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STATIC ARMORING AND DYNAMIC PAVEMENT

By

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ABSTRACT

When the sediment supply from the upstream region is suppressed, the bed surface composed of graded material becomes coarsened as self-adjustment of fluvial bed so as to make the bed-load discharge on it equivalent to the supplied load through non-equilibrium process of bed-load transport for each grain size. When no sediment is supplied, the bed is "armored"; while for finite but suppressed sediment supply, the bed is "paved". Under equilibrium states, no sediment is transported on the armored bed, but some on the paved bed. Although these two types of coarsened beds have different properties, both the processes of those formation can be reasonably described by a model constituted by pick-up rate and step length for each grain size. Calculations based on the present model clarify some properties of non-equilibrium transport of graded material and sorting of the fluvial bed. Moreover the several properties of bed coarsening empirically induced from laboratory-experiments and field-observations are discussed based on the present analytical model.

INTRODUCTION

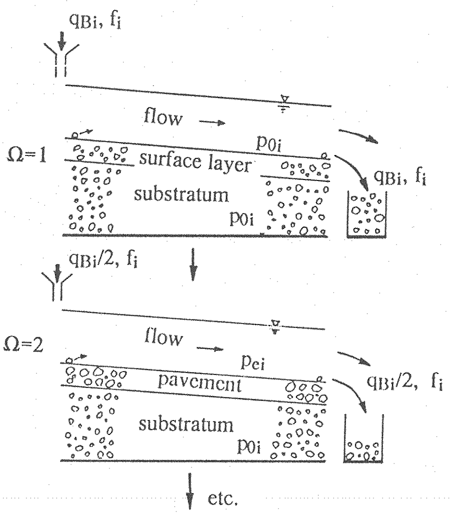
The sediment transport in a bed composed of graded materials cannot be argued independently of the bed surface composition, because the sediment transport is inevitably accompanied with sorting of the bed surface. Particularly, if the sediment supply is stopped as observed in the downstream part of a dam, the bed is coarsened appreciably. Such a phenomenon is termed "armoring", and a coarsened bed surface is called "armor coat".

Gessler (4) classified the bed degradation into two types: "parallel degradation" where the coarsening of the bed surface suppresses the degradation, and "rotational degradation" where the bed slope changes appreciably.

According to the concept of the critical tractive force for each grain size by Egiazaroff (3), the coarser sediment needs the larger tractive force to be transported. Based on this idea, Ashida & Michiue (1) reasoned as follows: The condition where the parallel degradation occurs or the armor coat is developed is that the bed shear stress should be smaller than the critical tractive force of the maximum size material and larger than that of the minimum size material. Then, the formation of armor coat was explained as follows: only the finer part of the bed material is transported away while the coarser part remains, and the latter covers the surface. On the other hand, if the bed shear stress is larger than the critical tractive force of the maximum size material, the difference of sediment transport properties due to grain size no longer appears and armoring never happens. In other words, then, the stop of the sediment supply brings a rotational degradation in such a case. If such reasons are right, the armor coats will be destroyed during floods, and on the descending stages of the floods they will develop again (Bayazit (2), Michiue & Suzuki (8)). However, some observations suggested that the armor coats still survived during floods (Harrison (5), Kellerhals (4)). It means that bed-load movement on the coarsened bed is possible to happen without destroying the coarsened bed composition.

Parker et al. (13, 14) found in the flume experiments the equilibrium state where the bed surface was coarsened but there was bed-load movement of all size-fractions of bed material over it. In case of armoring, the equilibrium transport rate is zero, and thus, they dared to term it "static armor." On the other hand, a coarsened bed with equilibrium non-zero bed-load transport was termed "mobile pavement." They emphasized the difference between them.

Figure 1 shows a thought experiment introduced by Parker (12). A stream which has a homogeneously distributed composition of bed material in the substratum is considered. p_{0i} represents the volumetric ratio of the i-th fraction, of which representative diameter is d_i, to the total bed material in the substratum. Originally, the surface layer has the same gradation distribution. When the equilibrium transport rate (q_{Bi}=bed-load transport rate of the i-th fraction sediment) determined by this bed-surface composition is supplied from the upstream end, the bed-surface composition is invariant (if the effect of the percolation of the finer material is neglected). What happens if the sediment supply is suppressed as q_{Bi}/Ω (Ω is an integer in Parker's thought experiments)? With the increase of Ω, the bed-surface layer becomes coarser. $\Omega \rightarrow \infty$ corresponds to the static armoring. When $\Omega > 1$ but is finite, the mobile pavement will be formed. Any way, Fig.1 suggests that the bed-surface composition changes as a "self-adjustment" so as the transport rate over it becomes equal to the supplied one in any fraction. In this sense, the bed surface becomes coarser for under-loading, while it becomes finer for over-loading. Actually the change of the bed slope ("rotational degradation" particularly for degradation) also contributes to adapt the transport rate to the sediment supply. Therefore, several variations of combination between the sorting of the bed surface and the change of the bed slope might be possible in rivers. In case that the bed is composed of uniform size material, the change of bed slope plays a role of adjustment of the transport rate. While, in case of graded bed material, the bed-surface sorting would do predominantly.



q_{Bi}=bed-load transport rate for each grain size;
p_{0i}=gradation of material of substratum;
p_{ei}=gradation of material of pavement (surface layer);
f_i=gradation of transported material.

