Journal of Hydroscience and Hydraulic Engineering
Vol. 7, No. 2, March, 1990, 1-25

NON-EQUILIBRIUM SEDIMENT TRANSPORT: A GENERALIZED MODEL

By

Tetsuro TSUJIMOTO

Department of Civil Engineering, Kanazawa University
2-40-20, Kodatsuno, Kanazawa, 920, Japan

Akio MORI

Department of Civil Engineering, Hokkaido University
Kita-13-Nishi-8, Kita-ku, Sapporo, 060, Japan

Takeshi OKABE

Department of Civil Engineering, Tokushima University
2-1, Nanjomishimamach, Tokushima, 770, Japan

and

Terunori OHMOTO

Department of Civil Engineering, Kyushu University
6-10-1, Hakosaki, Higashi-ku, Fukuoka, 812, Japan

ABSTRACT

Transient process of sediment transport is one of the currently fascinating topics in fluvial hydraulics. Direct effects of flow unsteadiness or/and nonuniformity and indirect effect due to "relaxation process" take place on transient process of sediment transport, but they are different each other in the mechanism as well as the effect. Sediment transport in relaxation might be called "non-equilibrium process of sediment transport." A convolution-integral model with an impulse response is effective as a perspective framework of describing a relaxation process or non-equilibrium sediment transport.

Although a lag distance of bed-load motion is as short as the order of the sediment size, it plays an important role not only on small scale phenomena but also on instability of bed undulation and bed-surface composition. With development of sand waves and sorting process, the lag of bed-load motion significantly increases to reach the order of the flow depth.

The adaptation length of non-equilibrium suspended sediment transport as for not only transport rate but also concentration profile is significantly long and often reaches a few hundred times the flow depth.

The transformation process of bed configurations is also a typical example of "relaxation process" in fluvial hydraulics, and it affects the sediment transport.

INTRODUCTION

In fluvial streams, non-equilibrium of sediment transport often appears due to a slow response of sediment transport and fluvial process to the change of flow or that of its boundary condition, and it severely affects the fluvial phenomena. There are various types of scales of non-equilibrium process, and they appear coupled together. Due to such a complexity of the phenomena, one has to look for a proper approach to describe each of them. Some of them cannot be described without any manipulation for non-equilibrium sediment transport but the others can be explained as a good approximation by using the equilibrium sediment transport formula. This study aims to show a kind of key to treat non-equilibrium fluvial process in a simple unified idea.

"Equilibrium sediment-transport rate" can be defined for given flow condition and bed material, and the sediment discharge in a sufficiently long stream under constant conditions is equal to it. In general, however, the boundary conditions of sediment transport and the flow condition are often variable in time and space, and an idealized "equilibrium" condition is difficult to be achieved but any unique relation is hardly expected between sediment-transport rate and flow parameters.

Recently, many researchers have fallen interested in the transient process of sediment transport under unsteady or/and non-uniform conditions. It was often chosen as one of major topics of many recent international symposia on fluvial hydraulics, in which several lectures or presentations emphasizing the importance of unsteadiness, non-uniformity and non-equilibrium effects on sediment transport (Jain (5), Di Silvio (2), Klaassen (6), Yen (13), Tsujimoto (12)).

As explained in Chapter 3 of this paper, in the transient process of sediment transport, direct effects of flow unsteadiness or/and non-uniformity should be distinguished from non-equilibrium process. The non-equilibrium process, which appears as a slow response of sediment transport or fluvial process to the change of flow or the boundary conditions, can be well described as a relaxation process, and such a concept may help understanding of the mechanism of non-equilibrium sediment transport and a unified description of non-equilibrium fluvial processes. In this paper, spatial or temporal lag as an elementary event of the non-equilibrium sediment transport or fluvial process is extracted, and it is represented by using the concept of an impulse response. Then, a unified description of the relaxation process by a convolution integration is attempted.

CONVOLUTION INTEGRAL MODELLING OF RELAXATION PROCESS

An independent variable and a dependent parameter in the fluvial processes (for example, one of the hydraulic parameters as an independent parameter and sediment-transport rate as a dependent one) are to be represented by the symbols, Φ and Ψ, respectively. These are in general functions of time or space (represented by ζ). Although any variables included in description of the fluvial phenomena are closely related and any of them is not necessarily an independent one in a rigorous sense, Φ and Ψ here are simply and formally distinguished from each other. A response of $\Phi(\zeta)$ to the spatial or temporal change of $\Psi(\zeta)$ is here investigated.

As the simplest case, for a rectangular (abrupt) change of Ψ as illustrated in Fig.1, the response of $\Phi_R(\zeta)$ behaves as a relaxation process due to an inherent lag system. For example, for an abrupt change of flow discharge, the geometry of bed forms shows such a relaxation process (Allen (1); Nakagawa & Tsujimoto (7)). When Ψ changes from Ψ_1 to Ψ_2 abruptly, Φ responds to it and it changes Φ_1 to Φ_2 gradually. In a relaxation process, in which Φ is non-equilibrium, a unique relation between Ψ and Φ is no longer expected. In other words, the equilibrium relation $\Phi_e=f(\Psi_e)$ is no longer valid in which the subscript e represents the equilibrium values. The gradual change of $\Phi_R(\zeta)$ can be often well approximated by the following mathematical expression.

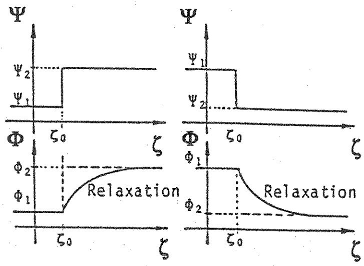

Fig.1 Relaxation process of outputdue to abrupt change of input process

$$\Phi_R(\zeta) - \Phi_2 = (\Phi_1-\Phi_2)\exp\left(-\frac{\zeta}{\Lambda_R}\right) \tag{1}$$

in which Λ_R=temporal or spatial scale of relaxation. The so-called adaptation length defined as a distance after which $[\Phi_R(\zeta)-\Phi_1]$ reaches 90 or 99% of $[\Phi_2-\Phi_1]$ is about 3~5 times Λ_R.

Equation 1 can be rewritten as

$$\frac{d\Phi}{d\zeta} = \frac{\Phi_e-\Phi}{\Lambda_R} \tag{2}$$

It is a linear approximation that the local (or instantaneous) rate of the change of Φ is proportional to the difference of the local value of Φ and its equilibrium value.

When $\Phi_R(\zeta)$ is known, the impulse response $g_R(\zeta)$ is obtained as follows (4, 7):

$$g_R(\zeta) = \frac{\frac{d\Phi_R}{d\zeta}}{\Phi_2-\Phi_1} \tag{3}$$

and the response of Φ to an arbitrary change of Ψ can be described by the following convolution integration:

$$\Phi(\zeta) = \int_0^\infty \Psi(\zeta-\varepsilon)g_R(\varepsilon)d\varepsilon \tag{4}$$

When $\Phi_R(\zeta)$ is written as an exponential function as Eq. 1, the impulse response is written as

$$g_R(\zeta) = \frac{1}{\Lambda_R}\exp\left(-\frac{\zeta}{\Lambda_R}\right) \tag{5}$$

If the mechanism of transient process or non-equilibrium process is well described, the impulse response can be analytically deduced. However, such a mechanism in fluvial process is often difficult to be clarified. Then, an experiment to know the behavior $\Phi_R(\zeta)$ will provide a powerful approach to describe more general non-equilibrium process through the framework mentioned above.

For simplicity, a fluctuation of Ψ is to be approximated as a simple sinusoidal curve as

$$\Psi(\zeta) = \Psi_0(1+a_\Psi \sin\kappa\zeta) \tag{6}$$

in which Ψ_0=the mean value of Ψ; $a_\Psi\Psi_0$=amplitude of fluctuation of Ψ; κ=angular wave number or frequency. By substituting Eq. 6 into Eq. 4, the following is obtained:

$$\Phi(\zeta) = \Phi_0[1+r_\Phi a_\Psi \sin(\kappa\zeta-\phi)] \tag{7}$$

in which Φ_0=equilibrium value for Ψ_0; $r_\Phi a_\Psi\Phi_0$=amplitude of Φ; ϕ=phase lag of Φ behind Ψ; and

$$r_\Phi = \frac{1}{\sqrt{1+(\kappa\Lambda_R)^2}}; \qquad \sin\phi = \frac{\kappa\Lambda_R}{\sqrt{1+(\kappa\Lambda_R)^2}}; \qquad \cos\phi = \frac{1}{\sqrt{1+(\kappa\Lambda_R)^2}} \tag{8}$$

The phase shift ϕ always belongs to the first quadrature. The relaxation process is characterized by this kind of phase lag. Thus the relation between Ψ and Φ draws a loop (represented by an equation of an ellipse) around the equilibrium relation between Ψ and Φ, and it explains an appearance of a hysteresis (Nakagawa & Tsujimoto (7)) in the relations among the alluvial stream parameters as pointed by Allen (1).

DIRECT EFFECT OF FLOW UNSTEADINESS OR/AND NON-UNIFORMITY AND NON-EQUILIBRIUM EFFECT ON SEDIMENT TRANSPORT

When the flow intensity (Ψ) varies temporally or spatially, the time derivative or the spatial derivative ($d\Psi/d\zeta$) may degenerate the relation between Ψ and Φ besides the non-equilibrium or the relaxation effect (explained in the preceding chapter). In order to investigate the essential difference between the "direct effect" of flow unsteadiness or non-uniformity and the "indirect effect" due to a relaxation, a simple behavior of Ψ as expressed by a sinusoidal curve and the response of Φ to it are employed (Tsujimoto, Graf and Suszka, (10, 11)).

$$\Psi(x,t) = \Psi_0[1+a_\Psi \sin(\omega t-\kappa x)] \tag{9}$$

in which κ=angular wave number; and ω=angular frequency.

Without any effects of unsteadiness, non-uniformity and relaxation, the equilibrium relation between Ψ and Φ is still valid, and thus,

$$\Phi(x,t) = \Phi_0[1+\beta a_\Psi \sin(\omega t-\kappa x)] \tag{10}$$

in which $\beta \equiv (d\Phi/d\Psi)_0(\Psi_0/\Phi_0)$ and it is evaluated for Ψ_0 by using the equilibrium relation between Ψ and Φ (for example, an equilibrium sediment-discharge formula). Then, the sediment-discharge variation is in phase to the variation of the flow intensity.

If the direct effect of flow unsteadiness and non-uniformity dominates, the relation between Ψ and Φ can be written as follows under a linear approximation:

$$\Phi(x,t) = \Phi_0\left[1+\beta\left(\frac{\Psi'}{\Psi_0}+p_1 h\frac{\partial\Psi}{\partial x}+\frac{p_2 h}{u_{*0}}\frac{\partial\Psi}{\partial t}\right)\right] \tag{11}$$

in which h=flow depth; u_*=shear velocity; the subscript 0 represents the equilibrium value; and ' means the perturbation. p_1 and p_2 should be determined as a function of Ψ_0 based on the equations of unsteady flow and sediment motion. Then the response of Φ to the sinusoidal fluctuation of Ψ is written as follows:

$$\Phi(x,t) = \Phi_0[1+\alpha_1\beta a_\Psi \sin(\omega t-\kappa x-\phi_1)] \tag{12}$$

in which

$$\alpha_1 = \sqrt{1+\lambda_0^2}\,; \qquad \phi_1 = \arctan\lambda_0\,; \qquad \lambda_0 = p_1\kappa - \frac{p_2\omega}{u_{*0}} \tag{13}$$

This result is illustrated in Fig.2 and characterized by an amplification of the amplitude ($\alpha_1>1$) and the phase shift (ϕ_1). The phase shift becomes either positive or negative depending upon the values of the parameters p_1 and p_2. The bed-load transport in a steep gravel bed at unsteady flow with a sharp hydrograph corresponds to this type. The experimental results at the laboratory (Graf & Suszka (3,9)) shown in Fig.3 are the good examples.

