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DETERMINATION OF DESIGN STORM PATTERN BY FREUND'S BIVARIATE PROBABILITY DENSITY FUNCTION

by

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SYNOPSIS

This paper describes a methodology for determining stochastic design storm patterns that preserve the joint probability of occurrence of total rainfall and peak rainfall by means of Freund's bivariate probability density function. The time distribution of the stochastic design storm is obtained from a conditional probability storm pattern given the total rainfall and the peak rainfall. This method takes into account not only the crosscorrelation coefficient between total rainfall and peak rainfall intensity but also the autocorrelation coefficient of heavy hourly sequences around the rainfall peaks causing flood peaks.

INTRODUCTION

According to the technical manual for river engineering and erosion control (a revised draft), Ministry of Construction, Japan, published in 1977, a design storm for flood-control and multi-purpose projects in river drainage areas in Japan is defined to have three characteristics, the total rainfall, time and spatial distributions. Considering the magnitude of drainage area, the rainfall and regional characteristics, we usually assume the design storm duration to be 1 to 3 days. In essence, the magnitude of project has been evaluated by the return period of the design total rainfall. Following the determination of the design total rainfall, the time distribution-shape may be determined to be almost same as the time distribution-shape of a historical heavy storm causing a large flood. This method is called the enlargement method of historical storms, and is easy and simple in application. However, this method often leads to overestimation of the peak rainfall intensity, so that some modification of the time distribution should be required.

As mentioned above, the current method for determination of a design storm pattern in Japan has been hardly supported by the theory of probability. Especially, the joint probability of occurrence of the design total rainfall and hourly rainfall intensities around the peak intensity governing the maximum discharge of flood has been hardly clarified, although it is very important for the determination of the design storm pattern.

This paper formulates the joint occurrence probability of the total rainfall and the peak rainfall intensity by means of the joint return period of two hydrologic variates associated with a Poisson process (Hashino, (2)). A method for determination of a design storm pattern is proposed using the conditional probability storm pattern (Hashino, (3)) given the design total rainfall and the peak rainfall intensity. Three probabilistic criteria of determination are established.

CONDITIONAL PROBABILITY STORM PATTERN GIVEN THE PEAK INTENSITY

Freund (1) defined a bivariate exponential probability density function $g(x,y)$ of a bivariate (x,y) with four parameters. For a special case the variables x and y have identical marginal distributions, so that Freund's probability density function g(x,y) becomes

$$g(x,y) = \begin{cases} \alpha\beta\exp\{-\beta x-(2\alpha-\beta)y\} & (0\le y\le x) \\ \alpha\beta\exp\{-\beta y-(2\alpha-\beta)x\} & (0\le x\le y) \end{cases} \tag{1}$$

Consider a single storm pattern with a peak of rainfall intensity x_p as shown in Fig. 1. Let x_i be the rainfall intensity at a discrete time i before or after the peak, measured to the left or right of the peak, respectively , and x_{i-1} be the rainfall intensity at the time(i-1) with a time interval Δt (see Fig. 1).

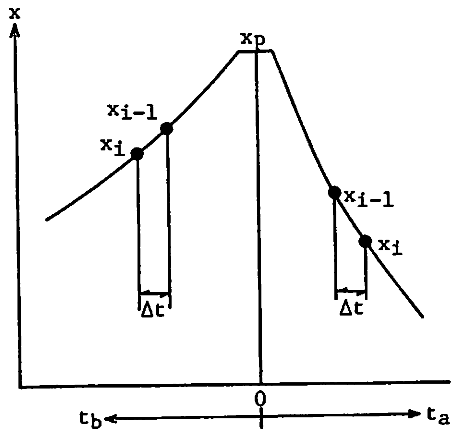

Fig. 1 A single storm pattern with a peak

By the definition of a single storm with a peak, the rainfall intensity decreases monotonously away from the peak; that is, $x_i<x_{i-1}$. By putting $x=x_i$ and $y=x_{i-1}$ and considering $x_i<x_{i-1}$ in Eq. 1, we can obtain the following condi

$$G(x_i|x_{i-1})= \begin{cases} \dfrac{1-\exp\{-(2\alpha-\beta)x_i\}}{1-2\{1-(\alpha/\beta)\exp\{-(2\alpha-\beta)x_{i-1}\}} & (2\alpha\neq\beta) \\ \beta x_i/(1+\beta x_{i-1}) & (2\alpha=\beta) \end{cases} \tag{2}$$

