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STOCHASTIC APPROACH TO SEDIMENT TRANSPORT PROBLEM*

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SYNOPSIS

Stochastic approach to sediment transport problem sometimes brings us fruitful results in describing or predicting alluvial phenomena. Bed load transport process can be well described by a stochastic model constituted by two parameters: rest period and step length, which are both random variables. The stochastic model of bed load motion makes it possible to describe reasonably sand wave formation and armoring process where non-equilibrium properties inherent to bed load transport appreciably dominate. Furthermore, a recent development of a stochastic model for bed material load transport, where suspended sediment exists with bed load, is perspectively reviewed.

INTRODUCTORY REMARKS

Alluvial phenomena are generally caused by unbalance of sediment transport, and then non-equilibrium sediment transport process should be reasonably described. Particularly, in bed load motion, the probabilistic and discrete properties of individual sediment motion easily bring about a non-equilibrium situation. Then, a sediment transport model based on stochastic consideration will be promising to describe reasonably and consistently various alluvial phenomena.

From the view point of the above-mentioned, a stochastic model for bed load transport of which constituent elements are sediment pick-up rate and step length is explained, and the essential characteristics of bed load motion are described. Next, in order to improve the applicability of the stochastic model to actual alluvial phenomena, an Eulerian stochastic model is presented and, making the most of the stochastic properties of bed load motion preserved in this model, incipient process of bed surface irregularity, processes of sand wave formation and armor coat propagation are explained as the typical examples which can be reasonably described by applying the Eulerian stochastic model.

Finally, in order to obtain an unified understanding for bed load and suspended load, the Eulerian stochastic model for bed load transport is to be extended to the non-equilibrium sediment transport process including the suspended load. As the first part of its development, saltation as bed load, random motion as suspended load, and transition between them are described by combining a stochastic approach with a deterministic approach.

STOCHASTIC MODEL FOR BED LOAD MOTION

Bed load transport is characterized as an ensemble set of probabilistic and intermittent motion of sand particles based on their incessant contacts with the loose boundary and the fluctuation of hydrodynamic forces acting on particles along the bed. By this reason, a stochastic model, in which bed load transport is considered as a stochastic process, is both visual and faithful to the actual phenomenon involving its essential characteristics.

Einstein (4) described a behavior of an individual bed load particle by a zigzag model as shown in Fig.1, which consists of two random variables, rest period $\{T\}$ and step length $\{X\}$. When $\{T\}$ is independent of $\{X\}$, a generating process of a random phenomenon can be described so that the occurrences of a particle dislodgement and stop are projected on the time axis and the distance axis, respectively. When p_s and p_d represent the probability density per unit time for a particle to be dislodged and that per unit distance to stop, respectively, the mean values of $\{T\}$ and $\{X\}$ are given by

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$$E[T] = 1/p_s \quad (1)$$

$$\Lambda \equiv E[X] = 1/p_d \quad (2)$$

in which Λ=mean step length; and the pick-up rate p_s is the reciprocal of the mean rest period. The step length is defined as a distance for a particle to travel from its incipient motion to the next definite stop.

The distribution of the rest period expressed by its probability of exceedence is shown in Fig.2, which has been obtained by tracer tests (16). It is recognized from this figure that the rest period follows approximately an exponential distribution and the process of particle dislodgement can be regarded as a Poisson process. The probability density function of rest period is approximated as follows:

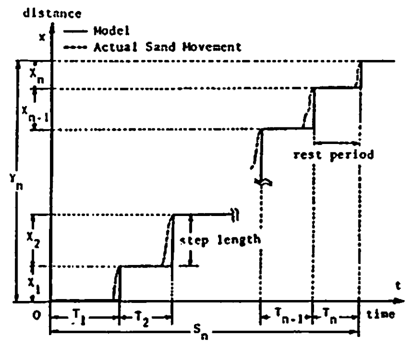

Fig.1 Definition sketch of stochastic model

$$f_T(\tau) = p_s \exp(-p_s\tau) \quad (3)$$

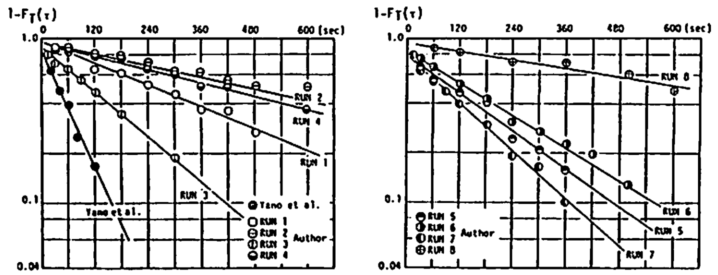

Fig.2 Distribution of rest period

Meanwhile, the distribution of step length obtained by tracer tests is shown in Fig.3, where they are expressed by its probability of exceedence. According to this figure, it is recognized that the step length follows an exponential distribution, and

$$f_X(\xi) = \frac{1}{\Lambda}\exp\left(-\frac{\xi}{\Lambda}\right) \quad (4)$$

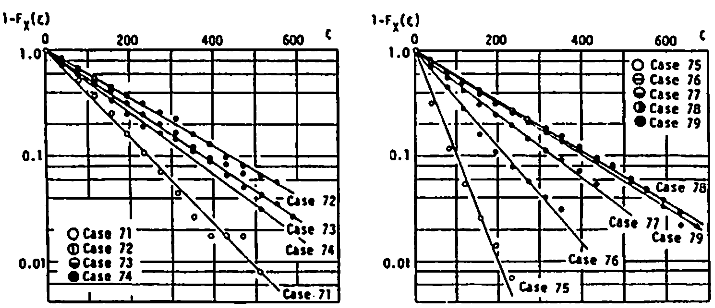

Fig.3 Distribution of step length

As for the sediment pick-up rate p_s and the mean step length Λ, a physically based analytical model to evaluate the relationships between these parameters and the bed shear stress was proposed by Nakagawa & Tsujimoto (15). According to this study on incipient motion of bed materials, dimensionless sediment pick-up rate is given by the following equation and shown in Fig.4.

$$p_{s*} \equiv \sqrt{d/(\sigma/\rho-1)g} = F_0 \tau_* \left(1 - \frac{\tau_{*c}}{\tau_*}\right)^3 \quad (5)$$

$$\tau_* \equiv \frac{\tau}{(\sigma-\rho)gd} \quad (6)$$

in which d=sand diameter; σ=density of sand; ρ=density of fluid; g=gravitational acceleration; F_0=experimental constant; τ=bed shear stress; τ_{*c}=dimensionless critical shear stress; and from a study of experimental data, it is approximated that F_0=0.03 and τ_{*c}=0.035. τ_{*c} adopted here corresponds to the condition that p_s=0.