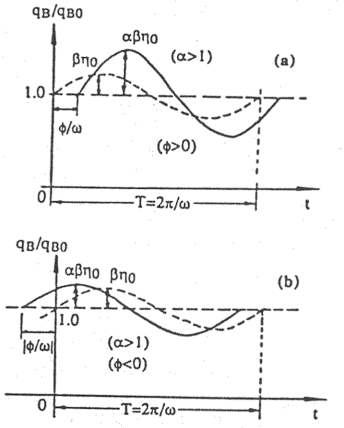

Fig. 2 Direct effect of flow unsteadiness on bed-load transport

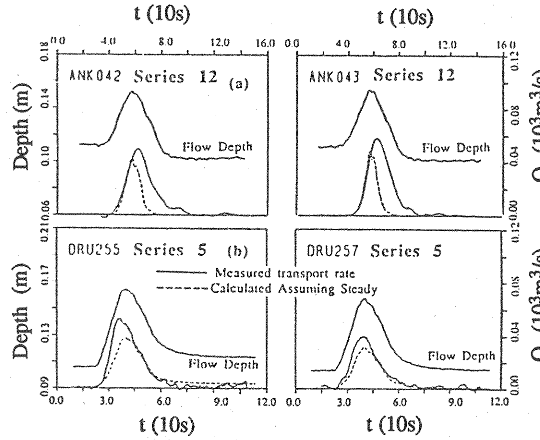

Fig. 3 Laboratory data (EPFL) of unsteady bed-load transport directly affected by flow unsteadiness

On the other hand, the situation where a relaxation effect dominates is investigated as follows. When the impulse response of the system is written as Eq. 5, the response of Φ to the sinusoidal fluctuation of Ψ is written as follows:

$$\Phi(x,t) = \Phi_0[1+\alpha_2\beta a_\Psi \sin(\omega t-\kappa x-\phi_2)] \tag{14}$$

in which

$$\alpha_2 = \frac{1}{\sqrt{1+(\kappa\Lambda_R)^2}}; \qquad \phi_2 = \arctan(\kappa\Lambda_R) \tag{15}$$

This means the suppression of the amplitude ($\alpha_2<1$) and a positive phase lag ($\phi_2>0$), as illustrated in Fig.4. When the sediment transport on a sand bed with dunes is focussed, the relaxation due to dune deformation dominates in the relation between the sediment discharge and the flow parameter. The analytical result is consistent with the experimental data of unsteady bed-load transport on a sand bed with dunes (Phillips & Sutherland (8)) as shown in Fig.5.

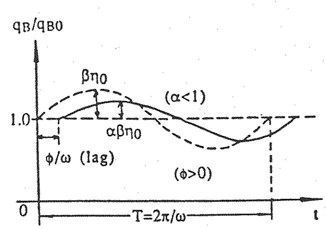

Fig.4 Relaxation effect on unsteady bed-load transport

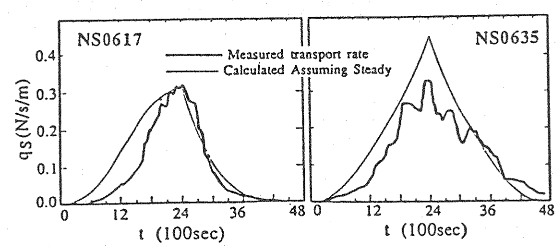

Fig.5 Laboratory data (Univ. of Canterbury) of unsteady bed-load transport dominantly affected by relaxation

As demonstrated in this chapter, the direct effect due to flow unsteadiness or/and non-uniformity and the indirect effect due to relaxation appear in different manners on the transient process of sediment transport or fluvial processes. In the cases shown in Figs.3 and 5, one of them appears independently, but in general, they are coupled together. In the study on the transient process of sediment transport or fluvial process, these two should be distinguished each other.

NON-EQUILIBRIUM BED-LOAD TRANSPORT

Non-Equilibrium Transport Law of Bed-Load

The sediment transport process is constituted by two phases: the pick-up of a sand particle from a bed and the deposition (or stop) of a moving sand particle on the bed. When the volume of sand particles dislodged from a bed and that of those depositing on the bed per unit area per unit time are represented by E and D, respectively, the following equation with respect to the spatial variation of bed-load transport rate ($q_B(x)$) is written based on a mass conservation law.

$$\frac{dq_B(x)}{dx} = E(x) - D(x) \tag{16}$$

E must depend on the local flow intensity, while D depends on the local discharge of sediment. D is affected by the upstream situation of sediment transport. Eq. 16 implies that the sediment transport inherits a (spatial) relaxation mechanism.

When the pick-up rate, defined as the probability density per unit time for a sand particle to be dislodged from a bed, is represented by p_s,

$$E = \left(\frac{A_3}{A_2}\right)p_s d \tag{17}$$

in which A_2, A_3=geometrical coefficients of sand; and d=sand diameter. On the other hand, Einstein (1950) expressed D as q_B/Λ, in which Λ=mean step length of bed-load motion. Based on it, Tsubaki & Saito (42), Tsuchiya & Iwagaki (43) and Hayashi & Ozaki (25) deduced the following differential equation with respect to the non-equilibrium bed-load transport rate.

$$\frac{dq_B(x)}{dx} = \left(\frac{A_3}{A_2}\right)p_s d - \frac{q_B}{\Lambda} \tag{18}$$

Because the equilibrium bed-load transport rate (q_{Be}) is written as $(A_3/A_2)p_s\Lambda d$ (Einstein (16)), Eq. 18 is rewritten as

$$\frac{dq_B}{dx} = \frac{q_{Be}-q_B(x)}{\Lambda} \tag{19}$$

which corresponds to Eq. 2. Fukuoka & Yamasaka (21, 22) also proposed Eq. 19 where $1/\Lambda$ was termed a "non-equilibrium parameter" without any argument on bed-load transport mechanics.

Nakagawa & Tsujimoto (32) deduced the following non-equilibrium bed-load transport formula based on a stochastic model of bed-load motion constituted by pick-up rate and step length:

$$q_B(x) = \frac{A_3 d}{A_2} \int_0^{\infty} p_s(x-\varepsilon) \int_{\varepsilon}^{\infty} f_X(\xi)d\xi \, d\varepsilon \tag{20}$$

in which $f_X(\xi)$=probability density of the step length. Eq. 20 satisfies the differential equation expressed by Eq. 18. Comparing Eq. 20 with Eq. 4, one can recognize that the distribution function of the step length is the impulse response of non-equilibrium bed-load transport process, as expressed as

$$g_R(\varepsilon) = \frac{A_3 d}{A_2} \int_e^{\infty} f_X(\xi)d\xi \tag{21}$$

Phenomenologically, the step length plays a role to transmit the upstream situation to the bed-load motion to the downstream reach.

Although a non-equilibrium transport formula should be deduced from the conservation laws of both mass and momentum, the above treatment is based only on the former, where the characteristics of pick-up rate and step length are regarded as the same as those under equilibrium. The momentum conservation law is used in estimation of these constituent elements under equilibrium.

Ashida & Michiue (15) divided the shear stress into that taken charge of by fluid and that by granular materials according to the Bagnold's theory (16), and they concluded that the latter shear stress is equal to the critical shear stress under equilibrium. Hasegawa (24) discussed the change of pick-up rate under non-equilibrium based on this concept, but the change of the pick-up rate under non-equilibrium could not be apparently realized in the laboratory experiments (see Fig.8, Nakagawa et al. (35)).

Parker (38) discussed non-equilibrium moving layer of sediment based on the momentum-conservation law for two-phase phenomenon. He attempted to explain a sand bed instability due to bed-load motion based on this concept, but could not successfully explain a sand bed instability in lower regime.

As above-mentioned, an application of the momentum-conservation law has not been succeeded in explanation of non-equilibrium bed-load transport and the subsequent fluvial processes up to date. On the contrary, a consideration of mass conservation under non-equilibrium has apparently succeeded in explanation of several fluvial processes.

The pick-up rate can be related to the bed shear stress. After the formulation by Nakagawa & Tsujimoto (32),

$$p_{s*} \equiv p_s\sqrt{d/(\sigma/\rho-1)g} = F_0\tau_*(1-k_2\tau_{*c}/\tau_*)^m \tag{22}$$

in which $\tau_* \equiv u_*^2/[(\sigma/\rho-1)gd]$=dimensionless bed shear stress; τ_{*c}=dimensionless critical tractive force (τ_{*c} does not rigorously correspond to the condition that p_s=0, and k_2<1.0); σ=mass density of sediment; ρ=mass density of fluid; g=gravitational acceleration; and the experimental constants in Eq. 22 are determined as follows: F_0=0.03; k_2=0.7; and m=3 (32).

According to the laboratory experiments under flat bed condition, the step length has a mean equal to 80~250 times sand diameter, and it follows an exponential distribution (32, 48). Hence the probability density function is written as follows:

$$f_X(\xi) = \frac{1}{\Lambda}\exp\left(-\frac{\xi}{\Lambda}\right) \qquad (23)$$

If the step length follows an exponential distribution, Eq. 20 perfectly coincides to Eqs. 16 and 18. No analytical attempts to evaluate the mean step length based on the equation of motion have succeeded up to date. However, if it is invariant even under non-equilibrium conditions, it can be estimated from the following equation which is deduced as an asymptotic equilibrium of Eq. 20:

$$\lambda \equiv \frac{\Lambda}{d} = \left(\frac{A_2}{A_3}\right)\frac{q_{Be*}}{p_{s*}} \qquad (24)$$

in which $q_{B*} \equiv q_B/\sqrt{(\sigma/\rho-1)gd^3}$; and the subscript e represents the value under equilibrium. q_{Be*} can be evaluated based on the equilibrium transport formula deduced from a more macroscopic approach (for example, Bagnold's formula (17) based on a macroscopic energetical approach); while p_{s*} can be evaluated by Eq. 22.

Figure 6 is an illustration of the laboratory experiment for the simplest non-equilibrium bed-load transport, where non-equilibrium bed-load motion was observed in a mobile bed in the downstream of a rigid bed by a 16mm film movie analysis (Nakagawa et al. (35)). Fig.7 depicts the relation between the local rate of bed-load transport and the distance from the origin of the mobile bed, and it can be well described by the following equation:

$$q_B(x) = q_{Be}\left[1-\exp\left(-\frac{x}{\Lambda}\right)\right] \qquad (25)$$

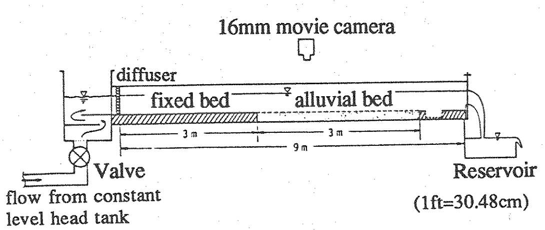

Fig.6 Flume experiment on non-equilibrium bed-load transport

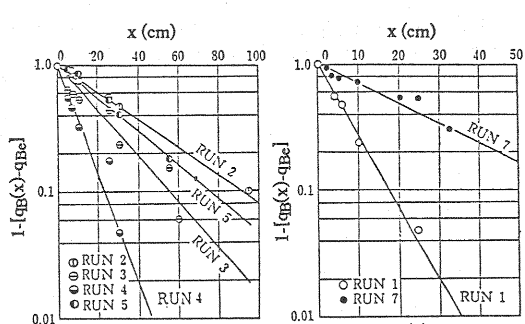

Fig.7 Spatial change of bed-load transport rate under non-equilibrium (Nakagawa & Tsujimoto)

The measured pick-up rate is almost constant even under non-equilibrium (see Fig.8), and thus Eq. 22 is a response of bed-load transport rate to a rectangular type change of the pick-up rate. When the step length follows an exponential distribution, Eq. 20 deduces Eq. 25. The slight increasing tendency may suggest that a momentum conservation should also be considered for more accurate argument. However, it may be difficult because the pick-up due to the collision increases more obviously than that due purely to the fluid force as shown in Fig.9. The pick-up rate due purely to the fluid force does not obviously increase even in the non-equilibrium reach as well as the speed of sand grains (see Fig.10). Another result is shown in Fig.11, where the relation between the deposit rate ($p_d \equiv (A_2/A_3d)\cdot D$) and the transport rate is inspected. The experimental data suggests a proportionality between them, which is an assumption when Eq.18 is deduced. The model constituted by pick-up rate and step length brings the following equation, and the deposit rate is proportional to the local transport rate of bed load if the step length follows an exponential distribution:

$$p_d(x) = \int_0^{\infty} p_s(x-\varepsilon) f_X(\varepsilon) d\varepsilon \qquad (26)$$

Thus, both Figs. 7 and 11 verify that the step length follows an exponential distribution, and the mean step lengths calculated from these figures are well consistent to those clarified by tracer tests (32, 48).

