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EFFECT OF SUSPENDED SEDIMENT ON FORMATION OF SAND WAVES

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SYNOPSIS

Stability of erodible beds in open channel flows with suspended sediment was analyzed and the effect of suspended sediment on the formation of sand waves was investigated. In the derivation of an equation for the bed load sediment in a non-equilibrium state, the effect of the suspended sediment was considered. The occurrences of two-dimensional sand waves were discussed as a problem of linear stability of erodible beds, and the new regime criteria for sand waves were proposed. The criteria indicate that on the finer sand bed the flat bed will occur in the lower values of the Froude number. The new regime criteria were compared with the data from the laboratory flumes and actual rivers. The theoretical results explain satisfactorily the regions of the occurrence of sand waves in alluvial streams with suspended sediment.

INTRODUCTION

Theoretical studies on the formation of sand waves in alluvial streams have been carried out by many researchers during the last twenty years. Most of them are analyzed as a problem of stability of the erodible bed, which are classified into two groups. One is the potential flow model developed by Kennedy (13), Hayashi (9) and others. The other is the shear flow model. In order to obtain the conditions of occurrence of sand waves, Kennedy (13) analyzed the sand bed instability in a two-dimensional potential flow by introducing the lag distance, δ. However, there remained a great uncertainty in the physical interpretation and evaluation of δ. Hayashi (9) tried to solve this uncertainty by considering the effect of the local bed inclination on the sediment transport. Engelund (3) described the two-dimensional flow by using a vorticity transport equation and a diffusion equation for the suspended sediment. Fredsøe (5) further developed by taking into account the influence of gravity on bed load transport. Using a one-dimensional analysis of the flow, Tsubaki and Saito (20) analyzed the two-dimensional stability of erodible bed. They introduced two factors of the bed instability; one is the local variation of bed shear stress due to the acceleration and the deceleration of flow and the other is the non-equilibrium of the transport process of bed load on the wavy bed. Subsequently, Tsubaki and Watanabe (21) discussed the formation of alternating bars by extending this theory to the three-dimensional stability.

Engelund (3) studied the effect of suspended sediment on the formation of sand waves and described that the stability boundaries shifted to smaller values of the Froude number for the finer material. Engelund and Fredsøe (4) confirmed this by using the experimental data of Guy, Simmons and Richardson (8).

The purpose of the present study is to clarify the effect of suspended sediment on the sand wave formation. In this paper, the two-dimensional stability of an erodible bed in a stream with suspended sediment is investigated by adopting the one-dimensional analysis of the flow. The local variation of bed shear stress and the non-equilibrium of the transport process of bed load are introduced as the instability factors for the bed. Furthermore, a bed load equation in a non-equilibrium state is developed by considering the suspension of bed load sediment and the deposition of suspended sediment. A linear analysis is adopted in order to examine the bed stability, and the effect of suspended sediment on the stability of a perturbed bed is clarified. Finally, a new diagram of regime criteria for sand waves is proposed. The theory is verified by the laboratory flume and river data.

BED SHEAR STRESS AND BED LOAD DISCHARGE ON SAND WAVES

The bed shear stress plays an important role to the stability of the erodible bed, because sand waves are the forms of the bed surface resulting from the local erosion and deposition produced by the irregularity of sediment transport in the direction of flow. First, the bed shear stress on the wavy bed is derived according to the theory developed by Tsubaki and Saito (20). Subsequently, an equation for the bed load discharge in a non-equilibrium state is described.

Bed Shear Stress

The flow over sand waves is periodically accelerating and decelerating owing to the convergence and divergence of the flow. In order to incorporate this flow pattern in the analysis, a parameter which denotes the acceleration and deceleration of flow is introduced into an equation for the velocity distribution.

For a uniform flow, an equation for the velocity distribution is obtained by assuming a constant eddy viscosity as

$$\frac{u}{u_*} = -\left(\frac{u_* h}{2\nu_t}\right)\zeta_1^2 + \frac{u_s}{u_*} \tag{1}$$

where u = the velocity at a point; u_s = the velocity at the water surface; u_* = the shear velocity; h = the water depth; ζ_1 = the dimensionless depth from the water surface; and ν_t = the eddy kinematic viscosity. Denoting the velocity defect at the bed by Δ, Eq. 1 is written as

$$u = u_s(1 - \Delta\zeta_1^2) \tag{2}$$

in which $\Delta = (u_* h/2\nu_t)(u_*/u_s)$. Then the mean velocity, u_m, over the cross section and the velocity, u_b, at the bed are written as

$$u_m = u_s\left(1 - \frac{\Delta}{3}\right) \; ; \qquad u_b = u_m\left(\frac{1 - \Delta}{1 - \Delta/3}\right) \tag{3}$$

From the above equations, the bed shear stress, τ_b, is expressed as

$$\frac{\tau_b}{\rho} = u_*^2 = \frac{1}{\phi^2}\left(\frac{1 - \Delta/3}{1 - \Delta}\right)^2 u_b^2 \tag{4}$$

where ρ = the mass density of water; and $\phi = u_m/u_*$.

The velocity distribution in the non-uniform flow over sand waves is assumed to be still quadratic as Eq. 2. In this case, however, the velocity defect, Δ, is considered to vary locally with the degree of convergence or divergence of flow, $\partial h/\partial x$. For simplicity, a linear relationship between Δ and $\partial h/\partial x$ is assumed as

$$\Delta = \Delta_o + \alpha\frac{\partial h}{\partial x} \tag{5}$$

where x = the coordinate in the downstream direction; Δ_o = the value of Δ for the uniform flow; and α = an empirical parameter. Tsubaki and Saito (20) has estimated

$\alpha \simeq 5.0$ for $\zeta_1 = 0.9$, by using the velocity distribution of the flow in the convergent and divergent pipes obtained by Nikuradse.