The relationship between dimensionless mean step length ($\lambda \equiv \Lambda/d$) and τ_* is shown in Fig.5, where the experimental data directly observed and those indirectly obtained by using the stochastic dispersion or transport model are shown separately. Under equilibrium flat bed condition, the mean step length can be given from the stochastic model for bed load transport rate by

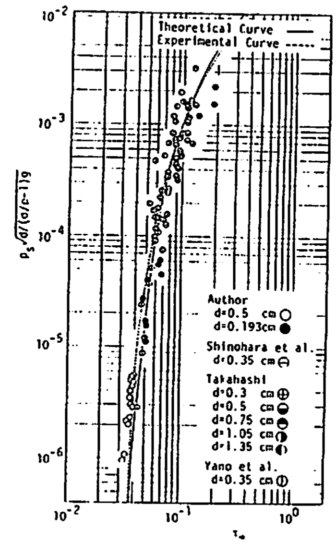

Fig.4 Sediment pick-up rate

$$\lambda \equiv \frac{\Lambda}{d} = \frac{A_2 q_{Be*}}{A_3 p_{s*}} \quad (7)$$

in which A_2, A_3=2- and 3-dimensional shape coefficients of sand; q_{Be}=bed load transport rate under equilibrium flat bed condition; and $q_{B*} \equiv q_B/\sqrt{(\sigma/\rho-1)gd^3}$=dimensionless expression of bed load transport rate q_B. Since q_{Be*} is generally connected to τ_* by transport formula, the relationship between λ and τ_* can be obtained from Eq.7. Four curves shown in Fig.5 have been obtained by the following transport formula: (A) Meyer-Peter & Müller's (14); (B) and (C) Bagnold's (2) for τ_{*c}=0.04 and 0.045, respectively; and (D) Ashida & Michiue's (1).

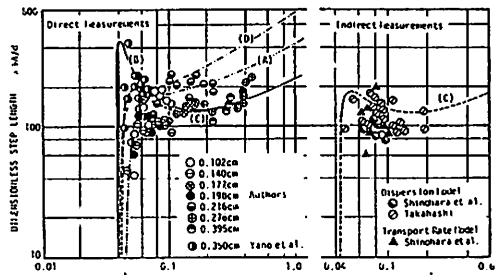

Fig.5 Mean step length of bed load

The above equations are valid only when the conditions surrounding sand movements are nearly same at any position and at any time as recognized under flat bed condition or conditions slightly shifted from flat bed. In this case, pick-up rate and step length are correlated with the scales of sediment particles (diameter and fall velocity), while these are affected by the scales of bed geometry when bed deformation becomes appreciable.

Since a succession of rest period and step length constitutes bed load transport process, a dispersion process can be described by using their probability density functions. Thus, study of the stochastic model has been put emphasis on description of sediment dispersion process and on investigation on distributions of step length and rest period not only in statistically homogeneous field but also in non-homogeneous field (Hubbell & Sayre (10), Yang & Sayre (31), Shen & Todorovic (23), Hung & Shen (11)).

These previous studies on stochastic model for bed load transport have been based on a Lagrangian consideration referred to a Brownian motion. In spite of a number of fine description of the bed load dispersion, the stochastic model still seemed to be deficient to describe various alluvial phenomena which are inherently to be analyzed by Eulerian consideration.

In order to improve the applicability of the stochastic model which can express the essential characteristics of bed load motion to the subsequent phenomena of bed load transport, Eulerian stochastic model was developed by Nakagawa & Tsujimoto (17).

FUNDAMENTAL EQUATIONS FOR NON-EQUILIBRIUM BED LOAD TRANSPORT

Although most of bed load transport formulae were derived under equilibrium conditions, even a slight change of the situation would generate non-equilibrium state of sediment transport. Sand wave initiation and early stage of its development must depend upon non-equilibrium as seen in many other alluvial processes such as armor coat formation, scouring and so on. Hence, it is necessary to establish non-equilibrium bed load transport formula.

In case of bed load motion, the phenomenon is quite probabilistic and discrete, and the moving period of one step is negligibly short compared with the rest period. Although the sediment pick-up rate at any point is uniquely determined by local shear stress, the local bed load transport rate is appreciably influenced by the sediment transport states in upstream reach by action of step length, particularly its distribution.

By making use of the stochastic model based on its Eulerian interpretation, the behavior of bed load motion and subsequent bed deformation process can be described as the following. Referring Fig.6, sediment deposit rate $p_d(x)$, which is defined in the same manner as pick-up rate, bed load transport rate $q_B(x)$, and the rate of bed deformation $\partial y(x)/\partial t$ can be expressed by

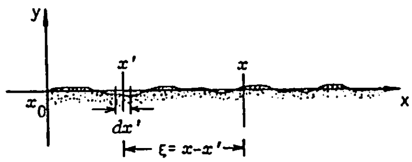

Fig.6 Definition sketch

$$p_d(x) = \int_0^{x-x_0} p_s(x-\xi) f_X(\xi) d\xi \tag{8}$$

$$q_B(x) = \int_{x_0}^{x} p_s(x') A_3 d^3 \frac{dx}{A_2 d^2} \int_{x-x'}^{\infty} f_X(\xi) d\xi \tag{9}$$

$$\frac{\partial y(x)}{\partial t} = [p_d(x) - p_s(x)] A_1 d \tag{10}$$

in which x_0=origin of sediment flow; and A_1=1-dimensional shape coefficient of sand.

For simplicity, the situation that x_0=0 and the bed shear stress or the pick-up rate is constant along the bed in the region $x>0$ is considered. In this case, Eqs.8, 9 and 10 become

$$p_d(x) = p_s F_X(x) \tag{11}$$

$$q_B(x) = \frac{A_3 p_s d}{A_2} \int_0^x [1 - F_X(x-x')] dx' \tag{12}$$

$$\frac{\partial y(x)}{\partial t} = p_s [1 - F_X(x)] \tag{13}$$

in which $F_X(x)$=distribution function of step length. When p_s is constant and the step length follows an exponential distribution, the bed load transport rate at any section becomes

$$q_B(x) = \frac{A_3 p_s \Lambda d}{A_2} \left[1 - \exp\left(-\frac{x}{\Lambda}\right)\right] \tag{14}$$

and, where $x \to \infty$, that for the equilibrium flat bed, q_{B0}, can be obtained as

$$q_{B0} = \frac{A_3 p_s \Lambda d}{A_2} \tag{15}$$

Then,

$$\frac{q_B(x)}{q_{B0}} = 1 - \exp\left(-\frac{x}{\Lambda}\right) \tag{16}$$

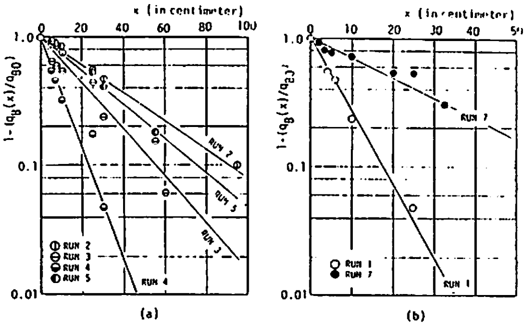

Fig.7 Spatial variation of local rate of bed load transport

And, the following relationship is obtained from Eq.11.

$$p_{d*}(x) = \frac{A_2 d}{A_3 \Lambda q_{B*}(x)} \tag{17}$$

in which $p_{d*} = p_d\sqrt{d/(\sigma/\rho-1)g}$.

The spatial variation of bed load transport rate obtained in the experiments by Nakagawa & Tsujimoto (17) is shown in Fig.7. In experiments, the alluvial part was connected with the fixed rough bed. The relationship between bed load transport rate and deposit rate in dimensionless forms is shown in Fig.8. From both figures, it is suggested that the step length follows an exponential distribution even when the bed condition is slightly shifted from equilibrium flat bed.

The mean step length inversely calculated from these figures by using the aforementioned equations is about 50~250 times sediment diameter and this result is identical to those obtained under equilibrium flat bed conditions. Thus, it can be concluded that the essential properties of individual sediment motion are almost preserved even in such non-equilibrium conditions as a slight shift from equilibrium.

