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EFFECT OF SURFACTANTS ON TWO-DIMENSIONAL LAMINER SURFACE FLOW

by

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SYNOPSIS

A mathematical model which describes surface hydrodynamics of surfactant solution is presented. In this model, equation of adsorption kinetics and relationship between surface tension and surfactant concentration are taken into consideration, in addition to ordinary basic equations such as conservation of mass and momentum for bulk fluid. In our theoretical model, the surface tension gradient resulting from surface concentration gradient acts as a boundary condition of fluid motion.

By making use of the boundary layer approximation and Langmuir-type adsorption equation, we investigated two simple and typical flow field, surface deceleration of laminar surface jet and surface acceleration induced by out flow over a weir at the downstream in the open-channel.

Taking SDBS as an example of surfactant, the effect of some factors such as surfactant concentration, flow rate of the surfactant solution and the value of the adsorption rate constant, on surface deceleration and acceleration are discussed, according to the numerical solution.

The validity of the numerical model is checked by laboratory experiments.

INTRODUCTION

This paper describes the surface hydrodynamics of surfactant solution. When the water contains surfactant its motion becomes more complicated one. In the contaminated water the local difference of surface tension acts as shear force at the water surface. This local difference of surface tension is given by the local difference of surface concentration of surfactant, which is affected by characteristic of the bulk flow.

This kind of phenomena is well known(17) as it has relations to many engineering problems, for example, bubble motion in water, foam separation, turbulence decay at the water surface, thin film stability, mass transfer at the gas-liquid interface, flow around rotating disk and so on.

To the mathematical model which describes these phenomena, however, there are not so many works, because the phenomena is so complicated. Cerro and Whitaker(4) and Whitaker(20) reported on the liquid film motion and Andrews et al.(1) studied the bubble motion in a liquid column. Imaishi et al (10,11) also investigated on the surface acceleration in an open channel flow caused by flow discharge over a weir at the downstream. Although these works explained well the phenomena they treated, they employed several assumptions and approximations to simplify the analysis. Thus, to the best of writers' knowledge, it is hard to find the mathematical model which allows universal description on the surface hydrodynamics of the surfactant solution.

The purpose of this paper is to investigate a mathematical model of these phenomena. We treated water surface deceleration and acceleration in a two-dimensional laminar steady flow, as a simple and typical case. In our mathematical model, adsorption kinetics and relationship between surface tension and surfactant concentration are considered in addition to usual basic equation, such as equation of motion for fluid, mass conservation of both fluid and surfactant.

THEORETICAL ANALYSIS

Basic Equations

The two simple phenomenon that we wish to analyze are illustrated in Fig.1. One is the surface flow retardation along the two dimensional surface jet issued into calm water(hereafter we write this phenomena as surface deceleration). The other is surface acceleration induced by out flow over the weir which is set at the downstream end of an open channel (hereafter we write this phenomena as surface acceleration).

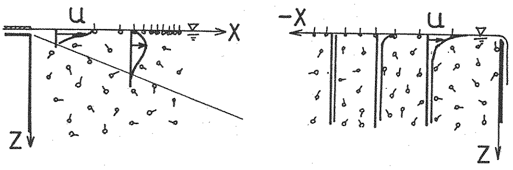

Fig.1 Surface acceleration and deceleration

By the usual boundary layer approximation, we assume that the governing equations can be written as

$$u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = \nu\frac{\partial^2 u}{\partial z^2} \qquad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \qquad (2)$$

$$u\frac{\partial c}{\partial x} + w\frac{\partial c}{\partial z} = D\frac{\partial^2 c}{\partial z^2} \qquad (3)$$

$$-\frac{d\sigma}{dx} = \mu\frac{\partial u}{\partial z}\Big|_{z=0} = \tau \qquad (4)$$

$$\sigma = \sigma_0 - K \ln \{ \Gamma^\infty/(\Gamma^\infty - \Gamma) \} \qquad (5)$$

$$\frac{d}{dx}(u_s\Gamma) = D\frac{\partial c}{\partial z}\Big|_{z=0} \qquad (6)$$

$$D\frac{\partial c}{\partial z}\Big|_{z=0} = K \{ c_s(\Gamma^\infty - \Gamma) - a\Gamma \} \qquad (7)$$

in which u=flow velocity in the x direction, w=flow velocity in the z direction, $\nu=\mu/\rho$ kinematic viscosity, μ= viscosity(0.001 Pa·s in the present work), ρ=density of surfactant solution(1000Kg/m^3), c=volume concentration of surfactant, D=diffusion coefficient, τ=surface stress of bulk fluid, σ_0=surface tension of clear water(74.7 mN/m), σ=surface tension of surfactant solution, Γ=concentration of surfactant at water surface(surface excess) and K,a,k_a are numerical constants. Subscript s means the value at the water surface. The advantage of using the boundary layer approximation is reduction of computational difficulties(5).