Assuming that the local friction factor in the non-uniform flow is equal to that in the uniform flow, the bed shear stress on the wavy beds is given by

$$\frac{\tau_b}{\rho} = \frac{1}{\phi_o^2}\left(\frac{1 - \Delta_o/3}{1 - \Delta_o}\right)^2 u_m^2\left(\frac{1 - \Delta}{1 - \Delta/3}\right)^2 \qquad (6)$$

where ϕ_o = the value of ϕ for the uniform flow. By integrating Eq. 1 and using 12 as an average value of $u_* h/\nu_t$ for open channel flows as suggested by Tsubaki (19), Δ_o is obtained as

$$\Delta_o = 6/(\ \phi_o + 2\ ) \qquad (7)$$

Equation for Bed Load Discharge

The bed material load is divided into the bed load which is the part of the sediment load transported by the tractive force in continuous contact with the bed and the suspended load which is supported by the turbulence and transported in suspension in the flow. In this paper, the flow section is divided into two parts; one is the bed layer where the sediment is transported as the bed load and the other is the suspension layer where the sediment as the suspended load.

On the basis of the model proposed by Einstein (2), an equation for bed load discharge in a non-equilibrium state is developed. In the derivation of this equation, two kinds of sediment flux at the boundary between bed layer and suspension layer are considered as shown in Fig. 1. One is an upward flux of sediment particles which are caught by the upward component of turbulent current. The other is a downward flux of suspended sediment particles which are settled into the bed layer.

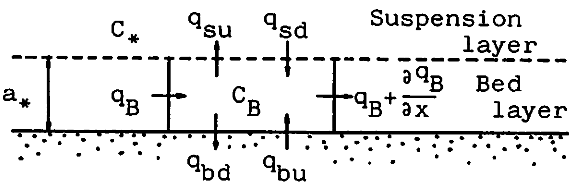

Fig. 1 Sediment fluxes in the bed layer

Thus the continuity equation for bed load sediment in a non-equilibrium state is expressed as

$$\frac{\partial(C_B a_*)}{\partial t} + \frac{\partial q_B}{\partial x} = q_{bu} - q_{bd} + q_{sd} - q_{su} \qquad (8)$$

where t = the time; q_B = the volumetric bed load discharge per unit time and width; C_B = the concentration of bed load; a_* = the thickness of bed layer; q_{bu}, q_{bd} = the erosion rate of sediment and the deposition rate of bed load sediment per unit time and unit area of bed surface, respectively; q_{sd}, q_{su} = the downward flux and the upward flux of sediment per unit time and unit area of the boundary surface between bed layer and suspension layer, respectively.

The fluxes at the bed surface are expressed as

$$q_{bd} = \frac{q_B}{\ell} \qquad (9)$$

$$q_{bu} = A_s\sqrt{sgd}\cdot P \qquad (10)$$

where ℓ = the average step length of a sediment particle; P = the probability of a particle being eroded; d = the diameter of sediment; s = the specific weight of sediment in the water; g = the gravitational acceleration; and A_s = an empirical constant. The average step length, ℓ, and the probability, P, are determined by the properties of sediment and the local conditions of flow. According to the Einstein's model,

$$\ell = \frac{\lambda_d d}{1 - P} \tag{11}$$

and

$$P = \frac{A_* \Phi_{Be}}{1 + A_* \Phi_{Be}} \tag{12}$$

where $\Phi_{Be} = q_{Be}/\sqrt{sgd^3}$ = the bed load function in the equilibrium state corresponding to the local dimensionless tractive force, $\Psi = u_*^2/sgd$; λ_d = dimensionless measure for a single step length of a particle; and $A_* = 1/(\lambda_d A_s)$. Substitution of Eq. 12 into Eqs. 10 and 11 yields

$$q_{bu} = \frac{\Phi_{Be}\sqrt{sgd^3}}{\lambda_d d(\ 1 + A_*\Phi_{Be}\ )} \quad ; \quad \ell = \lambda_d d(\ 1 + A_*\Phi_{Be}\ ) \tag{13}$$

Combining above two equations, a similar equation to Eq. 9 is obtained as

$$q_{bu} = \frac{q_{Be}}{\ell} \tag{14}$$

where q_{Be} = the bed load discharge in the equilibrium state. The dimensionless tractive force, Ψ, is obtained by rewriting Eq. 6 as

$$\Psi = \frac{1}{\phi_o^2}\left(\frac{1 - \Delta_o/3}{1 - \Delta_o}\right)^2 \frac{u_m^{\ 2}}{sgd}\left(\frac{1 - \Delta}{1 - \Delta/3}\right)^2 \tag{15}$$

With respect to the fluxes, q_{su} and q_{sd}, the model of Hirano (10) for the reference concentration of suspended sediment is adopted. Assuming that the probability density function for the vertical turbulence velocity is distributed following the normal error function, q_{su} and q_{sd} are expressed as

$$q_{su} = C_B\int_{w_0}^{\infty}(v' - w_0)f(v')dv' = C_B w_0\{\phi(\sigma) - F(\sigma)\} \tag{16}$$

$$q_{sd} = C_*\int_{-w_0}^{\infty}(v' + w_0)f(v')dv' = C_* w_0\{\phi(\sigma) + 1 - F(\sigma)\} \tag{17}$$

where v' = the vertical turbulence velocity; f(v') = the probability density function of v'; w_0 = the fall velocity of the sediment particle; and C_* = the reference concentration of suspended sediment. The functions, $\phi(\sigma)$ and $F(\sigma)$, stand for

$$\phi(\sigma) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{\sigma^2}{2}\right) \tag{18}$$

$$F(\sigma) = \int_{\sigma}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{t^2}{2}\right)dt \tag{19}$$

where $\sigma = w_0/\sqrt{\overline{v'^2}}$; and $\sqrt{\overline{v'^2}} = 0.93u_*$ (17).

