

SHORT-TERM FORECASTING FOR WATER LEVEL OF A FLOOD BY PRECIPITATION RADAR

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SYNOPSIS

A weather radar provides detailed real-time information over a wide range of areas with high time-space resolution. But the accuracy of the radar raingauge is not always high enough to use for flood forecasting.

In this paper, a forecasting method by using radar data is proposed, in which a parameter for the Z-R relationship of the radar raingauge is included in the system parameters and identified together by Kalman filtering method. The application of this method to the flood data in the Onga river basin shows that the forecasting of water satges by using radar data is possible with the same accuracy as that by rainfall data on the ground.

INTRODUCTION

Recently, rainfall observation systems using radar and telemeters are being fully utilized along the river basins throughout Japan. Flood forecasts using on-line information from these systems will provide effective ways to lessen or prevent damage due to floods. Several studies on real-time forecasting have been performed((1), (2), (3) and (4)), however, the further developent is required for more reliable forecasting.

The longer lead time in forecasts allows more time for preparing againstst a forthcoming flood, however, it will require rainfall predictions which are not always accurate enough to use for flood forecasting. In this situation, the short-term forecast without rainfall prediction may also be useful in making a more accurate forecast.

We propose a dynamic model for prediction of flood stages. In the model, the equations of continuity and momentum for unsteady flow are applied to a river flow and solved by the kinematic wave theory. The unit hydrograph is introduced for estimation of the lateral inflow from residual basin. From the viewpoint that the stage data are as much importance as rainfall data, a system model for the short-term forecasting of the flood stages is derived. By feeding on-line data regarding the rainfall and water levels into the model, the operating forecast of flood stages at a given site is possible. The Kalman filtering is used to identified the system parameters which are the unit hydrographs, the constants for kinematic wave theory and a parameter for the Z-R relationship of the radar raingauge.

The applicability of this system is examined using flood data, in the Onga River of Kyushu District.

DEVELOPMENT OF FORECASTING MODEL

The equations of continuity and momentum for unsteady flow in an open channel are

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_* \quad (1)$$

$$Q = \alpha A^\mu \quad (2)$$

where A = cross sectional area; Q = flow discharge, t = time; x = downstream distance; q_* = lateral inflow rate; $\alpha = a\sqrt{T_e}/n$; n = Manning's constant; a and μ = constants defined by $A \cdot R^{\frac{2}{3}} = \alpha \cdot A^\mu$; R = hydraulic radius; and I_e = energy slope.

Eqs. (1) and (2) are solved by the kinematic wave theory as

$$A_2(t) - A_1(t - \tau_{2,1}) = \int_{t-\tau_{2,1}}^t q_*(t) dt - \int_{t-\tau_{2,1}}^t A^P \frac{\partial K}{\partial x} dt \quad (3)$$

where τ = time that the flood wave travels from the upstream site to the downstream site; subscripts 1 and 2 denote the upstream and the downstream sites; P = constant defined by $P = 1/\mu$; and $K = I/\alpha$.

The second term in the right hand side of Eq. (3) can be approximated as

$$\int_{t-\tau_{2,1}}^t A^P \frac{\partial K}{\partial x} dt = \int_0^{L_{21}} \frac{A}{KP} \frac{\partial K}{\partial x} dx \cong \frac{L_{21}}{2} \left\{ \frac{A_2}{K_2 P_2} \left(\frac{\partial K}{\partial x} \right)_2 + \frac{A_1}{K_1 P_1} \left(\frac{\partial K}{\partial x} \right)_1 \right\} \quad (4)$$

$$= C_1 A_1(t - \tau_{2,1}) + C_2 A_2(t)$$

where L_{21} = length between the upstream and the downstream sites; and $C_i = L_i / (2 K_i P_i (\partial K / \partial x)_i)$.