When an adaptation length (if it is defined as the length for the bed-load transport rate reaches 95% of the equi-librium value, it is almost three times the mean step length) is almost 800 times sanddiameter at most, and it is still short. Thus, it can be often neglected in explanation of large scale fluvial processes. In other words, the non-equilibrium feature of bed-load transport need not be considered for fluvial processes with large submergence which have the scale larger than the flow depth, but one cannot neglect it in small scale model tests or fluvial phenomena in small flumes in laboratories.

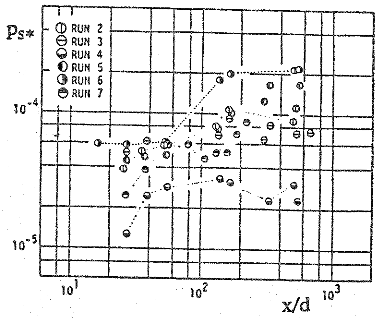

Fig.8 Spatial change of pick-up rate under non-equilibrium(Nakagawa et al.)

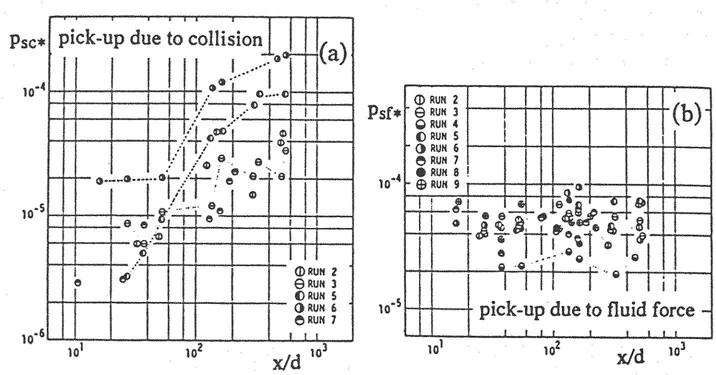

Fig.9 Spatial changes of pick-up rate due purely to fluid force and that due to collision by a moving particle under non-equilibrium (Nakagawa et al.)

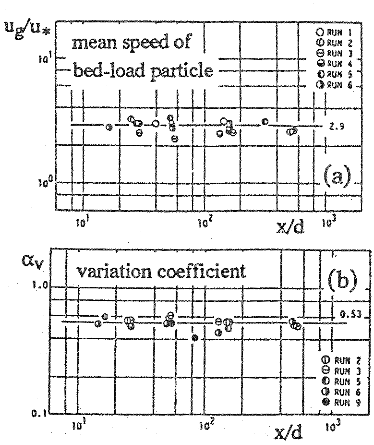

Fig.10 Spatial change of bed-load particle's speed under non-equilibrium (Nakagawa et al.)

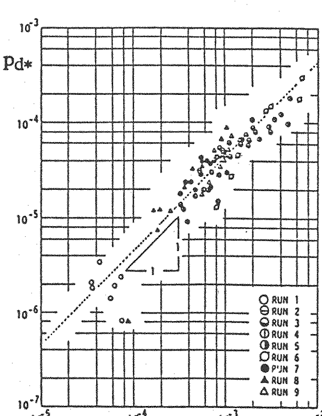

Fig.11 Relation between local rate of bed-load transport and deposit rate (Nakagawa et al.)

Reversely, one must exterpolate the prototype phenomena from model phenomena in laboratories with consideration on the non-equilibrium properties. Fig.12 demonstrates the spatial lag of the bed-load transport rate to the pick-up rate in a small flume with variable width (Tsujimoto (44)). Because the variation scale (L_B) of the stream width is not so large than the sediment diameter (d), the spatial lag is obvious. However, if the value of L_B/d is large enough as often observed in natural streams, such a lag would be negligibly small.

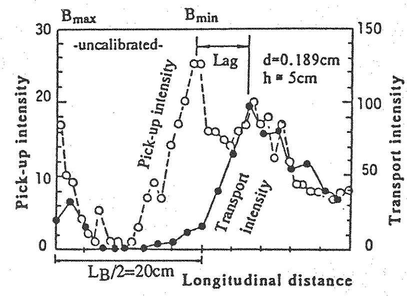

Fig.12 Phase shift between pick-up rate and transport rate in a stream with variable width (Tsujimoto)

Bed Forms and Non-Equilibrium Bed-Load Transport

Although the lag distance of bed-load transport is small in general, it plays an important role in small scale bed-form formation mechanism. When Eq. 20 is applied, the lag distance (δ) or the phase lag ($\kappa\delta$) of bed-load transport to the sinusoidal spatial variation of bed shear stress is obtained as follows (Nakagawa & Tsujimoto (32)), and it has given a significant progress of sand-bed instability analysis introduced firstly by Kennedy (29):

$$\sin\kappa\delta = \frac{\kappa\Lambda}{\sqrt{1+(\kappa\Lambda)^2}} \; ; \qquad \cos\kappa\delta = \frac{1}{\sqrt{1+(\kappa\Lambda)^2}} \tag{27}$$

According to the above result, the phase shift of the local bed-load transport rate to the bed shear stress belongs to the first quadrature. Because the phase shift of the bed shear stress to the bed form is almost π in the upper regime while it belongs to the forth quadrature in the lower regime, bed forms migrating upstream easily develop (antidunes) in the upper regime while those migrating downstream sometimes develop in the lower regime (dunes). Furthermore, since a convolution integration as seen in Eq. 20 becomes easy to analyze when it is transformed in a Fourier integration, the instability analysis has been applied to explain the spectral evolution of sand bed (Nakagawa & Tsujimoto (33)).

Figures 13 shows another experimental data for non-equilibrium bed-load transport rate (Bell & Sutherland (18)). According to the result, the adaptation length of bed-load transport rate increases with time and it reaches as long as 7~8 times the initial value of it. It implies a significance of non-equilibrium feature of bed-load transport (Jain (5), Di Silvio (2)). Such an increase of the adaptation length is caused by the developments of dunes and a scour hole at the upstream end of the mobile bed. In the experiments by Bell & Sutherland (18) a scouring and dunes developed with time; while they did not appreciably develop or the measurements were finished before their development in the experiments by Nakagawa & Tsujimoto (27) shown in Fig.7. Research works on the step length of bed-load motion over duned beds (Hung & Shen (25), Nakagawa & Tsujimoto (31)) suggests that the step length on a duned bed is proportional to the dune length, and thus, the adaptation length of bed-load transport is no longer of the scale of sand diameter but of the scale of flow depth (because the dune length is almost 5~10 times the flow depth). The distribution of the step length on a duned bed is no longer an exponential distribution but rather a gamma distribution of which shape parameter is almost 4.0 (31). Therefore, the impulse response of deposit rate and that of bed-load transport rate are different each other as seen from Eqs. 20 and 26, and thus the assumption that the deposit rate is proportional to the local transport rate is no longer valid for a duned bed.

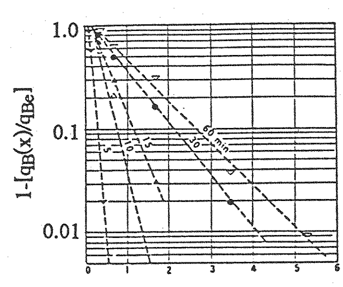

Fig.13 Spatial change of bed-load transport rate under non-equilibrium (Bell & Sutherland)

When the three-dimensional bed-forms are discussed, non-equilibrium bed-load transport promoted by the lateral bed slope should be focussed. Several analytical approaches to this topic were attempted by Hasegawa (24), Fukuoka & Yamasaka (21), Nakagawa et al. (35) and others.

Non-Equilibrium Bed-Load Transport of Graded Materials

When the bed materials are composed by several fractions of sediment size, non-equilibrium transport law of bed-load is written as follows for each fraction:

$$q_{Bi}(x) = \frac{A_3 d_i}{A_2} \int_0^\infty p_i(x-\varepsilon) p_{si}(x-\varepsilon) \int_\varepsilon^\infty f_{Xi}(\xi) d\xi \, d\varepsilon \qquad (28)$$

in which d_i=representative diameter of the i-th fraction of sediment; and p_i=volumetric ratio of the i-th fraction sand in the surface layer of the bed (often termed "exchange layer") to that of sand of all fractions in the surface layer. There are several works on the characteristics of bed-load transport for each size of graded materials (sediment mixtures) since Egiazaroff's idea (19) of the critical tractive force for each grain size. Among them, Nakagawa et al. (34, 37) proposed a modification of formulas for the pick-up rate and the step length of uniform size materials as follows, by which the pick-up rate and the step length for each grain size of sediment mixtures are evaluated:

$$p_{si*} \equiv p_{si}\sqrt{d_i/(\sigma/\rho-1)g} = F_0 \tau_{*i}(1-k_2\tau_{*ci}/\tau_{*i})^m \qquad (29)$$

$$f_{Xi}(\xi) = \frac{1}{\Lambda_i}\exp\left(-\frac{\xi}{\Lambda_i}\right) \quad ; \quad \Lambda_i = \lambda_G d_i \qquad (30)$$

in which $\tau_{*i} \equiv u_*^2/[(\sigma/\rho-1)gd_i]$, τ_{*ci}=dimensionless critical tractive force for each grain size; Λ_i =mean step length for each grain size; and the constant λ_G is almost 20~30 according to the tracer tests (37). τ_{*ci} is given by the following formula:

$$\frac{\tau_{*ci}}{\tau_{*cm}} = \left(\frac{\ln 19}{\ln 19\zeta_i}\right)^2 \quad (\zeta_i>0.4) \; ; \qquad \frac{\tau_{*ci}}{\tau_{*cm}} = \frac{0.85}{\zeta_i} \quad (\zeta_i \le 0.4) \qquad (31)$$

in which $\zeta_i \equiv d_i/d_m$; d_m=mean diameter of sand in the surface layer; and τ_{*cm}=dimensionless critical tractive force of sand of d_m in the mixture. Though τ_{*cm} is different from the dimensionless critical tractive force for uniform sand (τ_{*c0}), they are often identified with each other for simplicity. Eq. 31 was proposed by Ashida & Michiue (14) by modifying the Egiazaroff's formula (19).

When no sediment is supplied to a mobile bed composed of sand mixture, the bed surface of the region near the upstream end of the mobile bed becomes coarser than the original mixture (or the mixture in the "substratum" of the bed), and such a coarsened bed is termed "armor coat" or such a process is termed "armoring". Particularly when the flow intensity is so strong to pick up the minimum size material and not so strong to pick-up the maximum size material, an armor coat develops. Gessler (23) proposed a method to calculate the bed-surface constitution of an equilibrium armor coat. Under equilibrium of armoring, no sediment is picked-up from the armored area. In this sense, it is called "static armoring." In the process of armoring, bed-load transport is non-equilibrium, and thus the formation process of an armor coat cannot be described without non-equilibrium transport law for each grain size. Nakagawa et al. (34) proposed an analytical model to describe the formation and propagation of the armor coat, and this model could well explain the laboratory data obtained by Ashida & Michiue (14).