The boundary layer approximation is used on the basis of assuming

$$u \gg w \qquad ; \qquad \frac{\partial^2 u}{\partial x^2} \ll \frac{\partial^2 u}{\partial z^2}$$

and this assumpitions are valid for the most part of flow. As the surface accelerartion or deceleration increase, however, the flow reaches to the region where above assumpitons can't be employed. In those region our analysis is not satisfactory.

Eqs.1 and 2 are equations of conservation of momentum and mass for bulk fluid, respectively. Mass conservation for surfactant in bulk fluid is given by Eq.3 and mass conservation at the water surface is given in Eq.6. Eq.4 means momentum balance at the water surface, which corresponds to a boundary condition of equation of motion. The Szyszkowski relationship between surface tension and surface concentration is assumed (Eq.5).

$$\begin{tikzcd}
\text{Bulk Fluid} & & \text{Surface Fluid} \\
& \text{Momentum Balance} & \\
u\frac{\partial u}{\partial X} + w\frac{\partial u}{\partial Z} = \nu\frac{\partial^2 u}{\partial Z^2} \arrow[rr, leftrightarrow, dashed] & & \mu\frac{\partial u}{\partial Z}\Big|_{z=0} = -\frac{d\sigma}{dX} \\
\frac{\partial u}{\partial X} + \frac{\partial w}{\partial Z} = 0 & \text{Mass Balance} & \sigma = \sigma_0 - \kappa \ln\frac{\Gamma^\infty}{\Gamma^\infty - \Gamma} \arrow[u, dashed] \arrow[d, dashed] \\
u\frac{\partial C}{\partial X} + w\frac{\partial C}{\partial Z} = D\frac{\partial^2 C}{\partial Z^2} & D\frac{\partial C}{\partial Z}\Big|_{z=0} = Ka\,\{Cs\,(\Gamma^\infty - \Gamma) - a\Gamma\} \arrow[l, dashed] \arrow[r, dashed] & \frac{\partial}{\partial x}(us\Gamma) = D\frac{\partial C}{\partial Z}\Big|_{z=0}
\end{tikzcd}$$

Fig.2 Governing equations

On the adsorption kinetics of surfactant, there seems no clear universal theory so far, although there are many theoretical and experimental works (1,2,3,6,8,9,12,14,18). In this work, we intend to analyze the surface hydrodynamics of solution of surfactant such as, SDBS(Sodium dodecylbenzene sulphonate). To the adsorption kinetic model of surfactant, we employed the equation of Langmuir form, because this equation is often employed in other works and some of them reported that this equation can be well applied to these case(1,2,3,6,11,14). The value of Eq.7 means the discrepancy from the equilibrium state of adsorption, and it is the flux of surfactant to water surface.

Eqs.1-7 are dependent each other, mutual relations of the governing equations are given in Fig.2, following to Cerro and Whitaker (4) and Whitaker(20).

The boundary conditions at the upstream end are as follows,

$$u = f_1(z) \ ; \ c = f_2(z) \ , \ (0<z<\infty) \tag{8}$$

where $f_1(z)$ and $f_2(z)$ are the vertical profiles of flow velocity and surfactant concentration respectively. Once the value of Eq.7 is determined at the starting point of computation, then two concentrations of surfactant Γ and c_s are given by Eqs.7 and 8.

The value of Eq.7 at the starting point of computation has strong effect on total hydrodynamic phenomena under consideration, thus even very small difference in the values of Eq.7 at the starting point may result so large difference in surface velocity at the downstream.

In a case of surface acceleration, numerical computation starts from the point with a certain distance L upstream from the weir. So, the boundary condition of this starting point will be affected by the distance L, as well as another hydraulic conditions.

Simplification of the basic equations can be made by use of the von Mises transformation(Eq.14), which is also useful in decreasing numerical error. Under this transformation, our governing equation in dimensionless and refined form can be given as

$$\frac{\partial\bar{u}}{\partial\bar{x}} = \frac{1}{2}\frac{\partial^2\bar{u}}{\partial\bar{\psi}^2} \tag{9}$$

$$\frac{\partial\bar{c}}{\partial\bar{x}} = \bar{u}_s\frac{\partial^2\bar{c}}{\partial\bar{\psi}_1^2} \tag{10}$$

$$\frac{\partial\bar{u}}{\partial\bar{\psi}}\Big|_{\psi=0} = \frac{1}{\bar{u}_s}\frac{1}{1-\bar{\Gamma}}\frac{d\bar{\Gamma}}{d\bar{x}} \tag{11}$$