In the same way as mentioned above, the cross sectional area at the moment $t+I$ can be given by

$$A_2(t+I) - A_1(t+I - \tau_{2,1}) = \int_{t+I-\tau_{2,1}}^{t+I} q_* dt - C_1 A_1(t+I - \tau_{2,1}) - C_2 A_2(t+I) \quad (5)$$

Subtracting Eq. (3) from Eq. (5), one obtains

$$A_2(t+I) - A_2(t) = k \{ A_1(t+I - \tau_{2,1}) - A_1(t - \tau_{2,1}) \} + m \left\{ \int_{t+I-\tau_{2,1}}^{t+I} q_* dt - \int_{t-\tau_{2,1}}^t q_* dt \right\} \quad (6)$$

where $k = (1 - C_1)/(1 + C_2)$ and $m = 1/(1 + C_2)$.

Eq. (6) is the fundamental computing equation of the cross sectional area at the forecasting time I . The cross-sectional areas can easily be transformed into the water stages.

According to the theory of intermediate value, the lateral inflow rate is given by

$$\int_{t-\tau_{2,1}}^t q_* dt = \tau_{2,1} q_*(t - \tau_s) \quad (7)$$

where τ_s = time which lies between 0 and $\tau_{2,1}$.

The lateral inflow can be expressed by

$$q_*(t - \tau_s) = \int_0^{t-\tau_s} l u(t) f r(t - \tau_s - \tau) dt \quad (8)$$

where l = length of the slope; f = runoff coefficient; $u(t)$ = instantaneous unit hydrograph; and r = rainfall intensity.

Substitution of Eq. (8) into Eq. (6) yields

$$\begin{aligned}
A_2(t+I)-A_2(t) &= k\{A_1(t+I-\tau_{2,1})-A_1(t-\tau_{2,1})\} \\
&+ \int_0^{t-\tau_2} U(\tau)r(t+I-\tau_s-\tau)d\tau - \int_0^{t-\tau_2} U(\tau)r(t-\tau_s-\tau)d\tau \\
&= k\{A_1(t+I-\tau_{2,1})-A_1(t-\tau_{2,1})\} + \sum_0^{t-\tau_2} U(i)r(t+I-\tau_s-i) - \sum_0^{t-\tau_2} U(i)r(t-\tau_s-i) \quad (9)
\end{aligned}$$

where $U(i) = \frac{fI}{C_2} u(i)\Delta t$; and Δt = discrete time. The system parameters to be identified are k and $U(i)$.

If a river has N tributaries and each tributary has a water level station, then we can obtain

$$\begin{aligned}
A_{N+1}(t+I)-A_{N+1}(t) &= \sum_{i=1}^N [k_i\{A_i(t+I-\tau_i)-A_i(t-\tau_i)\}] \\
&+ \sum_{j=1}^{N+1} \left[\sum_{i=0}^{t-\tau_j} U_j(i)r_j(t+I-\tau_s-i) - \sum_{i=0}^{t-\tau_j} U_j(i)r_j(t-\tau_s-i) \right] \quad (10)
\end{aligned}$$

Next, let us consider the case where data from the precipitation radar are available. By applying equation $Z=Br^\beta$ which expresses the relation between the reflectivity factor Z (mm⁶/m³) and rainfall intensity r (mm/hour), the second term in the right hand side of Eq.(9) can be expressed by

$$\sum_0^{t-\tau_2} V(i)z(t+I-\tau_s-i) - \sum_0^{t-\tau_2} V(i)z(t-\tau_s-i) \quad (11)$$

where $V(i) = m\beta f \left[\frac{B_0}{B} \right]^{\frac{1}{\beta}} u(i)\Delta t$; β and B_0 = standard values which are used to estimate the rainfall on a precipitation radar; and z = reflectivity factor observed by the precipitation radar.

As the same procedure mentioned above, we obtain the following equation for a river with N tributaries.

$$\begin{aligned}
A_{N+1}(t+I)-A_{N+1}(t) &= \sum_{i=1}^N [k_i\{A_i(t+I-\tau_i)-A_i(t-\tau_i)\}] \\
&+ \sum_{j=1}^{N+1} \left[\sum_{i=0}^{t-\tau_j} V_j(i)z_j(t+I-\tau_s-i) - \sum_{i=0}^{t-\tau_j} V_j(i)z_j(t-\tau_s-i) \right] \quad (12)
\end{aligned}$$

The system parameters to be identified are k and $V(i)$.

