Journal of Hydroscience and Hydraulic Engineering
Vol. 6, No. 2, December, 1988, pp.17-27

ON THE DISPERSION PHENOMENA OF SUSPENDED SOLID IN TURBULENT OPEN-CHANNEL FLOW

by

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SYNOPSIS

The longitudinal dispersion of suspended particles in a turbulent open channel flow was studied. Special attention was directed to the effect of the settling of particles on the dispersion phenomena. In the theoretical analysis, we employed the similar method as Elder used in the analysis of the dispersion of neutral matter. The mean velocity and the dispersion coefficient of the sediment cloud are given as functions of Z and α, where Z is the fall velocity parameter and α is the bed absorbency coefficient. The theoretical results of this work include some results of the previous work, as its extreme case. Experimental data have been compared with the theoretical analysis obtained herein. The agreement between data and the theoretical results is found to be satisfactory.

INTRODUCTION

This paper describes the longitudinal dispersion of suspended sediment in a turbulent open channel flow. Much attention has been paid to the effect of the settling of sediment particles on the dispersion phenomena. Elder(2) applied the analysis developed by Taylor(6) to predict the longitudinal dispersion of matter in the two dimensional turbulent open channel flow. After Elder's work, many studies have been reported on this subject. Most of them, however, investigate the dispersion of the nonbuoyant matter.

When the dispersant has the fall velocity, its motion becomes more complicated than that of neutral one. Furthermore, it is very difficult to describe the particles motion near the bed of the flow. Whether a particle deposits on the bed or not depends on various factors such as bed roughness, hydrodynamic condition close to the flume bed and on the particles characteristics.

Sumer(5) studied the mean flow velocity and the longitudinal dispersion of heavy particles in an open channel flow. He treated the case when the particles stay in suspension all the time. In contrast to Sumer's work, Awaya and Fujisaki(1) studied another extreme case, where all particles settled onto the bed were assumed to remain there.

In this paper we have investigated more general case. The model introduced here allows partly reentrainment of the deposited particles into the bulk flow. In the theoretical analysis, we have employed the similar method originally used by Taylor(6). Laboratory experiments were carried out to examine the validity of the theoretical solution.

THEORETICAL INVESTIGATION

The governing equation is the equation of conservation of sediment particles. Taking x axis along the flume and y axis perpendicular to it, we obtain

$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}=\frac{\partial}{\partial y}\left(D\frac{\partial c}{\partial y}+w_0 c\right) \tag{1}$$

with the boundary conditions

$$D\partial c/\partial y+w_0 c=0 \qquad : \quad y=h \tag{2.1}$$

$$D\partial c/\partial y+(1-\alpha)w_0 c=0 \qquad : \quad y=y_0 \tag{2.2}$$

where u=flow velocity; c=sediment concentration; D= turbulent diffusivity; h=flow depth; w_0=fall velocity of particles; and y_0=height where u=0(see Eq.5).

To describe the motion of sediment particles near to the flume bed, we employ the idea of the bed absorbency coefficient in Eq.2.2. The coefficient α represents the probability that a sediment particle settled onto the bed remains there. Thus $\alpha=1$ corresponds to the perfect absorbing bed and $\alpha=0$ to the perfect reflection.

Although this concept is not accepted as the best boundary condition to be applicable to any case of flow, some researchers such as Sayre(4), Jobson and Sayre(3), Takamatsu et al(7) and Nakagawa et al(8) employed this idea to solve problems related sediment transport. Furthermore, to the best of writer's knowledge, there is no clear and universal description on the motion of sediment particles near the movable bed.

The physical meaning of the parameter α can also be explained as follows. According to Rouse's theory, the concentration distribution of particles near the flume bed is given by

$$c=c_*(y/y_0)^{-z}=c_* y_0^z\cdot y^{-z} \quad , \quad c_* : \text{const.} \tag{3}$$

This equation well represents the distribution of sediment particles in the case where some upward or downward flux exist in the flow. In Eq.3, only the value of $c_* y_0^z$ is important and the separated values of c_* or y_0 have no practical significance in itself. We assume that the settling flux $\alpha w_0 c_*$ at $y=y_0$ is proportional to $c_* y_0^z$. Then denoting the proportionality constant by β, we have

$$\alpha w_0 c_*=\beta\, c_* y_0^z, \quad \alpha=\beta y_0^z/w_0 \tag{4}$$

This expression means that the value of the parameter y_0 has no essential influence on the value of α. Therefore the generality of the expression of α is hold independently of the value of y_0. For convenience, we take the value of y_0 as the height where the velocity u=0 in Eq.5, and by using this y_0, we can also define α in Eq.4.