$$\frac{d}{d\bar{x}}(\bar{u}_s\bar{\Gamma}) = \frac{l*}{l}\,\bar{u}_s\,\frac{\partial\bar{c}}{\partial\bar{\psi}_1} \tag{12}$$

$$\frac{\partial\bar{c}}{\partial\bar{\psi}_1}\Big|_{\psi_1=0} = \frac{1}{\bar{u}_s}\bar{K}a\{\bar{c}_s(1-\bar{\Gamma})-\bar{\Gamma}\} \tag{13}$$

$$\psi = \int_0^{z_1} u\,dz \ ; \ \psi_1 = \int_0^{z_1} u_s\,dz_1 \tag{14}$$

$$l_* = \frac{\mu}{K}\sqrt{D\nu} \;;\quad \overline{k}a = l_* ka\Gamma^{\infty}/D \;;\quad l = \Gamma^{\infty}/a \tag{15}$$

$$\left.\begin{aligned} \overline{u} &= u\cdot\mu/K \;;\quad \overline{x} = xK/(\rho\nu^2) \;;\quad \overline{z} = zK/(\rho\nu^2) \\ \overline{z}_1 &= z_1/l_* \;;\quad \overline{c} = c/a \;;\quad \overline{\Gamma} = \Gamma/\Gamma^{\infty} \end{aligned}\right\} \tag{16}$$

Numerical Computation Procedure

Numerical solutions are obtained by using the ordinary finite difference method.

The phenomena which we treat here is, in principle, a two point boundary value problem to the longitudinal direction. The reason why this problem is a two point boundary value one, however, comes from the boundary conditions at the water surface. In order to get numerical solutions which satisfy the downstream boundary condition, we employed the shooting method. In our numerical computation, the value of Eq.6 at the starting point is used as an arbitrary parameter(shooting parameter), denoted by IP, which is also the flux of surfactant to the watersurface as given in Eqs.6 and 7.

After setting the value of shooting parameter IP at the starting point, numerical solutions are obtained by the following procedure(Fig.3). At first, the surface velocity at $\overline{x}+d\overline{x}$, $\overline{u}_{sa}$ is assumed. By making use of this velocity, the surface concentration Γ and concentration profile in a bulk fluid at $\overline{x}=\overline{x}+d\overline{x}$ are computed from Eqs.10,12 and 13. Making use of the surface tension gradient as a boundary condition, which is given by above computation, surface flow velocity $\overline{u}_s$ can be obtained from Eqs. 9 and 11. Iteration about $\overline{u}_s$ were made until the value of relative difference between $\overline{u}_{sa}$ and $\overline{u}_s$ becomes less than the previously determined value say 0.001.

The flow velocity in the concentration boundary layer is assumed to be constant in the vertical direction. Computing mesh sizes are, for example, $\Delta x=3.5\cdot10^2$, $\Delta\psi=25.0$, $\Delta\psi_1=2.5$, although these values are somewhat changed according to the hydrodynamic condition. As an example of surfactant, we take SDBS. Characteristic properties of SDBS and some other properties used in the computation are summarized in Table 1.

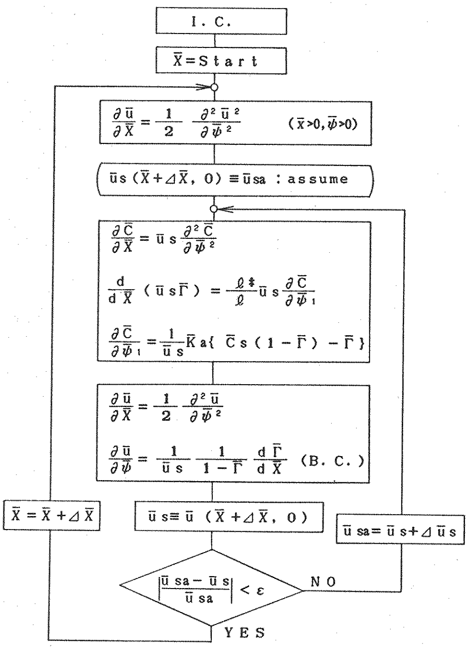

Fig.3 Computation procedure

Surfactant	SDBS	
Γ^{∞}	$1.59\cdot10^{-6}$	mol/m^2
a	0.647	mol/m^3
K	7.75	N/m
D	$4.2\cdot10^{-10}$	m^2/s

Table 1 Properties of surfactant solution

Numerical Solution

An example of surface deceleration is shown in Figs.4-7. These are solutions of the following case : jet momentum per unit width, $R_j=6\cdot10^{-3}$ Kg/s , surfactant concentration $c_\infty=0.01$mol/m^3, surfactant flux at x=0, $D\cdot dc/dz=3.0\cdot10^{-6}$ mol/s/m^2, and $k_a=3.0$ m^3/mol/s. In this case, for the initial flow velocity profile $f_1(z)$, we employed the flow velocity distribution of laminer surface jet at the 1 cm downstream from the virtual origin.

