic physical model to describe the seepage-velocity profile is developed based on an assumption of eddy viscosity.

EXPERIMENTAL APPROACH

Model Porous Medium

Several types of model porous layer composed of glass beads were prepared in a laboratory flume (8m long and 0.21m wide) with adjustable slope. Two kinds of glass beads were used, of which diameters (d) were 2.97cm and 1.75cm, and they were arrayed in constant thickness along the bed. The number of the arrayed layers of glass beads was changed from 1 through 3. The glass beads were arrayed as the most packed tetragonal-spheroidal pattern. Table 1 provides a summary of the experimental conditions, in which I_e=energy gradient; h=depth of free surface flow; U_m=depth-averaged velocity of free surface flow; H_s=thickness of porous medium; Re≡$U_m h/\nu$; and ν=kinematic viscosity. Figure 1 is a definition sketch.

Table 1 Experimental conditions

Run	I_e	h (cm)	U_m (cm/s)	Re	H_s (cm)
1	0.002	5.67	43.5	23510	8.91
2	0.002	8.61	48.7	38690	8.91
3	0.001	6.03	31.9	18080	8.91
4	0.001	9.10	34.9	30590	8.91
5	0.002	5.91	41.7	23390	11.90
6	0.002	7.59	33.2	23130	11.90
7	0.001	5.72	21.9	11880	11.90
8	0.001	7.95	20.3	14750	11.90
9	0.010	1.74	44.5	7750	5.45
10	0.010	2.13	47.6	10130	5.45
11	0.010	3.09	60.7	18680	5.45

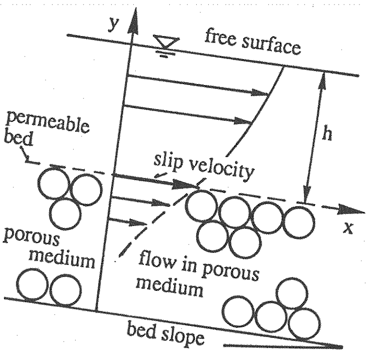

Fig.1 Definition sketch

In order to clarify the properties of the porous medium (without free surface flow), a permeability test was conducted. Figure 2 shows the experimental setup for the permeability test. Measurements of the specific discharge V (V=flow discharge per unit area of the porous medium projected to the the flow direction and simply termed the seepage velocity) and the pressure gradient along the closed conduit I_e were used to determine the resistance relation of Forchheimer's law as follows:

$$I_e = aV + bV^2 \tag{1}$$

in which a, b=resistance coefficients. According to the investigations of permeability of several porous media by Ward (9), these coefficients can be expressed as follows:

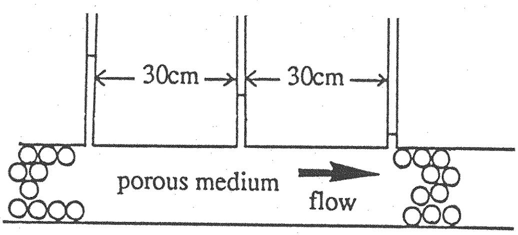

Fig.2 Setup of the permeability test

$$a = \frac{\nu}{gK}; \qquad b = \frac{c}{g\sqrt{K}} \qquad (2)$$

in which g=gravitational acceleration; K=intrinsic permeability; and c=dimensionless parameter. When b=0, Eq.1 represents the Darcy law, while Eq. 1 with non-zero b is often called non-Darcy law. Figure 3 shows the experimental relation between I_e and V, and the applicability of Eq.1 was confirmed. Table 2 lists the obtained data to represents the structural conditions of the porous layers, in which n=porosity of porous medium.