We assume that the flow velocity distribution is logarithmic and the shear stress distribution is linear, thus

$$\frac{u}{u_*}=\frac{1}{\kappa}\ln\frac{y}{y_0} \tag{5}$$

$$\nu_t=\kappa u_* y(1-y/h) \quad , \quad D\fallingdotseq\nu_t \tag{6}$$

where u_* =shear velocity, ν_t =coefficient of eddy viscosity and κ =von Karman's universal constant taken as 0.4 in this work. The turbulent eddy diffusivity D is assumed to be nearly equal to the coefficient of eddy viscosity($D=\nu_t$).

The non-dimensional expression of the governing equation is given by

$$\frac{\partial c}{\partial \bar{t}}+\bar{u}\frac{\partial c}{\partial \bar{x}}=\frac{\partial}{\partial \bar{y}}\left(\bar{D}\frac{\partial c}{\partial \bar{y}}+Z\,c\right) \tag{7}$$

$$\bar{y}(1-\bar{y})\frac{\partial c}{\partial \bar{y}}+Zc=0 \quad : \quad \bar{y}=1 \tag{8.1}$$

$$\bar{y}(1-\bar{y})\frac{\partial c}{\partial \bar{y}}+(1-\alpha)Zc=0 \quad : \quad \bar{y}=\bar{y}_0 \tag{8.2}$$

where

$$\left.\begin{array}{l}\bar{t}=t/(h/(\kappa u_*)),\ \bar{x}=x/h,\ \bar{y}=y/h,\ \bar{y}_0=y_0/h \\ \bar{u}=u/(\kappa u_*),\ \bar{D}=D/(\kappa u_* h)=\bar{y}(1-\bar{y}),\ Z=w_0/(\kappa u_*)\end{array}\right\} \tag{9}$$

In order to obtain the dispersion coefficient, it is assumed that the solution of Eq.7 can be approximated as

$$c(\bar{x},\bar{y},\bar{t})=\tilde{c}(\bar{y},\bar{t})(1+K\bar{x}_1)+\hat{c}(\bar{y},\bar{t}) \tag{10}$$

where

$$K=\frac{1}{<c>}<\frac{\partial c}{\partial x_1}>=\frac{1}{<c>}\frac{\partial <c>}{\partial x_1} \tag{11}$$

$$x_1=\bar{x}-\bar{U}_S\,\bar{t} \tag{12}$$

$$<*>\ \equiv \int_{\bar{y}_0}^{1} *\, d\bar{y} \tag{13}$$

$\tilde{c}$ =concentration distribution in case of $d/d\bar{x}=0$, in Eq.7. The term $\tilde{c}(1+K\bar{x}_1)$ is the first approximation due to the longitudinal concentration gradient. In order to obtain the solution which satisfy Eq.7, $\hat{c}$ is added to $\tilde{c}(1+K\bar{x}_1)$, as a corrction term. The mean velocity of the suspended particles is given by $\bar{U}s$. In the following, at first, we obtain $\tilde{c}$ and as a next step, making use of $\tilde{c}$ we seek $\hat{c}$.

Concentration distribution in case d/dx=0 : $\tilde{c}$

The concentration distribution $\tilde{c}$ is the solution of the following equation.

$$\frac{\partial \tilde{c}}{\partial \bar{t}}=\frac{\partial}{\partial \bar{y}}\left(\bar{D}\frac{\partial \tilde{c}}{\partial \bar{y}}+Z\tilde{c}\right) \tag{14}$$

This partial differential equation can be solved by the method of separation of variables

$$\tilde{c}=\tilde{c}_0(\bar{y})T(\bar{t}) \tag{15}$$

$$\bar{y}(1-\bar{y})\tilde{c}_0''+(1-2\bar{y}+Z)\tilde{c}_0'+\lambda_0\tilde{c}_0=0 \tag{16}$$

$$\bar{y}(1-\bar{y})\tilde{c}_0'+Z\tilde{c}_0=0 \quad : \quad \bar{y}=1 \tag{17.1}$$

$$\bar{y}(1-\bar{y})\tilde{c}_0'+(1-\alpha)Z\tilde{c}_0=0 \quad : \quad \bar{y}=\bar{y}_0 \tag{17.2}$$

$$T'/T=-\lambda_0 \tag{18}$$

where λ_0 =eigenvalue given by Eqs.16 and 17. From Eqs.16,17 and 18, we have

$$\tilde{c}=\sum_{i=0}^{\infty}A_i\,\tilde{c}_{0i}\exp(-\lambda_{0i}\bar{t}) \tag{19}$$

where c_{0i} =eigen function which correspond to the eigenvalue λ_{0i} , and A_i =numerical constant.