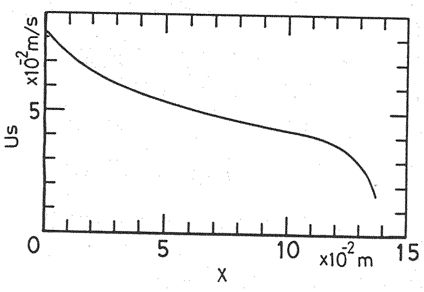

Fig.4 Surface velocity decrease

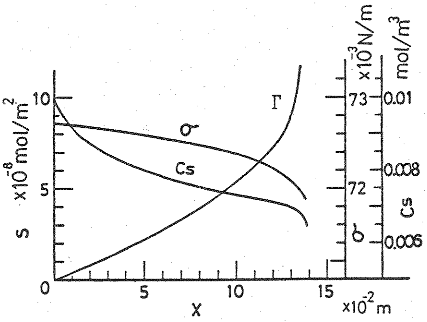

Fig.5 Variation of surfactant concentration and surface tension along the jet axis

Fig.4 shows the longitudinal surface velocity decrease. Surface concentration, volumetric concentration at the water surface and surface tension also varies along the jet axes and these are shown in Fig.5. Vertical profiles variation of flow velocity and surfactant concentration are also shown in Figs.6 and 7 respectively. From these figures, we can see the decreasing process in the surface velocity with the increase in surfactant concentration at water surface.

An example of surface acceleration is also given in Figs.8-11. These are solutions of the case : flow out volume per unit width $q=5.5\cdot10^{-4}m^2/s$, flow rate of surfactant solution $u_\infty=0.005m/s$, surfactant concentration $c_\infty=0.05$ mol/m^3 , $D\cdot dc/dz=6.0\cdot10^{-6}$ mol/s/m^2 ,and $k_a=5.0m^3$ /mol/s. In this case, uniform distribution for both flow velocity and surfactant concentration at the starting point are assumed.

Longitudinal variation of surface flow velocity, surface concentration and surface tension of this case are shown in Figs.8 and 9.

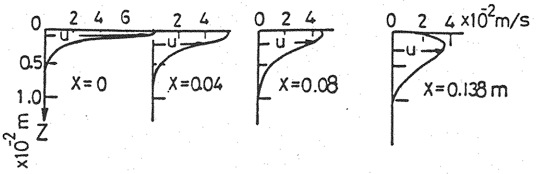

Fig.6 Flow velocity profiles along the jet axis

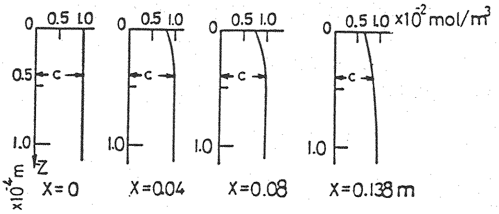

Fig.7 Concentration profiles along the jet axis

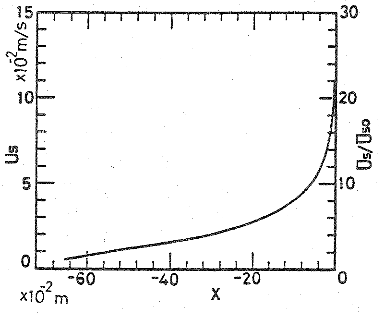

Fig.8 Surface velocity increase along the flow direction

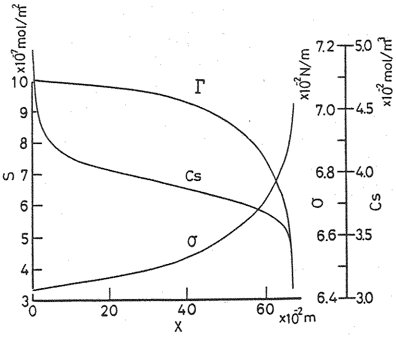

Fig.9 Variation of surfactant concentration and surface tension along the flow direction

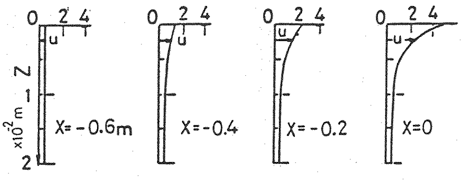

Fig.10 Streamwise variation of flow velocity distribution

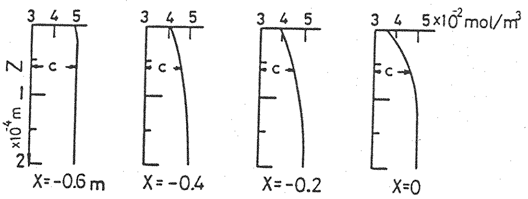

Fig.11 Streamwise variation of surfactant concentration distribution

Figs.10 and 11 show vertical profiles of flow velocity and concentration. In this case, the acceleration of surface flow starts at discharge point over the weir due to the surface elongation. From this point, the effect of surface elongation propagates upstream.

