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TRANSITION MECHANISM FROM SALTATION TO SUSPENSION IN BED-MATERIAL-LOAD TRANSPORT

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ABSTRACT

The interaction between bed-load motion and sediment suspension in bed-material load transport is characterized by the transition from bed-load motion to suspension. A reasonable description of bed-material load transport requires a qualitative description of the transition from bed-load motion to suspension. The trajectory of a particle in bed-load motion is rather deterministic. The turbulence gives a shift on the presumed trajectory, and according to the amplitude of the shift, the transition into suspension from bed-load motion takes place as an instability of particle's trajectory. Such a mechanism is described by a logistic equation deduced from the equation of particle's motion. Furthermore, based on the proposed model, the transition probability density per unit time from bed-load motion to suspension is evaluated against the shear velocity.

INTRODUCTION

The relation between the bed-material load discharge and the intensity of flow has been investigated based on almost independently arranged models for bed-load motion and sediment suspension. However, as seen from the Einstein's model (2) that the bottom reference concentration of suspended sediment must equal the bed-load concentration, bed load and suspended load coexist, and we can expect an active exchange between them. Hence, a model of the transition between bed-load motion and suspension to evaluate the transition probabilities between them will make a progress in understanding of bed-material load transport process and its qualitative prediction. That will make possible to distinguish the sediment suspension from the bed-load motion with a reasonable base. In this study, the transition from bed-load motion to suspension is investigated. The reverse process (suspension to bed-load motion) represents a deposition of a suspended particle, and it is already described in a model of sediment suspension.

This study deals with the transition mechanism from bed-load motion to suspension on a flat bed. Although the bed configuration affects the transition from bed-load motion to suspension, the fundamental mechanism may be reasonably well clarified without effects of bed undulations as similar as other mechanics of sediment transport.

In some previous studies on determining the bottom reference concentration of suspended sediment (Lane & Kalinske (6), Ashida Michiue (1), Hirano (4), Itakura & Kishi (5)), the concept of the transition from bed-load motion to suspension was taken into account more or less rather unconsciously. Tsujimoto & Nakagawa (15) treated the bed-material load transport by paying particular attention to the transition from saltation to suspension. They focussed the fact that the transition from bed-load motion to suspension appears with an extraordinary shift from the presumed rather deterministic trajectory of saltation, as confirmed through a video-film analysis. Modelling the transition mechanism, they evaluated the probability of transition from bed-load motion to suspension as the probability that an extraordinary shift in a saltation trajectory takes place. They empirically determined such an extraordinary shift as a threshold, by which a mere fluctuation of the trajectory and a transition into suspension are distinguished from each other. The authors' interest in this paper, however, lies on what is a phenomenological meaning of the threshold and how the threshold is determined related to the dynamics of a particle's motion. These have become a motivation of this study. The solution is obtained by regarding the transition as an instability of a saltation trajectory and by deducing a logistic equation from the equation of saltation motion. Based on the description of the transition from saltation to suspension, the probability density per unit time of transition from bed-load motion to suspension is estimated. The transition probability density is a key in a bed-material load transport model, by which the effects of the transition on bed-material load transport, or bed load and suspension composing the bed-material load respectively, are clarified.

MODEL OF BED-MATERIAL LOAD TRANSPORT PROCESS

According to the minute observation in the well-arranged laboratory flume through video-film analysis, bed-material load transport is described as follows: A particle dislodged from a bed moves as a bed-load particle at first, and it turns to suspension probabilistically if the condition for transition is satisfied. The suspended particle travels a probabilistic variable, "excursion length," subjected to the turbulence characteristics of the flow and the fall velocity of the particle. After such a probabilistic excursion length, the particle turns back to the bed, and it continues bed-load motion or stops on the bed. Then, we can postulate the following four susbsystems which constitute the bed-material load transport process (see Fig.1):

(A) Dislodgement of a sand particle from a bed;
(B) bed load motion as described by irregular successive saltation;
(C) transition from saltation to suspension; and
(D) suspension as described rather as a random motion.

The individual subprocesses should be represented by appropriate parameters respectively as follows: the pick-up rate (p_s) for the subprocess (A); the probability density of the step length ($f_X(x)$), the speed of bed-load particles (u_g), and the bed-load concentration (C_B) for (B); the transition probability from bed-load motion to suspension (p_T) for (C); and, the probability density distribution of the existence height of a suspended particle ($f_S(y)$; roughly similar to the suspended sediment concentration distribution) and that of excursion length of a suspended particle ($f_{XS}(x)$) for (D). The pick-up rate is defined as the probability density per unit time for a bed-material particle to be dislodged from a bed; the step length as a distance for a dislodged particle to travel as a bed load; and the transition probability from bed-load motion to suspension as the probability density per unit time for a bed-load particle to turn into a suspended one.

This study focuses on the subprocess (C). We already obtained some available results for the subprocesses (A), (B), and (D). (8, 14, 15, 16), and also as for the subprocess (C), we already started the study partially (13, 15).

The following formulation of bed-material load transport including both bed load and suspended load is proposed here, which is a modification of the non-equilibrium bed-load transport model established by Nakagawa & Tsujimoto (8).