As our concern is the case where the higher order terms of Eq.19 are negligible, the solution of Eq.14 can be written as

$$\tilde{c} = A_0 \tilde{c}_{00} \exp(-\lambda_{00}\bar{t}) \tag{20}$$

Hereafter λ_{00} is replaced by λ_0 and $\tilde{c}_{00}$ by $\tilde{c}_0$.

Figs. 1 and 2 show the value of $\tilde{c}_0$ for several values of α and Z. $\tilde{c}_0$ is the concentration profile where there is no concentration gradient in the flow direction. To the extreme case of α =0 and α =1, $\tilde{c}_0$ is given by Eqs.21 and 22 respectively

$$\tilde{c}_0 = \frac{\sin \pi z}{\pi z} \left(\frac{1-\bar{y}}{\bar{y}}\right)^z \tag{21}$$

$$\tilde{c}_0 = (1+Z)(1-\bar{y})^Z \tag{22}$$

In case of perfect absorbing bed (α=1), the value of λ_0 is given by

$$\lambda_0 = Z(Z+1) \tag{23}$$

Fig.3 shows the relationship among λ_0 , α and Z. As given in Eq.20, λ_0 means the measure of concentration decay due to settling.

The mean velocity of sediment particles $\bar{U}_s$ can be obtained from Eq.24.

$$\bar{U}_S = \int_{\bar{y}_0}^{1} \tilde{c}_0 \bar{u} d\bar{y} \Big/ \int_{\bar{y}_0}^{1} \tilde{c}_0 d\bar{y} \tag{24}$$

The value of $\bar{U}_s$ is shown in Fig.4.

Additional concentration : $\hat{c}$

Next, we seek the additional concentration $\hat{c}$, which is the essential term for dispersion. Considering the effect of longitudinal concentration gradient on the concentration decay, Eq.10 may be written in the form

$$c = \{\tilde{c}_0(1+Kx_1) + \hat{c}_0\} \exp(-\lambda\bar{t}) \tag{25}$$

$$\lambda = \lambda_0 + K\mu \tag{26}$$

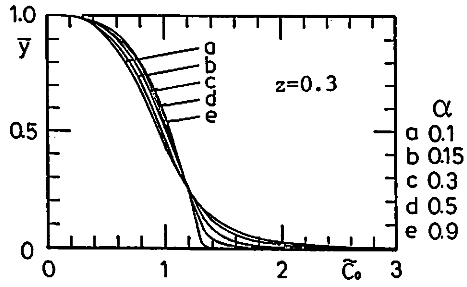

Fig.1 Concentration distribution $\tilde{C}_0$

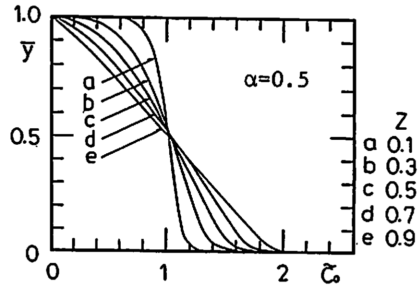

Fig.2 Concentration distribution $\tilde{C}_0$

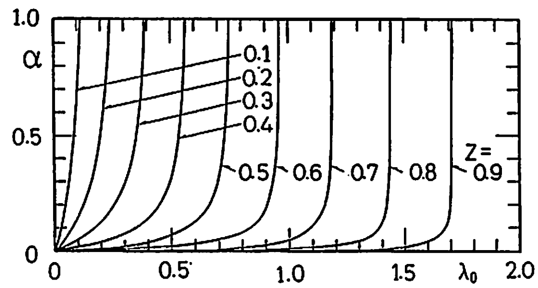

Fig.3 Relationship among α , λ_0 and Z

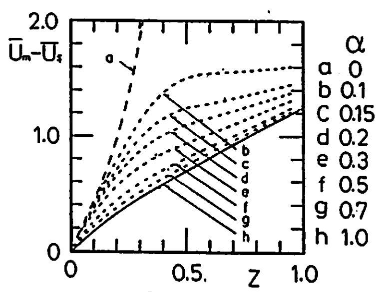

Fig.4 Mean velocity of suspended sediment in case of $d/d\bar{x}=0$ ($\bar{U}_m = \langle\bar{u}\rangle$)