Table 2 Structure of porous media

Run	D (cm)	n	K (cm^2)	c
1~8	2.97	0.38	0.0028	0.055
9~11	1.70	0.35	0.00069	0.290

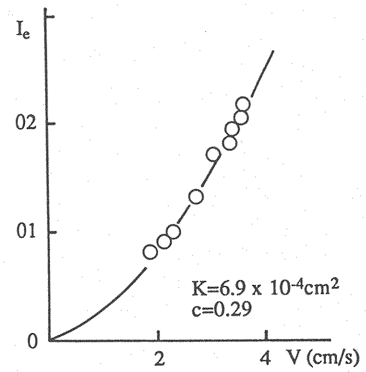

Fig.3 Relation between I_e and V obtained by permeability test

Velocity Measurements

The measurement of velocity distribution for free surface flow was made by a Pitot tube with a 2mm diameter. On the other hand, the seepage flow velocity was attained by the following method: Three probes of density-meters for salt water, A, B and C, were positioned at the same height, at streamwise intervals of 10cm in the porous medium (see Fig.4). The tracer (salt water) was instantaneously injected at the same height just upstream of the probe A (see Fig.4). The mean velocity at each height was estimated from the time difference of the peaks of the concentration recorded at the different probes. The examples of the recorded signals for the tracer concentration at the different probes are shown in Fig.5.

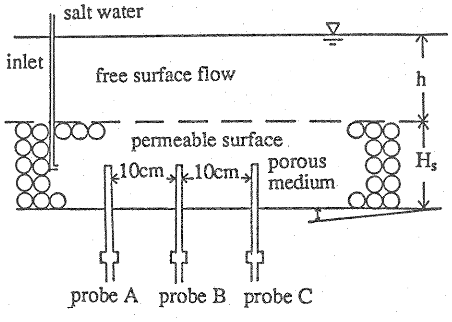

Fig.4 Measurement of seepage velocity

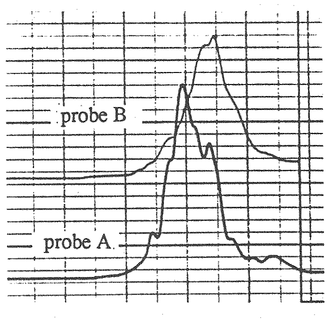

Fig.5 Example of recorded signal

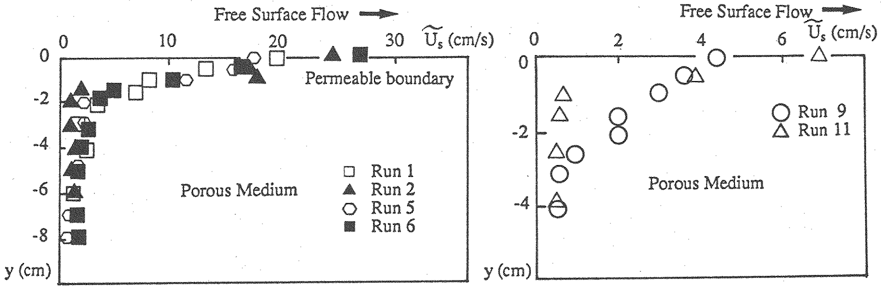

Fig.6 Velocity profile measured in porous medium

Several examples of the measured velocity profiles in the porous media are shown in Fig.6. The measured velocity is here represented by $\widetilde{U}_s$. This can be converted to the discharge per unit area of the porous medium projected to the longitudinal direction, U_s as $n\widetilde{U}_s$ (n=porosity of porous medium).

The seepage flow velocity is not constant but is appreciably dragged by the faster free surface flow. From the appreciable velocity gradient in the porous medium, one can expect the appearance of the Reynolds stress apparently. In other words, apparent turbulence brings about even in the porous medium not so far from the interface with the free-surface flow, and it contributes the additional momentum exchange. Because of a methodological reason, the Reynolds stress could not be directly measured and thus we could not discuss the eddy viscosity in this region (which will be guessed from the mass-dispersion test in the next section). The region far from the boundary with the surface-flow region, the seepage flow velocity is constant and is equal to that estimated by Eq. 1.