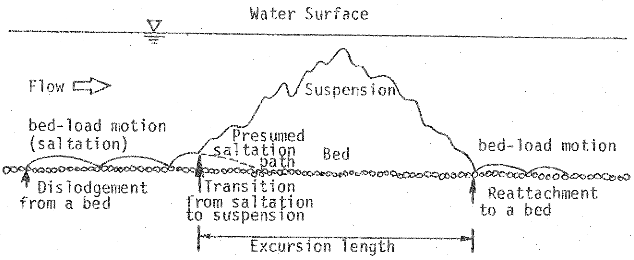

Fig.1 Illustration of bed-material load motion

$$q_B(x) = \frac{A_3 d}{A_2} \int_0^{\infty} \{ [p_s(x-\xi)+p_{ds}(x-\xi)] \cdot [\int_{\xi}^{\infty} f_X(\zeta) d\zeta] \cdot F_{TR}(\xi) \} d\xi \tag{1}$$

$$q_S(x) = \int_0^{\infty} \{ \frac{q_B(x-\xi)}{u_g(x-\xi)} \cdot p_T(x-\xi) \cdot [\int_{\xi}^{\infty} f_{XS}(\zeta) d\zeta] \} d\xi \tag{2}$$

in which x=longitudinal coordinate, q_B=bed-load transport rate (substantial volume per unit time per unit width); q_S=suspended load transport rate; A_2, A_3=geometric coefficients of sediment particle; $p_{ds}(x)$=number of suspended particles per unit time to deposit (not necessarily to stop) on the bed of the area occupied by one particle at x to turn into bed-load motion; $F_{TR}(\xi)$=the probability that a bed-load particle does not turn into suspended load during traveling a distance ξ after its incipient motion (dislodgement from a bed). $p_{ds}(x)$ is expressed as follows:

$$p_{ds}(x) = \frac{A_2}{A_3 d} \int_0^{\infty} [\frac{q_B(x-\xi)}{u_g(x-\xi)} \cdot p_T(x-\xi) \cdot f_{XS}(\xi)] d\xi \tag{3}$$

When the transition probability from bed-load motion to suspension is constant along x, $F_{TR}(\xi)$ is approximated as follows:

$$F_{TR}(\xi) = \exp\left(-\frac{p_T}{u_g}\xi\right) \tag{4}$$

When non-equilibrium transport on a flat alluvial bed at downstream of a rigid bed by uniform flow is taken for an example, non-equilibrium bed-load transport rate is expressed as follows:

$$q_B(x) = \frac{A_3 d}{A_2} \int_0^{\infty} \{ [p_s(x-\xi)+p_{ds}(x-\xi)] \cdot \exp\left(-\frac{\xi}{\Lambda_{BS}}\right) \} d\xi \tag{5}$$

in which the step length is assumed to follow the exponential distribution with the mean Λ; and

$$\Lambda_{BS} \equiv \Lambda\left(\frac{u_g}{u_g+p_T\Lambda}\right) \tag{6}$$

Under equilibrium which is obtained at the sufficiently downstream part of the stream, the following relations are valid.

$$q_{Be} = \frac{A_3 d}{A_2}(p_s+p_{dse})\Lambda_{BS} \tag{7}$$

$$p_{dse} = \frac{A_3 d}{A_2}\left(\frac{p_T}{u_g}\right)q_{Be} \tag{8}$$

$$q_{Se} = \left(\frac{p_T}{u_g}\right)\Lambda_S q_{Be} \tag{9}$$

in which the subscript e represents the values under equilibrium; and the excursion length is assumed also to follow the exponential distribution with the mean Λ_S. Then,

$$q_{Be} = \frac{A_3 d}{A_2} p_s \Lambda_{BS}\left(\frac{u_g}{u_g-p_T\Lambda_{BS}}\right) = \frac{A_3 d}{A_2} p_s \Lambda \tag{10}$$

Eq.10 implies that the equilibrium bed-load transport rate is invariant either with or without transition between bed-load motion and suspension. This fact assures that the total bed-material load is expressed as the sum of the bed-load transport rate and the suspended load transport rate.

The above argument emphasizes the importance of the transition probability from bed-load motion to suspension on describing bed-material load transport process.

Although the suspended sediment distributes along the flow depth, the above argument is one-dimensionalized. Because not only the quantitative concentration but also the concentration profile are degenerated under equilibrium condition, the two dimensional modelling will be required in future.

SEDIMENT SUSPENSION AFFECTED BY TRANSITION FROM BED-LOAD MOTION TO SUSPENSION

Since the transition from bed-load motion to suspension is the "production" of suspended sediment, the fundamental equation to determine suspended sediment concentration should have a production term as follows (see Fig.2):

$$w_0 C_S + \varepsilon_s \frac{dC_S}{dy} = \int_y^h S_S(y)dy \tag{11}$$

in which y=height from the bed; h=flow depth; C_S=suspended sediment concentration; w_0=terminal velocity of suspended particle; ε_s= diffusion coefficient of suspended sediment; and S_S=substantial volume of suspended sediment produced by the transition from bed-load motion per unit volume of water per unit time. Since the transition from suspension to bed-load motion is defined as "retouch" on the bed, the term of the "dissipation" need not be added for the region y>0 in Eq.11. Eq.11 is a non-homogeneous diffusion equation. Murphy (7) treated the same equation as Eq.11, and Tsujimoto (12) discussed an apparent change of the diffusion coefficient by the production term.