Against a static armor, an equilibrium coarsened bed where significant sediment transport exists are observed, and it is termed "pavement" (Parker & Klingeman (40, 41)) or often called "dynamic armoring." This concept becomes more important when one would describe morphological processes more appropriately (Parker & Andrews (39), Ikeda & Yamasaka (28)). The pavement process was described by applying Eq. 28 by Tsujimoto & Motohashi (47). According to this model, an increase or a decrease of the amount of sediment supply brings about a sorting of bed-surface constitution as a result of non-equilibrium and selective transport of sediment mixture. When the amount of sediment supply is suppressed, the bed is increasingly paved. On the pavement process, non-equilibrium characteristics of bed-load transport for each fraction varies with time, and finally the transport rate for each fraction becomes equal to the input bed-load discharge and an equilibrium pavement is accomplished. Fig.14 depicts the temporal spatial change of fractional local transport rate. It demonstrates that the adaptation length at the initial stage is certainly of the order of the step length but it appreciably increases with time. The adaptation length of bed-load motion of sand mixtures is negligibly small before sorting, but it increases appreciably with the progress of sorting.

Even if the spatial lag of bed-load motion is very short, it plays an important role on a sand bed instability (an instability of the fluctuation of the bed-surface elevation). Similarly, the spatial lag of fractional bed-load motion of sand mixture, which is different from each other according to the fractional size, plays a role in an instability of sorting. Unstable sorting brings about stripes due to the spatial difference of the local mean diameter of the bed-surface layer. A longitudinal alternation of coarser and finer parts, often called a "diffuse gravel sheet", was observed in the laboratories (Ikeda & Iseya (27), Kuhnle & Southerd (30), see Fig.15) and fields. A gravel sheet migrates downstream, and brings an appreciable temporal fluctuation of fractional bed-load transport rate (Kuhnle & Southerd (30), see Fig.16). The formation of a gravel sheet and its characteristics are well explained by the non-equilibrium trans-port model and an instability theory (Tsujimoto (45)). The longitudinal stripes as alternate lateral sorting due to cellular secondary current are also interesting and the formation process is well described by a similar manner (Tsujimoto (46)).

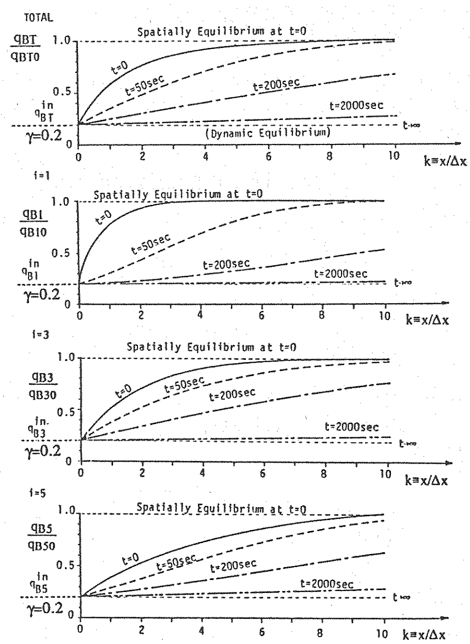

Fig.14 The temporal and spatial change of transport rate after sudden decrease of the sediment supply from from the initial equilibrium q_{Be} to γq_{Be}. γ=0.2 in this figure (q_{BT}=total bed-load transport rate; q_{Bi}= fractional bed-load transport rate.

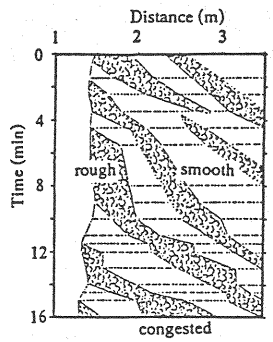

Fig.15 Diffuse gravel sheet as longitudinally alternating sorting (from Ikeda & Iseya's experiment)

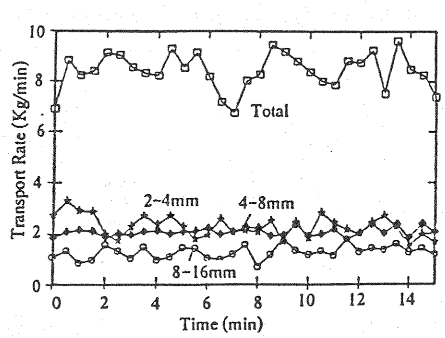

Fig.16 Temporal variation of fractional bed-load transport rate due to migration of diffuse gravel sheet (Kuhnle & Southard)

NON-EQUILIBRIUM TRANSPORT OF SUSPENDED SEDIMENT

Because the excursion length of a suspended particle (the travelling distance as a suspended load) is relatively long, the adaptation length becomes large and non-equilibrium feature of suspended load plays an important role on fluvial processes.

Relaxation Model

In the case of suspended sediment, the mass conservation law gives the following equation:

$$\frac{dq_S}{dx} = (C_a - C_{ae})w_0 \qquad (32)$$

in which q_S=suspended sediment transport rate; C_a=sediment concentration near the bed (the concentration at y=a, and simply called "bottom concentration" or "reference concentration"); w_0=terminal velocity of sediment; and the subscript e indicates the values under equilibrium. Under non-equilibrium condition, the response of the bottom concentration to the change of the bed shear stress is important. If the concentration profile and the velocity profile are similar to those under equilibrium respectively ($C/C_a=\phi_S(\eta)\equiv C_e/C_{ae}$ and $u/u_*=\psi_U(\eta)\equiv u_e/u_*$),

$$\frac{d}{dx}\left(\frac{C_a}{C_{ae}}\frac{u_*}{w_0}\right) = \frac{1}{\beta_{\phi\psi}}\left(1 - \frac{C_a}{C_{ae}}\right) \quad ; \qquad \beta_{\phi\psi} \equiv \int_{\eta_a}^{1} \phi(\eta)\psi(\eta)d\eta \qquad (33)$$

in which $\eta\equiv y/h$; and h=flow depth. For the simplest case such as the shear velocity changes abruptly, Eq. 33 becomes a simple ordinary differential equation with respect to C_a/C_{ae} and the following solution is obtained.

$$\frac{C_a - C_{ae}}{C_{a0} - C_{ae}} = 1 - \exp\left(-\frac{x}{\Lambda_{ST}}\right) \ ; \qquad \Lambda_{ST} \equiv \beta_{\phi\psi}\left(\frac{u_*}{w_0}\right) \qquad (34)$$

in which C_{a0}=bottom concentration before the change of the shear velocity; x=longitudinal distance from the abrupt change of u_*; and Λ_{ST}=relaxation length of suspended sediment discharge (the adaptation length of suspended sediment discharge is almost three times Λ_{ST}). Kalinske (59) and Ashida (49) proposed a simplified method to calculate the spatial change of C_a in almost the same manner as the above.

Figure 17 shows a development of suspended sediment concentration in the downstream mobile bed of a rigid bed measured by Yalin & Finlayson (75). Fig.18 shows the measured concentration distributions for the growing stage and the decaying stage of suspended sediment transport by Ashida & Okabe (52). These data demonstrate an importance of a degeneration of the concentration profile under non-equilibrium conditions. When the non-equilibrium sediment concentration profile is not similar to the equilibrium one, $\beta_{\phi\psi}$ becomes a function of x and Eq. 32 cannot be easily solved. In general, the relaxation is promoted by the degeneration of concentration profile and the relaxation length becomes shorter.

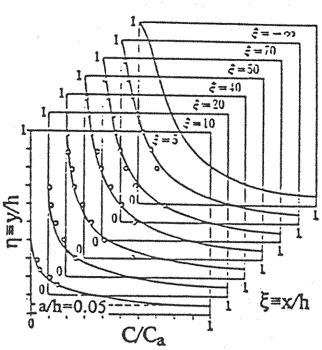

Fig.17 Development of suspended sediment concentration profile at downstream of a rigid bed (Yalin & Finlayson)

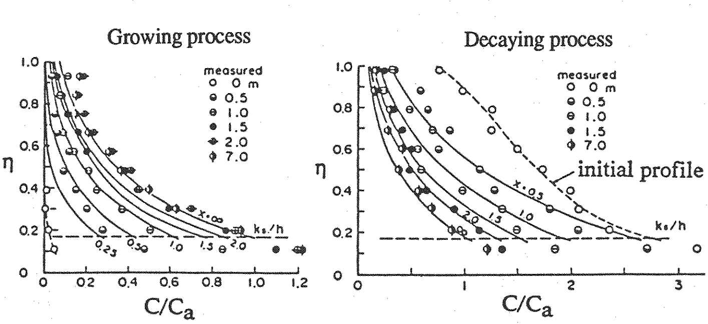

Fig.18 Growing stage and decaying stage of suspended sediment concentration profile (Ashida & Okabe)

Not only the sediment concentration profile but also the velocity distribution is a function of x in the transient process. In the transient region after an abrupt change of the bed roughness, the Reynolds stress-distribution shows a two-dimensional relaxation process as shown in Fig.19 (Jacobs (58)). The relaxation of the Reynolds stress is faster near the bottom and slower near the water surface. Such a characteristic can be well described by a convolution-integral model as follows (71):

$$\tau_T(\xi,\eta) = \int_0^\infty \tau_{Te}(\xi-\varepsilon,\eta) g_{RT}(\varepsilon \mid \eta) d\varepsilon \tag{35}$$

in which $\tau_T(\xi,\eta)$=Reynolds-stress distribution, $\tau_{Te}(\xi,\eta)=\tau_b(1-\eta)$=equilibrium Reynolds-stress distribution; $\tau_b=\rho u_*^2$=bed shear stress; $\xi \equiv x/h$; $g_{RT}(\varepsilon \mid \eta)$=impulse response of Reynolds stress at the dimensionless height η. The relaxation scale involved in $g_{RT}(\varepsilon \mid \eta)$ increases with η. Based on the data of a wind tunnel by Jacobs (58), $g_{RT}(\varepsilon \mid \eta)$ has been determined written as follows (71):

$$g_{RT}(\varepsilon \mid \eta) = \frac{1}{\lambda_T(\eta)} \exp\left(-\frac{\varepsilon}{\lambda_T(\eta)}\right) ; \qquad \lambda_T(\eta) = 20\eta(1+1.5\eta^2) \tag{36}$$

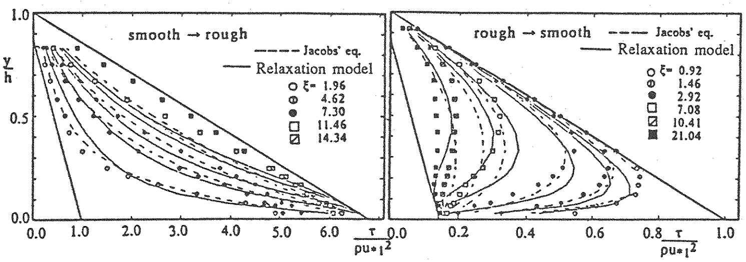

Fig.19 Transition process of Reynolds-stress distribution from Jacobs' wind-tunnel data and two-dimensional relaxation model

Tsujimoto et al. (71) reported Eq. 36 is still available for open channel flows by inspecting the data of the refined flow measurements in an open channel with LDA (Nakagawa et al. (67)). The velocity profile in the transient process can be well described by an integration of the following equation.

$$\frac{du}{dy} = \frac{\tau_T}{\rho \nu_T} ; \qquad \nu_T = \kappa_0 u_* y\left(1 - \frac{y}{h}\right) \tag{37}$$

in which ν_T=eddy kinematic viscosity; and κ_0=Kármán constant.