Mass Dispersion Test

The results of the velocity measurements in the porous media suggest an active momentum exchange there. According to the Reynolds' analogy, the mass-diffusion must be promoted in the same manner as the momentum exchange in this region. The salt-water tracer was also used for estimating the dispersion coefficient inside a porous medium. As shown in Fig.7, the salt water was continuously injected just at the level of the interface between the free-surface flow region and the porous medium (y=0). Three probes of density-meter were set at y=0, y=-0.5cm, and y=-2.5cm, respectively. The longitudinal distance from the inlets to the probes was 5cm. It would be favorable if this distance were as long as possible in order to approximate the phenomenon to a vertically one dimensional dispersion process, but the longer distance would make the concentration low at y=0. Another possibility would be a lot of inlets longitudinally distributed, but it would disturb the flow. With these probes, the time variations of density concentrations were measured.

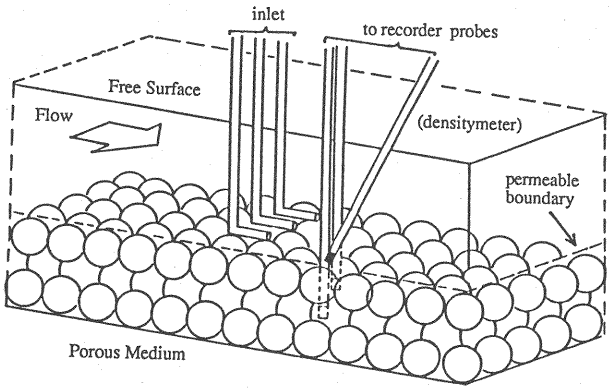

Fig.7 Setup of mass-dispersion test

The above-mentioned process (experiment) as recorded at the probe with time was here approximated as one-dimensional unsteady dispersion process, for simplicity. Then, the governing equation is written as follows:

$$\frac{\partial C(y,t)}{\partial t} = D_y \cdot \frac{\partial^2 C(y,t)}{\partial y^2} \qquad (3)$$

in which t=time; C=tracer concentration; and D_y=vertical dispersion coefficient. This equation is solved with the following initial and boundary conditions:

$$C(y,0) = 0 \qquad (0<y<-H_s) \qquad (4)$$

$$C(0,t) = C_0 \text{ (=const.)} \qquad (t>0) \qquad (5)$$

$$C(-H_s,t) = 0 \qquad (t>0) \qquad (6)$$

The third condition corresponds to the case $H_s \to \infty$. In general the porous layer is thicker than the turbulence-affected region. The resulting concentration profile, C(y,t), is as follows:

$$C(y,t) = C_0\left\{1+\frac{y}{H_s}+\frac{2}{\pi}\sum_{j=1}^{\infty}\frac{(-1)^j}{j}\cdot\exp\left(-\frac{j^2\pi^2 D_y}{H_s^2}t\right)\cdot\sin\left[\frac{j\pi(y+H_s)}{H_s}\right]\right\} \qquad (7)$$

The steady solution at y is written as

$$C(y,\infty) = C_0\left(1+\frac{y}{H_s}\right) \qquad (8)$$

Because a perfect line source was not accomplished in the experiment, the convection term could not be neglected against the approximated theory. Thus, $C(y,\infty)$ was not necessarily independent of x. However, one can guess that the temporal assymptotic behaviour of the concentration would be similar along x, and thus the following argument would be still available.

When the time scale at which the concentration reaches 1/e of the equilibrium concentration ($C(y,\infty)$) is represented by T_e, the following relation is valid:

$$C(y, T_e) = \frac{C_0}{e}\left(1+\frac{y}{H_s}\right) \qquad (9)$$

By equating Eq.9 with Eq.7,

$$\frac{C_0}{e}\left(1+\frac{y}{H_s}\right) = C_0\left\{1+\frac{y}{H_s}+\frac{2}{\pi}\sum_{j=1}^{\infty}\frac{(-1)^j}{j}\cdot\exp\left(-\frac{j^2\pi^2 D_y}{H_s^2}T_e\right)\cdot\sin\left[\frac{j\pi(y+H_s)}{H_s}\right]\right\} \qquad (10)$$

From this relation, D_y can be estimated for the experimentally measured T_e (see Fig.8).