The upward flux is given by the local concentration of bed load. On the other hand, the downward flux does not directly correspond to the local flow conditions. Since the sediment lifted into the flow in the upstream region settles scatteringly in the bed layer, the downward flux is assumed to be approximated by averaging along one wave length. In other words, the averaged downward flux is equal to the averaged upward flux. Thus,

$$q_{sd} - q_{su} = \overline{C_B w_0\{\phi(\sigma)-F(\sigma)\}}[1 - \frac{C_B w_0\{\phi(\sigma)-F(\sigma)\}}{\overline{C_B w_0\{\phi(\sigma)-F(\sigma)\}}}] \tag{20}$$

where $\overline{C_B w_0\{\phi(\sigma)-F(\sigma)\}}$ = the value of q_{su} averaged along one wave length. C_B is defined as

$$C_B = \frac{q_B}{\bar{u}_B a_*} \tag{21}$$

where $\bar{u}_B$ = the average velocity of the bed load sediment, which is given by Ashida and Michiue (1) as

$$\bar{u}_B = ku_*(1 - \frac{u_{*c}}{u_*}) \tag{22}$$

where k = 8.5 if one adopts the logarithmic law for the velocity distribution in the flow over the flat bed.

Thus, an equation for the bed load discharge in a non-equilibrium state is expressed as

$$\frac{\partial(C_B a_*)}{\partial t} + \frac{\partial q_B}{\partial x}$$
$$= \frac{1}{\ell}(q_{Be} - q_B) + \overline{C_B w_0\{\phi(\sigma)-F(\sigma)\}}[1 - \frac{C_B\{\phi(\sigma)-F(\sigma)\}}{\overline{C_B\{\phi(\sigma)-F(\sigma)\}}}] \tag{23}$$

STABILITY ANALYSIS

Continuity Equation for Sediment

In this analysis, the suspended load takes part not directly in the bed variation but in the transport rate of bed load in the non-equilibrium state. The change of the bed elevation is induced by the difference between the erosion rate, q_{bu}, and the deposition rate, q_{bd}, of sediment on the bed surface. Using Eqs. 9 and 14, the continuity equation for the sediment is expressed by

$$\frac{\partial z}{\partial t} + \frac{1}{1 - \varepsilon}\frac{1}{\ell}(q_{Be} - q_B) = 0 \tag{24}$$

where z = the elevation of the bed taken from the average bed surface with the slope $S_0 = \tan\theta$; and ε = the porosity of sediment.

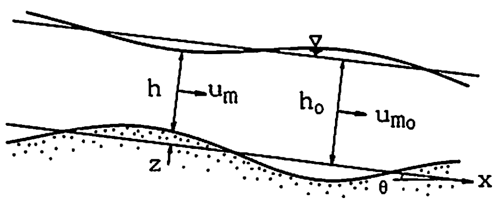

Fig. 2 Definition sketch of the perturbed flow

Equations for Flow

The continuity and momentum equations for the flow are expressed by

$$\frac{\partial h}{\partial t} + \frac{\partial(u_m h)}{\partial x} = 0 \tag{25}$$

$$\frac{\partial u_m}{\partial t} + u_m\frac{\partial u_m}{\partial x} = g\sin\theta - g\frac{\partial}{\partial x}(\lambda h\cos\theta + z\cos\theta) - \frac{\tau_b}{\rho h} \tag{26}$$

where λ = the Jaeger coefficient.

Stability Equation

In order to analyze the bed stability, we consider a small deformation imposed on the initial flat surface of the bed. The depth, the velocity and the transport rate of sediment are divided into undisturbed values and deviations as

$$h = h_o(1 + \eta) \;;\quad u_m = u_{mo}(1 + u') \;;\quad \Phi_B = \Phi_{Bo}(1 + \phi') \tag{27}$$

where the subscript, o, denotes quantities for the undisturbed flow. By considering small deviations and small steepness, substitution of Eq. 27 into Eqs. 23, 24, 25 and 26 yields the linearized equations for deviations. In the derivation of these equations, the following equation is used for the bed load function in the equilibrium state as

$$\Phi_{Be} = K_B(\Psi - \Psi_c)^m \tag{28}$$

where $\Psi_c = u_{*c}^2/sgd \simeq 0.05$ in which u_{*c} = the critical shear velocity; and K_B = a constant. The exponent, m, generally takes 3/2 for the bed load. Since the turbulence intensity over wavy beds does not directly correspond to the local shear stress, it is assumed to be given by using the mean shear stress herein, thus

$$\sigma = \sigma_o = w_0/(0.93u_{*o}) \tag{29}$$

Let

$$\zeta = z/h_o \;;\quad X = x/h_o \;;\quad T = t\cdot\Phi_{Bo}\sqrt{sgd^3}/\{(1-\varepsilon)h_o^2\}$$

then we obtain the following equations.