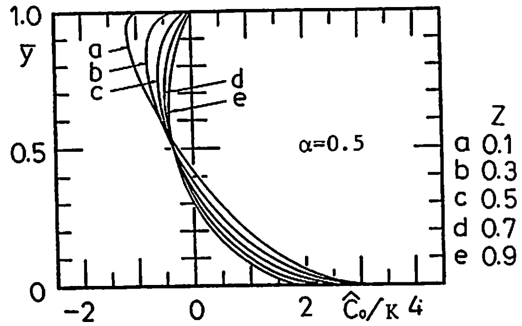

Fig.5 Additive concentration

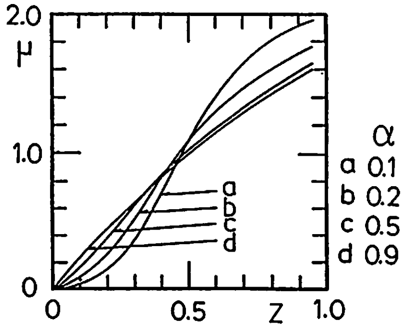

Fig.6 Relationship among α, Z and μ

Substituting Eq.25 into Eq.7 and using Eq.14, one obtains

$$\bar{y}(1-\bar{y})\frac{d^2\hat{c}_0}{d\bar{y}^2} + (1+Z-2\bar{y})\frac{d\hat{c}_0}{d\bar{y}} + \lambda\hat{c}_0 = K\hat{u}_s\tilde{c}_0 - (\lambda-\lambda_0)(1+K\bar{x}_1)\tilde{c}_0 \qquad (27)$$

where

$$\hat{u}_s = u - \bar{U}_s \qquad (28)$$

The solution of Eq.27 is given as

$$\hat{c}_0 = K\tilde{c}_0\int_{\bar{y}_0}^{\bar{y}} \frac{\tilde{c}_0^{-2}}{\bar{y}(1-\bar{y})}\left(\frac{1-\bar{y}}{\bar{y}}\right)^Z d\bar{y}\int_{\bar{y}_0}^{\bar{y}}\left(\frac{\bar{y}}{1-\bar{y}}\right)^Z(\hat{u}_s-\mu)\tilde{c}_0^2\, d\bar{y} + A_1\tilde{c}_0 \qquad (29)$$

$$\mu = \int_{\bar{y}_0}^1 \gamma(\bar{y})\hat{u}_s\tilde{c}_0\, d\bar{y} \Big/ \int_{\bar{y}_0}^1\gamma(\bar{y})\tilde{c}_0\, d\bar{y} \qquad (30)$$

$$\gamma(\bar{y}) = (\bar{y}/(1-\bar{y}))^Z \qquad (31)$$

or

$$\hat{c}_0 = K\sum_{i=1}^{\infty} a_i\tilde{c}_{0i} \qquad (32)$$

$$a_i = -\frac{1}{\lambda_0-\lambda_i}\,\frac{\int_{\bar{y}_0}^1\gamma\hat{u}_s\tilde{c}_0\tilde{c}_{0i}\, d\bar{y}}{\int_{\bar{y}_0}^1\gamma\tilde{c}_0^2\, d\bar{y}} \qquad (33)$$

The value of $\hat{c}_0/K$ and μ are shown in Fig.5 and Fig.6 respectively.

Dispersion coefficient and velocity of sediment cloud

The longitudinal dispersion coefficient D_L in the present analysis is defined by

$$\frac{D_L}{hu_*}\cdot\frac{\partial\langle c\rangle}{\partial x} = -K\langle\hat{u}_s\,\hat{c}\rangle \qquad (34)$$

Substituting Eq.29 into Eq.34, we have

$$\frac{D_L}{hu_*} = -K\int_0^1\hat{u}_s\tilde{c}_0\, d\bar{y}\int_{\bar{y}_0}^{\bar{y}}\tilde{c}_0\frac{\gamma^{-1}}{\bar{y}(1-\bar{y})}\, d\bar{y}\int_{\bar{y}_0}^{\bar{y}}\tilde{c}_0^2(\hat{u}_s-\mu)\gamma\, d\bar{y} \qquad (35)$$

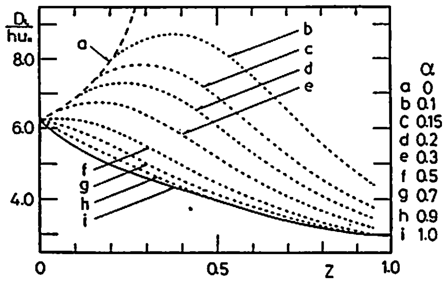

Fig.7 Longitudinal dispersion coefficient of suspended sediment

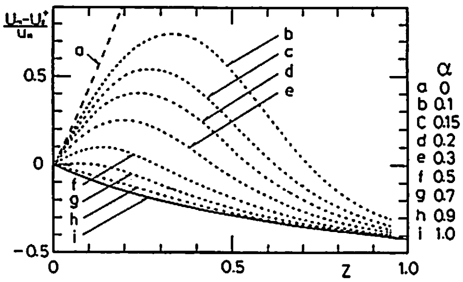

Fig.8 Difference of mean velocity between bulk flow and sediment

The value of D_L is shown in Fig.7 as a function of Z with α the parameter. The numerical solution shown in Fig.7 is in the case of Re=20000, where Re is the Reynolds number defined as Umh/ν . The effect of Re on dispersion phenomena will be discussed later.