Based on the Reynolds' analogy, the relaxation of the vertical flux of suspended sediment by turbulence $\Psi_T \equiv \overline{c'v'}$ has the same impulse response as that of the Reynolds stress, and thus it can be described as

$$\Psi_T(\xi,\eta) = \int_0^\infty \Psi_{Te}(\xi-\varepsilon,\eta) g_{RT}(\varepsilon \mid \eta) d\varepsilon \tag{38}$$

and the concentration distribution can be obtained by the following integration.

$$\frac{dC}{dy} = -\frac{\Psi_T}{\varepsilon_s} ; \qquad \varepsilon_s = \beta_s \nu_T \tag{39}$$

in which $\Psi_{Te}(\xi,\eta)=\Psi_T(\xi,\eta)$ under equilibrium; C=volumetric concentration of suspended sediment; ε_s=diffusion coefficient of suspended sediment; and β_s=reciprocal of turbulent Schmidt number. The boundary condition is that $C=C_a$ at y=a, but it cannot be determined independently. Reversely, the non-equilibrium profile is necessary to calculate the change of C_a. Under equilibrium, the flux $\Psi_{Te}(\xi,\eta)$ is equal to the falling flux due to the terminal velocity, and thus,

$$\Psi_{Te}(\xi,\eta) = -w_0\, C(\xi,\eta) \tag{40}$$

Eq. 39 with Eqs. 40 brings the Rouse equation or the Lane-Kalinske equation as an equilibrium concentration profile of suspended sediment.

Eq. 36 suggests the turbulent flux of suspended sediment adapts the new condition faster near the bed and more slowly near the water surface. This is supported by the measured data of c'v' under the transient process by Sukegawa et al. (69) and Kanda et al. (60). Moreover, Eq. 39 with Eq. 38 implies that the concentration profile adapts its equilibrium gradient $(dC/dy)_e$ faster near the bed and more slowly near the water surface, and this is also consistent with many data in the transient process (Yalin & Finlayson (75) (see Fig.17), van Rijn (74) (see Fig.20), Ashida & Okabe (52) (see Fig.18), Kanda et al. (60)).

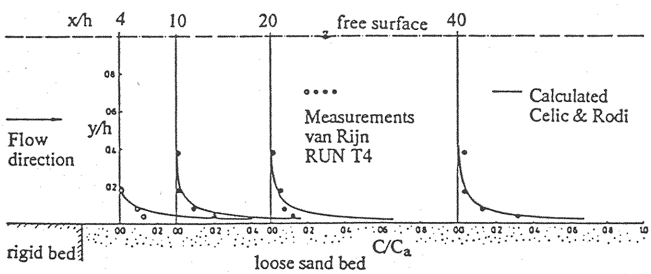

Fig.20 Development of suspended sediment concentration profile at downstream of a rigid bed (experiment by van Rijn, and calculated by Çelic & Rodi)

Approach by Diffusion Equation

A conventional approach to non-equilibrium suspended sediment transport is one based on a diffusion equation with a convection term. The fundamental equation for non-equilibrium suspended sediment transport under two-dimensional steady uniform flow is written as follows:

$$u\frac{\partial C}{\partial x} = \frac{\partial}{\partial y}\left(\varepsilon_s \frac{\partial C}{\partial y}\right) + w_0\frac{\partial C}{\partial y} \tag{41}$$

The following boundary conditions are adopted.

$$\left(\varepsilon_s \frac{\partial C}{\partial y} + w_0 C\right)_{y=h} = 0 \;; \qquad \left(\varepsilon_s \frac{\partial C}{\partial y} + w_0 C\right)_{y\to 0} = w_0(C_a - C_{ae}) \tag{42}$$

The bottom boundary condition is based on the fact that the turbulent flux of suspended sediment at the bottom responds immediately to the change of the bed shear stress.

When an above-explained approach is adopted, one has to know the spatial change of C_a but it depends on the change of the profile as mentioned precedingly, and the equations with respect to the velocity would be solved simultaneously.

The numerical calculation based on this approach was attempted by Kerssens et al. (57), Michiue et al. (63), Çelic & Rodi (54) and others. The computed example by Çelic & Rodi (54) is shown in Fig.20, where the computed results well explains the development of suspended sediment concentration profile in a mobile bed downstream of a rigid bed investigated by van Rijn (74). Michiue et al. (63) computed the change of the concentration profile to an abrupt change of the shear velocity (see Fig.21) and moreover computed an adaptation length of the concentration profile in the case that the shear velocity changes abruptly from u_{*1} to u_{*2}, as shown in Fig.22.

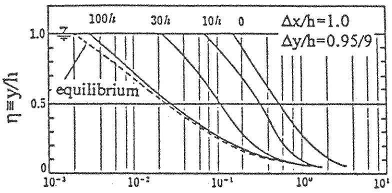

Fig.21 Change of suspended sediment concentration after abrupt decrease of shear velocity (calculated by Michiue et al.)

Kerssens et al. (57) computed an adaptation length of the concentration profile in the case that the depth-averaged concentration of the supplied sediment transport changes (see Fig.23), and they also applied the result to river morphological computation. Although a numerical method enabled a computation of non-equilibrium suspended sediment transport based on a diffusion equation, approaches based on it could not necessarily clarify the characteristics of non-equilibrium transport systematically.

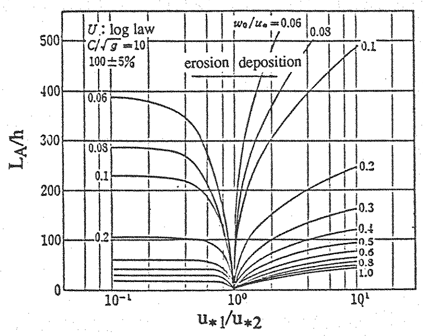

Fig.22 Adaptation length of suspended sediment concentration profile due to abrupt change of shear velocity (calculated by Michiue et al.)

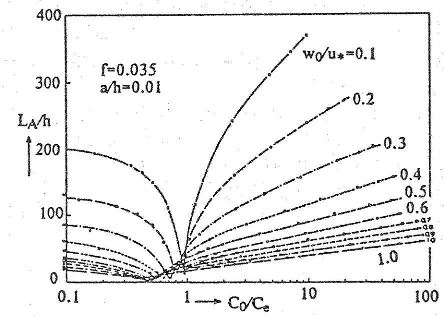

Fig.23 Adaptation length of suspended sediment profile for different concentration of supplied suspended sediment (calculated by Kerssens et al.)

A numerical calculation of turbulent flow has recently developed remarkably by an introduction of a k-ε model. For a sediment laden flow, Fukushima (56, 57), Kanda et al. (60) published some computed examples.

Kuroki et al. (61) obtained an approximated concentration distribution of non-equilibrium suspended sediment without solving Eq. 41 directly. They assumed a functional form of the concentration distribution as follows:

$$C(\eta) = C_a \exp(-R_E \eta) + K_1 \eta \cdot \exp(-K_2 \eta) \qquad (43)$$

in which $R_E \equiv w_0 h / \varepsilon_s$. Applying the boundary conditions indicated in Eq. 42, they determined the constants K_1 and K_2 as follows.

$$K_1 = R_E (C_a - C_{ae}) \; ; \qquad K_2 = R_E + 1 \qquad (44)$$

If C_a is estimated reasonably, Eq. 43 well approximates the non-equilibrium concentration profile of suspended sediment as shown in Fig.24. Although the second term represents the relaxation or non-equilibrium effect, the spatial change is included only in the constant K_2 but not in the functional form.

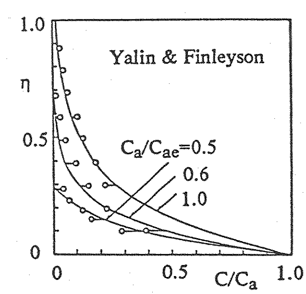

Fig.24 Applicability of simplified method by Kuroki et al. to the data of Yalin & Finlayson

Stochastic Approach

Another approach to non-equilibrium suspended sediment transport is stochastic or probabilistic one. Yalin & Krishnappan (76) made a good progress of a stochastic approach to suspended sediment transport. They deduced a concentration profile by a stochastic approach (simulation). However, a stochastic approach is more effective to non-equilibrium conditions. A stochastic simulation will easily draw non-equilibrium concentration profiles. Several attempts based on such an idea were conducted by Bechteler & Fäber (53), Ashida & Fujita (50), and others. Fig.25 is an example of stochastic simulation for development of suspended sediment concentration profile at the downstream of a rigid bed (53). Furthermore, such an approach has been

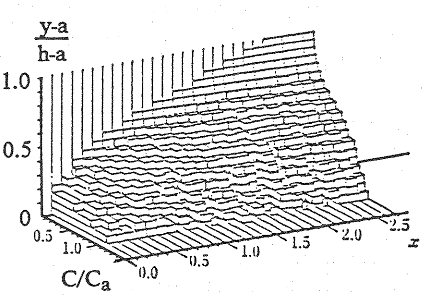

Fig.25 Example of stochastic simulation of development process of suspended sediment concentration by Bechteler & Fäber.

applied to non-equilibrium suspended sediment transport over dunes (Nakagawa et al. (64)).

According to a stochastic model, an excursion length of a suspended particle corresponds to the step length of bed-load transport in a concept, and thus an adaptation length of suspended sediment transport might be a few times the excursion length. The distribution of the excursion length was investigated by a stochastic simulation (Tsujimoto &Yamamoto (73), Ohmoto et al. (68)), and the results suggested that the mean excursion length varies from a few times the flow depth to a few dozens of the flow depth with the increase of (u_*/w_0). The variation coefficient of the excursion length decreases from 2.0 to less than 1.0 with the increase of (u_*/w_0).

In stochastic simulation, there are still many difficult problems to evaluate constituent parameters (for example, the time step of stochastic trials), which should be reasonably solved. Conveniently, they are determined so as to obtain the Rouse distribution under equilibrium condition. Furthermore, a problem how to determine the bottom concentration still remains for a stochastic model. In order to solve it, a transport model of bed-material load including both bed load and suspended load should be established. An attempt was conducted by Nakagawa et al. (65, 72). They defined the bed load motion as sediment motion incessantly contacting a bed and the suspension as random sediment motion subjected to the turbulence. Based on the detailed observation of bed-material load transport in the laboratory flume, they imagined a bed-material particle's motion as follows: A bed-material particle is picked up from a bed and starts a bed-load motion which is expressed as "irregular successive saltation". The turbulence sometimes makes a saltating particle entrained into the body of the flow, and it is a transition from bed-load motion into suspension. After a relatively long drift as a suspended particle, it returns to the bed again. This is a transition from suspension to bed-load motion. Based on the above image, the following non-equilibrium transport law of bed material-load was proposed:

$$q_B(x) = \frac{A_3 d}{A_2}\int_0^{\infty}\{[p_s(x-\xi)+p_{ds}(x-\xi)]\cdot[\int_{\xi}^{\infty} f_X(\zeta)d\zeta]\cdot F_{TR}(\xi)\}d\xi \tag{45}$$

$$q_S(x) = \int_0^{\infty}\{\frac{q_B(x-\xi)}{u_g(x-\xi)}\cdot p_T(x-\xi)\cdot[\int_{\xi}^{\infty} f_{XS}(\zeta)d\zeta]\}d\xi \tag{46}$$

$$p_{ds}(x) = \frac{A_2}{A_3 d}\int_0^{\infty}[\frac{q_B(x-\xi)}{u_g(x-\xi)}\cdot p_T(x-\xi)\cdot f_{XS}(\xi)]d\xi \tag{47}$$

in which $F_{TR}(\xi)$=the probability that a bed-load particle does not turn into suspended load during traveling a distance ξ after its incipient motion (dislodgement from a bed); $p_{ds}(x)$=number of suspended particles per unit time to deposit (not necessarily to stop) on the bed of the area occupied by one particle at x to turn into bed-load motion; u_g=bed-load particle's speed; p_T=probability density of transition from bed-load motion to suspension per unit time; and $f_{XS}(\xi)$=probability density function of the excursion length. $F_{TR}(\xi)$ might be approximated as follows:

$$F_{TR}(\xi) = \exp(-\frac{p_T}{u_g}\xi) \tag{48}$$

In this modelling, the transition from bed-load to suspension is a key, and it has been recently analyzed as an instability of the saltation trajectory by using a logistic equation (66, 70).