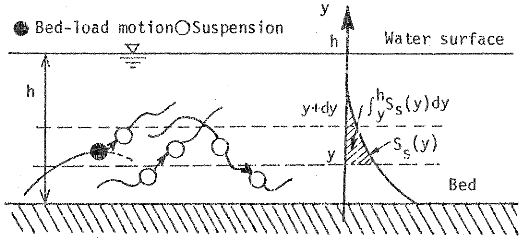

Fig.2 Schematic figure of transition from bed-load motion to suspension

When the diffusion coefficient of suspended sediment is assumed to be constant along the flow depth (the depth-averaged value of the diffusion coefficient is used), the solution of Eq.11 is written as

$$\frac{C_S(\eta)}{C_S(0)} = \gamma\phi_1(\eta) + (1-\gamma)\phi_2(\eta) \tag{12}$$

in which

$$C_S(0) \equiv C_1 + C_2 = C_1 + \frac{\int_0^h S_S(y)dy}{w_0} \tag{13}$$

$$\phi_1(\eta) = \exp\left(-\frac{w_0\eta}{u_*\varepsilon_{s*}}\right) \tag{14}$$

$$\phi_2(\eta) = \frac{\left[\int_\eta^1 \Psi_S(\eta)d\eta + \phi_1(\eta)\right] \cdot \int_0^h [\Psi_S(\eta)/\phi_1(\eta)]d\eta}{\int_0^1 \Psi_S(\eta)d\eta} \tag{15}$$

$\gamma \equiv C_1/(C_1+C_2)$; $\eta \equiv y/h$; $\varepsilon_{s*} \equiv \varepsilon_s/(u_*h)$; $\Psi_S(\eta h) \equiv S_S(y)/S_S(0)$; and u_*=shear velocity. ϕ_1 is a "homogeneous solution"; while ϕ_2 is a solution due to "non-homogeneity". The contribution of the non-homogeneous solution to the concentration profile is represented by the parameter γ (see Fig.2).

As seen from Eq.13, the bottom concentration of suspended sediment is determined partially by the transition from bed-load motion to suspension. In the previous studies, ϕ_1 was used, and the bottom concentration was estimated by a model for the transition. Without the transition from bed-load motion to suspen- sion (there is no exchange between bed-load motion and suspension), the concentration profile would be expressed only by ϕ_1 as similar as in the previous studies, but the bottom concentration would be determined by the integral constant C_1 which is independent of the transition. C_2 is certainly related to the transition, and it is evaluated by the following equation when p_T is known.

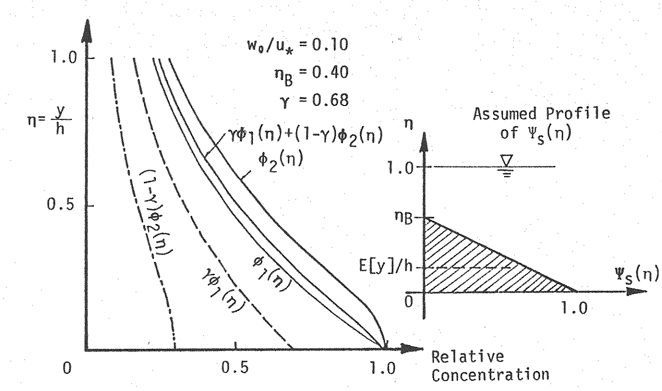

Fig.3 Effect of transition from bed-load motion to suspension on concentration-distribution profile of suspended sediment

$$C_2 = \frac{\int_0^h S_S(y)dy}{w_0} = \frac{p_T q_B}{w_0 u_g} \tag{16}$$

By multiplying the flow velocity (rigorously saying, the speed of the suspended particles) to the suspended sediment concentration and integrating the result through the flow depth, the following relation is obtained:

$$\frac{q_s}{u_* h} = C_1 I_1 + C_2 I_2 \tag{17}$$

$$I_1 = \int_0^1 \left[\frac{u(\eta)}{u_*}\right]\phi_1(\eta)d\eta \ ; \qquad I_2 = \int_0^1 \left[\frac{u(\eta)}{u_*}\right]\phi_2(\eta)d\eta \tag{18}$$

in which u=local velocity of flow. Then, C_1 is expressed as follows:

$$C_1 = \frac{p_T q_B}{w_0 u_g}\left[\left(\frac{\Lambda_S}{h}\right)\left(\frac{w_0}{u_*}\right) - I_2\right] \tag{19}$$

Eq.19 implies that C_1 cannot be determined without information of the excursion length of suspended particles. It is quite reasonable but the excursion process has not been taken into account on determination of the bottom concentration up to date.

The parameter γ can be determined without an evaluation of p_T, as follows (Tsujimoto & Yamamoto (16)):

$$\gamma = \frac{(\frac{\Lambda_S}{h})(\frac{w_0}{u_*}) \cdot I_1}{I_1 + (\frac{\Lambda_S}{h})(\frac{w_0}{u_*}) \cdot I_2} \tag{20}$$

According to Tsujimoto & Yamamoto (16), γ increases with (w_0/u_*) and it varies from 0.2 to 0.7 in the range of (w_0/u_*) from 0.1 to 1.0 (see Fig.4). When (w_0/u_*) is small and thus the suspended sediment dominates, γ is so small that the concentration profile is affected by non-homogeneity and the bottom concentration is correlated to the transition probability from bed-load motion to suspension. On the other hand, for the large value of (w_0/u_*) where the bed load dominates, the concentration profile is approximated by a familiar homogeneous solution and the bottom concentration is subjected to the excursion length of suspended sediment.