In applying the longitudinal dispersion coefficient to the prediction of the dispersion phenomena, we have the following equation, which is derived by integration of Eq.7 over the flow depth.

$$\frac{\partial\langle c\rangle}{\partial t}+\overline{U}_s\frac{\partial\langle c\rangle}{\partial x}=-\frac{\partial}{\partial x}\langle\hat{u}_s c\rangle-\lambda\langle c\rangle=\overline{D}_L\frac{\partial^2\langle c\rangle}{\partial x^2}-(\lambda_0+K\mu)\langle c\rangle \qquad (36)$$

Substituting Eq.11 into Eq.36, we have

$$\frac{\partial\langle c\rangle}{\partial t}+(\overline{U}_s+\mu)\frac{\partial\langle c\rangle}{\partial x}=\overline{D}_L\frac{\partial^2\langle c\rangle}{\partial x^2}-\lambda_0\langle c\rangle \qquad (37)$$

Eq.37 shows that the mean velocity of sediment cloud is $\overline{U}_s+\mu$ not $\overline{U}_s$. The value of $\overline{U}_s^+=\overline{U}_s+\mu$ is shown in Fig.8. We will discuss later on the physical meaning of this virtual increase in the velocity of the sediment cloud.

In the above discussion, when we put α=0, the results correspond to the case of no particles deposition, in this case the value of λ_0 becomes 0. This solution is the same one that Sumer(5) had reported by using the method of moment transformation. When α=1, the solution reduces to the solution of the perfect absorbing bed, which we investigated previously(Awaya and Fujisaki(1)).

EXPERIMENTAL INVESTIGATION

Experimental Set Up

Experiments were conducted in a 8m long, 0.4m width, straight, tilting and recirculating laboratory flume(Fig.9). It has glass walls and a smooth steel floor. A centrifugal pump was used and flume discharge was controlled by two valves. The values for flow velocities were chosen between 0.2m/s and 0.5m/s, and the water depth were between 0.02m and 0.05m.

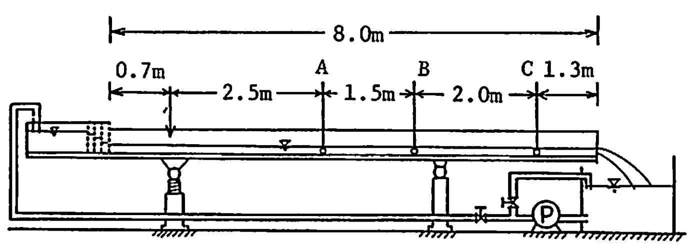

Fig.9 Laboratory flume

Polyvinylchloride resin powder and fine sand were used as the sediment dispersant. The resin powder had specific gravity of 1.22, and was sized by sieves to three classes. The mean fall velocity of these powders were 0.31cm/s, 0.26cm/s, and 0.17cm/s. The specific gravity of the sand particles was 2.65 and mean fall velocity 0.54cm/s.