$$k_0\phi_o E\chi\frac{\partial\phi'}{\partial T} + E\frac{\partial\phi'}{\partial X} = M(2u' - q\alpha\frac{\partial\eta}{\partial X}) - \phi' - EW\phi' \tag{30a}$$

$$E\frac{\partial\zeta}{\partial T} + M(2u' - q\alpha\frac{\partial\eta}{\partial X}) - \phi' = 0 \tag{30b}$$

$$\chi\frac{\partial\eta}{\partial T} + \frac{\partial u'}{\partial X} + \frac{\partial\eta}{\partial X} = 0 \tag{30c}$$

$$F^2\chi\frac{\partial u'}{\partial T} + F^2\frac{\partial u'}{\partial X} + 2S_0u' + \frac{\partial\zeta}{\partial X} + \lambda_1F^2\frac{\partial^3\zeta}{\partial X^3} - S_0\eta + (1 - S_0q\alpha)\frac{\partial\eta}{\partial X} + \lambda_2F^2\frac{\partial^3\eta}{\partial X^3} = 0 \tag{30d}$$

in which

$$\left.\begin{aligned}
F &= \frac{u_{mo}}{\sqrt{gh_o\cos\theta}}\\
\chi &= \frac{\Phi_{Bo}\sqrt{sgd^3}}{(1-\varepsilon)u_{mo}h_o} = \frac{1}{(1-\varepsilon)}\frac{q_{Bo}}{u_{mo}h_o}\\
E &= \frac{\lambda_d d}{h_o}(1 + A_*\Phi_{Bo})\\
W &= \frac{k_0w_0}{\xi_*u_{*o}}\{\phi(\sigma) - F(\sigma)\} \;;\quad k_0 = \frac{1}{k(1-u_{*c}/u_{*o})} \;;\quad \xi_* = \frac{a_*}{h_o}
\end{aligned}\right\} \tag{31}$$

$$
\left.
\begin{aligned}
M &= \frac{m\Psi_o}{\Psi_o - \Psi_c} \\
q &= \frac{4/3}{(1-\Delta_o)(1-\Delta_o/3)}
\end{aligned}
\right\}
$$

Parameters λ_1 and λ_2 represent the centrifugal effects of curvilinearity of bed surface and water surface, respectively, which are given by Iwasa and Kennedy (12).

Sinusoidal perturbations are considered as

$$\zeta = \hat{\zeta}e^{\gamma T+i\beta X}; \quad \eta = \hat{\eta}e^{\gamma T+i\beta X}; \quad u' = \hat{u}e^{\gamma T+i\beta X}; \quad \phi' = \hat{\phi}e^{\gamma T+i\beta X} \tag{32}$$

where $\hat{\zeta}$, $\hat{\eta}$, $\hat{u}$, $\hat{\phi}$ = the amplitudes at $T = 0$; $\gamma = \gamma_1 + i\gamma_2$ = the dimensionless complex propagation velocity; and β = the dimensionless wave number. Substitution of Eq. 32 into Eq. 30 yields

$$(k_0\phi_o\chi\gamma E + 1 + EW + i\beta E)\hat{\phi} - M(2\hat{u} - i\beta q\alpha\hat{\eta}) = 0 \tag{33a}$$

$$\gamma E\hat{\zeta} + M(2\hat{u} - i\beta q\alpha\hat{\eta}) - \hat{\phi} = 0 \tag{33b}$$

$$i\beta\hat{u} + (\chi\gamma + i\beta)\hat{\eta} = 0 \tag{33c}$$

$$(F^2\chi\gamma + 2S_0 + i\beta F^2)\hat{u} + i\beta(1 - \lambda_1\beta^2F^2)\hat{\zeta} + \{-S_0 + i\beta(1 - S_0q\alpha - \lambda_2\beta^2F^2)\}\hat{\eta} = 0 \tag{33d}$$

Elimination of the amplitudes from the above four equations reduces to the stability criterion for the bed as

$$
\begin{aligned}
& i\beta\frac{1}{\beta^2}F^2(k_0\phi_oE)^2\chi^4\gamma^5 \\
& + \left[-2F^2(k_0\phi_oE)^2 + i\beta\frac{2}{\beta^2}(k_0\phi_oE)\{F^2(1+EW) + S_0k_0\phi_oE\} \right]\chi^3\gamma^4 \\
& + \left[\begin{aligned} & 2M(k_0\phi_o)^2E(1-\lambda_1\beta^2F^2)\chi - k_0\phi_oE\{4F^2(1+EW) + 3S_0k_0\phi_oE\} \\ & + i\beta[\ \frac{1}{\beta^2}F^2\{\beta^2E^2+(1+EW)^2\} + \frac{4}{\beta^2}S_0k_0\phi_oE(1+EW) \\ & \qquad + (k_0\phi_oE)^2\{1-S_0q\alpha-(1+\lambda_2\beta^2)F^2\}] \end{aligned} \right]\chi^2\gamma^3 \\
& + \left[\begin{aligned} & Mk_0\phi_o(1-\lambda_1\beta^2F^2)\{E(2W-k_0\phi_o\beta^2q\alpha) + 2(1+EW)\}\chi \\ & \qquad - 2F^2\{\beta^2E^2+(1+EW)^2\} - 6S_0k_0\phi_oE(1+EW) \\ & + i\beta[\ 2M(k_0\phi_o)^2E(1-\lambda_1\beta^2F^2)\chi \\ & \qquad + \frac{2}{\beta^2}S_0\{\beta^2E^2+(1+EW)^2\} + 2k_0\phi_oE(1+EW)\{1-S_0q\alpha-(1+\lambda_2\beta^2)F^2\}\ ] \end{aligned} \right]\chi\gamma^2 \\
& + \left[\begin{aligned} & M(1-\lambda_1\beta^2F^2)\{\beta^2(2E-k_0\phi_oEq\alpha W) + (1+EW)(2W-k_0\phi_o\beta^2q\alpha)\}\chi - 3S_0\{\beta^2E^2+(1+EW)^2\} \\ & + i\beta[\ -M(1-\lambda_1\beta^2F^2)\{E(2W-k_0\phi_o\beta^2q\alpha) + k_0\phi_oE(\beta^2q\alpha-2W) - 2(k_0\phi_o+1)(1+EW)\}\chi \\ & \qquad + \{\beta^2E^2+(1+EW)^2\}\{1-S_0q\alpha-(1+\lambda_2\beta^2)F^2\}\ ] \end{aligned} \right]\gamma \\
& - \beta^2M(1-\lambda_1\beta^2F^2)[\ \beta^2[\{E+(1+EW)/\beta^2\}q\alpha + 2/\beta^2] + i\beta\{q\alpha-2E-2(1+EW)W/\beta^2\}] \\
& = 0
\end{aligned}
\tag{34}
$$