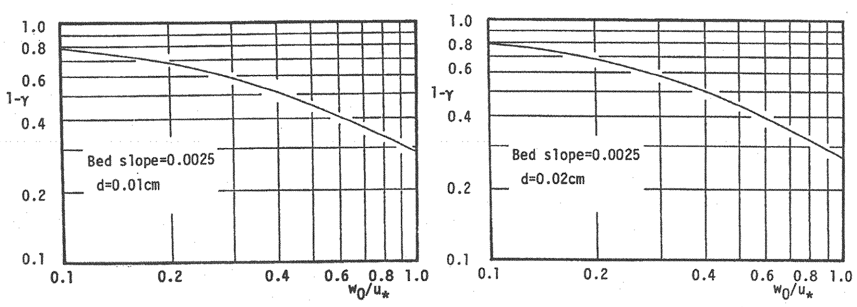

Fig.4 Non-homogeneous index γ versus (w_0/u_*)

INSTABILITY OF SALTATION TRAJECTORY

Tsujimoto & Nakagawa (15) pointed out based on the minute observation of bed-material load transport through a video-film analysis that a bed-load particle turns its motion into suspension when the rather deterministic trajectory of saltation is deviated more than some criterion by turbulence. Tsujimoto (13) tried to explain the existence of the threshold which distinguishes the transition from bed-load motion to suspension from mere fluctuations and to explain why the behavior of a particle is quite different depending on whether the shift of the trajectory exceeds such a threshold or not. He noticed of the properties of a logistic equation which could be deduced from the equation of saltation motion, and regarded the transition from bed-load motion to suspension as the instability of saltation trajectory. Such an idea is developed in the present study.

The irregularity of bed-load motion implies the widely distributed sizes of individual saltations, which are brought about random repulsions at the bed surface. The individual saltation trajectory is rather deterministic. However, it is subjected to the action of turbulence, and fluctuates. When the deviation of the trajectory from the presumed path is expressed by ξ, the vertical component of the relative speed for a particle to deviate from the presumed path is formally written as

$$v_g = \frac{d\xi}{dt} \equiv f(\xi) \tag{21}$$

When the deviation brought about by a "unit of turbulence action" is ξ_i, which is termed "initial deviation" in the following, and the particle's behavior after the initial deviation is represented by v_g, Taylor's expansion of Eq.21 yields

$$v_g = f(\xi) = f(\xi_i) + (\xi-\xi_i)\, f'(\xi_i) + \frac{(\xi-\xi_i)^2}{2} f''(\xi_i) + \cdots\cdots\cdots \tag{22}$$

When the terms of the higher order of ξ than ξ^3 are neglected, Eq.21 is rewritten as

$$\frac{d\xi}{dt} = a+b\xi+c\xi^2 \tag{23}$$

in which a, b and c can be expressed as follows, and they are determined by using the equation of motion for a particle moving in the flowing fluid.

$$a = v_{gi}\ ;\qquad b = \frac{1}{v_{gi}}\frac{dv_{gi}}{dt}\ ;\qquad c = \frac{1}{2v_{gi}}\left[\frac{d^2v_{gi}}{dt^2} - \frac{1}{v_{gi}}\left(\frac{dv_{gi}}{dt}\right)^2\right] \tag{24}$$

in which all quantities are defined at $\xi=\xi_i$.

Equation 23 is a kind of logistic equation. Based on the mathematical properties of Eq.23, ξ_c as a threshold is determined as

$$\xi_c = \frac{-b+\sqrt{b^2-4ac}}{2c} \tag{25}$$

The particle deviates from the presumed path rapidly if $\xi_i>\xi_c$; while, it converges to the gentle behavior to return the presumed path if $\xi_i<\xi_c$ (see Fig.5). The former behavior must be the transition from bed-load motion to suspension.

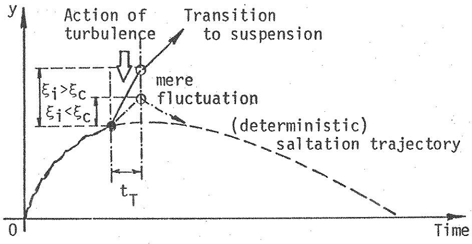

Fig.5 Schematic figure of transition from saltation to suspension and mere fluctuation of saltation trajectory

A logistic equation has the threshold and the solution behaves differently depending on whether the initial value exceeds it or not. The behavior of saltation particle is represented by a logistic equation, and the property that the transition from bed-load motion to suspension takes place if the shift of the trajectory exceeds some criterion well corresponds to the mathematical property of a logistic equation.