Fig.1 Parker's thought experiments of pavement formation

In this paper, the aforementioned story is mathematically described by using the non-equilibrium bed-load transport law, which is characterized by pick-up rate and step length defined for each grain size of graded material. Such a modeling of bed-load transport will clarify the mechanism of the bed-surface sorting, particularly coarsening, and will give a reasonable description of formation and propagation processes of both static armoring and mobile (dynamic) pavement.

NON-EQUILIBRIUM BED-LOAD TRANSPORT OF GRADED MATERIAL

When the bed material is composed of several fractions of sediment size, non-equilibrium transport often takes place with the bed-surface sorting. For uniform size material, Nakagawa & Tsujimoto (9) proposed a non-equilibrium transport model characterized by pick-up rate and step length, and applied it successfully to describe several alluvial processes brought about by non-equilibrium bed-load motion such as the formation of small-scale bed form. Moreover, Nakagawa et al. (10) modified it in order to describe the formation and propagation of armor coat. Based on this idea, a generalized non-equilibrium model for graded material is developed in the following. The non-equilibrium bed-load transport rate for each grain size of graded material is written as follows :

$$q_{Bi}(x) = \frac{A_3 d_i}{A_2} \int_0^{\infty} p_i(x-\varepsilon) p_{si}(x-\varepsilon) \int_{\varepsilon}^{\infty} f_{Xi}(\xi) d\xi \, d\varepsilon + q_{Bi}^{in} \int_x^{\infty} f_{Xi}(\xi) d\xi \tag{1}$$

in which q_{Bi}=fractional bed-load transport rate in the subjected reach of the fluvial bed; p_i=volumetric ratio of the i-th fraction material to the total material in the surface layer; p_{si}=pick-up rate of the i-th fraction of bed material; $f_{Xi}(\xi)$=probability density of step length of the i-th fraction of bed material;

q_{Bi}^{in}=fractional bed-load transport rate supplied at x=0 (the upstream end of the subjected reach); A_2, A_3=geometrical coefficients of sediment particles; and the ratio of the area at the surface shared by the i-th fraction material to the total area has been identified with p_i as far as only the parallel degradation is considered.

The change of the bed-surface composition brought about by non-equilibrium bed-load transport which is different for each fraction is calculated as follows: For convenience' sake of explanation, the subjected reach of the bed is divided into $N\Delta x$, and the unit width is considered. Δx is favorably chosen to be smaller than the mean step length of the minimum-size material and larger than the maximum sediment size. The number of particles of the i-th fraction exposed at the surface of the k-th interval (k=1,..., N) per unit width is written as

$$n_{ik} = \frac{p_{ik}\Delta x}{A_2 d_i^2} \tag{2}$$

in which $p_{ik}=p_i$ at $x=k\Delta x$. When the numbers of particles of the i-th fraction dislodged from the bed and depositing on the bed of the k-th interval per unit width during the time interval $(t, \Delta t)$ are represented by $\Delta M_{ik}(t)$ and $\Delta Q_{ik}(t)$, respectively, the change of n_{ik} during this time interval is expressed as follows:

$$n_{ik}(t+\Delta t) = n_{ik}(t) - \Delta M_{ik}(t) + \Delta Q_{ik}(t) + p_{0i}\left\{\sum_{j=1}^{N}[\Delta M_{jk}(t)-\Delta Q_{jk}(t)]\left(\frac{d_j}{d_i}\right)^2\right\} \tag{3}$$

in which the number of particles newly exposed at the surface due to the difference between the number of dislodged particles and that of depositing ones has been taken into account, and the part of the newly exposed surface layer is assumed to have the same gradation with the substratum (p_{0i}). In the above equation, $\Delta M_{ik}(t)$ is written as

$$\Delta M_{ik}(t) = n_{ik}(t)\bullet p_{sik}(t)\bullet\Delta t \tag{4}$$

in which p_{sik}=pick up rate of the i-th fraction of bed material in the k-th interval of the bed. On the other hand, ΔQ_{ik} is composed of the component due to the dislodged particles from the j-th regions ($j<k$) and that due to the sediment supply from the region of $x<0$, and thus $\Delta M_{ik}(t)$ is written as follows:

$$\Delta M_{ik}(t) = \sum_{s=1}^{k-1}[\Delta M_{ik}(t)\bullet\mu_{i,k-s}] + \frac{q_{Bi}^{in}\Delta t}{A_3 d_i^3 \Lambda_i}\bullet\mu_{i,k-1} \tag{5}$$

in which Λ_i=mean step length of the i-th fraction of bed load; and $\mu_{i,s}$ defined as follows:

$$\mu_{i,s} \equiv \int_{s\Delta x}^{(s+1)\Delta x} f_{Xi}(\xi)d\xi \tag{6}$$

By coupling Eq.1 and Eq.3 with Eqs.4 and 5, the temporal and spatial variations of the fractional transport rate and the bed-surface composition are simultaneously described.