APPLICATION

Study basin

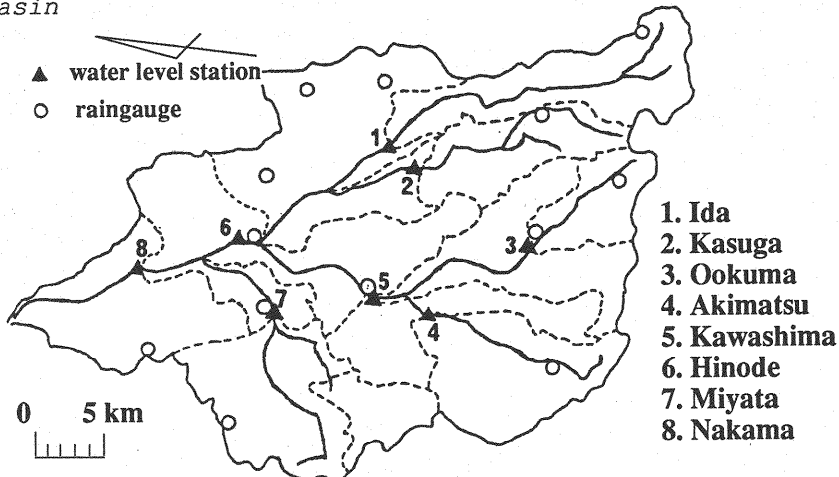


Fig.1 Onga River Catchment

The model is applied to flood data in the Onga river whose basin is illustrated in Fig.1. There are 8 water level stations and 13 raingauges on the ground. The total area of this basin is 926km².

Data

The rainfall data used in this study are from the raingauges on the ground and a precipitation radar. Both kinds of data are observed by the Ministry of Construction. This radar site locates on Mt.Shakadake in Kyushu island and covers the area in the northern part of Kyushu island as shown in Fig.2. The whole catchment area of the Onga river is covered by this radar. The data observed by this radar are recorded on magnetic tapes every 5 minutes, and spatial resolution is 3km for the radius direction and 2.8125(360/128) degrees for the horizontal angle. The rainfall data are accumulated for an hour on each catchment area in the Onga river.

Figure3 shows an example of average rainfall intensity data observed by radar and rain gauge on the ground in Akimatu basin. The standard values of parameters in Z-R relationship are taken as $\beta=1.58$ and $B_0=224.4$ which are used by the Ministry of Construction Japan for this precipitation radar. As shown in Fig.3, however, these values of rainfall intensity are under-estimated comparing with the rainfall intensity from the rain gauge on the ground.

Water level is recorded every 1 hour at 8 stations.

Identification of System Parameters

To identify the system parameters, the Kalman filtering method is used as follows:

The state equation and measurement function are written as

$$X(j+1)=X(j)+w(j) \quad (13)$$

and

$$Y(j+1)=M(j+1)X(j+1)+v(j) \quad (14)$$

where X = state vector; j = discrete time whose interval is unit; w = system noise; Y = measurement vector; M = measurement matrix; and v = measurement noise.

For example, since Eq.(9) corresponds to the measurement function, the terms of Eqs.(13) and (14) are expressed as

$$\begin{aligned} Y(j+1) &= A_2(t+1) - A_2(t) \\ M(j+1) &= [A(t+1) - A(t), R(1), R(2), \dots] \\ X(j+1) &= [k, U(0), U(1), \dots]^T \end{aligned} \quad (15)$$