The governing equation of v_g is as follows:

$$\rho\left(\frac{\sigma}{\rho}+C_M\right)A_3d^3\frac{dv_g}{dt} = -\frac{1}{2}C_D\rho\,|v_g|v_gA_2d^2 - \rho g\left(\frac{\sigma}{\rho}-1\right)A_3d^3 \tag{26}$$

in which ρ=mass density of fluid; σ=mass density of sediment particle; C_M=added mass coefficient; C_D=drag coefficient; the lift force and the Magnus effect are neglected; and it is assumed that the particle speed in the longitudinal direction becomes equal to the flow velocity immediately. The following Rubey's equation is used to evaluate C_D.

$$C_D = \frac{24\nu}{v_gd} + 2.0 \tag{27}$$

in which ν=kinematic viscosity. By calculating a, b and c based on Eq.26, Eq.25 can determine the threshold ξ_c as a criterion of the transition from bed-load motion to suspension.

TRANSITION PROBABILITY FROM BED-LOAD MOTION TO SUSPENSION

When the initial deviation of the particle's path from the presumed trajectory (ξ_i) is given by the vertical component of turbulence v' acting during the time interval t_T,

$$\xi_i = k\alpha v' t_T \tag{28}$$

in which α=coefficient with respect to the response of particle's motion to turbulence; and k=correction factor due to averaging during the transition process. The relative speed of the particle in the vertical direction after deviating ξ_i is written as

$$v_{gi} = \beta_1 v' \tag{29}$$

in which β_1 is another coefficient with respect to the response of particle's motion to turbulence. v' is the vertical component of the turbulence at the height from which the deviation starts, and its standard deviation (v'_{rms}=turbulence intensity in the vertical direction) varies along the flow depth as follows (Nezu (10)):

$$\frac{v'_{rms}}{u_*} = 1.27\exp\left(-\frac{y}{h}\right) \tag{30}$$

The transition probability density per unit time from saltation to suspension might be expressed as follows:

$$p_T = \int_{v_c'}^{\infty} \frac{k\alpha v'}{\xi_c} \phi_v(v')dv' \tag{31}$$

in which $\phi_v(v')$=probability density function of the turbulence intensity in the vertical direction; and v_c'=the value of v' such that the displacement of the height of the particle during t_T is ξ_c ($v_c'=\xi_c/(k\alpha t_T)$). In other words, p_T is regarded as the expected value of the reciprocal of the time required for a particle to deviate the critical value from the presumed trajectory.

In calculation of p_T, one has to evaluate α and β_1 reasonably. For this purpose, the following governing equation for the particle motion in turbulent is used.

$$\rho\left(\frac{\sigma}{\rho}+C_M\right)A_3 d^3 \frac{dv_p}{dt} = -\frac{1}{2}C_D\rho\,|v'-v_p|\cdot(v'-v_p)A_2 d^2 - \rho g\left(\frac{\sigma}{\rho}-1\right)A_3 d^3 + \rho(1+C_M)A_3 d^3 \frac{dv'}{dt} \tag{32}$$

in which v_p=particle's speed in the vertical direction. An immediate response of particle's speed to the flow velocity in the longitudinal direction is for simplicity assumed, and thus the one-dimensional equation of motion is used here. Moreover, the Basset term has been neglected in Eq.32.

A unit of turbulence action (called "a unit turbulence" in the following) is idealized by a half period of sinusoidal variation of v' (amplitude=$\sqrt{2}v'_{rms}$; period=T), and Eq.32 is solved under the initial condition that v_p=0 at t=0. Then, α is defined as the ratio of the maximum displacement of particle's height by a unit turbulence (A_{pmax}) to the displacement of the fluid lump initially near the particle (A_w), while β as the ratio of the maximum vertical speed which the particle reaches when a unit turbulence acts (v_{pmax}) to the maximum vertical fluid velocity ($\sqrt{2}v'_{rms}$). A_w is given as

$$A_w = \frac{2v'_{rms}T}{\pi} \tag{33}$$

NUMERICAL RESULTS AND DISCUSSION

According to the procedure explained in this paper, the criterion and the probability density per unit time of the transition from bed-load motion to suspension are calculated.

Firstly, the maxima of the displacement and the speed of the particle in the vertical direction due to a unit turbulence are calculated, and the results are shown in Fig.6. When the dimensionless bed shear stress ($\tau_* \equiv u_*^2/[(\sigma/\rho-1)gd]$) is given, the average existing height of saltating particles are calculated (Tsujimoto & Nakagawa (14)), and the representative turbulence intensity in the "bed-load layer" is estimated by using Eq.30. Then, the calculated results are expressed as a function of the period of the velocity fluctuation (T) with τ_* as a parameter. T is identified with t_T.