PICK-UP RATE AND STEP LENGTH FOR EACH GRAIN SIZE OF GRADED MATERIAL

There are several works on the characteristics of bed-load transport for each size of graded materials (sediment mixtures) since Egiazaroff's work (3) on the critical tractive force for each grain size. As for the critical tractive force for each grain size of sediment mixture, τ_{*ci}, Egiazaroff derived a formula, and Ashida & Michiue (1) modified it slightly for the finer part of graded material. The result explained the difference of the incipient motion for each grain size of graded bed material, and thus it was often applied to predict the fractional bed-load transport rate under equilibrium (Hirano (6), Ashida & Michiue (1)). Hirano and Michiue & Suzuki succeeded in explaining the composition of the armor coat based on such a fractional transport formulae. Moreover, Nakagawa et al. (10) described the formation and propagation process of armor coat by using this formula and the experimentally investigated step length. On the other hand, recent observations in paved beds (Parker et al. (15)) and experiments under dynamic equilibrium (Michiue & Suzuki (8)) pointed out that the Egiazaroff's formula might overestimate the difference of the critical tractive force according to sediment size. Moreover, the analysis of Nakagawa et al. (11) using a numerical simulation of the bed surface constitution also suggested that the Egiazaroff's formula overestimates the critical tractive force for the coarser part of graded materials. In the present study, however, the classic formula derived by Egiazaroff (3) and modified by Ashida & Michiue (1) is adopted for the sake of the conformity to the analysis of armoring by Nakagawa et al. (10). If the formula is revised based on the recent suggestions, the following analysis deduces little different conclusions. Then, the critical tractive force of the i-th fraction of bed material is estimated as

$$\frac{\tau_{*ci}}{\tau_{*cm}} = \left[\frac{\ln 19}{\ln 19\zeta_i}\right]^2 \ (\text{for } \zeta_i > 0.4)\ ; \qquad \frac{\tau_{*ci}}{\tau_{*cm}} = \frac{0.85}{\zeta_i} \quad (\text{for } \zeta_i \leq 0.4) \tag{7}$$

in which $\tau_{*i} \equiv u_*^2/[(\sigma/\rho-1)gd_i]$, u_*=bed shear stress; σ=mass density of bed material; ρ=mass density of fluid; g=gravitational acceleration; τ_{*ci}=dimensionless critical tractive force of the i-th fraction of bed material; τ_{*cm}=dimensionless critical tractive force of sediment with the mean diameter in the mixture; $\zeta_i \equiv d_i/d_m$; and d_m=mean diameter. $\alpha_{cm} \equiv \tau_{*cm}/\tau_{*c0}$ (τ_{*c0}=dimensionless critical tractive force for uniform bed material and it is almost 0.05 when the grain-size Reynolds number is larger than 100) is in general a function of the gradation of sediment mixture (Nakagawa et al. (11)), but it is often assumed to be 1.0 conveniently or approximately. The mean diameter is here defined as follows:

$$d_m \equiv \sum_{i=1}^{N} d_i p_i \tag{8}$$

By using the critical tractive force for each grain size, a pick-up rate formula proposed for uniform sand (Nakagawa & Tsujimoto (9)) is applied to each grain size of graded bed material, as follows:

$$p_{si*} \equiv p_{si}\sqrt{\frac{d_i}{(\sigma/\rho-1)g}} = F_0 \tau_{*i}\left(1 - \frac{k_2 \tau_{*ci}}{\tau_{*i}}\right)^m \tag{9}$$

in which the empirical constants are assumed to have the same values as those determined for uniform sand, and then, F_0=0.03, k_2=0.7 and m=3. Fig.2 shows a good agreement between Eq.9 with Eq.7 and the experimental data. A better agreement is achieved by an improvement of Eq.7 (see (17)).

On the other hand, the step length of graded material was experimentally investigated by Nakagawa et al. (11) The results are as follows: On a flat bed, the distribution of the step length for each grain size is approximated by an exponential distribution (see Fig.3). The dimensionless mean step length for each grain size ($\lambda \equiv \Lambda_i/d_i$) is almost constant but λ is smaller (10~30) than the value for uniform size material (80~250).

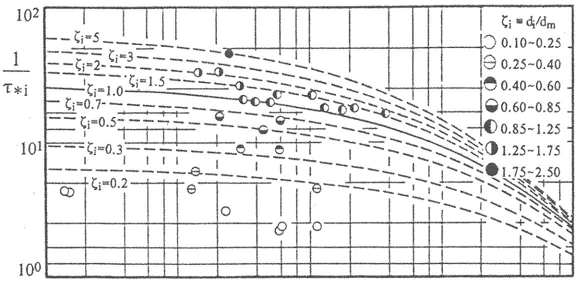


Fig.2 Pick-up rate for each grain size of sediment mixture

$$f_{Xi}(\xi) = \frac{1}{\Lambda_i}\exp\left(-\frac{\xi}{\Lambda_i}\right) \quad ; \quad \Lambda_i = \lambda d_i \tag{10}$$

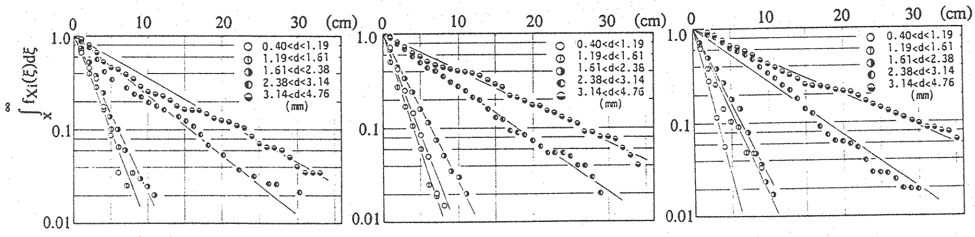


Fig.3 Distribution of step length for each grain size of sediment mixture

DESCRIPTION OF COARSENING PROCESS OF BED SURFACE

The purpose of this chapter is a unified description of (static) armoring and (dynamic) pavement, and then, the thought experiments illustrated in Fig.1 is here described mathematically. For this purpose, the sediment transport rate supplied at the upstream end of the subjected reach is expressed as follows:

$$q_{Bi}^{in} = \beta\, q_{B0i} = \beta\, \frac{A_3}{A_2}\, p_{0i} p_{s0i} \Lambda_i d_i \tag{11}$$

in which q_{B0}=equilibrium bed-load transport rate corresponding to the original bed-surface composition (p_{0i}); and β indicates the suppression of the sediment supply (β<1). Eq.11 indicates that the ratio of the transport rate supplied at the upstream end to the original equilibrium transport rate is constant for any fraction. In general, this ratio may be defined for each fraction (β_i). β corresponds to the reciprocal of Ω defined in Parker's thought experiments ($\beta \equiv 1/\Omega$).

Figure 4 depicts the temporal variation of the bed-surface composition calculated based on the present model. In the following, the calculations were conducted in the scales of laboratory experiments, and the results were shown with dimensions. However, if these quantities are normalized by the reference scales such as d_{m0} for the length, $\sqrt{d_{m0}/(\sigma/\rho-1)g}$ for the time and u_{*cm0} for the velocity (d_{m0}=mean diameter of the substratum of the bed; u_{*cm0}=shear velocity corresponding to the critical tractive force of bed material with the diameter d_{m0} in the original surface layer), the relations are expected to be universal. u_*=6.36cm/s in the calculation implies the condition slightly larger than the critical tractive force for the maximum size grain in the graded bed material. According to Fig.4, the more appreciable coarsening of the surface layer takes place with the smaller value of β (the severer suppression of sediment supply). This fact is more obviously understood in Fig.5, where the relation between the mean diameter of the equilibrium coarsened bed and $\Omega \equiv 1/\beta$. Fig.5 suggests that the bed

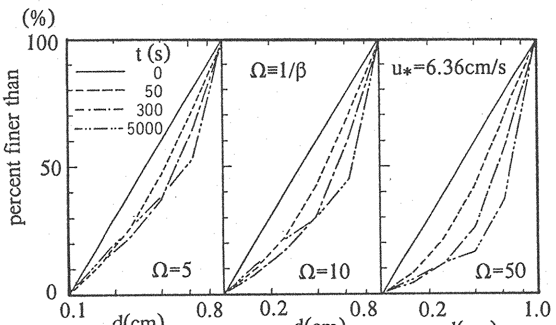


Fig.4 Pavement formation

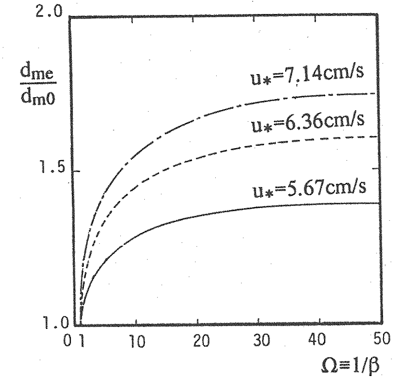


Fig.5 Mean diameter of paved bed-surface

becomes more appreciably coarsened with the increase of the shear velocity. However, the bed is no longer coarsened if the shear velocity increases and excess some limit, because no difference would be expected under the so-called rotational degradation.

Figures.4 and 5 show the results of the calculations of the pavement formation at $x=\Delta x$ (k=1). With the increase of k, in the more downstream intervals, the progress of coarsening of the bed becomes slower. This fact is recognized in Fig.6, where the temporal changes of the mean diameter of the surface layer at several longitudinal locations (k=1, 5, 10) are compared with each other.

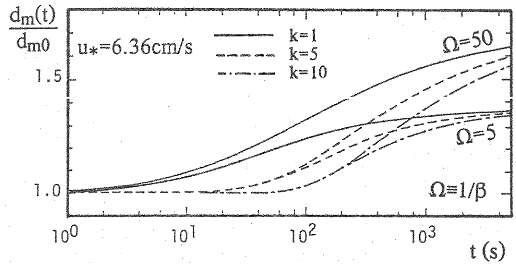


Fig.6 Temporal variation of mean diameter of surface layer

Figure 7 depicts the temporal and spatial variation of the fractional and the total bed-load transport rates. In this figure, the smaller i means the finer material. According to Fig.7, the appreciable spatial relaxation of non-equilibrium transport to the equilibrium one corresponding to the original composition of bed material appears at the early stage because of the difference between the spatially equilibrium transport rate and the supplied transport rate, and the relaxation distance corresponds to the step length. The relaxation distance of non-equilibrium transport rate for the finer material is shorter because the step length is almost proportional to the diameter. With the time elapsed, the coarsened parts propagates downstream and the spatially equilibrium transport rate decreases. Finally, the spatially equilibrium transport rate becomes equal to the supplied rate for any fraction, and a "dynamic" equilibrium is accomplished. The transport rate to adapt the difference between the spatial equilibrium transport rate and the supplied rate decreases with coarsening of the bed, and thus, the spatial relaxation-length increases with time and becomes much longer than the step length.