Verification of Stability Equation

The stability equation (Eq. 34) is so complicated that it is difficult to discuss the physical meanings. The authors try to simplify the equation by estimating the order of each term. Since the value of χ is on the order of 10^{-3} at the highest, we can neglect the terms containing χ compared with other terms in the order of unity keeping a sufficient accuracy. This means that the flow can be assumed quasi-uniform. Thus, γ can be simply expressed as

$$\gamma = \frac{\beta^2 M(1-\lambda_1\beta^2F^2)\{\beta^2(E_*q\alpha+2/\beta^2) + i\beta(q\alpha-2E_*)\}}{\{\beta^2E^2+(1+EW)^2\}[-3S_0+ i\beta\{1-S_0q\alpha-(1+\lambda_2\beta^2)F^2\}]} \tag{35}$$

in which

$$E_* = E + (1+EW)W/\beta^2 \tag{36}$$

From Eq. 35, the real part, γ_1, and the imaginary part, γ_2, of γ are derived as

$$\gamma_1 = A(1-\lambda_1\beta^2F^2)[\{1-S_0q\alpha-(1+\lambda_2\beta^2)F^2\}(q\alpha-2E_*) - 3S_0(E_*q\alpha+2/\beta^2)] \tag{37}$$

$$-\frac{\gamma_2}{\beta} = A(1-\lambda_1\beta^2F^2)[\{1-S_0q\alpha-(1+\lambda_2\beta^2)F^2\}(E_*q\alpha+2/\beta^2) + 3S_0(q\alpha-2E_*)/\beta^2] \tag{38}$$

in which

$$A = \frac{\beta^2 M}{\{\beta^2E^2+(1+EW)^2\}[\{1-S_0q\alpha-(1+\lambda_2\beta^2)F^2\}^2+9S_0^2/\beta^2]} \tag{39}$$

The sign of γ_1 identifies stable and unstable conditions. Since the propagation velocity of the perturbation is expressed by $-\gamma_2/\beta$, the waves migrate downstream when this value is positive, and upstream when it is negative.

This theory is characterized by three parameters α, E and W. The parameter α represents the deformation of velocity distribution and the asymmetric distribution of the bed shear stress due to the acceleration and deceleration of flow over wavy beds. The parameter E denotes the non-equilibrium of the transport of bed load. The parameter W represents the effect of the suspension of sediment which decreases rapidly against w_0/u_{*0}.

If $\alpha = 0$, $E = 0$, $\lambda_1 = 0$ and $\lambda_2 = 0$ according to the method by Iwagaki (11) in the analysis of the bed stability, then Eqs. 37 and 38 yield

$$\gamma_1 = -A'\{(1-F^2)W + 3S_0\} \tag{40}$$

$$-\gamma_2/\beta = A'\{(1-F^2) - 3S_0W/\beta^2\} \tag{41}$$

in which

$$A' = \frac{2M}{(1-F^2)^2 + 9S_0^2/\beta^2} \tag{42}$$

When $W = 0$, $-\gamma_2/\beta > 0$ for $F < 1$ and $-\gamma_2/\beta < 0$ for $F > 1$. These roughly coincide the actual directions of movement of dunes and antidunes, respectively. However, the sign of γ_1 is always negative, that is, the bed is stable and sand waves do not occur. This means that when the flow near the bed is determined by the averaged conditions as the case of the disturbance with a long wave length ($\beta \rightarrow 0$), the bed is stable as is evident from Eq. 37. When $W \neq 0$, the bed can become unstable for the supper-critical flow.

Meanwhile, according to the Yalin's simple expression (23), the wave length L is given as

$$\left.\begin{aligned} L &\approx 5h_0 && \text{for dunes} \\ L &\approx 1000d && \text{for ripples} \end{aligned}\right\} \tag{43}$$

Therefore the value of β lies in the order of unity for the disturbance with a short wave length such as sand waves. Moreover, considering that the value of S_0 rarely exceed 10^{-2}, we can neglect the terms containing S_0 in Eqs. 37 and 38. From the above discussion, it is clarified that α, E and W are important parameters for the bed stability.

From Eq. 37, the following three curves for the neutral are obtained:

$$1 - \lambda_1\beta^2F^2 = 0 \qquad \text{or} \qquad F^2 = Fc^2 = \frac{1}{\lambda_1\beta^2} \tag{44a}$$

$$1 - (1 + \lambda_2\beta^2)F^2 = 0 \qquad \text{or} \qquad F^2 = Fa^2 = \frac{1}{1 + \lambda_2\beta^2} \tag{44b}$$

and

$$q\alpha - 2E - \frac{2}{\beta^2}(1+EW)W = 0 \tag{44c}$$

Since Fa is less than Fc for the range of $\beta < 2.45$ which value is obtained from Eqs. 44a and 44b by using the values of $\lambda_1 = 1/2$ and $\lambda_2 = 1/3$ (Iwasa and Kennedy (12)), the following conditions can be deduced:

When $F < Fa$:

if $q\alpha > 2E + \frac{2}{\beta^2}(1 + EW)W$, then $\gamma_1 > 0$ (unstable);

if $q\alpha < 2E + \frac{2}{\beta^2}(1 + EW)W$, then $\gamma_1 < 0$ (stable);

and

$-\gamma_2/\beta > 0$ (downstream).