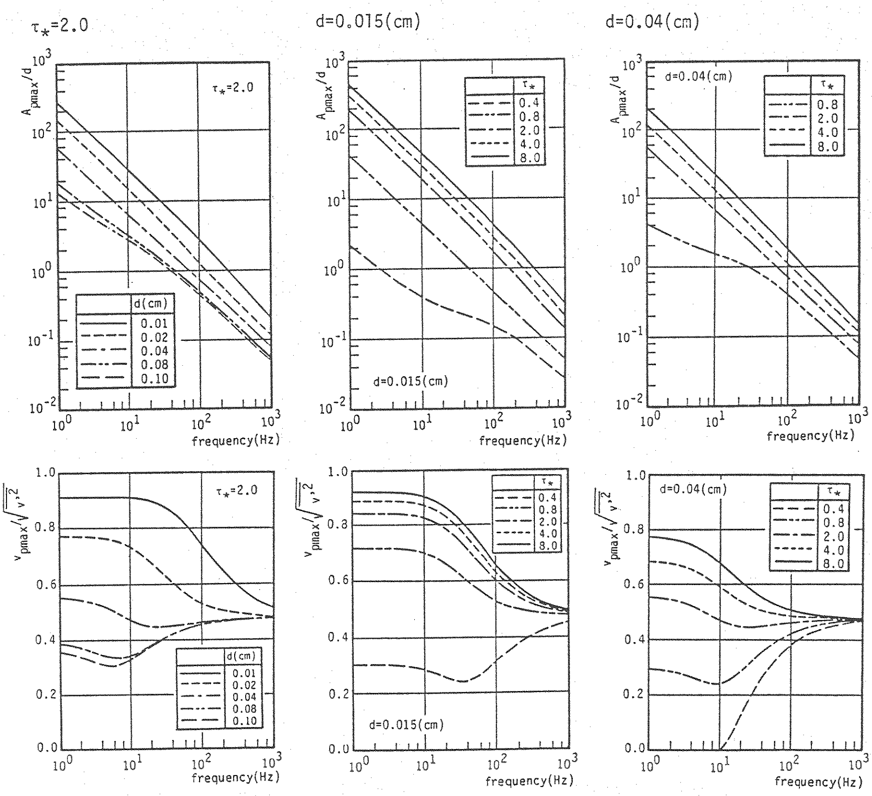

Fig.6 Maximum displacement and responding speed of a particle for simulated turbulence

Then, α and β_1 can be calculated. When β_1 is known, v_{gi} and ξ_c are evaluated successively. It might be reasonable to determine t_T such that the maximum vertical displacement of the particle's position is as large as ξ_c. Hence, the following equation is reasonably assumed based on the calculated result and a consideration of the similarity criterion:

$$\Pi_t \equiv t_T\sqrt{\left(\frac{\sigma}{\rho}-1\right)\frac{g}{d}} = 1.64 \tag{34}$$

Moreover, the calculated result suggests that k=0.3. For natural sand of σ/ρ=2.65 and d=0.015cm, Eq.34 reveals that t_T=0.005s. Although it looks very short period, it corresponds, in fact, to the sharp phenomenon of the transition as observed through the film analysis. Tsujimoto & Nakagawa (15) conducted an experiment by using polystyrene particles, and found that $t_T \cong 0.067$s. Then $\Pi_t \cong 1.0$ and it is almost consistent with Eq.34.

Now that the time scale for the initial displacement, α, β_1, ξ_c and p_T can be calculated along the presumed saltation trajectory. Fig.7 shows examples of the calculated results of them. In the figures, ξ_c and p_T are expressed in dimensionless versions as follows:

$$\xi_{c*} \equiv \frac{\xi_c}{d} \quad ; \quad p_{T*} \equiv p_T\sqrt{\frac{d}{(\sigma/\rho-1)g}} \tag{35}$$

Any of α, β_1, ξ_{c*} and p_{T*} varies gradually along the saltation path except the initial stage where a particle moves up rapidly, and particularly in the descending stage it shows almost constant. Furthermore, it is recognized that the variation of any of α, β_1, ξ_{c*} and p_{T*} along the saltation path is almost similar for different value of τ_*.

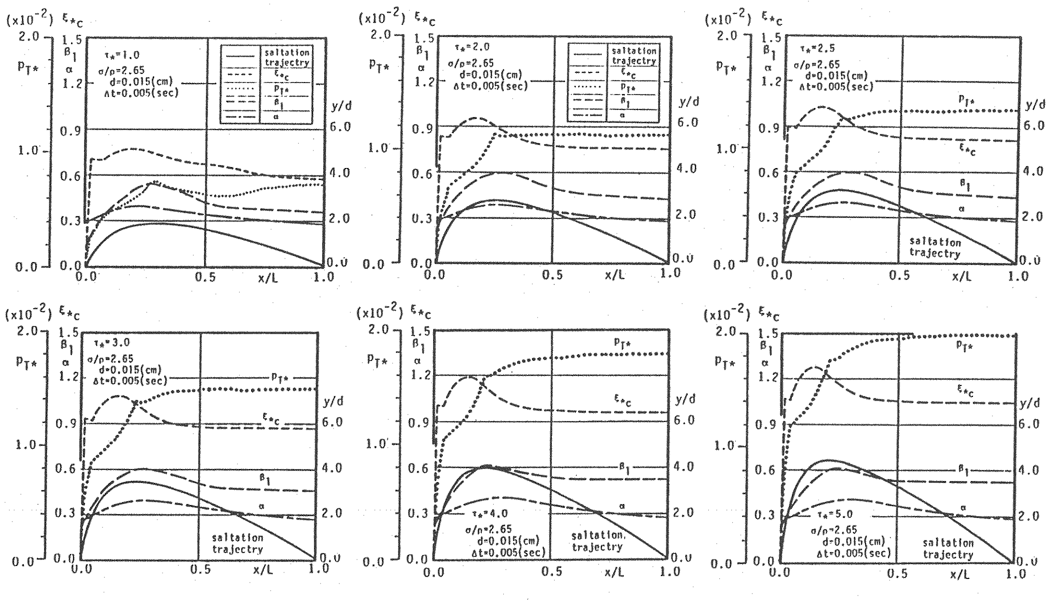

Fig.7 Calculated results of parameters on response of particle's motion, α and β_1; threshold value of ξ; and probability density per unit time of transition p_T; along one salttaion period