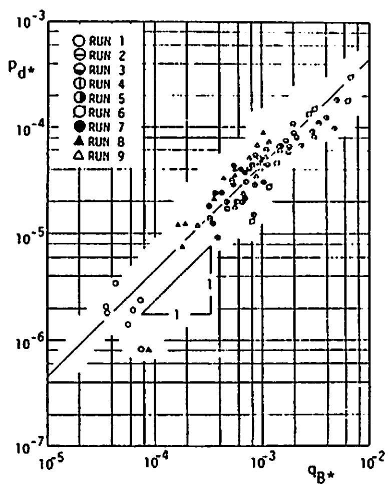

Fig.8 Relation between q_{B*} and p_{d*}

INCIPIENT PROCESS OF SAND BED IRREGULARITY

For a long time hydraulic researchers or geologists have been interested in sedimentary wave formation and its mechanism has been rigorously investigated. Among these studies, the work of Kennedy (12) which introduced the linear instability analysis into alluvial problems is the most suggestive one. However, any answer has not been found to the question how the bed surface disturbance initially assumed in instability analysis is generated. Based on the properties of bed load motion preserved in the stochastic model, incipient process of bed surface irregularity, namely initiation of sand waves can be explained by stochastic techniques (24).

When $\{N_s(x,t)\}$ represents the number of particles dislodged from the bed surface occupied by one sand aprticle at x during the time interval (0,t), the following equation can be obtained by the similar consideration to Eqs.8 and 10.

$$y(x,t) = \left[\int_0^\infty N_s(x-\xi,t) f_X(\xi) d\xi - N_s(x,t)\right] A_1 d \tag{18}$$

And, the spectrum of sand bed can be given by

$$S(\kappa,t) = \overline{F[y(x,t)] \cdot F^*[y(x,t)]} \tag{19}$$

in which $S(\kappa)$=wave number spectral density function of sand bed; κ=angular wave number. $F(\cdot)$ denotes the Fourier transformation from x-space to κ-space; $F^*(\cdot)$ represents the complex conjugate; and $\overline{\Omega}$ indicates the ensemble mean of Ω.. Hence, the Fourier transformation of Eq.18 yields

$$S(\kappa,t) = (A_1 d)^2 \overline{\{F[f_X]-1\} \cdot \{F^*[f_X]-1\}} \cdot S_N(\kappa,t) \tag{20}$$

in which $S_N(\kappa,t)$=wave number spectrum corresponding to the spatial fluctuation of $\{N_s(x,t)\}$. As for the spatial correlation of $\{N_s(x,t)\}$, an exponential correlation model is adopted here for convenience' sake. Namely,

$$R_{NN}(\xi,t) = \mathrm{Var}[N_s(x,t)] \cdot \exp(-\alpha_0 \xi) \tag{21}$$

in which $R_{NN}(\xi,t)$=autocorrelation of the spatial fluctuation of $\{N_s(x,t)\}$; α_0=reciprocal of relaxation distance; and $\mathrm{Var}[N_s(x,t)]$=variance of $\{N_s(x,t)\}$. $\mathrm{Var}[N_s(x,t)]$ is given by

$$\mathrm{Var}[N_s(x,t)] = E[\{N_s(x,t)\}^2] - \{E[N_s(x,t)]\}^2 \tag{22}$$

$$E[\{N_s(x,t)\}^k] = \int_0^t F^{-1}[(1-\phi_T) \sum_{n=0}^{\infty} n^k \phi_T^n] d\tau \tag{23}$$

$$\phi_T \equiv F[f_T(\tau)] \tag{24}$$

in which $F(\cdot)$ represents here a Fourier transformation with respect to τ, and F^{-1} represents its inverse transformation. Under flat bed condition, $f_T(\tau)$ is given by Eq.3 and then

$$\mathrm{Var}[N_s(x,t)] = p_s t \tag{25}$$

Meanwhile, $f_X(\xi)$ is given by Eq.4. Consequently, the following can be obtained.

$$S(\kappa,t) = \frac{4\alpha_0 (A_1 d)^2 p_s t \cdot \kappa^2}{(\kappa^2+\alpha_0^2)(\kappa^2+1/\Lambda^2)} \tag{26}$$

This indicates that $S(\kappa,t)$ can increase even from zero in proportion to time and $S(\kappa,t) \sim \kappa^{-2}$ in the range of higher wave number, while $S(\kappa,t) \sim \kappa^{-3}$ for fully developed sand waves as clarified by Hino (9). In Fig.9, a wave number spectrum calculated by Eq.26 in which $\alpha_0 = 1/\Lambda$ is compared with an experimental result of bed-elevation change at initial stage. It is also clarified from Eq.26 that the variance of bed elevation, σ_y^2, increases in proportion to the elapsed time from initially flattened bed.

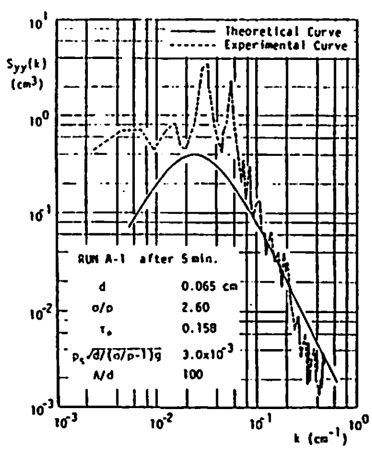

Fig.9 Wave number spectrum of sand bed irregularity at earlier stage

As a result of this analysis, it is concluded that random repetitions of sand dislodgement and deposition bring about an initial bed disturbance as a germ or a trigger of sand waves, and it is considerably irregular.

SAND BED INSTABILITY DUE TO BED LOAD MOTION

Instability Analysis of Erodible Sand Bed

The preceding analysis has been indicated that initial bed disturbance appears due to random motion of bed materials, and it must depend upon bed instability mechanism whether this disturbance would grow up to sand waves or not.

Since Kennedy's instability analysis (12), it has become an established view that sand wave formation is due to instability on the interaction between the bed surface and the flow field. Kennedy introduced the lag distance between bed load transport rate and bed shear stress. But, it was not a physically definite but ambiguous quality.

The lag distance can be evaluated reasonably based on the stochastic model which is applicable to such non-equilibrium situation as when sand waves are initially formed (17).

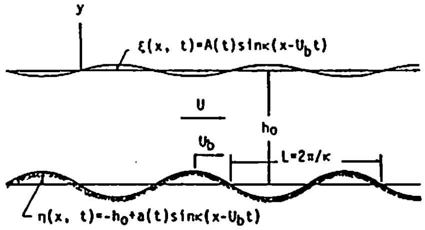

Fig.10 Definition sketch

Now, the bed surface is assumed to be expressed by

$$y(x) = a \cdot \sin\kappa(x-U_b t) \qquad (27)$$

in which a=amplitude of the perturbation of bed surface; κ=angular wave number; and U_b=propagation celerity (Fig.10). When we use a linear stability theory, aκ is so small that its higher-order quantities will be neglected.

Then, the properties of the flow over a wavy bed, the sediment movement and the bed surface can be expressed as the following sinusoidal waves, of which only amplitude and phase shift are different respectively.

$$\psi_R(x,t) = r_R a \cdot \sin\{\kappa(x-U_b t)-\phi_R] \qquad (28)$$

in which ψ=perturbation non-dimensionalized by the undisturbed quantity. Then, the local rate of bed load transport can be written as

$$q_B(x) = q_{B0}\{1+r_B a(t) \cdot \sin[\kappa(x-U_b t)-\phi_B]\} \qquad (29)$$

in which q_{B0}=undisturbed bed load transport rate; $r_B a q_{B0}$=amplitude of the perturbation of bed load transport rate; and ϕ_B=phase lag of bed load transport rate for y(x). Substituting Eqs.27 and 29 into the following continuity equation of sediment transport,

$$\frac{\partial y}{\partial t}+\frac{1}{1-\rho_0}\frac{\partial q_B(x)}{\partial x} = 0 \qquad (30)$$

we can obtain

$$\frac{1}{a}\frac{\partial a}{\partial t}= -\frac{r_B q_{B0}}{1-\rho_0}\sin\phi_B \; ; \qquad \kappa U_b = \frac{r_B q_{B0}}{1-\rho_0}\cos\phi_B \qquad (31)$$

in which ρ_0=porosity of sand. From Eq.31, the stability and the direction of wave propagation can be determined by the phase lag ϕ_B as shown in Table 1.