NON-EQUILIBRIUM DUE TO RELAXATION OF BED FORMS GEOMETRY

As mentioned in the preceding chapter, the bed form affects the sediment transport and it requires a relatively long lag time to be adapted to the change of flow condition. Fig.26 shows a hysteresis appearing in the relation between the dune geometry and the flow discharge in an actual river (Allen (77)). Such a hysteresis is a result of a relaxation, as illustrated in Fig.27. Fredsφe (4) and Nakagawa & Tsujimoto (7) analyzed such a relaxation process by a linear approximation. Fig.28 demonstrates the phase shift and the amplification of the fluctuating amplitude of the dune height for sinusoidally fluctuating flow discharge obtained in the analysis by Nakagawa & Tsujimoto (7), in which ω=angular frequency of the fluctuation; and T_{EX}=time scale for dunes to grow from an initially flattened bed to the fully developed state. In this analysis, the following impulse response was adopted.

$$g_{RD}(t) = \frac{H_e}{q_0 T_{EX}} \exp\left(-\frac{t}{T_{EX}}\right) \tag{49}$$

in which H_e=equilibrium dune height for q_0; and q_0=average flow discharge per unit width. Then, the response of dune height to the fluctuation of the flow discharge q(t) is described as follows:

$$H(t) = \int_0^{\infty} q(t-\delta) g_{RD}(\delta) d\delta \tag{50}$$

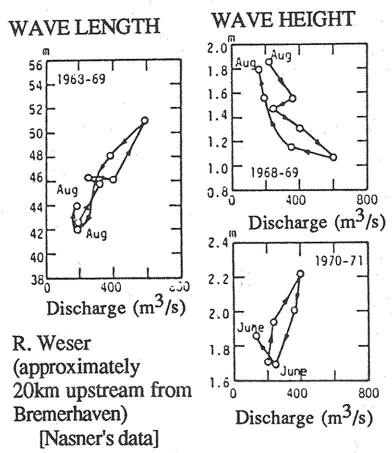

Fig.26 Data in an actual river of bedform response to the flow-doscharge fluctuation

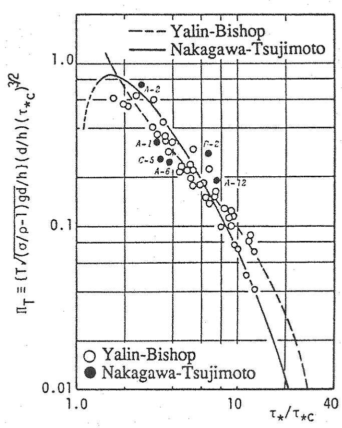

Fig.29 Time scale for dunes to reach the fully-developed dimension from initially flattened bed

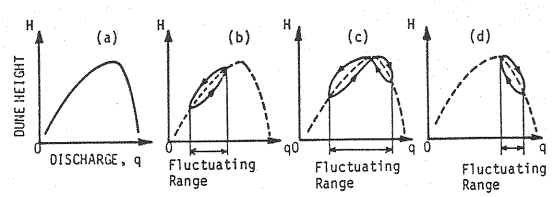

Fig.27 Relation between dune height and flow discharge and expected hysteresis loop

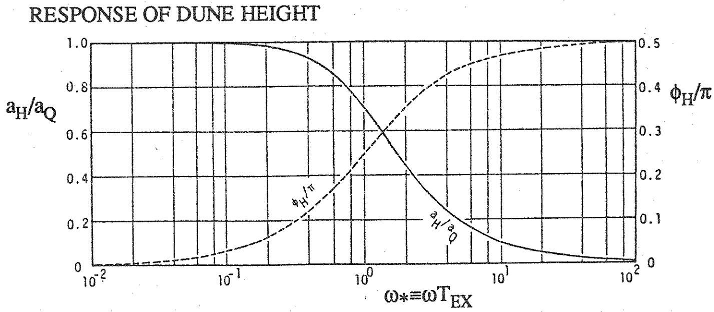

Fig.28 Response properties of dune height to the fluctuation of flow discharge(a_H=amplitude of bed undulation; a_Q=amplitude of fluctuation of flow discharge; ϕ_H=phase shift of duhe height to flow discharge)

The investigation of dune development from an initially flattened bed contributes to an estimation of the impulse response. Eq. 49 does not necessarily well correspond to the previous results of laboratory experiments (Yalin & Bishop (83); Nakagawa & Tsujimoto (79)), but Eq. 49 is easy for analytical calculations. Yalin & Bishop (83) obtained an empirical equation to express the dune development and estimated T_{EX} based on it. Nakagawa & Tsujimoto (79) proposed an analytical model for dune development, and exterpolated the time required for dunes to reach the fully-developed geometry from an initially flattened bed, T_D, based on their model ($T_D \cong 2T_{EX}$). The results are shown in Fig.29.

When the amplitude of fluctuation is not so small, the impulse response varies with time. Then, a numerical calculation describe a phenomenon, and the instantaneous growth rate (the time derivative of the bed-form geometry) becomes important. Several experimental data (Nakagawa et al. (79, 80); Ashida & Sawai (78), Wijbenga & Klaassen (82)) can be used for determination of the process rate. An example of empirically formulated process rate is as follows (Tsujimoto & Nakagawa (81)):

$$\frac{dL}{dt} = \frac{\beta_L q_B}{(1-\rho_0)H}\left[1-\frac{L-L_0}{L_e-L_0}\right] \quad ; \qquad \frac{dH}{dt} = \frac{\beta_H q_B}{(1-\rho_0)L}\left[1-\frac{H}{H_e}\right] \tag{51}$$

in which L=dune length; β_L, β_H=empirical constants (β_H=0.36 and β_L=0.24 for growing stage of dunes; while β_H=0.72 and β_L=0 for decaying stage of dunes); ρ_0=porosity of sand; L_0=minimum dune length which appears initially in the dune-development process from an initially flattened bed. The dune length and height in the transient process can be computed as follows:

$$L(t+\Delta t) = L(t) + \left(\frac{dL}{dt}\right)_t \Delta t \quad ; \qquad H(t+\Delta t) = H(t) + \left(\frac{dH}{dt}\right)_t \Delta t \tag{52}$$

Fig.30 is a computed example (81), where an 8-figure loop appears which cannot appear in a linear analysis. Fig.31 shows a comparison between the computed results and the laboratory experiments (81).

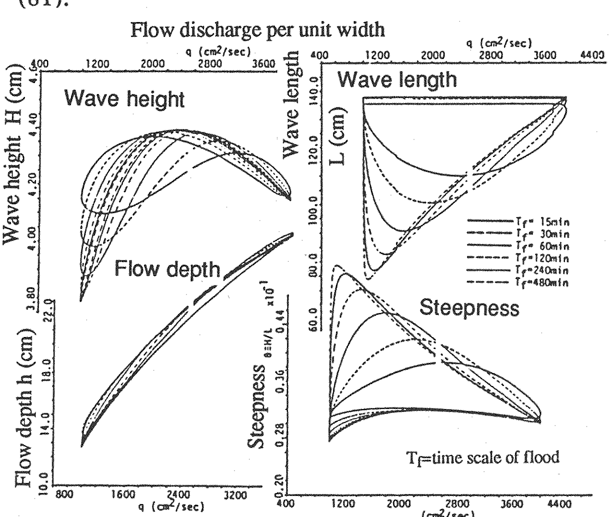

Fig.30 Calculated examples of response of dune-dimensions and flow depth to fluctuation of flow discharge (Tsujimoto & Nakagawa)

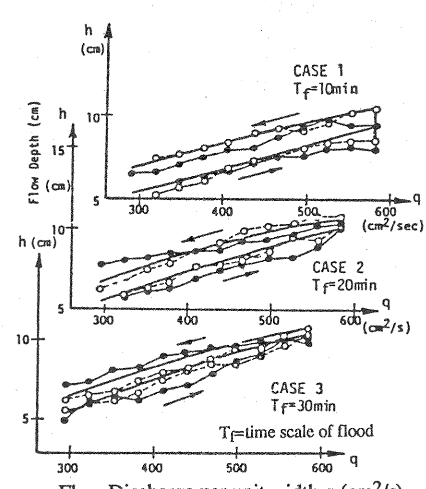

Fig.31 Hysteresis loop in the relation between depth and discharge of unsteady flow with dunes (flume experiment and calculated examples by Tsujimoto & Nakagawa)

CONCLUSIONS

The transient process of sediment transport is affected not only by flow-unsteadiness or/and nonuniformity but also by non-equilibrium characteristics. The former effects might be expressed as functions of the temporal or/and spatial derivative of the flow parameters. While, the latter effects appear even when these derivatives are zero, and they are often called hysteresis. In this paper, they are termed relaxation processes, and a relaxation of sediment transport is termed non-equilibrium sediment transport. Besides the non-equilibrium sediment transport itself, there are several examples of relaxation processes in fluvial phenomena such as bed-form deformation. The sediment transport with such a relaxation is also treated in this paper because it might be described in the same manner.

As a simplified and generalized description of relaxation processes, a convolution-integral model is proposed, where the impulse response is a key. An impulse response expressed by an exponential function often approximates several relaxation processes. This model has been explained in Chapter 2. In Chapter 3, the essential difference between the direct effects of flow-unsteadiness and non-uniformity on sediment transport and the indirect effect due to relaxation has been demonstrated in a linear mode. The former brings about a phase shift (either positive or negative) and an amplification of the amplitude; while the latter brings about a phase lag and a suppression of the amplitude.

In Chapter 4, non-equilibrium transport of bed load has been discussed. Several proposals have been explained in comparison. Particularly, the model constituted by the pick-up rate and the step length is the most effective one for description of several fluvial processes due to non-equilibrium transport characteristics. The bed-load motion is essentially non-equilibrium which is caused by the step length. Because the mean step length is the order of the sediment diameter, non-equilibrium feature of bed-load transport is often neglected. However, it shoud be considered on exterpolating the prototype phenomena from the small scale model tests, and it often plays an important role on the instability of sand bed elevation (sand-wave formation) and that of bed-surface constitution (alternate sorting stripes). The transport of graded materials often becomes non-equilibrium with its selective transport characteristics for each size. The non-equilibrium transport law can be written for each grain size of sediment mixtures. The adaptation length of non-equilibrium bed-load transport is certainly short, but it becomes longer to reach the order of the flow depth with the growth of bed undulation or with the progress of sorting.

In Chapter 5, non-equilibrium suspended sediment transport has been discussed. In the case of the suspended sediment, not only the transport rate but also the concentration profile become non-equilibrium and they are interacted each other. Firstly, an application of a relaxation model has been attempted. With a comparison to the transient process of the Reynolds-stress distribution, it has been suggested that the turbulent flux of sediment adapts the new equilibrium faster near the bottom and more slowly near the water surface. Moreover, it has been pointed out that the relaxation of the concentration profile promotes that of the bottom concentration of suspended sediment. Besides, the recent works by an approach based on a diffusion equation and a stochastic approach have been explained. As for the former, some diagrams for the adaptation length of suspended sediment transport for sudden change of the bed shear stress or the concentration of the supplied sediment are introduced. As for the latter, an attempt of modelling bed-material load transport including both bed load and suspended sediment has been introduced.

In Chapter 6, the relaxation process of dune geometry has been discussed based on a convolution-integral model. The deformation process of dunes brings about a temporal lag and it might cause a temporally non-equilibrium sediment transport.

The main purpose of this paper has been to clarify what is non-equilibrium sediment transport, and thus several problems on non-equilibrium processes in fluvial streams have been treated as similarly as possible. A convolution-integral modelling is an effective one of the means to satisfy this purpose. For respective non-equilibrium phenomena, several other approaches have been also introduced but the relation with the common framework has been focussed. The proposed model is certainly powerful, but some rigorous mechanics involved in the non-equilibrium processes has been rather slighted. Fundamental studies would be added to this paper in future, and also more applied aspects should be developed.

Most part of this paper is translated from the Japanese paper (12), though several parts are revised.