EXPERIMENT

Experimental Set Up and Method

The channel is made of polyacryl plate with 2m length, 0.10m width and 0.15m depth(Fig.12). Surfactant used is SDBS(molecular weight=348.48, CMC=3.1mol/m^3). Tap water is used and its temperature is kept 20°C during experiment. By making use of the DuNoüy surface tension meter, the relationship between concentration and static surface tension was checked to be given by the Szyszkowski formula.

Surface deceleration experiment was conducted by injecting the water from jet nozzle of 1mm opening to the water surface of the channel(Fig.12). Injected water has the same surfactant concentration as those of bulk fluid, and flows out of the downstream end of the flume. Our interest is the brake action of surface velocity due to accumulation of surfactant in water surface. Therefore, we set no additional attachment on the water surface, although there are several works on surface film growth with a barrier on surface(4,8,15,19).

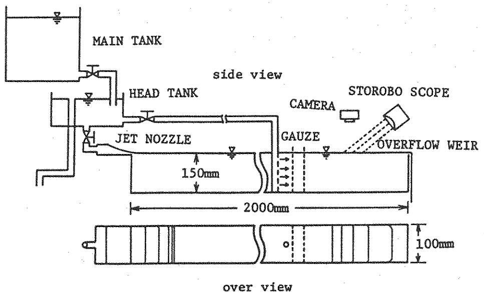

Fig.12 Experimental set-up

Furthermore, in order to avoid the effect of downstream condition, experimental observation is made within a very short time after injection, say 1 minute. Long time experiment brings circulatory secondary flow at the water surface(7,13) which is more complicated phenomena than the one we are going to solve.

Surface acceleration experiment was conducted at the downstream side of this flume(Fig.12). As water approaches to the weir, surface velocity increases. This brings surface elongation and surface tension gradient, which results the acceleration of surface flow.

To get uniform flow velocity over the flow section and equilibrium state of surfactant concentration as upstream conditions, wire gauzes were set at the distance 0.7m from the discharge weir. Gauzes were set carefully, so that the wire of the gauze does not act as a surface barrier.

The measurement of the surface velocity was based on the measurement of the velocities of very fine powders of polyvinylchloride(density=1400Kg/m^3, mean diameter =81μm). The powder deposited on the water surface from a thread which is set laterally at the height of 5.0 mm over the water surface. The stroboscopic light was also used for discrete high intensity lightening. To check this flow velocity measurement, we measured the surface flow velocity of two-dimensional laminar surface jet, forthecase of uncontaminated water. The agreement between the theory(16) and experiment indicated the validity of this tracer method.

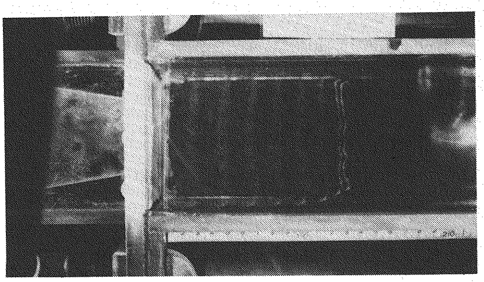

Photo.13-1 c_∞=0.007 mol/m^3,
q=1.17cc/cm/s, 240flashes/min

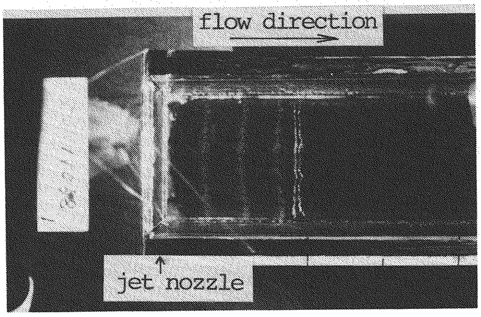

Photo.13-2 c_∞=0.010 mol/m^3,
q=1.24cc/cm/s, 240flashes/min

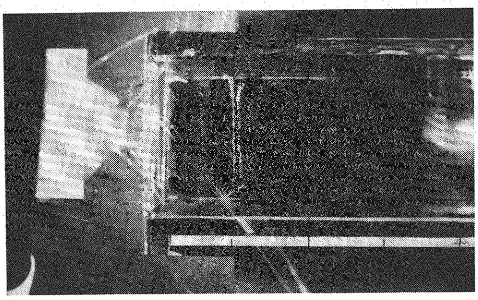

Photo.13-3 c_∞=0.013 mol/m^3,
q=1.32cc/cm/s, 240flashes/min

Fig.13 Photographs of surface deceleration
(Over view)
(q=jet issuing volume per unit width)