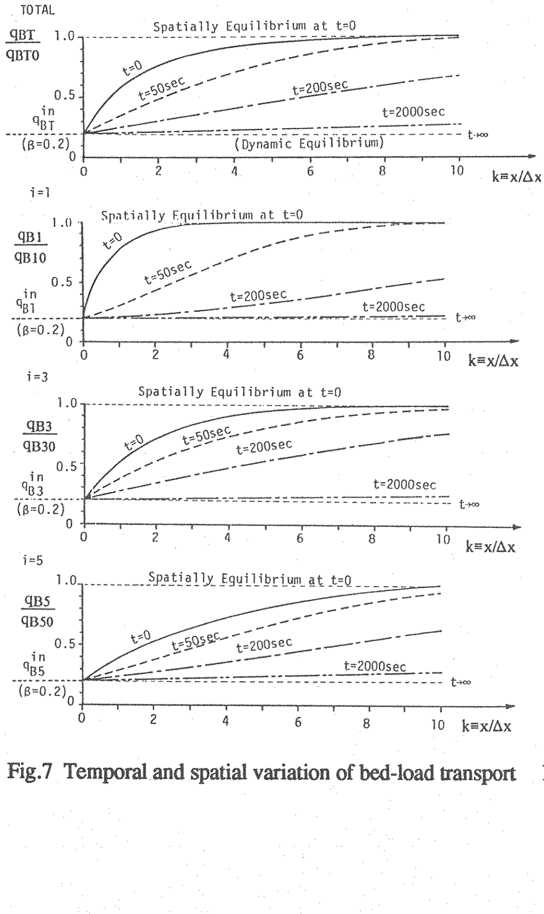


Fig.7 Temporal and spatial variation of bed-load transport

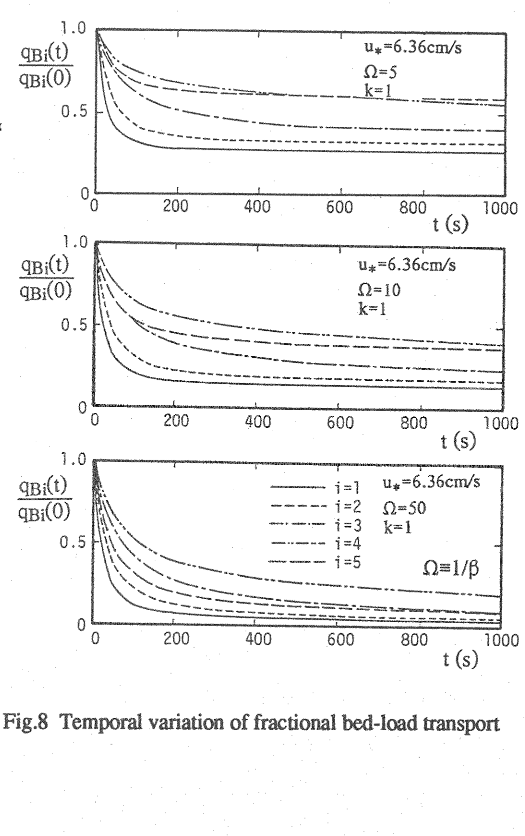


Fig.8 Temporal variation of fractional bed-load transport

Figure 8 shows the decrease of bed-load transport rate with time at $x=\Delta x$ (k=1). If the location is more downstream, the decrease of bed-load transport rate delays more. The result implies that the relaxation time of the finer material is smaller.

Figure 9 shows the time variation of gradation curve of transported material at $x=\Delta x$ (k=1). In the earlier stage, the transported material is finer than the original bed material, but it is then coarsened and the gradation curve of the transported material becomes relatively similar to that of the original bed material. This is consistent to the experimental results. Parker (12) induced from the results of the experiments by Harrison (5), Proffitt (16), and Parker (13) that the gradation of the transported material was approximately the same as that of the substratum. The present analytical result has provided the theoretical basis to the empirically induced interesting property.

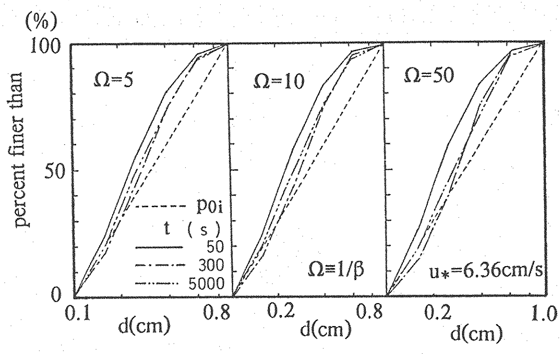


Fig.9 Temporal change of gradation curve of bed load

The process of bed-surface coarsening to reach the dynamic equilibrium are well described based on the non-equilibrium transport model. If β is set zero, the model can easily describe the formation and propagation process of armor coat. In other words, the present model can describe both static armoring and dynamic pavement. These two phenomena have different properties but it is not necessary to distinguish each other when one would analyze them.