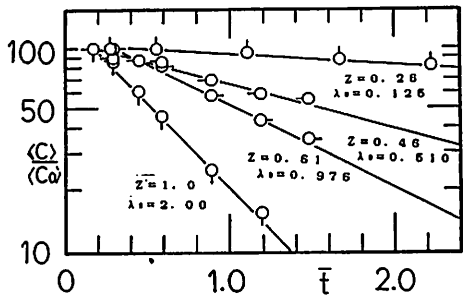

Fig.10 Concentration decay with time

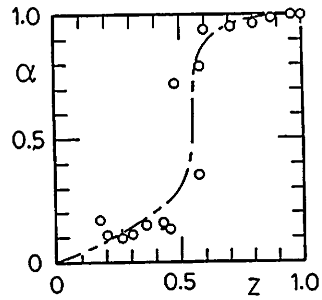

Fig.11 Bed absorbency coefficient

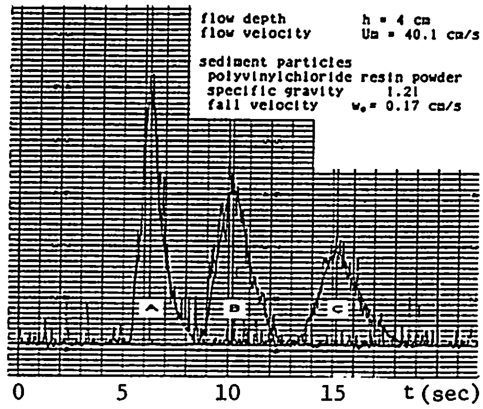

Fig.12 Output of the turbidity meter

A handmade turbidity meter was used which is composed of photo-coupler and the electric amplifier. Output of the amplifier was recorded by an analog data recorder and was also monitored by a pen recorder. Three probes of turbidity meter were all set at the height of a half depth of the bulk flow. The distance of each probe were 2.5m, 4m, and 6m from the dispersant injection point, which was located 0.70m downstream from the inlet of the flume.

Two types of experiment were carried out. One is the experiment for concentration decay(Experiment A), and the other was for sediment dispersion(Experiment B). We measured the concentration decay with flow distance, by injecting suspension continuously from the fixed point.

In Experiment B, sediment particles were injected by pouring the 20ml suspension into the flow from the height of 1cm over the water surface. The solid concentration of this suspension was 0.07g/ml. Small amount of salt was also added to the injecting suspension and the flow velocity of the injected liquid was measured using conductivity meter. From these experimental data, we obtained the velocity difference between the solid particles and injected liquid.

Experimental Results

Experiment A

Examples of the concentration decay with $\bar{t}=\bar{x}/\bar{U}_m$ are shown in Fig.10. The value of λ_0 is obtained as the slope of the lines in Fig.10. By using this values of λ_0 and Z, which is given by the experimental condition, we obtained the value of α from the theoretical α-λ_0-Z curves given in Fig.3. Thus, we determined the relationship between α and Z as shown in Fig.11. The experiments are carried out in the range of 10000<Re<30000. Experimental results, however, did not show clear dependence on Re within a range of these experiments. Therefore we assumed that the α-Z relationship given in Fig.11 is hold for 10000<Re<30000. The line in Fig.11 is a fitted curve to the experimental data. This curve will be used later to predict the dispersion phenomena.

Experiment B

An example of signals from the turbidity meter is shown in Fig.12. From these concentration distributions of the sediment cloud, the dispersion coefficient is given by

$$D_L = \frac{1}{2}\frac{d\sigma^2}{dt} \qquad (38)$$

where σ = standard deviation of the concentration distribution obtained from the

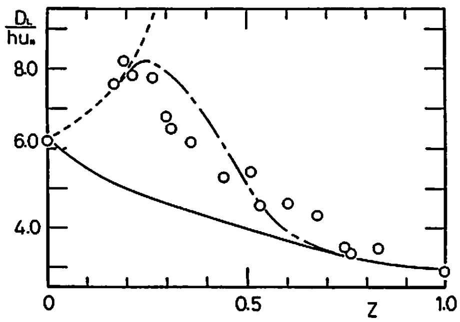

Fig.13 Comparison of predicted dispersion coefficient with experimental data

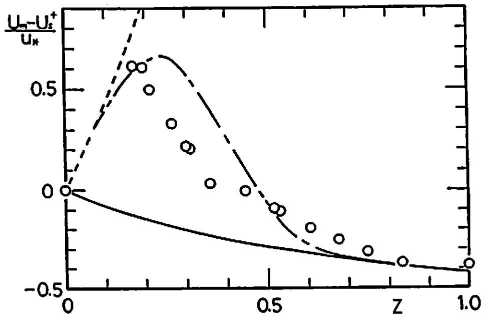

Fig.14 Comparison of predicted velocity difference with experimental data

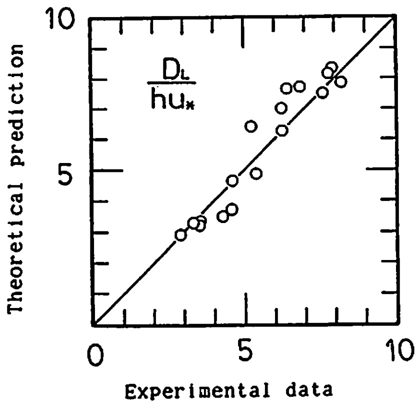

Fig.15 Comparison of the theoretical prediction and experimental value (Dispersion coefficient)