ACKNOWLEDGEMENTS

This paper is a progress report of the research project "Mechanics of non-equilibrium sediment transport and its application." encouraged by Hydraulic Committee of JSCE. The research group started in 1986, when Prof. Hiroji Nakagawa (Kyoto University) and Prof. Shoji Fukuoka (Tokyo Institute of Technology) helped its inauguration. Prof. Masanori Michiue (Tottori University) was the advisor of this research group. All the member of this group express their gratitude to them.

REFERENCES

Transient or Relaxation Process:

1. Allen, J.R.L : Reaction, relaxation and lag in natural sedimentary system: General principles, examples and lessons. *Earth Science Review*, Vol.10, pp.263-342, 1974.
2. Di Silvio, G. : Adaptation processes through the surface of sedimentary systems. *Proc. EUROMECH 215, Mechanics of Sediment Transport in Fluvial and Marine Environments*, St. Margherita, Italy, 1987.
3. Graf, W.H. and L. Suszka : Unsteady flow and its effect on sediment transport. *Proc. 21st IAHR Cong.*, Merbourne, Australia, Vol.3, pp.539-544, 1985.
4. Fredsϕe, J. : Unsteady flow in straight alluvial streams - Modification of individual dunes. *Jour. of Fluid Mech.*, Vol.91, Part 3, pp.497-512, 1979.
5. Jain, S.C. : Sediment transport under nonequilibrium conditions. *Proc. NATO Workshop on Movable Bed Physical Models*, De Voorst, The Netherlands, 1987.
6. Klaassen, G.J. : On time scale of transient processes in rivers. *Proc. Int. Conf. on Fluvial Hydraul.*, Budapest, Hungary, pp.343-360, 1988.
7. Nakagawa, H. and T. Tsujimoto : Time-lag appearing in unsteady flow with sand waves. *Jour. of Hydroscience and Hydraul. Eng.*, JSCE, Vol.1, No.1, pp.83-95, 1983.
8. Phillips, B.C. and A.J. Sutherland : Numerical modelling of spatial and temporal lag effects in bed load transport. *Proc. 21st Congress of IAHR*, Melbourne, Australia, Vol.3, pp.571-576, 1985.
9. Suszka, L. and W.H. Graf : Sediment transport in steep channels at unsteady flow. *Proc. 22nd IAHR Cong., Fluvial Hydraul.*, Lausanne, Switzerland, pp.166-170. 1987.
10. Tsujimoto, T. : Effect of unsteadiness, non-uniformity and relaxation on bed load transport process. *Proc. Int. Conf. on Fluvial Hydraul.*, Budapest, Hungary, pp.70-75, 1988.
11. Tsujimoto, T., W.H. Graf and L. Suszka : Bed-load transport in unsteady flow. *Proc. 6th APD-IAHR Cong.*, Kyoto, Japan, Vol.2, pp.15-22, 1988.
12. Tsujimoto, T., A. Mori, T. Okabe and T. Ohmoto : Non-equilibrium sediment transport. *Proc. 33rd Jap. Conf. on Hydraul.*, JSCE, pp.445-461, 1989 (in Japanese).
13. Yen, B.C. : Significance of nonuniform, unsteady and nonequilibrium conditions in fluvial channels. *Proc. Int. Conf. on Fluvial Hydraul.*, Budapest, Hungary, pp.112-118, 1988.

Bed-Load Transport:

14. Ashida, K. and M. Michiue : Studies on bed load transportation for nonuniform sediment and river bed variation. *Annuals*, Disas. Prev. Res. Inst., Kyoto Univ., No.14B, pp.259-273, 1971 (in Japanese).
15. Ashida, K. and M. Michiue : Study on hydraulic resistance and bedload transport rate in alluvial streams. *Proc. JSCE*, No.206, pp.59-69, 1972 (in Japanese).
16. Bagnold, R.A. : The flow of cohesionless grains in fluids. *Phil. Trans. of Royal Soc. of London*, Vol.249, 1957.
17. Bagnold, R.A. : An approach to the sediment transport problem from general physics. *U.S. Geol. Survey, Prof. Paper* 422-J, pp.1-37, 1966.
18. Bell, R.G. and A.J. Sutherland : Nonequilibrium bedload transport by steady flows. *Jour. of the Hydraul. Div.*, ASCE, Vol.109, pp.351-367.
19. Egiazaroff, I.V. : Calculation of nonuniform sediment concentration. *Jour. of the Hydraul. Div.*, ASCE, Vol.91, HY4, pp.73-80, 1965.
20. Einstein, H.A. : The bed-load function for sediment transportation in open channel flows. *Tech. Bulletin*, No.1026, U.S. Dep. of Agr., Soil Conservation Service, pp.1-78, 1950.
21. Fukuoka, S. and M. Yamasaka : Alternating bars in a straight channel. *Proc. 27th Jap. Conf. on Hydraul.*, JSCE, pp.703-708, 1983 (in Japanese).
22. Fukuoka, S., M. Yamasaka and K. Abe : Nonequilibrium processes in alluvial channels. *Proc. 28th Jap. Conf. on Hydraul.*, JSCE, pp.679-684, 1984 (in Japanese).
23. Gessler, J. : Self stabilizing tendencies of alluvial channels. *Jour. of the Waterways, Harbors and Cosatal Eng. Div.*, Vol.96, WW 2, pp.235-248, 1970.
24. Hasegawa, K. : Bank-erosion discharge based on a non-equilibrium theory. *Proc. JSCE*, No.316, pp.37-50, 1981 (in Japanese).
25. Hayashi, T. and S. Ozaki : On the unit step length of saltation of sediment particles in the bed-load layer. *Proc. 3rd Int. Symp. on Stochastic Hydraul.*, Tokyo, Japan, pp.337-346, 1980.
26. Hung, C.S. and H.W. Shen : Statistical analysis of sediment motions on dunes. *Jour. of the*

Hydraul. Div., ASCE, Vol.105, HY3, pp.213-227, 1979.
27. Ikeda, H. and F. Iseya : Longitudinal sorting process in heterogeneous sediment transportation. *Proc. 30th Jap. Conf. on Hydraul.*, JSCE, pp.217-221, 1986 (in Japanese).
28. Ikeda, S., M. Yamasaka and M. Chiyoda : Stable bed topography and sorting in uniformly-curved channels with heterogeneous bed materials. *Proc. JSCE*, No.375, pp.151-160, 1986 (in Japanese).
29. Kennedy, J.F. : The mechanics of dunes and antidunes in erodible-bed channels. *Jour. of Fluid Mech.*, Vol.16, Part 4, pp.521-554, 1963.
30. Kuhnle, R.A. and J. Southard : Bed load transport fluctuations in gravel bed laboratory channel. *Water Resources Res.*, Vol.24, No.2, pp.247-260, 1988.
31. Nakagawa, H. and T. Tsujimoto : Characteristics of sediment transport process on duned beds analyzed by stochastic approach. *Bulletin*, Disas. Prev. Res. Inst., Kyoto Univ., Vol.29, pp.45-63, 1979.
32. Nakagawa, H. and T. Tsujimoto : Sand bed instability due to bed load motion. *Jour. of the Hydraul. Div.*, ASCE, Vol.106, HY12, pp.2029-2051, 1980.
33. Nakagawa, H. and T. Tsujimoto : Spectral analysis of sand bed instability. *Jour. of Hydraul. Eng.*, ASCE, Vol.110, No.4, pp.467-483, 1984.
34. Nakagawa, H., T. Tsujimoto and T. Hara : Armoring an alluvial bed composed of sediment mixtures. *Annuals*, Disas. Prev. Res. Inst., Kyoto Univ., No.20B-2, pp.355-370, 1977 (in Japanese).
35. Nakagawa, H., T. Tsujimoto and H. Miyamoto: 16mm film analysis of characteristic quantities of bed load transport. *Annuals*, Disas. Prev. Res. Inst., Kyoto Univ., No.21B-2, pp.407-421, 1978 (in Japanese).
36. Nakagawa, H., T. Tsujimoto and S. Murakami : Non-equilibrium bed load transport along side slope of an alluvial stream. *Proc. 3rd Int.. Symp. on River Sedimentation*, Jackson, Mississippi, USA, pp.885-893, 1986.
37. Nakagawa, H., T. Tsujimoto and S. Nakano : Characteristics of sediment motion for respective grain sizes of sediment mixtures. *Bulletin*, Disas. Prev. Res. Inst., Kyoto Univ., Vol.32, Part 1, pp.1-32, 1982.
38. Parker, G. : Sediment inertia as cause of river antidunes. *Jour. of the Hydraul. Div.*, ASCE, Vol.101, HY2, pp.211-221, 1975.
39. Parker, G. and E.D. Andrews : Sorting of bed load sediment by flow in meander bends. *Water Resources Res.*, Vol.21, No.9, pp.1361-1373, 1985.
40. Parker, G. and P.C. Klingeman : On why gravel bed streams are paved. *Water Resources Res.*, Vol.18, No.5, pp.1409-1423, 1982.
41. Parker, G., P.C. Klingeman and D.G. McLean : Bedload and size distribution in paved gravel-bed streams. *Jour. of the Hydraul. Div.*, ASCE, Vol.108, HY4, pp.544-571, 1982.
42. Tsubaki, T. and T. Saito : Regime criteria for sand waves in erodible-bed channels. *Annuals*, Fac. of Eng., Kyushu Univ., Vol.40, No.5, pp.741-748, 1967 (in Japanese).
43. Tsuchiya, Y. and Y. Iwagaki : On the mechanism of the local scour from flows downstream of an outlet. *Proc. 12th IAHR Cong.*, Fort Collins, Colorado, USA, Vol.3, 1967.
44. Tsujimoto, T. : Non-uniform bed load transport and equilibrium bed profile. *Proc. 22nd IAHR Cong.*, *Fluvial Hydraulics*, Lausanne, Switzerland, pp.177-182, 1987.
45. Tsujimoto, T. : Instability of bed-surface composition due to sorting in a stream composed of sand and gravel. *Proc. Int. Symp. on Sediment Transport Modeling*, New Orleans, USA, S12, 1989.
46. Tsujimoto, T. : Longitudinal stripes of alternate lateral sorting due to cellular secondary currents. *Proc. 23rd IAHR Cong.*, Ottawa, Canada, Vol.2, pp.17-24, 1989.
47. Tsujimoto, T. and K. Motohashi : Sorting process and dynamic equilibrium of graded material transport in open channels. *Mem.*, Fac. of Tech., Kanazawa Univ., Vol.21, No.2, pp.9-17, 1988.
48. Yano, K., Y. Tsuchiya and M. Michiue : On the stochastic characteristics of transport mechanism of sand in a stream. *Annuals*, Disas. Prev. Res. Inst., Kyoto Univ., No.11B, pp.61-73, 1968 (in Japanese).