ϕ_B is obtained as the sum of the phase lag ϕ_τ of bed shear stress τ(x) for bed surface y(x), and the phase lag $\phi_{B\tau}$ of $q_B(x)$ for τ(x), as follows:

$$\phi_B = \phi_\tau + \phi_{B\tau} \qquad (32)$$

Table 1 Stability of wavy bed

ϕ_B	c	$\mathring{a}/a$	Stability of Wavy Bed
$0 \sim \pi/2$	+	-	decay
$\pi/2 \sim \pi$	-	-	decay
$\pi \sim (3/2)\pi$	-	+	growth,moving upstream
$(3/2)\pi \sim 2\pi$	+	+	growth,moving downstream

The lag distance δ_{BT} is related to ϕ_{BT} as $\delta_{BT}=\phi_{BT}/\kappa$. ϕ_T and ϕ_{BT} can be evaluated by using the flow model over a wavy bed and the sediment transport model, respectively.

Flow Properties over Wavy Bed

The term ϕ_T has been investigated by several researchers but has not been clarified sufficiently. The authors (17) adopt a modified potential flow model, referred to the previous works of Kennedy (12) and Hayashi (7).

Now, local flow depth and local flow velocity are to be expressed respectively by

$$h(x,t) = h_0[1+r_s a\cdot\sin[\kappa(x-U_b t)-\phi_s]\} \tag{33}$$

$$U(x,t) = U_0[1+r_u a\cdot\sin[\kappa(x-U_b t)-\phi_u]\} \tag{34}$$

in which h_0=undisturbed flow depth; $r_s a h_0$=amplitude of perturbation of flow depth; ϕ_s=phase shift between free surface and bed surface; U_0=undisturbed flow velocity; $r_u a U_0$=amplitude of perturbation of flow velocity; and ϕ_u=phase shift between flow velocity and bed form. According to the theory of potential flow model, ϕ_s and ϕ_u are o or π, and the following relationships can be obtained.

$$R_* \equiv r_s\cos\phi_s = \frac{F^2\kappa h_0 \mathrm{sech}\kappa h_0}{F^2\kappa h_0-\tanh\kappa h_0} \tag{35}$$

$$F_* \equiv r_u\cos\phi_u = \frac{1-F^2\kappa h_0\tanh\kappa h_0}{\tanh\kappa h_0-F^2\kappa h_0} \tag{36}$$

in which $F \equiv U_0/\sqrt{gh_0}$=Froude number of undisturbed flow.

Eq.35 shows whether the free surface of flow and the bed surface are in phase or out of phase, and this is a criterion whether bed forms are dunes and/or ripples, or antidunes if the bed surface perturbation can grow up. The critical Froude number is given by

$$F = \frac{\tanh\kappa h_0}{\kappa h_0} \tag{37}$$

On the other hand, by putting $F_*=0$, the following equation is obtained.

$$F = \frac{1}{\kappa h_0\tanh\kappa h_0} \tag{38}$$

By Eqs.37 and 38, the F versus κh_0 plane is divided into three regions as shown in Fig.11.

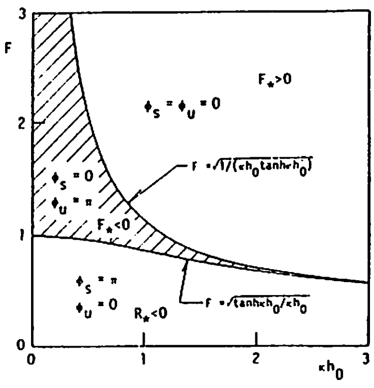

Fig.11 Phase lags of flow velocity and free surface for bed form

Since bed shear stress $\tau(x)$ is more significant than local flow velocity for bed load movement, $\tau(x)$ on a wavy bed may be related to $U(x)$ as

$$\tau(x) = \beta\rho U^2\left(1-\alpha\frac{\partial h}{\partial x}\right) \tag{39}$$

in which β=resistance coefficient; and α=correction factor. In Eq.39, the effect of flow divergence and convergence is considered. From $(\partial h/\partial x)$ and perturbation of flow velocity u(x), the amplitude and the phase lag of perturbation of bed shear stress can be obtained as

$$r_\tau^2 = \alpha\kappa(R_*-1)^2+(2\kappa F_*)^2 \tag{40}$$

$$r_\tau \sin\phi_\tau = \alpha\kappa(R_*-1)\ ; \qquad r_\tau \cos\phi_\tau = 2\kappa F_* \tag{41}$$

Lag Distance of Bed Load Transport

When the response of bed load transport rate to bed shear stress is to be regarded as a non-equilibrium transport process, the bed load transport formula given by Eq.9 can be applied to investigate the lag distance. Because the lag distance of sediment pick-up rate for bed shear stress can be neglected, $p_s(x)$ is to be expressed by

$$p_s(x) = p_{s0}\{1+r_p a\cdot\sin[\kappa(x-U_b t)-\phi_\tau]\} \tag{42}$$

in which p_{s0}=average sediment pick-up rate along the bed; and $r_p a p_{s0}$=amplitude of perturbation of pick-up rate.

Substituting Eq.42 and Eq.4 into Eq.9 where $x_0 \rightarrow -\infty$ and comparing with Eq.29, the lag distance $\delta_{B\tau}$ can be obtained as follows:

$$\sin\kappa\delta_{B\tau} = \frac{\kappa\Lambda}{\sqrt{1+(\kappa\Lambda)^2}}; \quad \cos\kappa\delta_{B\tau} = \frac{1}{\sqrt{1+(\kappa\Lambda)^2}}; \quad \frac{r_B}{r_p} = \frac{1}{\sqrt{1+(\kappa\Lambda)^2}} \tag{43}$$

Therefore, $0<\kappa\delta_{B\tau}<\pi/2$. In Fig.12, the lag distance is shown against κh_0 with a parameter $\gamma_1 = \Lambda/h_0$.

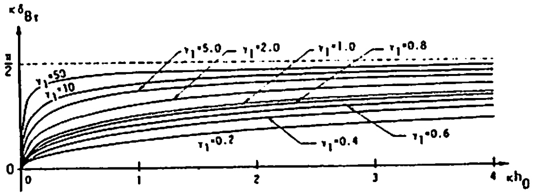

Fig.12 Lag distance for bed load transport

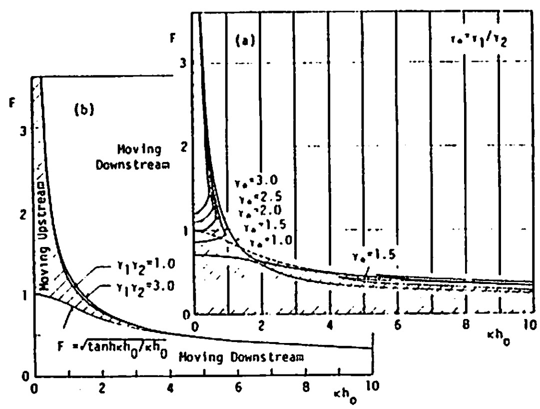

Fig.13 Stability and propagation direction of bed surface disturbance

As above investigated, the phase lag ϕ_{BT} and ϕ_T are clarified and thus it is possible to conduct the instability analysis of erodible bed. The analytical results obtained by the authors (17) are shown in Fig.13, where $\gamma_1 \equiv \Lambda/h_0$; $\gamma_2 \equiv \alpha F^2$; and $\gamma_* \equiv \gamma_1/\gamma_1$.