The value of the dispersion coefficients (D_y) obtained at y=-2.5cm is about 1~10cm²/s and it looks larger than the ordinary value of the diffusion coefficient usually we use. Comparisons of the present data with Nagaoka & Ohgaki's data (3) obtained by the similar method in the manner of Fig.9, however, suggest the possibility of such large values for the present hydraulic conditions. In Fig.9, U_m=mean velocity of the free-surface flow.

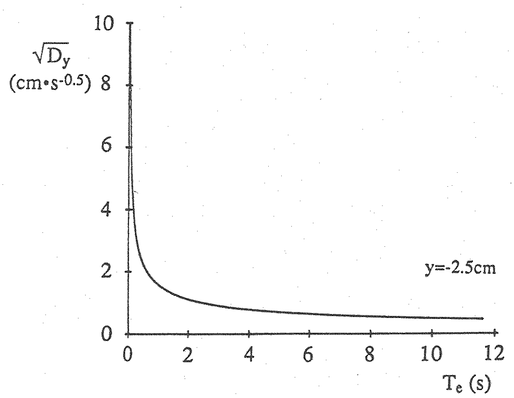

Fig.8 Relation between $\sqrt{D_y}$ and T_e

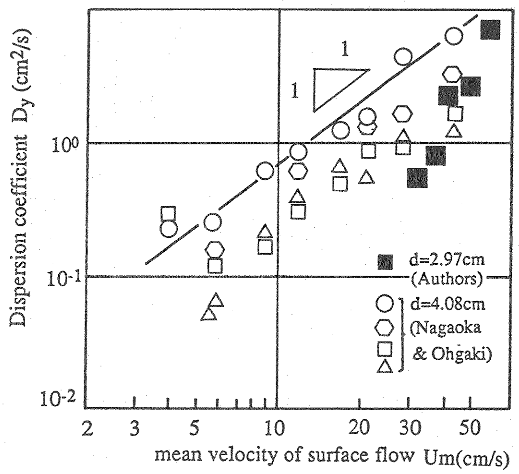

Fig.9 Relation between dispersion coefficient and mean velocity of surface flow

MACROSCOPIC MODELLING OF FLOW IN POROUS MEDIUM

A steady uniform two-dimensional flow over and in a porous layer with constant thickness is supposed here. The balance of forces acting on a control volume in the porous medium (see Fig.10) in the streamwise direction is expressed as

$$\left[\left(\tau_s+\frac{d\tau_s}{dy}\delta y\right)-\tau_s\right]\delta x+(\rho g\sin\theta-f)\delta x\delta y=0 \tag{11}$$

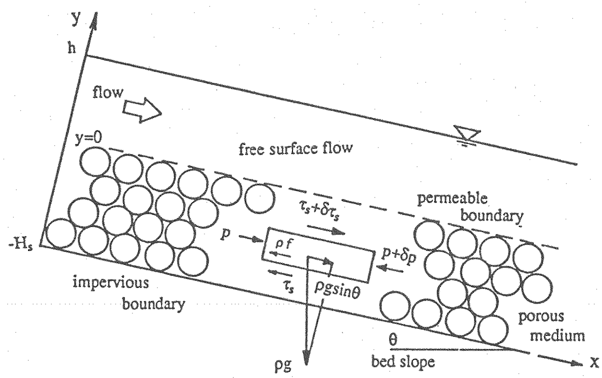

Fig.10 Force balance in porous medium

in which ρ=mass density of fluid; τ_s=shear stress induced by the presence of the free surface flow; θ=bed slope; and f=drag force per unit volume of the porous matrix. Since the flow in the porous medium near the interface with the free surface flow is expected to be turbulent, the resistance law expressed by Eqs. 1 and 2 is applied. Then,

$$f=\left(\frac{\nu}{K}\right)U_s+\left(\frac{c}{\sqrt{K}}\right)U_s^2 \tag{12}$$

in which U_s=seepage flow velocity in the porous medium. Substituting Eq.12 into Eq.11, we obtain the following equation.