The aforementioned arguments have been based on the non-equilibrium model, but the dynamic equilibrium pavement can be treated by an equilibrium model of bed-load transport in principle, as follows: The equilibrium fractional transport rate is written as

$$q_{Bei} = \frac{A_3}{A_2} p_{ei} p_{sei} \Lambda_i d_i \tag{12}$$

in which the subscript e represents the values under equilibrium. When the shear velocity is given, p_{sei} is evaluated as a function of $\zeta_i \equiv d_i/d_{me}$ and d_{me} is a function of p_{ej}. Under equilibrium, the transport rate is equal to the supplied rate and thus q_{Bei} is already known. Therefore, Eq.12 can be solved with respect to p_{ei}, or the equilibrium bed surface composition can be obtained without calculation for non-equilibrium process, in principle. Eq.12 implies that the dynamic equilibrium is determined by the properties of supplied sediment but never by the gradation of the substratum, though the process to reach the dynamic equilibrium is influenced by the gradation of the substratum. Eq.12 means, in fact, N equations (N=number of fractions of bed material), and the individual equation with respect to i is not independent of each other because p_{sei} is a function of p_{ej}. Then, even when the equilibrium is needed to be solved, the calculation based on the equilibrium equation is often more difficult than the calculation of non-equilibrium process. Furthermore, Eq.12 has no solution unless the range of the gradation curve of the supplied sediment is same as that of the substratum. Particularly, in case of static armoring, the right hand term is zero and thus Eq.12 becomes meaningless. Hence, if we insist an approach based on the equilibrium equation, we cannot but distinguish the static armor from the mobile pavement. Fig.10 is an example of successful results directly calculated from the equilibrium transport formula. In the figure, f_i=ratio of the transport rate of the i-th fraction to the total bed-load transport rate supplied at the upstream end.

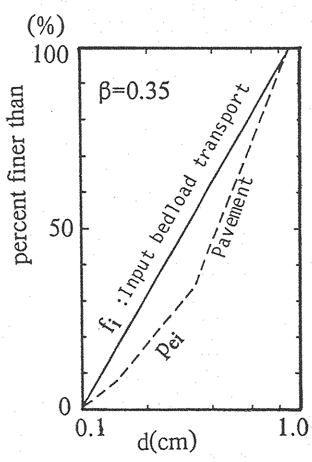


Fig.10 Gradation curves of input bed-load and paved bed-surface

COMPARISON OF THE PRESENT MODEL WITH EXPERIMENTAL DATA AND FIELD DATA

Figure 11 depicts the comparison between the calculated results of the present model and the experimental data by Ashida & Michiue (1) with respect to the formation and propagation process of armor coat. The data of Ashida & Michiue detected the propagation of the armor coat clearly, and the present model described the process fairly well. On calculation, Δx=5cm, and Δt was set so as $p_{si}\Delta t$ did not exceed 0.1.

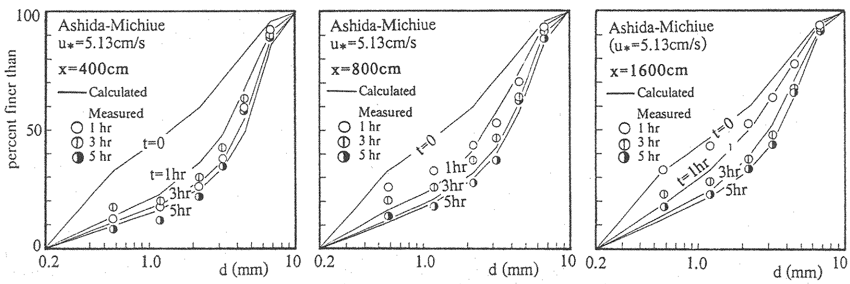


Fig.11 Formation and propagation of armoring

Few systematic experiments were conducted for bed-load transport of graded material or bed-surface coarsening with non-zero variable sediment supply, though a lot of experiments of armor-coat formation were conducted. Parker et al. (14) conducted an experiment of bed-surface coarsening with sediment supply as a model test of the Oak Creek where the bed was paved. In their experiment, the gradation curves of the substratum and the paved bed material (p_{0i} and p_{ei}) and the fractional bed-load discharge (q_{Bei}) were measured. In Figs.12 and 13, the gradation curve of the pavement (p_{ei}) and that of transported material (f_{ei}) are depicted, respectively. f_{ei} is the ratio of the transport rate of the i-th fraction to the total bed-load transport under dynamic equilibrium and is calculated as follows:

$$f_{ei} = \frac{q_{Bei}}{\sum_{j=1}^{N} q_{Bej}} \tag{13}$$

The spatially equilibrium transport rate for the initial bed (q_{B0i}) is written as

$$q_{B0i} = \frac{A_3}{A_2} p_{0i} p_{s0i} \Lambda_i d_i \tag{14}$$

Since the dynamic equilibrium bed-load transport (q_{Bei}) is equal to the supplied rate at the upstream end (q_B^{in}), the index of the suppression of sediment supply β_i is obtained for each fraction as follows:

$$\beta_i = \frac{q_{Bei}}{q_{B0i}} \tag{15}$$

The numerical values of β_i are shown in Fig.12, and they are almost constant except the value for the maximum size. β in the figure indicates the mean obtained by excluding the value for the maximum size.

The ratio of the initial spatially equilibrium transport rate of the i-th fraction to that of the total material, f_{0i}) is calculated as follows:

$$f_{0i} = \frac{q_{B0i}}{\sum_{j=1}^{N} q_{B0j}} \tag{16}$$

The results obtained by the present model through the calculation of non-equilibrium processes are shown also in Figs.12 and 13, and they are well consistent with the observed ones. If the value of β is adopted for all fractions, the calculated result is a good approximation of that by using respective values of β_i, as shown in Fig.12. Furthermore, the result of the direct calculation by using the equilibrium equation is also depicted in Fig.13, and it is perfectly same as the result with the non-equilibrium equation.

The comparison between the gradation curve of the transported material and that of the substratum in Fig.13 suggests that a coincidence of them as Parker (1986) expects is poor in this example.