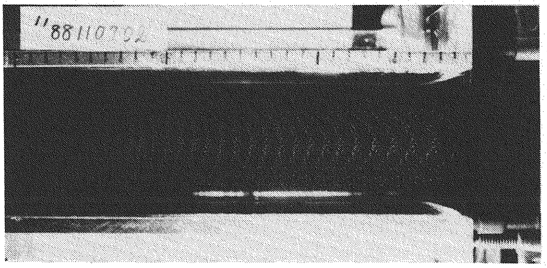

Photo.14-1 c_∞=0.0 mol/m^3,
q=0.67cc/cm/s, 20flashes/min

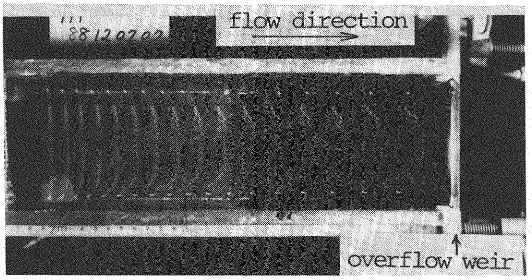

Photo.14-2 c_∞=0.050 mol/m^3,
q=0.67cc/cm/s, 100flashes/min

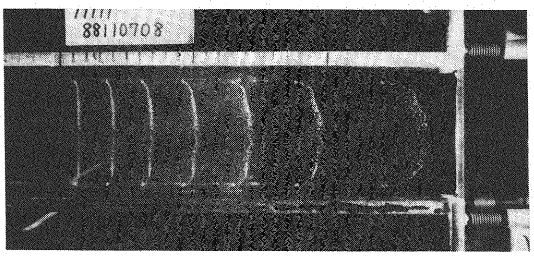

Photo.14-3 c_∞=0.20 mol/m^3,
q=0.69cc/cm/s, 100flashes/min

Fig.14 Photographs of surface acceleration
(Over view)
(q=discharge volume per unit width)

Experimental Results

Fig.13 are photographs which show the surface velocity decreasing process. The effect of surfactant on surface deceleration is shown so distinctly, although jet injection momentum are almost the same in these three experiment. To avoid the effect of secondary circulation flow at water surface, these photographs were taken at about 30 second after jet injection.

Photographs of the surface acceleration are shown in Fig.14. Especially, in the case of 0.2 mol/m^3 surfactant solution, the surface flow velocity accelerated so strongly near the discharging weir, which is brought by local surface tension difference. It is said, from these photographs, that the effect of surface stiffness(19) on surface flow is not so large in our experiment.

Let us compare the theoretical prediction with the experimental results obtained from these photoes.

Our mathematical model has two parameters, k_a in Eq.7 and the iteration parameter IP. The value of k_a seems to be determined by properties of surfactant and surfactant solution, such as co-existing ion. Thus, in the comparison of theory with experiment, it is reasonable that the parameter k_a has unique one value in the present work. So the value, k_a=5.0 m³/mol /s was employed as a fitting parameter. We will discuss about this parameter later on.

As the flow develops downstream, there is a point where the surface deceleration or acceleration becomes so large, that the boundary layer approximation cannot be applied, and the theoretical values are neither obtained nor can predict the experiment. We call this piont as "bursting point" and the longitudinal distance from the starting point to this point as "acceleration or decelaration distance".

This distance is very sensitive to the value of the iteration parameter IP. Therefore, to get the numerical solutions for comparison with experiment, the value of the iteration parameter had to be determined mostly from the value of the experimental "acceleration or deceleration distance" mentioned above.

Comparison of theoretical predictions to experimental data are shown in Figs.15 and 16. From Figs.15 and 16, it can be said that the computed values well agree with experiment over the wide range of flow and the validity of the numerical model is confirmed.

DISCUSSION

In this section we discuss, at first, effect of some properties on surface flow according to the numerical solution.

The relationship between surface flow decay and jet momentum are shown in Fig.17. In the case of large jet momentum, the increase in the surfactant concentration can't affect the flow field at the early stage of flow, and the brake action to surface velocity suddenly occurs after the surface velocity decays to sufficiently small value. On the other hand, for a given jet momentum, as the concentration of surfactant increases, the surface deceleration due to surfactant appears from the early stage of jet, as shown in Fig.18.