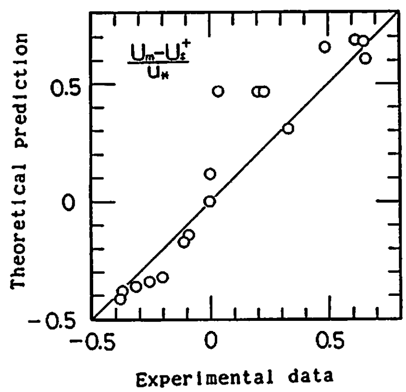

Fig.16 Comparison of the theoretical prediction and experimental value (Velocity difference)

experiments and τ = elapsed time after the tracer injected. Dispersion coefficient obtained from the experiments are plotted in Fig 13. The velocity difference between the bulk flow and sediment cloud is also plotted in Fig.14. The theoretical predictions given in Figs.13 and 14 are obtained with use of both α-Z relation and the numerical solutions given in Figs.7 and 8. The curves shown in Figs.13 and 14 are theoretical prediction for Re=20000. The effect of Re on experimental results is small, so experimental results for $15000<Re<28000$ are plotted in Figs.13 and 14.

The broken line in Figs.7,8,13 and 14 are the Sumer's solution(5) and the solid line are the result of Awaya and Fujisaki(1). Figs.13 and 14 show that the broken line can predict the experimental values for the small values of Z. On the other hand, the solid line fits the experimental results for the large values of Z

When the value of Z is small, the effect of turbulence of the bulk flow on the motion of sediment particles near the flume bed becomes more predominant than the effect of gravity, so particles hardly settle down to the flume bed. This is the case studied by Sumer. When the value of Z becomes large, deposited particles may hardly be picked up to the bulk flow. This is the case we have formerly investigated.

Figs.15 and 16 also show the comparison of the theory and experiment. As the nature of experiment, experimental data is rather scattered, but it can be concluded that the agreement between the predicted values and the experimental data are satisfactory.

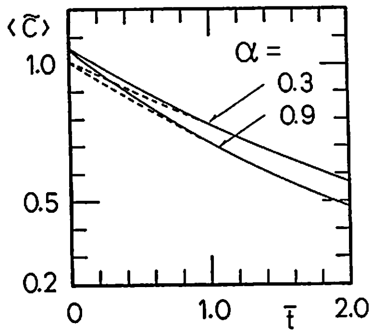

Fig.17 Effect of higher order terms on the concentration decay

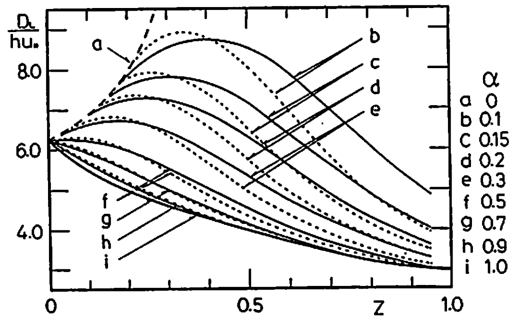

Fig.18 Effect of Reynolds number Re on Dispersion coefficient
——— Re=10000, ------Re=100000

DISCUSSION

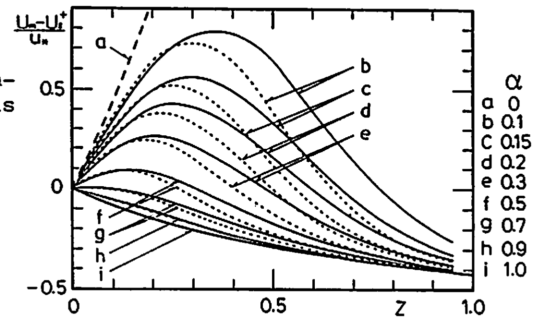

Fig.19 Effect of Reynolds number Re on velocity difference
——— Re=10000,------Re=100000

Effect of the higher order term

Broken lines in Fig.17 show concentration decay with elapsed time, which is given by,

$$\left.\begin{aligned} \langle c\rangle = A_0\langle c_{00}\rangle \exp(-\lambda_{00}\bar{t}) \\ + A_1\langle c_{01}\rangle \exp(-\lambda_{01}\bar{t}) \\ + A_2\langle c_{02}\rangle \exp(-\lambda_{02}\bar{t}) \end{aligned}\right\} \quad (39)$$

Numerical values of A_0, A_1 and A_2 are given by the initial condition, $\langle c\rangle = 1$ at $t=0$. Solid lines are obtained only from 0th order term in Eq.39. In the theoretical analysis, we neglected the higher order terms in Eq.19. When $\bar{t}>0.2$, the numerical value of the sum of higher order terms is less than 3%, in this example. Fig. 17 shows that the effect of higher order term in Eq.39 is negligible small for large values of $\bar{t}$. By using the finite difference method, previously we also discussed the numerical error of the similar problems(Awaya and Fujisaki(1)).