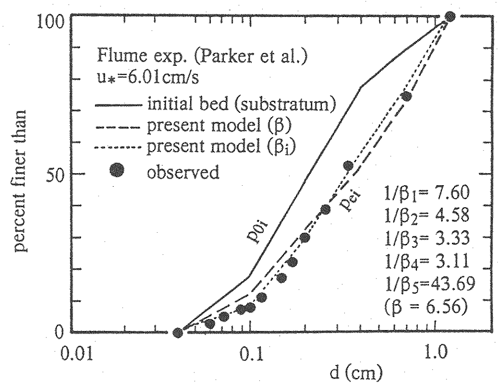


Fig.12 Size-distribution of substratum and paved bed-surface

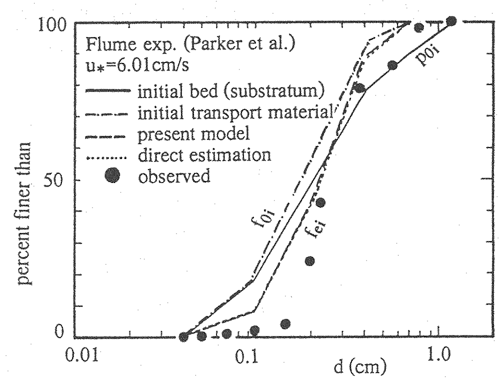


Fig.13 Gradation curve of bed-load over a paved bed

Parker et al. collected the field data in the Oak Creek and some other rivers as for the gradation of the pavement and that of the substratum. For some rivers, no data but 50% and 90% diameters (50% diameter is the diameter which 50% of material in volume basis is finer than) were obtained. As for these examples, the gradation curves have been extrapolated as linear curves in a semi-logarithmic paper as shown in Fig.14. The shear velocity was obtained or predicted as indicated in Table 1.

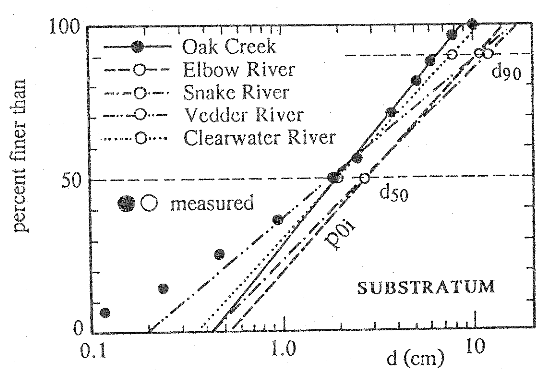


Fig.14 Size-distribution of substrata of paved streams

Since p_{0i} and u_* are known, the relation between (d_{me}/d_{m0}) and β can be obtained by the present model. The calculated results for respective rivers are shown in Fig.15. Since (d_{me}/d_{m0}) is known for each river as the observed data, Fig.15 gives an expected value of β for each river. The obtained value of β is also shown in Table 1. As the results, the coarsened beds in the Oak Creek, the Elbow River and the Snake River are classified into static armor, while those in the Clearwater River and the Vedder River into mobile pavement.

Table 1 Some parameters for paved streams

Rivers	u_* (cm/s)	d_{me} (cm)	Ω=1/β
Oak Creek	19.5	4.91	10.5
Elbow River	23.4	7.43	9.0
Snake River	23.5	6.77	3.7
Clearwater River	16.4	7.77	∞
Vedder River	16.9	4.90	49.5

The process that the pavement develops in the Oak Creek is predicted by a simulation based on the present model. The result shown in Fig.16 suggests that an appreciable pavement develops within a few hours if the substratum is exposed at the surface.

The gradation curve of the transported material over the paved bed of the Oak Creek was estimated by the present model. The result is compared with the gradation curve of the substratum in Fig.17. As far as this example, the "hypothesis" that the gradation curve of transported material and that of the substratum are the same is valid, which Parker et al. (12, 15) proposed from the data and based on which they deduced formulas to predict bed-surface composition and bed-load discharge. However, they do not always coincide each other, as clarified by the present model.

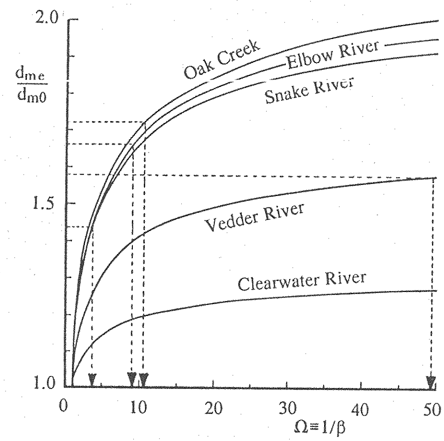


Fig.15 Estimation of decrease of bed-load supply at the upstream end of the paved-stream