For the measurement noise $v=100$ is used and covariance matrix P is given as

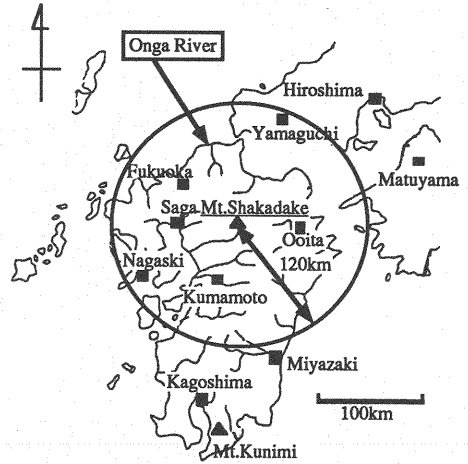


Fig.2 The covered area of radar

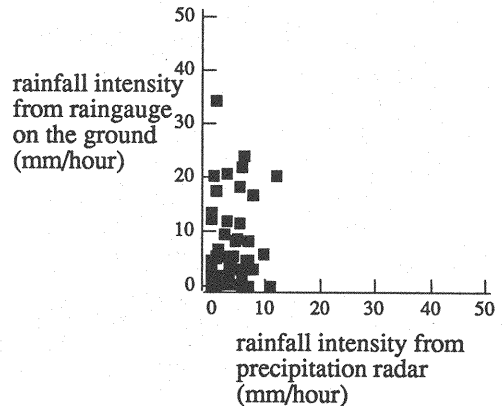


Fig.3 Rainfall intensity of ground rain gauge VS Radar

$$P = \begin{bmatrix} P_1 & P_0 & P_0 \\ P_0 & \ddots & \ddots \\ P_0 & \ddots & P_1 \end{bmatrix}$$

where $P_i = 0.1$; and $P_0 = 0.01$.

Application of Equations

The water level at Nakama station is predicted by the procedure as follows:

- 1) The water level is translated to the cross-sectional area.
- 2) The cross-sectional area at Kawashima station is computed by Eq.(10) or (12) using cross-sectional area at upper stations which are Chikuma and Akimatsu stations.
- 3) The cross-sectional area at Hinode station is computed by Eq.(10) or (12) using cross-sectional area at upper stations, Kawashima, Ida and Kasuga stations.
- 4) The cross-sectional area at Nakama station is computed by Eq.(10) or (12) using cross-sectional area at upper stations, Hinode and Miyata stations.
- 5) The computed value of cross-sectional area is converted into water level.

To examine the accuracy of the forecasted water level by using the data from the precipitation radar, it is compared with the water level computed by using the data from the raingauge on the ground. The data observed in August 1980 are applied to this model. In Figs.4 and 5, the predicted values of water stage with lead time of 3 hours are compared with the observed ones at Nakama station. In these computations, the

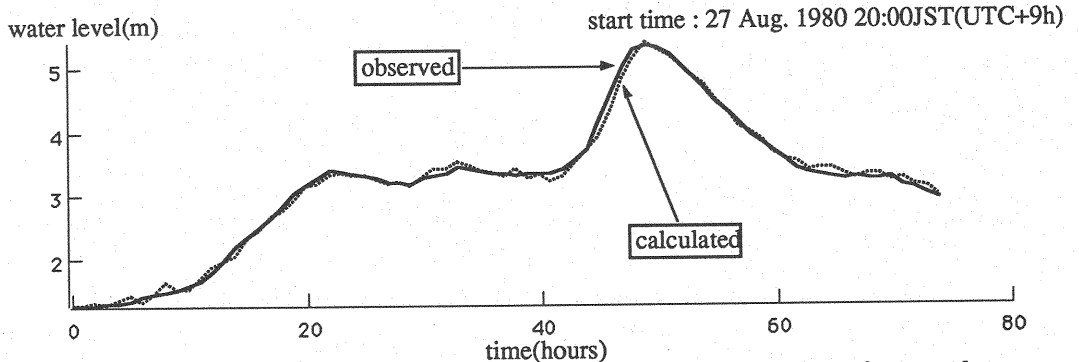


Fig.4 Three-hour prediction by using precipitation radar (Nakama station)