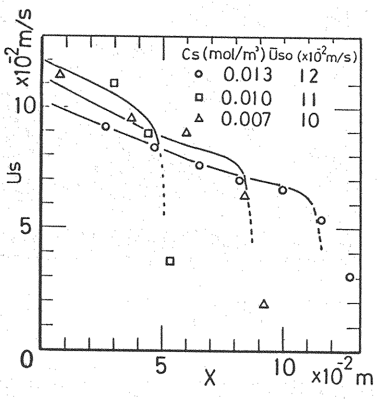

Fig.15 Comparison of theoretical prediction with experimental value (Surface deceleration)

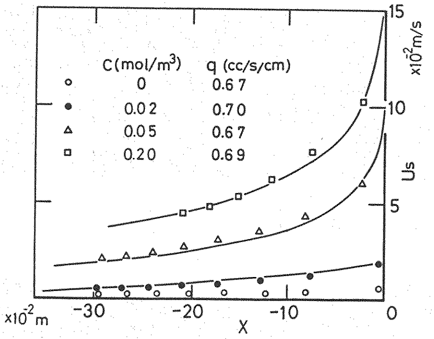

Fig.16 Comparison of theoretical prediction with experimental value(Surface acceleration)

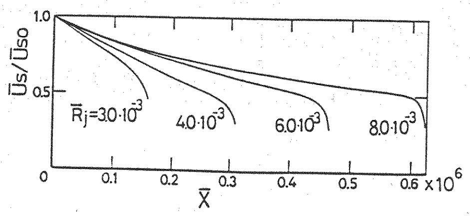

Fig.17 Effect of jet momentum on surface deceleration ($\bar{c}_\infty$=1.0, $\bar{k}_a$=4.0·10^{-5} IP=5.35·10^{-5})

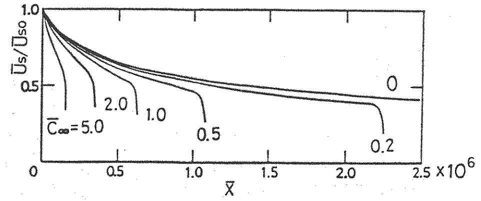

Fig.18 Effect of surfactant concentration on surface deceleration ($\bar{R}j$=8·10^{-3} k_a=4.0·10^{-5}, IP= 5.35·10^{-5} $\bar{c}_\infty$)

In the case of surface acceleration, the effect of surface elongation propagates to more upstream with the decrease both in the bulk flow velocity and in the concentration of surfactant (Fig.19 and 20). In those cases, the values of the shooting parameter IP are determined such a way that the flow out volume per unit width of flume ($\bar{q}=q/u_*/d_*$) has the same value.

To the adsorption kinetics of surfactant, we employed the Langmuir type adsorption equation. The parameter k_a of this equation have a meaning of a rate constant of the adsorption kinetics. Thus, when the value of k_a becomes large, the surface adsorption of surfactant controlled by diffusion in a liquid phase becomes predominant and the limiting case of diffusion controlled adsorption is realized when $k_a=\infty$. There are several works on the relation between the surfactant concentration and surface age by making use of the Langmuir equation(9,14, 18). however, sufficient investigation on the effect of the value of k_a on hydrodynamic behavior is not made so far.

Effects of the value of k_a on surface deceleration and acceleration are given in Figs.21 and 22. As the value of k_a becomes large, both surface deceleration and acceleration take place more sharply. In particular surface deceleration is so sensitive as shown in Fig.21.

In the comparison of theory with experiment, we assumed that k_a=5.0, but this does not mean that the value is universal. As our objective is to present a mathematical model of the surface hydrodynamic of surfactant solution, we avoid more discussion on this matter, although further studies must be needed.

Imaishi et al(11) also treated the surface acceleration in an open channel flow, by making use of elegant and simple mathematical model. In their work, the second term on the left hand side of Eq.1 and the first term on the left hand side of Eq.3 were omitted, thus the numerical procedure became so easy. The good agreement between the numerical results and experiment was reported. In case of surface acceleration is small, numerical solution obtained by Imaishi et al gives very near value to our solution, on the other hand, in the case where the acceleration effect is large, the model given by Imaishi et al' presents rather smaller acceleration than our model.