The presented model can explain the bed instability fairly well without any assumption for the phase lag and this model may be successful in classification of bed forms. However, it is not appropriate to apply the results of this analysis to the classification of fully-developed sand waves because the wave length and the resistance coefficient for fully-developed sand waves are different from those assumed in the analysis.

SPECTRAL ANALYSIS OF SAND BED INSTABILITY

An incipient bed disturbance is so irregular that a sand bed instability analysis of the Fourier transformed version is more appropriate to explain the phenomenon. Such a technique was first introduced by Jain & Kennedy (13), in which the lag of sediment transport was not adequately treated, and then the authors developed this idea by applying the Eulerian stochastic model for non-equilibrium bed load transport (18).

As the Fourier transformation of Eq.10 into which Eq.18 is substituted, the following equation can be obtained.

$$\frac{\partial Y}{\partial t} = A_1 d \cdot r_p p_{s0} \cdot \mathcal{F}[r_T \psi_T(x)]\{\mathcal{F}[f_X(\xi)]-1\} \tag{44}$$

in which $Y(\kappa,t) \equiv \mathcal{F}[y(x,t)]$; and the perturbation of sediment pick-up rate $p_s'(x)$ is expressed as follows:

$$p_s'(x) \equiv r_p r_T p_{s0} \psi_T(x) \tag{45}$$

in which $r_T \psi_T(x) \tau_0$=perturbation of bed shear stress; and r_p is easily evaluated by Eq.5 as

$$r_p = \left.\frac{\partial p_{s*}}{\partial \tau_*}\right|_{\tau_{*0}} \tag{46}$$

The density function of wave number spectrum of a sand bed is defined as Eq.19. Then, if the perturbation of bed shear stress can be expressed by Eq.40 and the step length follows an exponential distribution (Eq.4), the spectral evolution due to instability mechanism is given by the ensemble mean of the solution of Eq.44 and its complex conjugate, as follows:

$$S(\kappa,t) = S(\kappa,0) \cdot \exp[2B^* \Gamma^*(\kappa h_0) \cdot p_{s0} t] \tag{47}$$

in which $B^* \equiv A_1 d r_p/\Lambda$; and

$$\Gamma^*(\kappa h_0) = -\left[\frac{(\kappa\Lambda)^2}{1+(\kappa\Lambda)^2}\right][\alpha\kappa(R_*-1)^2+(2\kappa F_*)^2] \tag{48}$$

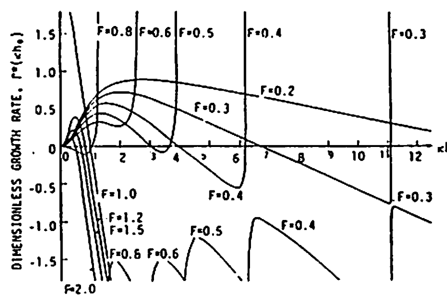

Fig.14 Spectral growth rate for each wave number

$\Gamma^*(\kappa h_0)$ is regarded as a dimensionless growth rate of sand wave spectrum, and the calculated result (γ_1=1.0 and γ_2=1.2) is shown in Fig.14. According to this figure, the waves of some wave number range grow up but the other waves decay. Furthermore, the growth rate of waves at definite wave number (corresponding to the Airy wave) becomes infinite and this corresponds well to the spectral peak at an early stage of sand wave development from initially flattened bed. And it is suggested that considerably regular waves will appear at an early stage.

STOCHASTIC ANALYSIS OF ARMORING PROCESS

Non-equilibrium transport process of sediment mixture accompanied with the subsequent formation and propagation of armor coat is another example which can be explained by the Eulerian stochastic model for sediment motion. In general, the river bed is composed of bed materials of widely distributed size of sand grains, and a probabilistic approach is required for more appropriate description of the phenomena. In other words, the behavior of sediment mixtures should be regarded as an ensemble set of that of sediment of each grain size.

Although many studies on the transport of sediment mixtures and on the prediction of armor coat formation have been conducted up to the present, they are not sufficient to estimate the transport rate reasonably and to describe the propagation process of armor coat, because non-equilibrium state of sediment transport has not adequately been considered.

If there is no sediment inflow from the upstream region, for example, the river bed downstream of the dam reservoir, an appreciable armor coat is formed and it propagates downstream. Nakagawa et al. (19) applied the stochastic model of bed load transport to such a situation as called "parallel degradation" by Gessler (6).

For the convenience' sake, the alluvial part is divided into some intervals of finite length, Δx, as shown in Fig.15. This interval length should be sufficiently longer than the maximum sand diameter and shorter than the mean step length of the minimum size of sand mixtures. Variables used in the following have two subscripts: one ($1 \le i \le N$) represents the class of sediment size; and the other ($1 \le k \le K$) represents the interval of the bed.

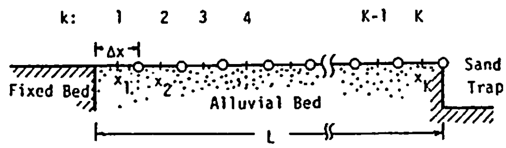

Fig.15 Definition sketch

If the number of particles of the i-th class exposed on bed surface per unit width in the k-th interval is represented by n_{ik}, the number-basis size distribution q_{ik} and the volume-basis size distribution p_{ik} can be given by

$$q_{ik}(t) = n_{ik}(t) / \sum_{j=1}^{N} n_{jk}(t) \quad (49)$$

$$p_{ik}(t) = q_{ik}(t) d_i^2 / \sum_{j=1}^{N} [q_{jk}(t) d_j^2] \quad (50)$$

If the pick-up rate for each grain size, p_{sik}, can be evaluated for the hydraulic condition and the bed constitution, the number of particle dislodgements of the i-th class sand from the k-th interval during the time interval Δt, ΔM_{ik}, can be expressed as

$$\Delta M_{ik}(t) = n_{ik}(t) p_{sik}(t) \Delta t \quad (51)$$

Meanwhile, the number of particle depositions of the i-th class on the k-th interval during Δt, ΔQ_{ik}, can be expressed as

$$\Delta Q_{ik}(t) = \sum_{s=1}^{k-1} [\Delta M_{is}(t) \cdot \mu_{i,k-s}] \quad (52)$$

in which $\mu_{i,k-s}$=probability that the step length of the i-th class sand is longer than $(k-s)\Delta x$ and shorter than $(k-s+1)\Delta x$. Thus, $\mu_{i,k-s}$ is given by

$$\mu_{i,k-s} = \int_{(k-s)\Delta x}^{(k-s+1)\Delta x} f_{Xi}(\xi)d\xi \qquad (53)$$

in which $f_{Xi}(\xi)$=probability density function of the step length of the i-th class sand.