$$\frac{d\tau_s}{dy}+\rho gI_e-\left(1+\frac{1}{cRe_s}\right)\frac{\rho c}{\sqrt{K}}U_s^2=0 \tag{13}$$

in which $Re_s \equiv U_s\sqrt{K}/\nu$="seepage flow Reynolds number." When $c \cdot Re_s$ is very large, Eq.13 is approximated as follows:

$$\frac{d\tau_s}{dy}+\rho gI_e-\frac{\rho c}{\sqrt{K}}U_s^2=0 \tag{14}$$

As suggested by the velocity measurements of flow in the porous medium with free surface flow, the flow inside the porous medium can be divided into two parts, U_{s1} and U_{s2}, where U_{s1} is independent of the free surface flow while U_{s2} is the induced velocity, such that

$$U_s=U_{s1}+U_{s2} \tag{15}$$

U_{s1} is obtained as a solution of Eq.14 with dropping the first term, and it is written as follows:

$$U_{s1}=\sqrt{\frac{gI_e\sqrt{K}}{c}} \tag{16}$$

When an eddy-viscosity assumption is applied to the momentum flux τ_s ("turbulent" shear stress in the porous medium as an effect of the free surface flow),

$$\tau_s = \rho \varepsilon_p \frac{dU_{s2}}{dy} \tag{17}$$

in which ε_p=eddy kinematic viscosity in the porous medium. The eddy kinematic viscosity is in general expected to be proportional to a product of a characteristic length scale representing the momentum exchange and a scale of velocity characterizing the velocity fluctuation. The characteristic length scale is assumed to be proportional to the square root of the permeability (K) because the motion of fluid lump must be constrained by the geometrical scale of the porosity; while it is assumed that the scale of velocity of eddy momentum exchange is proportional to the induced seepage velocity (U_{s2}). Thus,

$$\varepsilon_p = \alpha_0 \sqrt{K} \cdot U_{s2} \tag{18}$$

The coefficient of proportionality, α_0, will be determined by experiments.

ε_p might have the same order to the mass dispersion coefficient, D_y, discussed in the mass dispersion tests. Fig.11 shows the relation between D_y obtained by means of the mass-dispersion tests and U_{s2}, and it suggests a linear relation. Nagaoka & Ohgaki's data (3) shown in Fig.12 also supports the above conjecture because the proportionality coefficient in the relation between D_y and U_m decreases against the distance from the interfacial boundary.

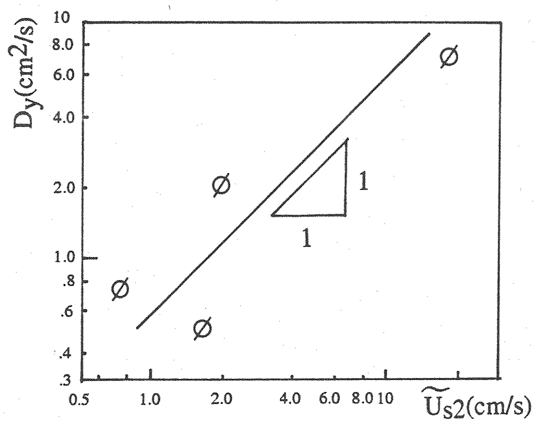

Fig.11 Relation between D_y and $\widetilde{U_{s2}}$

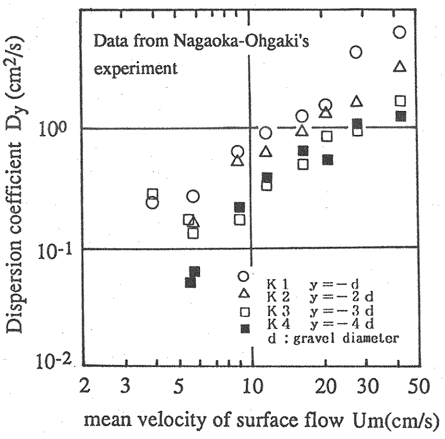

Fig.12 Relation between dispersion coefficient and surface flow velocity

Substituting Eqs.15, 17, 18 into Eq.14, we obtain the following equation with respect to U_{s2}:

$$U_{s2}\frac{d^2U_{s2}}{dy^2} + \left(\frac{dU_{s2}}{dy}\right)^2 - \frac{c}{\alpha_0 K} U_{s2}{}^2\left(1+2\frac{U_{s1}}{U_{s2}}\right) = 0 \tag{19}$$

The boundary conditions are as follows:

$$U_{s2}=U_{slip}-U_{s1} \qquad \text{at} \quad y=0 \tag{20}$$

$$U_{s2}=0 \qquad \text{at} \quad y=-H_s \rightarrow -\infty \tag{21}$$

in which U_{slip}=slip velocity=velocity at the interface between the free surface flow and the porous medium. Under the assumption that U_{s2}>>U_{s1}, the solution of Eq.19 is obtained as follows:

$$U_{s2} = (U_{slip}-U_{s1})\cdot\exp(\beta y) \tag{22}$$

in which U_{s1} is given by Eq.16; and

$$\beta \equiv \sqrt{\frac{c}{2\alpha_0 K}} \tag{23}$$

In Fig.13, the measured distribution of seepage velocity (U_s) are compared with Eq.22. The values of β and U_{slip} were determined by fitting Eq.22 to the measured profiles, and the calculated U_s was converted to $\widetilde{U}_s$ as U_s/n. Eq.22 was deduced under the assumption that $c\cdot Re_s<<1$ and $U_{s2}>>U_{s1}$. In the region -2.0cm<y<0 (turbulence-affected region in the porous medium), the experiments reveals that $U_{s2}/U_{s1}\cong 10$ and $c\cdot Re_s\cong 3\sim 8$. Hence, Eq.22 is available in the significant region in the porous medium. Fig.13 suggests that the present model is applicable if β and U_{slip} are reasonably evaluated.

From the values of β used in Fig.3, the values of the eddy kinematic viscosity ε_p were reversely calculated by using Eqs.18 and 23, as follows:

$$\varepsilon_p = \alpha_0\sqrt{K}(U_{slip}-U_{s1})\cdot\exp(\beta y) \tag{24}$$

The averaged values of ε_p through the range $-3<\beta y<0$ are plotted against the dispersion coefficient D_y for the respective experimental runs in Fig.14. This figure suggests that the momentum diffusion coefficient could be reasonably estimated from the mass dispersion coefficient. This implies that the alternation of transpiration (injection and suction) through the interfacial boundary between the flow in the porous medium and the free surface flow, that is the mass exchange, brings about the momentum exchange. Thus an induced Reynolds stress appears in the porous medium. The induced Reynolds-stress distribution in the porous medium is reversely obtained from Eqs.17, 18 and 22, as follows:

$$\frac{\tau_s(y)}{\tau_{s0}} = \exp(2\beta y) \tag{25}$$

in which τ_{s0}=the induced Reynolds stress at the interfacial boundary. τ_{s0} is given as

$$\tau_{s0} = \sqrt{\frac{\alpha_0 c}{2}}\,\rho(U_{slip}-U_{s1})^2 \tag{26}$$

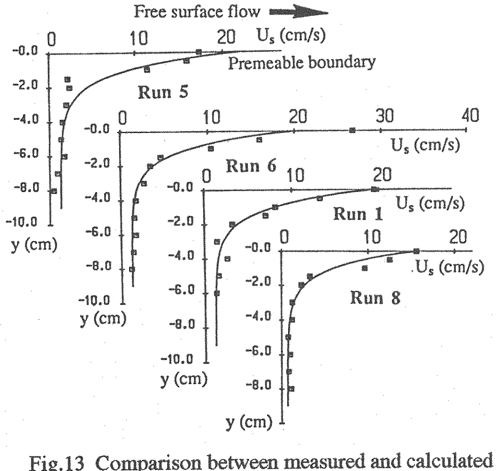

Fig.13 Comparison between measured and calculated velocity distribution in porous medium