When $Fa < F < Fc$:

if $q\alpha > 2E + \frac{2}{\beta^2}(1 + EW)W$, then $\gamma_1 < 0$ (stable);

if $q\alpha < 2E + \frac{2}{\beta^2}(1 + EW)W$, then $\gamma_1 > 0$ (unstable);

and

$-\gamma_2/\beta < 0$ (upstream).

These indicate that dunes formation is possible for $F < Fa$ and antidunes formation for $Fa < F < Fc$. Therefore, Eqs. 44a and 44b yield the region of the occurrence of antidunes and Eq. 44c gives a criterion to distinguish dunes regime from flat bed regime.

By using the relations of $\Psi_o = h_oS_0/sd$ and $\phi_o{}^2 = F^2/S_0$, E is rewritten as

$$E = \frac{\lambda_d}{s}\frac{1}{\phi_o^2}\frac{1 + A_*\phi_{Bo}}{\Psi_o}F^2 \tag{45}$$

Therefore, in a stream with a moderately low Froude number ($F << Fa$), the bed is unstable owing to the role of the parameter α, and ripples and/or dunes will occur. As the value of F increases, the parameter E acts as a stabilizing factor, and flat bed may occur. When $F > Fa$, 2E is mostly larger than $q\alpha$. Therefore, the bed becomes unstable and antidunes will occur. Further increase of F beyond Fc results in a stable flat bed. By introducing the centrifugal effect, an upper limit for the region of occurrence of antidunes is obtained.

Since the parameter W plays a similar role to E, the smaller the value of w_0/u_{*o}, the lower the Froude number at which the transition from dunes to flat bed occurs.

REGIONS OF OCCURRENCE OF SAND WAVES

Since E in Eq. 44c is a function of Ψ_o and F from Eq. 45, we can show the regions of occurrence of sand waves in the (Ψ_o, F) plane as shown by Garde and Albertson (6). First, we discuss the transition from dunes to flat bed. The values for various parameters contained in Eq. 44c are determined as follows. Since Δ_o is a function of ϕ_o from Eq. 7, q defined by Eq. 31 is also determined by ϕ_o. For the value of α, we take 5.0 as previously mentioned. With respect to the values of λ_d and A_*, Einstein (2) gave 100 and 43.5, respectively. Shinohara and Tsubaki (16), however, reported from their tracer experiments that the rate of increase of the erosion probability of a sediment particle with the tractive force, Ψ, was overestimated in the Einstein's bed load formula, that is, the A_* value was smaller than that given by Einstein. On the other hand, they concluded that the λ_d value was about 100. Herein $\lambda_d = 100$ and $A_* = 10$ are adopted as proposed by Tsubaki and Saito (20). Except the extremely small value of β, there is little effect of β on the stability. Hence, $\beta = 1$ is adopted by referring to Eq. 43. Considering the sand grain for the bed material, s = 1.65. Five percent of the flow depth is often used as the bed layer thickness, that is, $\xi_* = 0.05$. Consequently, the boundary between dunes regime and flat bed regime is expressed as a function of Ψ_o and F with parameters ϕ_o and w_0/u_{*o}.

Here, we have rewritten Eq. 44c by using the relation of Eq. 45, as

$$\frac{F^2}{\phi_o^2 q\alpha} = \frac{1}{2}\frac{s}{\lambda_d}\frac{\Psi_o}{1 + A_*\Phi_{Bo}}\frac{1 - \frac{2}{\beta^2 q\alpha}W}{1 + \frac{1}{\beta^2}W^2} \qquad (46)$$

In the case that W = 0 (no suspended sediment, or, $w_0/u_{*o} = \infty$), the right-hand side of this equation is a function of Ψ_o only, and thus the boundary curve between dunes and flat bed can be expressed as a single curve in the (Ψ_o, $F/(\phi_o\sqrt{q\alpha})$) plane.

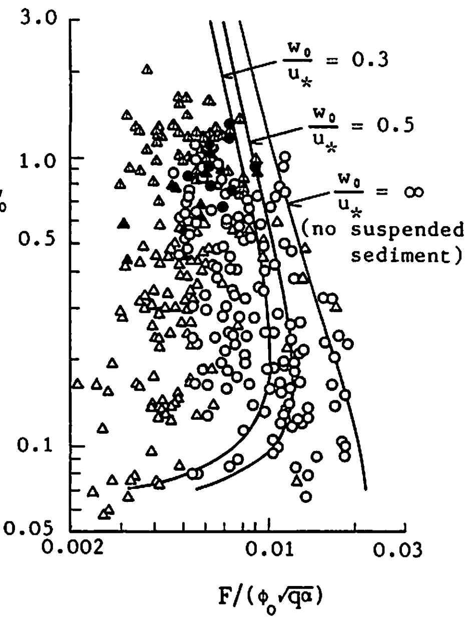

Fig. 3 Comparison between theory and experiments (ripples(△),dunes(○))
△,$w_0/u_* < 0.3$; ▲,●,$0.3 < w_0/u_* < 0.5$;
△,○, $w_0/u_* > 0.5$

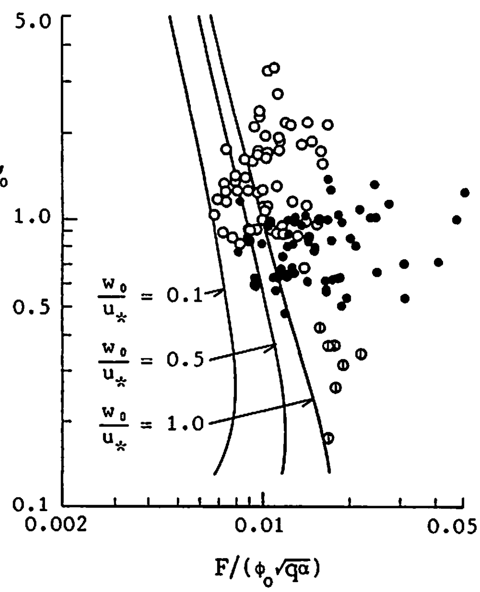

Fig. 4 Comparison between theory and experiments (flat bed)
○,$0.1 < w_0/u_* < 0.5$; ●,$0.5 < w_0/u_* < 1.0$;
Φ,$w_0/u_* > 1.0$

The effect of q (or ϕ_0) on the value of right-hand side of this equation is negligibly small. Thus, the parameters which determine the value of the right-hand side of Eq. 46 reduce to w_0/u_{*0} only. This means that the neutral curve in this plane moves with the value of w_0/u_{*0}.