Figure 8 shows the relation between p_{T*} and the height of the particle along a saltation path, and it draws a hysteresis loop. The dashed line is the averaged relation of p_{T*} and y/d by taking account of the duration times of rising and the descending stage of saltation, and the chained line shows the average through a saltation path. If the rising and descending stages were not distinguished from each other, the transition probability from bed-load motion to suspension would hardly change against the height from the bed, and the probability averaged through the saltation could be applied. Nevertheless, the production term of suspended load, S_S, increases with approaching to the bed because of the distribution of saltation size.

Furthermore, the diameter of sand was changed and the relation between the averaged probability of transition from bed-load motion to suspension and the bed shear stress was investigated. The results

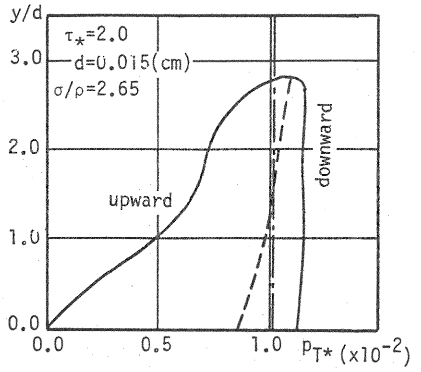

Fig.8 Variation of transition probability with the vertical distance from the bed

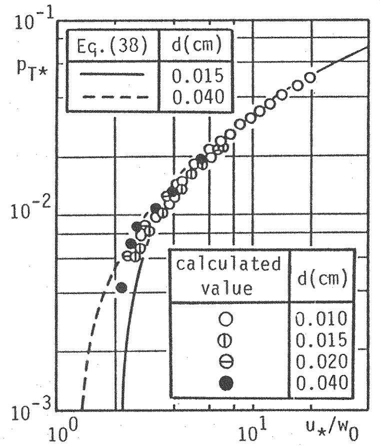

Fig.9 Relation between transition probability density and dimensionless shear velocity (u_*/w_0)

were plotted as the relation between p_{T*} and (u_*/w_0) in Fig.9. As the best fit curve, the following equation was proposed:

$$p_{T*} \equiv p_T\sqrt{\frac{d}{(\sigma/\rho-1)g}} = F_{0T}\left(\frac{u_*}{w_0}\right)^n\left[1-\frac{(u_*/w_0)_c}{(u_*/w_0)}\right]^m \tag{36}$$

The constants in Eq.36 were determined as follows: F_{0T}=0.0175; n=0.4; m=1.10. $(u_*/w_0)_c$ is the value of (u_*/w_0) corresponding to p_{T*}=0 (critical value for suspension), and it decreases against the sediment diameter (or $d_* \equiv (\sigma/\rho-1)gd^3/\nu^2$) as a dimensionless parameter) as shown in Fig.10.

Although we are able to argue some problems more important from engineering viewpoint such as the reference concentration of suspended sediment based on the results of the present study, several problems such as where is a reasonable reference level still remain and more accurate evaluation of the excursion length is needed.

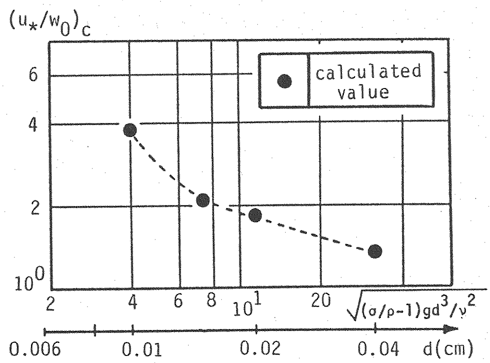

Fig.10 Critical value of (w_0/u_*) for transition

CONCLUSIONS

In this paper, the transition from bed-load motion to suspension has been investigated. The reverse process, the transition from suspension to bed-load motion, is already described by the model of sediment suspension which determines the excursion length of a suspended particle. The results obtained in this paper are summarized below:

(1) A mathematical description of bed-material load transport has been established. The model emphasizes the importance of the study on the transition mechanism from bed-load motion to suspension.

(2) The effect of the transition phenomenon on the sediment suspension is evaluated by proposing a non-homogeneous diffusion equation which includes the production term to represent the transition from bed load to suspension. When the ratio of the suspended sediment to the total bed-material load is large, the non-homogeneous effect dominates and the bottom concentration of suspended sediment is subjected to the transition from bed load to suspension. While, when the bed load dominates in bed-material load transport, the concentration profile is well approximated by the solution of the homogeneous diffusion equation but the bottom concentration of suspended sediment cannot be determined without considering the excursion length of suspended particles.

(3) The transition from bed-load motion to suspension appears as an extraordinary shift of the saltation trajectory due to turbulence. This phenomenon has been represented by a logistic equation deduced from the equation of the particle's motion. The threshold exists and it distinguishes unstable trajectory from stable one. The instability of saltation trajectory just represents the transition from bed-load motion to suspension. Thus, the criterion of the transition has been analyzed as the threshold of the logistic equation.