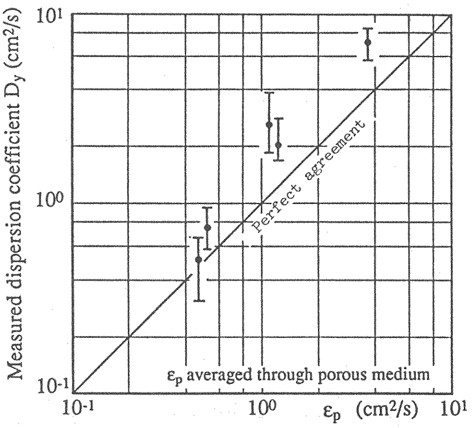

Fig.14 Comparison between measured dispersion coefficient and eddy kinematic viscosity

Though the quantitative reliability on the obtained Reynolds-stress distribution is difficult to argue because it has not been measured by methodological reasons, the profiles of the induced Reynolds stress expressed by Eq.25 is consistent to the theoretical result by Gelhar-Chu's model (1,4,5).

CONCLUSIONS

The results obtained in this study are summarized as follows:

(1) In the case of a highly permeable porous medium, the resistance law for seepage flow cannot be expressed by the ordinary Darcy law. Then, the characteristics of the flow in the porous medium with the free surface flow above it is appreciably modified due to the drag effect of the higher velocity of the free surface flow.

(2) The velocity measurements using the tracer method clarified that the velocity profile in the porous medium modified by the higher velocity of the free surface flow. The relatively large velocity gradient appears in the porous medium near the interface with the free surface flow, but far from the interfacial boundary the seepage velocity becomes uniform. The velocity gradient suggests an apparent turbulent momentum flux like the Reynolds stress.

(3) The vertical dispersion process of salt water in the porous medium from the interfacial boundary with the free-surface flow was investigated in the laboratory flume. As a result, the vertical dispersion coefficient was estimated.

(4) A macroscopic modelling based on the momentum equation for the flow in the porous medium with the induced shear distribution was proposed, and the velocity profile was deduced from it. The approximate profile is expressed as an exponential function. One of the two parameters which determine the velocity profile is related to the eddy kinematic viscosity, and it might be possible to estimate its order from the mass dispersion coefficient easily obtained by the tracer tests. The other parameter is the slip velocity, which must be determined based on the argument including the degeneration of the free surface flow due to the alternation of transpiration through the permeable boundary. That remains unresolved.

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APPENDIX - NOTATION

The following symbols are used in this paper:

a, b	= resistance coefficients involved in Forchheimer's law;
C	= tracer concentration
c	= dimensionless parameter;

D_y	= vertical dispersion coefficient;
f	= drag force per unit volume of porous matrix;
d	= diameter of glass beads;
g	= gravitational acceleration;
H_s	= thickness of porous medium;
h	= depth of free surface flow;
I_e	= energy gradient;
K	= intrinsic permeability;
n	= porosity;
p	= pressure;
Re_s	= seepage flow Reynolds number;
U_m	= depth-averaged velocity of free surface flow;
U_s	= velocity in porous medium;
U_{s1}	= seepage velocity without effect of free surface flow;
U_{s2}	= induced seepage velocity due to the presence of free surface flow;
U_{slip}	= slip velocity (velocity at y=0);
V	= seepage velocity (flow discharge per unit area projected to the flow direction);
y	= height measured from the boundary between porous medium and free surface flow region;
α_0	= proportional constant;
β	= dimensional parameter;
ε_p	= eddy kinematic viscosity in the porous medium;
ν	= kinematic viscosity;
ρ	= mass density of fluid;
θ	= bed slope; *and*
τ_s	= shear stress in the porous medium induced by free surface flow.

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