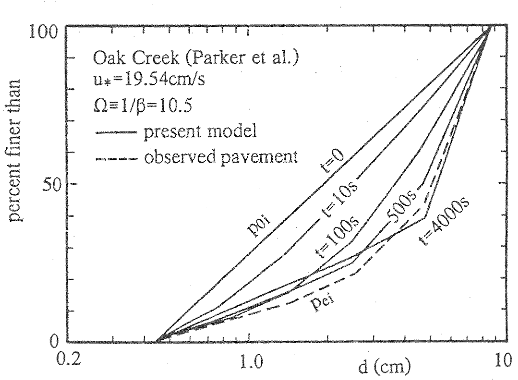


Fig.16 Formation process of pavement in the Oak Creek

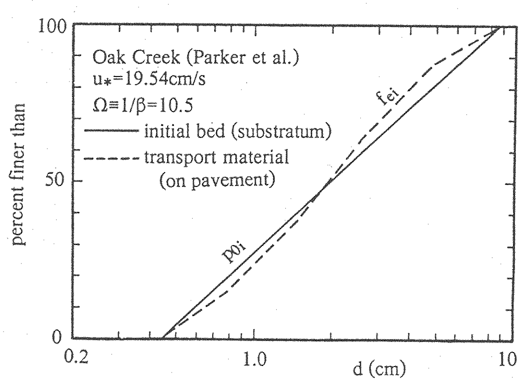


Fig.17 Gradation curves of substratum and bed-load over paved bed

CONCLUSIONS

The results obtained in this study were summarized below:

(1) Non-equilibrium bed-load transport model characterized by pick-up rate and step length for each grain size of graded materials and a model to describe the change of the bed-surface composition due to non-equilibrium and selective transport of graded material have been established. Based on these models, the variation of bed-load transport rate as a spatial and temporal relaxation process is reasonably described. It helps an understanding of the self-adjusting of the bed-surface composition so as the transport rate over the bed becomes equal to the supplied transport rate.

(2) The present model clarifies the difference of the properties between static armoring and mobile or dynamic pavement, but these two processes are described in the same analytical framework. In other words, these two processes are distinguished from each other according to the extent of suppression of the supplied sediment.

(3) The pick-up rate and the step length for each grain size can be well predicted by the proposed formulas.

(4) The calculations based on the proposed model clarifies that the transported material becomes finer than the material of the substratum initially but it becomes coarser with the progress of pavement formation so as the gradation of the transported material approaches to that of the substratum. The

hypothesis that the gradation of the transported material under dynamic equilibrium equals that of the substratum is sometimes valid, but not always.

(5) The formation and propagation process of armor coat observed in the laboratory flume has been well explained by the present model.

(6) The pavement formation observed in the laboratory flume has been also explained by the present model. As the result of comparing the properties of pavement deduced from the present model with the field data obtained in the Oak Creek and some rivers, the applicability of the present model to real rivers has been confirmed.

The content of this paper was already published in the Japanese paper of the authors (19).

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APPENDIX - NOTATION

The following symbols are used in this paper:

A_2, A_3 = two- and three-dimensional geometrical coefficients of sand;

d	= diameter of sand;
d_m	= mean diameter of sand mixture;
F_0	= empirical constant of pick-up rate formula;
f_i	= ratio of the transport rate of the i-th fraction to the total bed-load transport rate;
$f_X(x)$	= probability density function of the step length of bed-load motion;
g	= gravitational acceleration;
k_2, m	= empirical constants of pick-up rate formula;
N	= number of fractions of bed material;
n_{ik}	= number of particles of the i-th fraction exposed at the surface of the k-th interval per unit width
p_{0i}	= volumetric ratio of the i-th fraction sand to that of sand of all fractions in the substratum of the bed or the initial surface layer;
p_i	= volumetric ratio of the i-th fraction sand to that of sand of all fractions in the surface layer;
p_s, p_{s*}	= pick-up rate and its dimensionless form ($\equiv p_s\sqrt{d/(\sigma/\rho-1)g}$);
q_B, q_{B*}	= bed-load transport rate and its dimensionless form ($\equiv q_B/\sqrt{(\sigma/\rho-1)gd^3}$);
q_{B0}	= initial spatially equilibrium bed-load transport rate;
q_{Bi}^{in}	= fractional bed-load transport rate supplied at x=0 (the upstream end of the subjected reach);
t	= time;
u	= flow velocity;
u_*	= shear velocity;
x	= longitudinal distance from the upstream end of the subjected reach of a fluvial bed;
α_{cm}	$\equiv \tau_{*cm}/\tau_{*c0}$;
β	= the ratio of the supplied bed-load transport rate to the initial spatially equilibrium bed-load transport rate;
$\Delta M_{ik}(t)$	= number of particles of the i-th fraction dislodged from the bed of the k-th interval per unit width during the time interval (t, Δt);
$\Delta Q_{ik}(t)$	= number of particles of the i-th fraction depositing on the bed of the k-th interval per unit width during the time interval (t, Δt);
Δt	= duration of the time step on calculation;
Δx	= length of the segment of the bed;
ζ_i	$\equiv d_i/d_m$;
Λ	= mean step length of bed-load motion;
$\mu_{i,s}$	= probability that the step length of the i-th fraction material is in the range ($s\Delta x$, $(s+1)\Delta x$);
λ	= dimensionless mean step length of bed-load motion (Λ/d);
ρ	= mass density of fluid;
σ	= mass density of sand;
τ_*	= dimensionless bed shear stress ($\equiv u_*^2/[(\sigma/\rho-1)gd]$);
τ_{*c0}	= dimensionless critical tractive force for uniform size material;
τ_{*ci}, τ_{*cm}	= dimensionless critical tractive force for each grain size and that for sand of mean diameter in the mixture; *and*

Ω	$= 1/\beta$;

Subscripts

0	= substratum of the bed or the initial surface layer;
e	= equilibrium conditions;
i	= the i-th fraction of graded material whose diameter is d_i; *and*
k	= the k-th interval of the subjected reach of the fluvial bed whose length is Δx.

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