Effect of y_0

We have discussed the effect of Z and α on the sediment dispersion. Another parameter is y_0 given in Eq.5. As we assumed that the flume bottom is smooth, the following relationship is given

$$\frac{u_m}{u_*} = \frac{1}{\kappa}\left\{ \ln\frac{1}{\bar{y}_0} - 1\right\} = \sqrt{8/f} \quad (40)$$

where f=frictional factor and it is the function of Reynolds number. Mean velocity and dispersion coefficient of sediment cloud are computed for several Reynolds numbers. The values of dispersion coefficient shown in Fig.18 differs less than 25%, within a range of $10^4<Re<10^5$. In order to obtain the dispersion coefficient, in addition to the numerical results shown in Fig.18, we must also use the relation between α and Z such as shown in Fig.11. Therefore it can be said that the effect of Re on the finally predicted values of dispersion coefficient is not so large. The similar matter can be said to the velocity difference shown in Fig.19.

Relation between α and Z

To describe the motion of sediment particle near the flume bed, we employed bed absorbency coefficient and determined the functional relation $\alpha=\alpha(Z)$(Fig.11). This relation, however, is obtained only from our laboratory experiment. Therefore somewhat different results may be given by other experiments. Further elaborate experimental study may be required to determine the exact and universal relation among α , Z, and Re.

Virtual increase in the mean velocity of sediment

The second term of the left-hand side of Eq.37 means that there is the virtual increase in the mean velocity of sediment cloud. When the sediment concentration decrease due to settling is expressed in terms of $\exp(-\lambda_0 \bar{t})$, the velocity of the center of gravity of a sediment cloud becomes greater to the amount of μ than the real mean velocity of particles $\overline{U}_s$. This stems from the local difference of the sediment flux to the flume bed, i.e., the sediment flux of the rear part of cloud is greater than that of front part.

SUMMARY AND CONCLUSION

We have investigated the longitudinal dispersion of suspended sediment particles in a turbulent open channel flow. Special attention has been directed to the effect of the settling of particles on the dispersion phenomena.

In the theoretical investigation, we employed the same method as Elder used in his analysis of the dispersion of neutral matter. The virtual velocity and the dispersion coefficient of the sediment cloud are given as a function of Z and α, where Z is the fall velocity parameter of particles and α is bed absorbency coefficient. It is also shown that there is a virtual increase in the velocity of the sediment cloud, when the sediment cloud flows downstream with the decrease of its concentration due to settling.

We have previously investigated the dispersion phenomena of sediment particles in the case of perfect absorbing bed, and Sumer's study(5) corresponds to the another extreme case. The results of these two theoretical works are included in the present work as a special case.

Numerical solutions of the theoretical work show reasonable agreement with the laboratory experimental data.

ACKNOWLEDGMENT

We wish to express our thanks to graduate student Noriyuki Yoshitake by his help in performing experiments.

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APPENDIX-NOTATION

The following symbols are used in this paper

c	=	volume concentration of sediment particles;
$\tilde{c}(y)$	=	concentration distribution given by Eqs.14 and 15;
$\hat{c}(y)$	=	additional concentration given by Eq.25;
D	=	turbidity transfer coefficient;
$\bar{D}$	=	non-dimensional turbidity transfer coefficient as defined by Eq.7;
D_L	=	longitudinal dispersion coefficient;
h	=	flow depth;
K	=	longitudinal concentration gradient as defined by Eq.11;
t	=	time;
$\bar{t}$	=	$t/(h/\kappa u_*))$ non-dimensional time;
Um	=	mean velocity of bulk flow;
Us	=	mean velocity of sediment particles in case d/dx=0;
$\bar{U}_s^+$	=	velocity of sediment cloud;
u	=	flow velocity in the x direction;
$\bar{u}$	=	$u/(\kappa u_*)$ non-dimensional flow velocity;
u_*	=	shear velocity of bulk flow;
w_0	=	fall velocity of sediment particles;
x	=	coordinate in the horizontal direction;
$\bar{x}$	=	x/h;
y	=	coordinate in the vertical direction;
$\bar{y}$	=	y/h;
y_0	=	height from the flume bed where u=0;
$\bar{y}_0$	=	y_0/h;
Z	=	$w_0/(\kappa u_*)$ particles fall velocity parameter;
α	=	probability for a particle to deposit on the bed;
κ	=	0.4 von Karman's universal constant;
λ_0	=	eigenvalue of the differential Eq.16;
λ	=	parameter given by Eq.25;
μ	=	virtual increase in velocity of sediment cloud;
σ	=	standard deviation of sediment concentration distribution;

notation

$<*>$: cross-sectional average of $*$.