In Figs. 3 and 4, the theoretical results are compared with the experimental data for fully developed sand waves presented by Laursen (14), Simons and Richardson (15), Garde and Raju (7), Guy, Simons and Richardson (8) and Willis, Coleman and Ellis (22), and those collected by the Committee on Hydraulics and Hydraulic Engineering of JSCE (18). The data are grouped by the value of w_0/u_*, and the computed curves are illustrated with solid lines for each value of w_0/u_{*0} corresponding to each group. A constant value $\phi_0 = 18$ is used in these diagrams, since they are less sensitive to the change of ϕ_0. In order to estimate the value of ϕ_0 from the data, the Manning-Strickler's formula was used. It is clearly seen

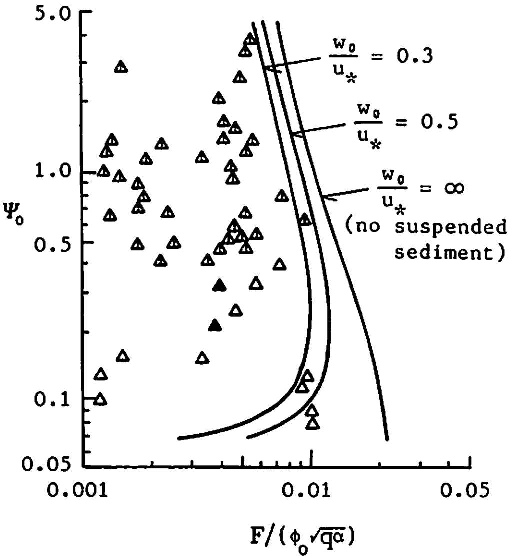

Fig. 5 Comparison between theory and river data (ripples)
△, $w_0/u_*<0.3$; ▲, $0.3<w_0/u_*<0.5$; △, $w_0/u_*>0.5$

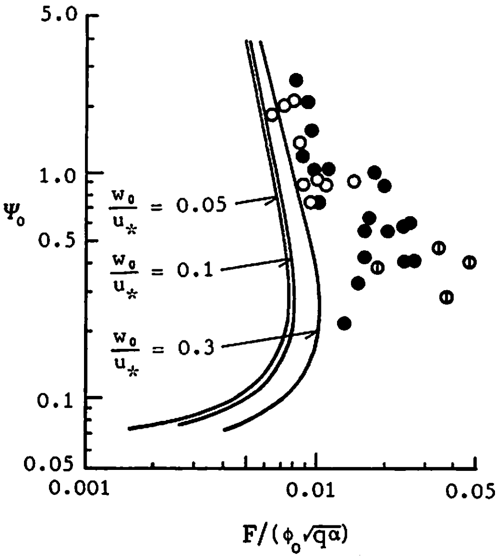

Fig. 6 Comparison between theory and river data (flat bed)
○, $0.05<w_0/u_*<0.1$; ●, $0.1<w_0/u_*<0.3$; ⦶, $w_0/u_*>0.3$

in these figures that the smaller the value of w_0/u_{*0}, the smaller the value of $F/(\phi_0\sqrt{q\alpha})$ at which the transition from dunes to flat bed occurs. This means that for the finer sediment the boundary curve between the lower and upper regimes moves to the lower values of the Froude number, as previously mentioned. The predicted regime criteria from Eq. 46 seem to agree well with the actual behavior of occurrence of ripples, dunes and flat bed for each value of w_0/u_*.

Figures 5 and 6 show the comparisons between the theoretical results and the river data collected by Garde and Albertson (6). Ripples and flat bed are both formed in the regions predicted by this theory.

Conversely, the unstable region in upper flow regime is bounded by two curves; F = Fa, Eq. 44a and F = Fc, Eq. 44b. It can be determined by F and β independently of w_0/u_*, as shown by the solid lines in Fig. 7, which is similar to the results predicted from the potential theory of Hayashi (9) shown by the two dotted lines. In this figure the experimental data corresponding to antidunes are plotted and also classified by the values of w_0/u_*. Almost all of antidunes are formed in the predicted region. No effect of w_0/u_* on the region for antidunes is seen in this figure, though suspended sediment is dominant there.