(4) The time scale governing the transition from bed-load motion to suspension has been determined by investigating the response properties of particle's motion to turbulence, and then the probability density per unit time that a bed-load particle turns into suspension (the transition probability) has been deduced. Based on the present model, the relation between the transition probability and the shear velocity has been obtained. An empirical formula to approximate this relation has been proposed.

The results obtained in this study might be available for describing various type of alluvial processes brought about by bed-material load transport, but several problems remains to be investigated more reasonably for the applications, though they are able to be treated roughly from engineering viewpoint.

Most of the contents of this paper was already published in Japanese (9).

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APPENDIX - NOTATION

The following symbols are used in this paper:

A_2, A_3 = two- and three-dimensional geometrical coefficients of sand;

A_{pmax} = maximum displacement of particle's height by a unit turbulence;

A_w =displacement of the fluid lump initially near the particle;

C_1, C_2 = integral constants of non-homogeneous diffusion equation;

C_B = bed-load concentration;

C_D, C_M = drag coefficient and added mass coefficient;

C_S = suspended sediment concentration;

d = diameter of sand;

d_* $\equiv (\sigma/\rho-1)gd^3/\nu^2$;

F_{T0} = empirical constant of transition-probability formula;

$F_{TR}(x)$ = probability for a bed-load particle does not turn into suspension within x;

$f_S(y)$	= probability density function of the existence height of suspended particle;
$f_X(x)$	= probability density function of step length of bed-load particle;
$f_{XS}(x)$	= probability density function of excursion length of suspended particle;
g	= gravitational acceleration;
h	= flow depth;
I_1, I_2	= integrations defined by Eq.18;
k	= correction factor due to averaging during the transition process;
m, n	= empirical constants of transition-probability formula;
$p_{ds}(x)$	= number of suspended particles to deposit on the bed of the area occupied by one particle at x to turn into bed-load motion;
p_s	= pick-up rate of bed-material particle;
p_T, p_{T*}	= probability density of transition per unit time from bed-load motion to suspension and its dimensionless form $(\equiv p_T\sqrt{d/(\sigma/\rho-1)g})$;
q_B, q_{B*}	= bed-load transport rate and its dimensionless form $(\equiv q_B/\sqrt{(\sigma/\rho-1)gd^3})$;
q_S, q_{S*}	= suspended load transport rate and its dimensionless form $(\equiv q_S/\sqrt{(\sigma/\rho-1)gd^3})$;
S_S	= substantial volume of suspended sediment produced by the transition from bed-load motion per unit volume of water per unit time;
T	= period of a model turbulence;
t	= time;
t_T	= time scale of transition phenomenon from bed-load motion to suspension;
u	= flow velocity;
u_g	= bed-load particle's speed in the longitudinal direction;
u_*	= shear velocity;
$(u_*/w_0)_c$	= the value of (u_*/w_0) corresponding to $p_{T*}=0$ (critical value for suspension);
v', v'_{rms}	= turbulence in the vertical direction and its root-mean square (turbulence intensity);
v_c'	=the value of v' such that the deviation of the particle's height during t_T is ξ_c $(v_c'=\xi_c/(k\alpha t_T))$
v_g	= vertical speed of a particle in saltation $(d\xi/dt)$;
v_p, v_{pmax}	= particle's speed responding to turbulence in the vertical direction and its maximum value;
w_0	= terminal velocity of sand;
x, y	= longitudinal and vertical coordinates;
α, β_1	= parameters with respect to particle's motion responding to turbulence;
γ	= index of the magnitude of homogeneity of sediment suspension $(\gamma\equiv C_1/(C_1+C_2))$;
ε_s, ε_{s*}	= turbulent diffusion coefficient of suspended sediment and its dimensionless form $(\varepsilon_{s*}\equiv\varepsilon_s/(u_*h))$;
η	= relative flow depth $(\equiv y/h)$;
κ	= Kármán constant;
Λ	= mean step length of bed-load motion;

Λ_{BS}	=defined by Eq.6;
Λ_S	= mean excursion length of suspended particles;
ν	= kinematic viscosity;
ξ, ξ_*	= deviation of the particle's height from presumed saltation trajectory and its dimensional expression ($\xi_* \equiv \xi/d$);
ξ_c	= threshold value of ξ by which the transition from saltation to suspension is distinguished from mere fluctuation of saltation trajectory;
ξ_i	= initial deviation of the particle's height from presumed saltation trajectory ;
Π_T	= dimensionless form of t_T ($\Pi_T \equiv t_T \sqrt{(\sigma/\rho-1)g/d}$);
ρ	= mass density of fluid;
σ	= mass density of sand;
τ_*	= dimensionless bed shear stress ($\equiv u_*^2/[(\sigma/\rho-1)gd]$);
$\phi_1(\eta)$, $\phi_2(\eta)$	= relative concentration profiles of suspended sediment as solutions of homogeneous and non-homogeneous diffusion equations, respectively; *and*
Ψ	$\equiv S_S(\eta h)/S_S(0)$.

Subscripts

e	= equilibrium states; *and*
i	= initial deviation from the presumed saltation trajectory.

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