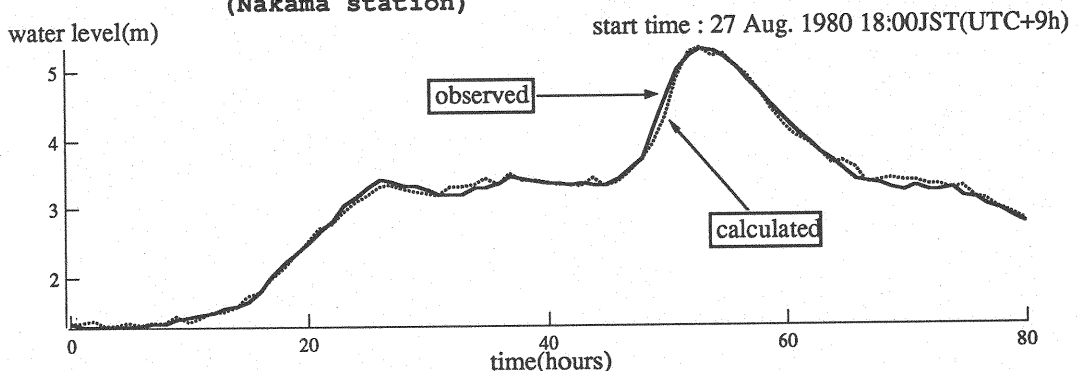


Fig.5 Three-hour prediction by using raingauge on the ground (Nakama station)

values of lag times of rainfall for each basin were determined so as to avoid rainfall predictions. Though the radar gives the smaller values of rainfall than the ground raingauges, the forecasting values by using radar data gives the same accuracy with the ones by ground rainfall data. Fig.6 shows the estimated unit hydrograph at Ida basin for the flood at Aug. 1980. The estimated unit hydrograph using radar data is about 2 times as large as one using raingauge data. This means that the identification by the Kalman filter supplements the underestimating of rainfall intensity using radar data.

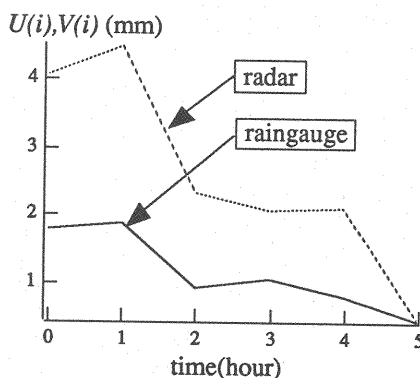


Fig.6 estimated $U(i)$ and $V(i)$

CONCLUSION

A dynamic model for real time forecasting of the water stages has been developed from the basic equations of unsteady flow. The inputs into the model are the water stages of the upper stages and data from the raingauges on the ground or precipitation radar. The Kalman filtering technique is used to identify the system parameters of the model.

The model was applied to Onga River by using the data from the raingauges on the ground as well as from the precipitation radar. In both cases, the forecasting water levels with a lead time of 3 hours showed close agreement with the observed ones, even though there is significant difference between the both rainfall data.

ACKNOWLEDGEMENTS

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APPENDIX - NOTATION

The following symbols are used in this paper:

- a = constant defined by $a R_1^2 = \alpha \cdot A^4$
- A = cross sectional area;
- B = radar constant;

- B_o = standard value of radar constant;
 $C_i = L_i/2K_i P_i (\partial k/\partial x)_i$;
 f = runoff coefficient;
 I = forecasting time (lead time);
 I_e = energy slope;
 j = discrete time whose interval is unit;
 k = constant defined by $k = (1-C_1)/(1+C_2)$;
 $K = 1/\alpha$;
 l = length of the slope;
 L_{21} = length between the upstream and the downstream sites;
 M = measurement matrix;
 n = Manning's constant;
 N = number of tributaries;
 P = constant defined by $P=1/\mu$;
 q_e = lateral inflow;
 Q = flow discharge;
 r = rainfall intensity;
 R = hydraulic radius;
 t = time;
 $u(t)$ = instantaneous unit hydrograph;
 $U(i)$ = function defined by $U(i) = \frac{\int_0^i u(i) \Delta t}{1+C_2}$;
 $V(i)$ = function defined by $V(i) = m \int_0^i [B_o/B]^\beta u(i) \Delta t$;
 w = system noise;
 x = downstream distance;
 X = state vector;
 Y = measurement vector;
 Z = reflectivity factor (mm^6/m^3)
 α = value defined by $\alpha = \alpha \sqrt{T_e}/n$;
 β = radar constant;
 Δt = discrete time
 μ = constant decided by cross section i.e. defined by $\alpha R^3 = \alpha \cdot A^\mu$;
 v = measurement noise;
 τ = time that the flood wave travels from the upstream site to the downstream site; the subscripts 1 and 2 denote the upstream and the downstream sites; and
 τ_i = the lag time which lies between 0 and $\tau_{2,1}$.

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