As a result of incessant repetition of dislodgement and deposition, the lower layer is newly exposed in the case of degradation, and it is now assumed that the lower layer has the initial distribution of bed materials, p_{i0}. Considering this event, the number of particles of the i-th class sand exposed at the bed surface of the k-th interval at $(t+\Delta t)$ can be written by

$$n_{ik}(t+\Delta t) = n_{ik}(t) - \Delta M_{ik}(t) + \Delta Q_{ik}(t) + p_{i0} \sum_{j=1}^{N} \{[\Delta M_{jk}(t) - \Delta Q_{jk}(t)](\frac{d_j}{d_i})^2\} \qquad (54)$$

This can be rewritten by the following differential equation after substituting Eqs.51 and 52.

$$\frac{dn_{ik}}{dt} = - \sum_{j=1}^{N} \{[p_{i0}(\frac{d_j}{d_i})^2 - \delta_{ij}] \cdot \sum_{s=1}^{k} [\mu^*_{i,k-s} \cdot p_{sjs}(t) n_{js}(t)]\} \qquad (55)$$

in which δ_{ij}=Kronecker's delta; and $\mu^*_{i,k-s}=\mu_{i,k-s}$ for s<k, -1 for s=k, and 0 for s>k. Based on Eq.55, the temporal and spatial variations of bed constitution can be calculated in succession.

By the way, transport rate should be also treated for each grain size, as already suggested by Einstein (5), Ashida & Michiue (1) and others. In addition, however, the consideration on non-equilibrium property is also necessary. And these two important points in transport process can be reasonably treated by the Eulerian stochastic model, and the following non-equilibrium transport rate formula for each grain size at just downstream of the K-th interval is derived.

$$q_{Bi}(t|k\Delta x) = \frac{A_3 d_i}{A_2} \sum_{k=1}^{K} [p_{sik}(t) n_{ik}(t) \Delta x \cdot \sum_{s=K-k}^{\infty} \mu_{i,s}] \qquad (56)$$

When the Eulerian stochastic model is applied, the pick-up rate and the step length should be reasonably evaluated for each grain size. Although the characteristics of behaviors of sediment mixtures are very complicated, the following simplified model may be available from the practical view point.

As for the incipient motion of bed materials, the most different point between sediment mixtures and uniform sand is the critical tractive force, and the pick-up rate for each grain size can be estimated by applying Eq.5 as follows, if the critical tractive force can be approximately evaluated for each grain size.

$$p_{s*i} \equiv p_{si}\sqrt{d_i/(\sigma/\rho-1)g} = F_0 \tau_{*i}(1-\frac{0.7\tau_{*ci}}{\tau_{*i}})^3 \qquad (57)$$

in which τ_{*ci}=dimensionless critical tractive force for each grain size. In Fig.16, the pick-up rate curve for each grain size obtained by Eq.57 are compared with the experimental results (19).

If the idea proposed by Egiazaroff (3) and some modification for finer sand by Ashida & Michiue (1) are adopted, τ_{*ci} can be evaluated as

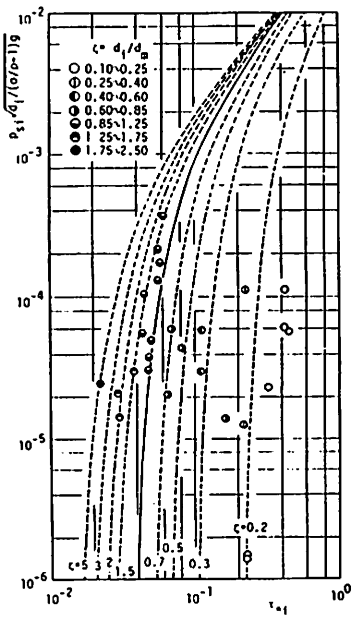

Fig.16 Sediment pick-up rates for non-uniform bed materials

$$\frac{\tau_{*ci}}{\tau_{*cm}} = \begin{cases} \left[\dfrac{\ln(30.1a)}{\ln(30.1a\zeta_i)}\right]^2 & (\zeta_i > 0.4) \\ \dfrac{0.8}{\zeta_i} & (\zeta_i \leq 0.4) \end{cases} \quad (58)$$

in which $\zeta_i = d_i/d_m$; d_m=mean diameter; τ_{*cm}=dimensionless critical tractive force for sand of mean diameter in sediment mixtures. The representative height of bed particles of each grain size is assumed to be ad ($a \cong 0.5$).

Substituting Eq.58 into Eq.57, we can evaluate the pick-up rate for each grain size. τ_{*cm} is identified with the dimensionless critical tractive force of uniform sand with mean diameter, though it is not necessarily a reasonable assumption (21).

Meanwhile, the mean step length (Λ_i) for each grain size is almost well expressed by λd_i, though λ is slightly smaller than the value of uniform sand.

In Fig.17, the experimental data for armor coat propagation process obtained by Ashida & Michiue (1) are shown with the theoretical predictions based on the stochastic model, and the agreements between the both are comparatively well.

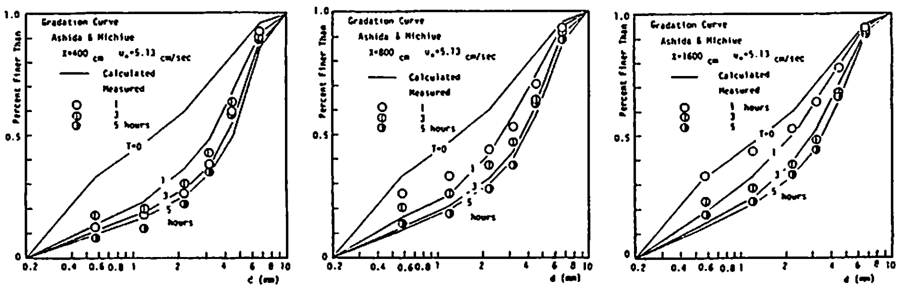

Fig.17 Propagation process of armor coat

DEVELOPMENT OF STOCHASTIC MODEL FOR BED MATERIAL LOAD

Bed Load and Suspended Load

In most of the previous formulas for the transport rate of bed material load, the bed load and the suspended load were treated separately, though they were not always easy to distinguish from each other. Considering the situation in which both sediment loads exist simultaneously under a non-equilibrium condition and a sediment particle moves sometimes as bed load and sometimes in suspension, the probabilistic behaviors of bed material particles should be considered more minutely in order to unify understanding for bed load and suspended load. Here, the outline of the authors model based on the above consideration is explained and for further details the recently published papers ((20), (25), (26), (27)) may be referred to.

The transport process of bed material load appears to be constituted by the following subsystems as illustrated in Fig.18, according to the detailed observation of sediment particles through the video film analysis in the laboratory:

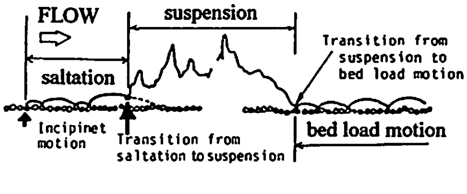

Fig.18 Illustration of bed material load transport process

(i) Dislodgement of a bed material particle from bed;
(ii) bed load motion as described by deterministic equation of motion;
(iii) transition from bed load motion to suspension; and
(iv) suspension described as a random motion.

The bed load is defined as the bed material load of which the average trajectory is uniquely determined in terms of the equation of motion and is characterized by the irregular successive saltation. On the other hand, the suspended load is defined as the bed material load which follows the stochastic path due to the effect of turbulence. According to this definition, a clear citerion to distinguish between these two modes of particle motion can be formulated. The following three subtopics should be clarified and appropriate models should be obtained respectively: (i) irregular successive saltations; (ii) behavior of suspended particle as random motion; and (iii) transition mechanism from bed load motion to suspension or from suspension to bed load.