Suspended-Load Transport:

49. Ashida, K. : Study on the suspended sediment transportation under non-equilibrium condition. *Annuals*, Disas. Prev. Res. Inst., Kyoto Univ., No.13B-2, pp.161-270, 1970 (in Japanese).
50. Ashida, K. and M. Fujita : A stochastic model to calculate equilibrium and non-equilibrium suspended sediment load in open channel. *Proc. JSCE*, No.375, pp.107-116, 1986 (in Japanese).
51. Ashida, K. and M. Michiue : Study on suspended sediment. *Annuals*, Disas. Prev. Res. Inst., Kyoto Univ., No.13B-2, pp.63-79, 1970 (in Japanese).
52. Ashida, K. and T. Okabe : On the calculation method of the concentration of suspended sediment

under non-equilibrium condition. *Proc. 26th Jap. Conf. on Hydraul.*, JSCE, pp.153-157, 1982 (in Japanese).
53. Bechteler, W. and K. Fäber : Stochastic model of suspended solids dispersion. *Jour. of Hydraul. Eng.*, Vol.111, No.1, pp.64-78, 1985.
54. Çelic, I. and W. Rodi : A deposition-entrainment model for suspended sediment transport. *Sonderforschungsbereich*, 215, Univ. of Karlsruhe, 1984.
55. Di Silvio, G. and A. Armanini : Influence of the upstream boundary conditions of the erosion-deposition processes in open channel. *Proc. 14th IAHR Cong.*, A(a)22, New Delhi, India, 1981.
56. Fukushima, Y. : Analysis of non-uniform flow with suspended sediment by using k-ε turbulence model. *Proc. 3rd Int. Symp. on Refined Flow Modelling and Turbulence Measurements*, Tokyo, Japan, pp.577-584, 1988.
57. Fukushima, Y. and M. Fukuda : Analysis of turbulent structure of open-channel flow with suspended sediment. *Jour. of Hydroscience and Hydraul. Eng.*, JSCE, Vol.4, No.2, pp.47-54, 1986.
58. Jacobs, W. : Umformung eines turbulenten Geschwindigkeits-Profils. *ZAMM* 19, pp.87-100, 1939 (see *Boundary-Layer Theory*, 6th ed. by H. Schlichting, pp.615-616, McGraw-Hill, 1968).
59. Kalinske, A. : Suspended material transportation under non-equilibrium conditions. *Trans. A.G.U.*, 1940.
60. Kanda, K., Y. Muramoto, Y. Fujita and T. Yada : Transition process of turbulent structure and suspended sediment concentration in open channel with sudden bottom roughness change. *Annuals*, Disas. Prev. Res. Inst., Kyoto Univ., No.32B-2, pp.619-639, 1989 (in Japanese).
61. Kuroki, M., Y. Shi and T. Kishi : Distribution of suspended sediment concentration in nonequilibrium state. *Proc. 32nd Jap. Conf. on Hydraul.*, JSCE, pp.407-412, 1988 (in Japanese).
62. Kerssens, P.J.M., A. Prins and L.C. van Rijn : Model for suspended sediment transport. *Journal of the Hydraul. Div.*, ASCE, Vol.105, HY5, pp.461-471, 1979.
63. Michiue, M., M. Hirai and M. Umiga : Concentration distribution of suspended sediment under nonequilibrium condition. *Proc. 36th Annual Conf. of JSCE*, II-303, 1981 (in Japanese).
64. Nakagawa, H., S. Murakami and H. Gotoh : Structure of flow and dispersion process of suspended particles over two-dimensional dunes. *Proc. Hydraul. Eng.*, JSCE, Vol.34, pp.523-528, 1990 (in Japanese).
65. Nakagawa, H., T. Tsujimoto and S. Murakami : Convolution-integral modeling of non-equilibrium sediment transport. *Proc. Int. Symp. on Sediment Transport Modeling*, New Orleans, USA, S23, 1989.
66. Nakagawa, H., T. Tsujimoto, S. Murakami and H. Gotoh : Transition mechanism of saltation bed particles into suspension. *Proc. 32nd Jap. Conf. on Hydraul.*, JSCE, pp.529-534, 1988 (in Japanese).
67. Nezu, I., H. Nakagawa, K. Seya and Y. Suzuki : Response of velocity distribution and bed shear stress of open channel flow with abrupt change of bed roughness. *Proc. Hydraul. Eng.*, JSCE, Vol.34, pp.505-510, 1990 (in Japanese).
68. Ohmoto, T., M. Hirano and H. Ishimaru : Numerical simulation of particle trajectories in open channel flow. *Proc. 31st Jap. Conf. on Hydraul.*, JSCE, pp.593-598, 1987 (in Japanese).
69. Sukegawa, N., W. Nishikiori, H. Sugiyama, H. Simomura and T. Nakagawa : Numerical analysis of sedimentation of suspended load. *Proc. 26th Jap. Conf. on Hydraul.*, JSCE, pp.147-152, 1982 (in Japanese).
70. Tsujimoto, T. : Sediment suspension and transition from saltation to suspension. *Proc. EUROMECH 215, Mechanics of Sediment Transport in Fluvial and Marine Environments*, St. Margherita, Italy, pp.18-21, 1987.
71. Tsujimoto, T., A. Cardoso and A. Saito : Open channel flow with variable bed shear stress. *Proc. Hydraul. Eng.*, JSCE, Vol.34, pp.433-438, 1990 (in Japanese).
72. Tsujimoto, T. and H. Nakagawa : Saltation and suspension. *Proc. 3rd Int. Symp. on River Sedimentation*, Jackson, Mississippi, USA, pp.777-786, 1986.
73. Tsujimoto, T. and T. Yamamoto : Sediment suspension affected by transition from bed-load motion into suspension. *Jour. of Hydroscience and Hydraul. Eng.*, JSCE, Vol.6, No.1, pp.35-45, 1988.
74. Van Rijn : Entrainment of fine sediment particles; Development of concentration profiles in a steady, uniform flow without initial sediment load. *Rep.*, M1531, Delft Hydraulic Laboratory, 1983.
74. Yalin, M.S. and G.D. Finlayson : On the development of the distribution of suspended load. *Proc. 16th IAHR Cong.*, Istanbul, Turkey, Vol.1, pp.287-294, 1973.
75. Yalin, M.S. and B.M. Krishnappan : A probabilistic method for determining the distribution of suspended solids in open channels. *Proc. Int. Symp. on River Mech.*, Bangkok, Thailand, A52, pp.1-12, 1973.

Relaxation of Bed Forms:

76. Allen, J.R.L : Time-lag of dunes in unsteady flows; An analysis of Nasner's data from the River Weser, Germany. *Sedimentary Geol.*, Vol.15, pp.309-321, 1976.
77. Ashida, K. and K. Sawai : Transformation process of sand waves. *Annuals*, Disas. Prev. Res. Inst., Kyoto Univ., No.23B-2, pp.457-473, 1980 (in Japanese).
78. Nakagawa, H. and T. Tsujimoto : Experimental study on evolution of sand waves from an initially flattened bed. *Annuals*, Disas. Prev. Res. Inst., Kyoto Univ., No.19B-2, pp.289-309, 1976 (in Japanese).
79. Nakagawa, H., T. Tsujimoto and A. Yada : On relation between bed configuration and sediment transport and deformation process of sand waves. *Annuals*, Disas. Prev. Res. Inst., Kyoto Univ., No.21B-2, pp.385-405, 1978 (in Japanese).
80. Tsujimoto, T. and H. Nakagawa : Unsteady behaviour of dunes. *Channels and Channel Control Structures*, edited by K.V.H. Smith, Springer-Verlag, pp.85-99, 1984.
81. Wijbenga, J.H.A. and G.J. Klaassen : Changes in bedform dimensions under unsteady flow conditions in a straight flume. *Pub.* No.260, Delft Hydraulic Laboratory, 1981.
82. Yalin, M.S. and C.T. Bishop : On the physical modelling of dunes. *Proc. 17th IAHR Cong.*, Baden-Baden, West Germany, Vol.1, pp.25-32, 1977.

APPENDIX - NOTATION

The following symbols are used in this paper:

A_2, A_3	= two- and three-dimensional geometrical coefficients of sand;
$a_\Psi \Psi_0$	= amplitude of fluctuating process of Ψ;
C	= suspended sediment concentration;
C_a	= reference (bottom) concentration of suspended sediment;
D	= volume of bed-load particles depositing on the bed per unit time per unit area;
d	= diameter of sand;
d_m	= mean diameter of sand mixture;
E	= volume of bed-load particles dislodged from the bed per unit time per unit area;
F_0	= empirical constant;
$F_{TR}(x)$	= probability for a bed-load particle does not turn into suspension witin x;
$f_X(x)$	= probability density function of step length of bed-load particle;
$f_{XS}(x)$	= probability density function of excursion length of suspended particle;
g	= gravitational acceleration;
$g_R(x)$	= impulse response of Φ to Ψ in the fluvial system;
$g_{RB}(x)$	= impulse response of bed-load transport rate;
$g_{RD}(t)$	= impulse response of dune height;
$g_{RT}(\zeta \mid \eta)$	= impulse response of Reynolds-stress distribution;
H	= dune height;
h	= flow depth;
k_2	= empirical constant;
L, L_0	= dune length and its initial value on grwoth from flattened bed;
L_B	= variation scale of stream width;
L_A	= adaptation distance of suspended sediment concentration distribution;
m	= empirical constant;
p_1, p_2	= parameters to represent the flow-unsteadiness;

p_d, p_{d*}	= deposit rate and its dimensionless form ($\equiv p_d\sqrt{d/(\sigma/\rho-1)g}$);
$p_{ds}(x)$	= number of suspended particles to deposit on the bed of the area occupied by one particle at x to turn into bed-load motion;
p_i	= volumetric ratio of the i-th fraction sand to that of sand of all fractions in the surface layer;
p_s, p_{s*}	= pick-up rate and its dimensionless form ($\equiv p_s\sqrt{d/(\sigma/\rho-1)g}$);
p_T	= probability density of transition per unit time from bed-load motion to suspension;
q	= flow discharge per unit width;
q_0	= temporal averag of the fluctuating flow discharge per unit width;
q_B, q_{B*}	= bed-load transport rate and its dimensionless form ($\equiv q_B/\sqrt{(\sigma/\rho-1)gd^3}$);
R_E	$\equiv w_0h/\varepsilon_s$;
$r_\Phi a_\Psi \Phi_0$	= amplitude of the fluctuation of Φ;
T_D	= time required for dunes to reach the fully-developed dimension from initially flattened bed;
T_{EX}	= time scale for dune development;
t	= time;
u	= flow velocity;
u_g	= bed-load particle's speed;
u_*	= shear velocity;
w_0	= terminal velocity of sand;
x, y	= longiyudinal and vertical coordinates;
α_1, α_2	= direct effect of flow unsteadiness and relaxation effect on the amplitude;
β	$\equiv (d\Phi/d\Psi)_0(\Psi_0/\Phi_0)$;
β_H, β_L	= empirical constants for process rates of dune geometries;
β_s	= reciprocal of turbulent Schmidt number;
$\beta_{\phi\psi}$	$\equiv \int_{\eta_a}^{1}\phi(\eta)\psi(\eta)d\eta$;
δ_B	= lag distance of bed-load motion;
ε_s	= turbulent diffusion coefficient of suspended sediment;
ζ_i	$= d_i/d_m$;
η	= relative flow depth ($\equiv y/h$);
κ	= angular wave number;
κ_0	= Kármán constant;
Λ	= mean step length of bed-load motion;
Λ_R	= temporal or spatial relaxation scale;
Λ_{ST}	= relaxation length of suspended sediment discharge;
λ, λ_G	= dimensionless mean step length of bed-load motion (Λ/d and Λ_i/d_i);
$\lambda_T(\eta)$	= dimensionless relaxation length of Reynolds stress at respevctive relative height;
ν_T	= eddy kinematic viscocity;
ξ	$\equiv x/h$;

ρ	= mass density of fluid;
ρ_0	= porosity of sand;
σ	= mass density of sand;
τ_b	= bed shear stress;
τ_T	= Reynolds stress;
τ_*	= dimensionless bed shear stress ($\equiv u_*^2/[(\sigma/\rho-1)gd]$);
τ_{*c}	= dimensionless critical tractive force;
τ_{*ci}, τ_{*cm}	= dimensionless critical tractive force for each grain size and that for sand of mean diameter in the mixtures;
Φ	= output process of the fluvial system;
Φ_0	= mean value of the fluctuating process of Φ;
ϕ_1, ϕ_2	= phase shifts due to direct effect of flow unsteadiness and relaxation effect;
Φ_R	= output process responding to an abrupt change of the input process Ψ;
ϕ	= phase shift of Φ to Ψ;
$\phi_S(\eta)$	= normalized equilibrium concentration profile of suspended sediment (C_e/C_{ae});
Ψ	= input process of the fluvial system;
Ψ_0, Ψ'	= mean value and perturbation of the fluctuating process of Ψ;
Ψ_T	= upward flux of suspended sediment due to turbulence;
$\psi_U(\eta)$	= normalized equilibrium velocity profile of flow (u_e/u_*); *and*
ω	= angular wave number;

Subscripts

e	= equilibrium states; *and*
i	= the i-th fraction of sediment mixture whose diameter is d_i.