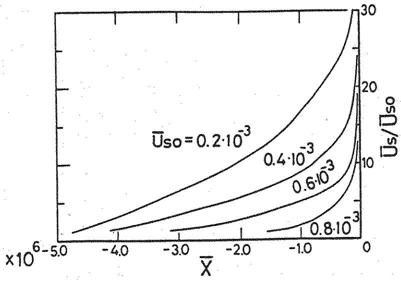

Fig.19 Effect of flow rate of surfactant solution $\bar{u}_{so}$ on surface acceleration ($\bar{c}_\infty$=2.5, $\bar{q}$=200, $\bar{k}_a$=2.0 10^{-5})

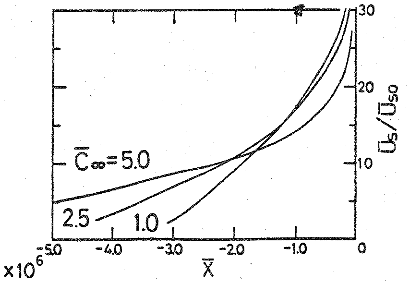

Fig.20 Effect of surfactant concentration $\bar{c}_\infty$ on surface acceleration ($\bar{q}$=250, $\bar{u}_{so}$ =2.0·10^{-4}, $\bar{k}_a$=2.0·10^{-5})

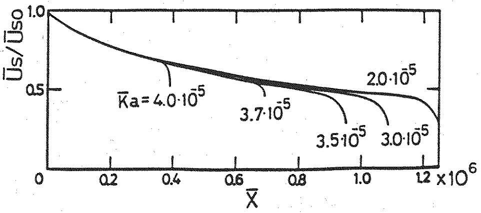

Fig.21 Effect of the value of adsorption rate constant $\bar{k}_a$ on surface deceleration ($\bar{R}j$=6, $\bar{c}_\infty$=0.5, IP =1.07·10^{-5})

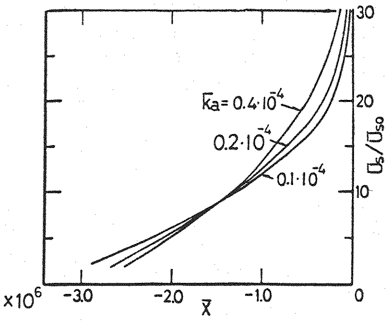

Fig.22 Effect of the value of adsorption rate constant $\bar{k}_a$ on surface acceleration ($\bar{q}$=200, $\bar{u}_{so}$=2 10^{-4}, $\bar{c}_\infty$= 2.5)

CONCLUDING REMARK

This paper describes surface hydrodynamics of surfactant solution. In this phenomena, difference of local surface tension acts as a boundary condition of fluid motion. The surface tension is given by the surface concentration of surfactant, which is influenced by the bulk fluid motion through the mass conservation of surfactant. As our objective is to obtain a mathematical model of this complicated phenomena, we studied very simple two-dimensional surface flows. These are surface flow deceleration of surface jet and surface acceleration which is induced by flow out over the weir set at the downstream end of the flume.

In our mathematical model the following factors were considered: mass and momentum conservation for fluid, mass conservation for surfactant both in bulk fluid phase and water surface, surface adsorption kinetics, and relationships between surface tension and surfactant concentration.

Making use of the boundary layer approximation to the equation of motion and employing the Langmuier isotherm adsorption equation as adsorption kinetic equation, numerical solutions of the basic equation were obtained. Taking SDBS as an example of surfactant, the effect of some factors such as surfactant concentration, flow rate of the surfactant solution and the value of the adsorption rate constant, on surface deceleration and acceleration were discussed, according to the numerical solution.

Numerically predicted results were compared with laboratory experiment and the agreement was reasonable, so this mathematical model will be applicable to many related problems.

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APPENDIX-NOTATION

The following symbols are used in this paper:

a = constant in Langmuir equation, Eq.7;
D = diffusion coefficient;
c = concentration of surfactant in bulk fluid;
K = constant in Szyszkowski equation, Eq.5;
k_a = constant in Langmuir equation, Eq.5;
$f_1(z)$ = vertical profile of flow velocity at starting point;
$f_2(z)$ = vertical profile of surfactant at starting point;
q = flow discharge volume over a weir
Rj = jet momentum
u = flow velocity in the x direction;
u_s = computed flow velocity at water surface;
u_{sa} = assumed flow velocity at water surface;
w = flow velocity in the z direction
x = longitudinal coordinate;
z = vertical coordinate
Γ = surface concentration of surfactant(surface excess);
Γ^∞ = constant in Langmuir equation, Eq.7;
μ = coefficient of viscosity;
ρ = density of bulk fluid;
σ = surface tension of surfactant solution;
σ_0 = surface tension of clear water;
τ = surface stress of bulk flow;
$\bar{\Psi}$ = dimensionless stream function; and
$\bar{\Psi}_1$ = dimensionless stream function in concentration boundary layer;

suffix
s = value at water surface.