Concentration Distribution of Bed Material Load

Both the bed load discharge and the suspended load discharge are expressed here as an integration of the product of the respective concentration distribution and velocity distribution along the flow depth.

$$q_B = \int_0^h C_B(y)u_g(y)dy \tag{59}$$

$$q_S = \int_0^h C_S(y)u_p(y)dy \tag{60}$$

in which q_B, q_S=transport rate of bed load and that of suspended load; C_B, C_S=concentration distributions of bed load and suspended load, respectively; u_g, u_p=speed of bed load particle and longitudinal speed of suspended particle; and h=flow depth.

When the existence probability density of particles in bed load motion in the vertical direction is represented by $f_B(y)$, the concentration distribution of bed load can be given by

$$C_B(y) = v_* d^3 f_B(y) \tag{61}$$

in which $v_* \equiv v_g A_3 d^3 = (A_3/A_2)p_s T_m$; v_g=dimensionless number density of particles in bed load motion; T_m=mean duration time of successice saltation (mean moving period during the so-called step length). The relationship between p_s and bed shear stress can be given by Eq.5, while $f_B(y)$ and mean moving period T_m can be clarified by stochastic analysis of irregular successive saltations.

As for suspended load, a particle which has reached the height $y=y_B$ by bed load motion changes the state of motion into suspension due to turbulence. The existence probability density function of a particle in random motion, which starts from $y=y_B$, $f_S(y|y_B)$, may be formulated by probabilistic modelling. The concentration distribution of suspended load $C_S(y|y_B)$ can be expressed by

$$C_S(y|y_B) = \gamma_0(y)\cdot C_B(y)\cdot\frac{f_S(y|y_B)}{f_S(y_S|y_B)} \tag{62}$$

in which $\gamma_0(y)$=weighted function in consideration of the difference of the mean duration time as suspended load due to the difference of the height at which a particle starts suspended motion and expresses the ratio of the concentration of suspended sediment to that of bed load at $y=y_B$. When $f_T(y_B)$ represents the probability density function that a particle in suspension has started from $y=y_B$, $C_S(y)$ can be given by

$$C_S(y) = \int_0^h [C_S(y|y_B)\cdot f_T(y_B)]dy_B \tag{63}$$

When the probability that a saltating particle turns into suspension at $y=y_B$ is written as $p_T(y_B)$, $f_T(y_B)$ is expressed as

$$f_T(y_B) = \frac{p_{T^*}(y_B)}{f_B(y_B)} \tag{64}$$

$$p_{T^*}(y_B) = \frac{f_T(y_B)}{\int_0^h [p_T(y_B) \cdot f_B(y_B)] dy_B} \tag{65}$$

Substituting Eqs.61 and 64 into Eq.63, we obtain

$$C_S(y) = v_* A_3 d^3 \int_0^h \{p_T(y_B) \cdot \gamma_0(y) \cdot [f_B(y_B)]^2 \cdot f_{S^*}(y|y_B)\} dy_B \tag{66}$$

in which

$$f_{S^*}(y|y_B) = f_S(y|y_B)/f_S(y_B|y_B) \tag{67}$$

Stochastic Approach to Dynamics of Irregular Successive Saltation

Though bed load motion is constituted by various types of motion (such as rolling, sliding, saltation and so on), it is significant to be represented by saltation from the view point of unification of bed load and suspended load. Recent studies on saltation in stochastic aspects (Yalin & Krishnappan (30), Hayashi & Ozaki (8)) have noticed only the irregular hydrodynamic lift force. In bed load motion, the successive saltation with irregularity, caused by irregular collisions with a bed, is more noticeable phenomenon.

The equation to subject a saltation in the vertical direction can be expressed as

$$M\frac{dv_g}{dt} = \pm D - W \qquad \text{(+ for descending ; - for rising motion)} \tag{68}$$

in which M=virtual mass of a sand particle; D=drag force; W=submerged weight of a sand particle; and v_g=vertical speed of a saltating particle. The temporal variation of the existence height of a particle during one saltation, which is a solution of Eq.68, can be written as

$$y = \vartheta(t|v_0) + \frac{d}{2} \tag{69}$$

in which v_0=initial speed of a particle in the vertical direction. The mathematical expression of ϑ has been given by the authors (25). As shown by Eq.69, individual saltations are determined for respective value of v_0.

The conditional existence probability density functions for given v_0, $f_B(y|v_0)$, is obtained by

$$f_B(y|v_0) = 1/\left[\left|\frac{d\vartheta(t|v_0)}{dt}\right|_y \cdot T_s(v_0)\right] \tag{70}$$

in which $T_s(v_0)$=duration time of an individual saltation. Incidentally, v_0 is statistically distributed due to the irregularity of repulsive conditions on the bed. If the probability density functions of v_0, $g_0(v_0)$, is clarified, the existence probability of bed load particles can be obtained by

$$f_B(y|v_0) = \frac{\int_0^\infty [T_s(v_0) f_B(y|v_0) g_0(v_0)] dv_0}{\int_0^\infty \int_0^\infty [T_s(v_0) f_B(y|v_0) g_0(v_0)] dv_0 dy} \tag{71}$$

Generally, the initial conditions of repulsion, incidence angle and speed, are determined by the preceding saltation. The results of the repulsion become the initial conditions of the successive saltation. Because of such a closed system, $g_0(v_0)$ is difficult to be obtained analytically. Thus, by numerical simulation model, the characteristics of successive saltations such as the number of successions, moving period and step length can be clarified as shown in Fig.19.

As for the existence height of a saltating particle, some examples of its probability distribution, the mean value $E[y^*]$, and the variation coefficient α_y are shown in Figs.20 and 21, respectively, where

$$y^* = \left[\frac{C_D A_2}{2A_3(\sigma/\rho+C_M)}\right]\left(\frac{y}{d}-0.5\right) \tag{72}$$

C_D=drag coefficient; and C_M=added mass coefficient.

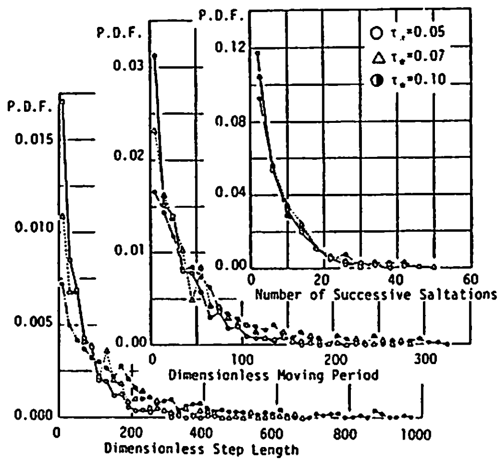

Fig.19 Distribution of number of succession, moving period and step length

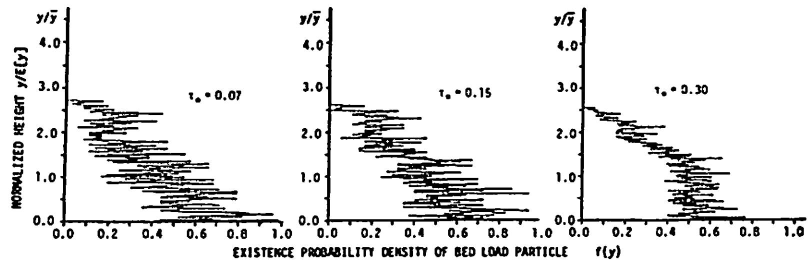

Fig.20 Existence probability distributions of saltating particle

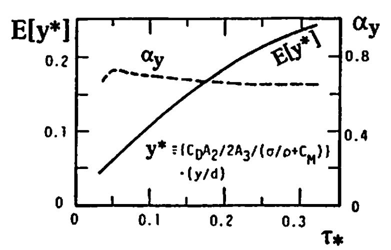

Fig.21 Mean height of saltating particle and its variation coefficient

Probabilistic Model for Particle Suspension

Since suspension is random motion due to turbulence, it is appropriate to be described by a probabilistic model. Yalin & Krishnappan (29) first proposed such a model on the assumption that the vertical displacement of the suspended particle, $\{\eta\}$, follows a normal distribution throughout the flow depth. And he obtained the concentration distribution relative to the so-called reference concentration.