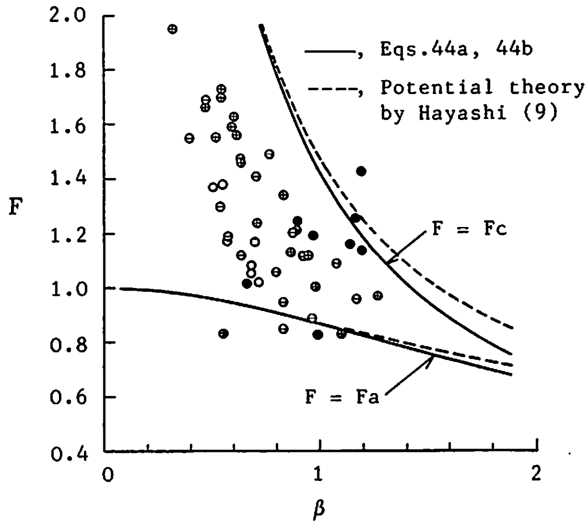

Fig. 7 Comparison between theory and experiments (antidunes)
○, $w_0/u_* < 0.3$; ⊖, $0.3 < w_0/u_* < 0.5$; ⊕, $0.5 < w_0/u_* < 1.0$; ●, $w_0/u_* > 1.0$

CONCLUSIONS

The instability of erodible beds was studied by considering the effect of suspended sediment. In order to analyze the bed stability, two factors for instability were introduced; one is the asymmetry of the distribution for bed shear stress and the other is the non-equilibrium of bed load transport on sand waves. The results obtained herein are summarized as follows:

1. In a flow with a moderately low Froude number, the bed is unstable owing to the asymmetric distribution of the bed shear stress. As the value of Froude number increases, the value of stability factor increases as a result of the non-equilibrium of bed load transport leading to the stable flat bed. The boundary between the lower and upper regimes for sand waves is shown very well in the $(\Psi_o, F/(\phi_o\sqrt{q\alpha}))$ plane.

2. Suspended sediment contributes to the non-equilibrium of bed load transport. It is clarified that w_0/u_* is an important parameter for the transition from the lower regime to the upper regime. That is, for a smaller value of w_0/u_*, the region for flat bed extends to a smaller value of $F/(\phi_o\sqrt{q\alpha})$.

3. No effect of w_0/u_* on the region of occurrence of antidunes is seen.

This theory explains satisfactorily the regime criteria for sand waves such as ripples, dunes, and antidunes.

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APPENDIX - NOTATION

The following symbols are used in this paper:

a_* = thickness of bed layer;

A_* = constant in the equation of bed load function by Einstein;

C_B = concentration of bed load;

C_* = reference concentration of suspended sediment;

d = diameter of sediment particle;

E = parameter denoting the non-equilibrium of bed load transport;
$f(v')$ = probability density function of v';
F = Froude number, defined by $F = u_{mo}/\sqrt{gh_o \cos\theta}$;
$F(\sigma)$ = function of σ, defined by Eq. 19;
g = gravitational acceleration;
h = water depth;
k = constant;
K_B = coefficient in the equation for bed load function in equilibrium state;
ℓ = average step length of sediment particle;
L = wave length;
m = exponent in the equation for bed load function in equilibrium state;
M = $m\Psi_o/(\Psi_o-\Psi_c)$;
P = probability of a particle being eroded;
q = $4/3/\{(1-\Delta_o)(1-\Delta_o/3)\}$;
q_{bd} = deposition rate of bed load sediment on the bed;
q_{bu} = erosion rate of sediment on the bed;
q_{sd} = deposition rate of suspended sediment to the bed layer;
q_{su} = pick up rate of bed load sediment into the suspension layer;
q_B = bed load discharge in non-equilibrium state;
q_{Be} = bed load discharge in equilibrium state;
s = specific weight of sediment in the water;
S_0 = slope of the undisturbed bed;
t = time;
T = dimensionless time, defined by $T = t \cdot \Phi_{Bo}\sqrt{sgd^3}/\{(1-\varepsilon)h_o^2\}$;
u = velocity at a point;
u' = dimensionless deviation from u_{mo};
$\hat{u}$ = amplitude of u';
u_b = velocity at the bed;
u_m = mean velocity over the cross section;
u_s = velocity at the water surface;
$\bar{u}_B$ = average velocity of the bed load sediment;
u_* = shear velocity;
u_{*c} = critical shear velocity;
v' = vertical turbulence velocity;
w_0 = fall velocity of sediment particle;
W = parameter denoting the effect of suspension of sediment;
x = coordinate in the downstream direction;
X = x/h_o;
z = elevation of the bed taken from the average bed;
α = parameter denoting the asymmetric distribution of bed shear stress;

β = dimensionless wave number;

γ = dimensionless complex propagation velocity;

γ_1 = real part of γ;

γ_2 = imaginary part of γ;

Δ = velocity defect at the bed, defined by $\Delta = (u_* h/2\nu_t)(u_*/u_s)$;

ε = porosity of sediment;

ζ = z/h_o;

$\hat{\zeta}$ = amplitude of ζ;

ζ_1 = dimensionless depth from the water surface;

η = dimensionless deviation from h_o;

$\hat{\eta}$ = amplitude of η;

θ = slope angle of the undisturbed bed;

λ = Jaeger coefficient;

λ_1, λ_2 = parameters denoting the centrifugal effects of curvilinearity of bed surface, and water surface, respectively;

λ_d = constant relating to a single step length of a particle;

ν_t = eddy kinematic viscosity;

ξ_* = dimensionless thickness of the bed layer , defined by $\xi_* = a_*/h_o$;

ρ = mass density of the water;

σ = $w_0/\sqrt{\overline{v'^2}}$;

τ_b = bed shear stress;

ϕ = u_m/u_*;

ϕ' = dimensionless deviation from Φ_{Bo};

$\hat{\phi}$ = amplitude of ϕ';

$\phi(\sigma)$ = function of σ, defined by Eq. 18;

Φ_B = bed load function, defined by $\Phi_B = q_B/\sqrt{sgd^3}$;

χ = concentration of bed load in the flow section, defined by $\chi = q_{Bo}/(1-\varepsilon)/(u_{mo}h_o)$ in Eq. 31;

Ψ = dimensionless tractive force, defined by $\Psi = u_*^2/sgd$; and

Ψ_c = dimensionless critical tractive force, defined by $\Psi_c = u_{*c}^2/sgd$.

Subscript o represents the value corresponding to the undisturbed flow.