The mean of $\{\eta\}$, $E[\eta]$, and its standard deviation, $\sigma_\eta(y)$, may be expressed by

$$E[\eta] = -w_0\Delta t\,; \qquad \sigma_\eta(y) = \psi_0(y)u_*\Delta t \tag{73}$$

in which w_0=terminal velocity of a particle; u_*=frictional velocity; Δt=time interval; and $\psi_0 \equiv \sqrt{v'^2}/u_*$. If we keep the consistency with the diffusion theory, the following relation should be valid.

$$\varepsilon_s = \frac{\sigma_\eta{}^2}{2\Delta t} \tag{74}$$

in which ε_s=turbulent diffusion coefficient of suspended particle which may be approximately identified with eddy kinematic viscosity. Since ε_s and ψ_0 generally depend on y, then the dominant parameter in the model, $k_T(\equiv u_*\Delta t/h)$, should be a function of y. Moreover, the presence of a free surface and a bed surface is to restrict the vertical motion of suspended sediments and consequently the probability density function of particle displacement will be distorted near the boundaries. These factors make the calculation based on the probabilistic model complicated.

In Fig.22, the relative existence probability density of suspended particles, calculated by using the modified probabilistic model, is compared with the relative concentration distribution given by Rouse's equation (22) and the experimental data obtained by Vanoni (28).

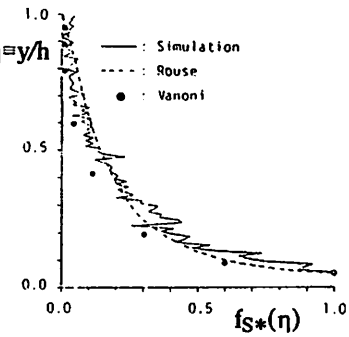

Fig.22 Relative existence probability of a suspended particle

Transition from Saltation to Suspension

If the initial particle speed v_0 is given, an individual saltation can be perfectly determined, and thus the speed in the vertical direction at the level y, $v_g(y|v_0)$, is also determined. If the vertical component of the fluctuating velocity v' acts on a particle during the time interval Δt, the additional displacement of the particle (the shift from the saltation path), ξ, can be expressed as

$$\frac{\xi}{d} = \left[\frac{C_D A_2}{4A_3(\sigma/\rho+C_M)}\right]\left(\frac{h}{d}\right)^2(\Delta t_*)^2\left[\,|\omega-v_*|(\omega-v_*)+|v_*|v_*\right] \tag{75}$$

in which $\Delta t_* \equiv u_*\Delta t/h$; $\omega \equiv v'/u_*$; and $v_* \equiv v_g/u_*$. In case of $v'>v_g$, Eq.75 can be written as

$$\xi_* \equiv \frac{\xi}{d} = K_\xi[\omega^2-2v_*\omega+v_*{}^2(1+|v_*|v_*)] \tag{76}$$

$$K_\xi \equiv \left[\frac{C_D A_2}{4A_3(\sigma/\rho+C_M)}\right]\left(\frac{h}{d}\right)^2(\Delta t_*)^2 \tag{77}$$

If the transition is to be defined as $\xi_*>\xi_{*c}$ and the value of ω to bring about the transition is to be expressed by ω_c, then

$$\omega_c = v_*\left\{1 \pm \sqrt{[1/(K_\xi v_*{}^2)]\mp 1}\right\} \tag{78}$$

Thus, the conditional probability that a particle in a saltation turns into suspension at the level y, $p_T(y|v_0)$ is given as the probability that $\omega > \omega_c(y)$. If ω follows a normal distribution with zero mean of which standard deviation is $\psi_0(y)u_*$,

$$p_T(y|v_0) = \frac{1}{\sqrt{2\pi}} \int_{\zeta_c}^{\infty} \exp\left(-\frac{\zeta^2}{2}\right) d\zeta \tag{79}$$

$$\zeta_c = \frac{\omega_c(y|v_0)}{\psi_0(y)u_*} \tag{80}$$

For example, according to the study on turbulence of open channel flow,

$$\psi_0(y) = 1.27 \exp\left(-\frac{y}{h}\right) \tag{81}$$

$p_T(y)$ can be obtained after releasing condition as for v_0 as follows:

$$p_T(y) = \int_0^{\infty} p_T(y|v_0) g_0(v_0) dv_0 \tag{82}$$

Some examples of video-film analysis as for transition from saltaion to suspension in flume experiments using polystyrene particles (d=0.128cm, σ/ρ=1.03, I=energy slope=1/500, h=3~5cm) are shown in Fig.23. The histogram indicates the measured heights of transition from bed load motion to suspension which must correspond to $f_T(y)$. On the other hand, $f_T(y)$ calculated by use of $f_B(y)$ and $p_T(y)$ for ξ_{*c}=0.2 is shown by a dotted line, while $f_T(y)$ calculated by use of the observed data on the probability density of saltation height is shown by a solid line. In spite of various phases of transition included, the transition phenomenon can be well described by this model.

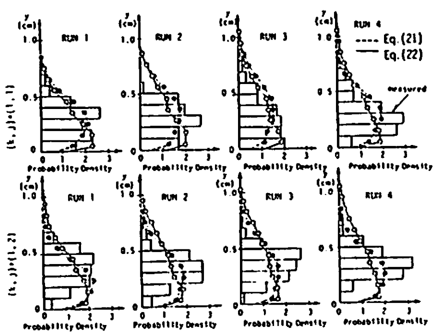

Fig.23 Probability density of transition height for suspension

CONCLUDING REMARKS

In this paper, various alluvial phenomena have been described on the basis of the concept that the probabilistic and discrete properties inherent to individual sediment motion play an important role on appearance of non-equilibrium situation of alluvial process. And thus, the sediment transport process has been treated as stochastic process and the superiority on the applicability of the Eulerian stochastic model has been clarified by a reasonable description of sand wave formation and armoring in transport of sediment mixtures. Furthermore, in order to obtain an unified stochastic model for bed material load transport, the subsystems of transport process are discussed by combining a stochastic approach with an approach based on dynamics, though the present model has not been enough to describe completely non-equilibrium transport process as bed material load.

On the other hand, there are many alluvial phenomena to which the characteristics of sediment on a grain size level as considered in the aforementioned model are scarcely contributive. For instance, the variation of channel boundary such as contraction, expansion or channel bend will also bring about non-equilibrium state of alluvial bed. When the variation of channel geometry has a predominant scale on the phenomenon, it will be admissible and rather available only to consider the change of sediment transport due to change of channel boundaries, as seen in usual analysis of river bed deformation of large scale. In general, as far as the scale of bed deformation under consideration is larger than that of non-equilibrium of sediment transport, we can obtain a practically allowable solution by use of the conventional formula for equilibrium sediment discharge. In word, it depends on the level of "graining" for a phenomenon whether we treat it as equilibrium or not.

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