The rainfall intensity x_i is transformed into a reduced variate z_i as

$$z_i = (x_i - u_z) / \sigma_z \qquad (0<u_z<x_i) \tag{3}$$

where u_z = base intensity level ; and σ_z = standard deviation of the exceedance (x_i-u_z). We can define the same probability density function as Eq. 1 for the reduced bivariate(z_i,z_{i-1}), and obtain the same conditional probability distribution function $G(z_i|z_{i-1})$ as Eq. 2. Considering that the variance of the reduced variate z_i equals to unity, and rewriting Eq. 2, in which x_i and x_{i-1} are replaced by z_i and z_{i-1}, with respect to z_i, we have the following equation (Hashino, (3)).

$$\begin{aligned} &\exp(-\lambda z_i)=2G\cdot(1-k)\exp(-\lambda z_{i-1})+(1-G) && (k\neq1/2, 0<k\le1) \\ &z_i = G\cdot z_{i-1} + (2\sqrt{7}/7)\cdot G && (k=1/2) \end{aligned} \tag{4}$$

where

$$k \equiv \alpha/\beta \tag{5}$$

$$\lambda \equiv (2k-1)\sqrt{3k^2+1}\ /2k \tag{6}$$

with G denoting a given value of the conditional probability distribution function $G(z_i|z_{i-1})$. The ratio $k \equiv \alpha/\beta$ governs the autocorrelation coefficient of z_i :

$$\rho = (1-k^2)/(1+3k^2) \tag{7}$$

It is clear from Eq. 7 that $\rho=0$ for $k=1$, $\rho \to 1$ for $k \to 0$, and $\rho \to -1/3$ for $k \to \infty$. In practice, an appropriate value of ρ in the range of $0 \le \rho < 1$ will be adopted, so that the ratio k may be in the range of $0 < k \le 1$. We call k the autocorrelation index. For the purpose of stochastic formulation of design storm patterns, the conditional probability G may be assumed to be time-invariant before and after the peak. Hence, the values of G before and after the peak are denoted as G_b and G_a, respectively. In summary if the autocorrelation index k, the conditional probability G and the reduced variate z_p $(\equiv(x_p-u_z)/\sigma_z)$ of the peak intensity x_p are given, a sequence of z_i or a discrete hyetograph in the time inteval Δt can be calculated from Eq. 4.

It is inconvenient that the time distribution of z_i calculated by Eq. 4 is discrete. Thus, we transform Eq. 4 into an equivalent equation with respect to the reduced rainfall intensity z $(\equiv(x-u_z)/\sigma_z)$ at the continuous time t. Considering the general solution of a first order ordinary linear differential equation: $dZ/dt = A + B \cdot Z$, we can obtain the equivalent equations for the continuous one-sided pattern before or after the peak and the corresponding constraint conditions as follows (Hashino, (3)).

(a) For $0<k<1$, $k \neq 1/2$ $(0<\rho<1$ and $\rho \neq 3/7)$,

$$Z-\delta = (Z_p-\delta)\exp\{Bt-B(\Delta t/24)\}$$

$$Z \equiv \exp(-\lambda z) \; ; \; Z_p \equiv \exp(-\lambda z_p)$$

$$\delta \equiv (1-G)/\{1-2G(1-k)\} \; ; \; B \equiv (1/\Delta t)\ln\{2G(1-k)\} \tag{8a}$$

$$\lambda \equiv (2k-1)\sqrt{3k^2+1}\,/(2k) \; ; \; -(1/\lambda)\ln\delta<z<z_p \quad (\Delta t/24<t)$$

$$0<G \le 1/\{2(1-k)\} \quad (0<k<1/2) \; ; \quad 0<G<1 \quad (1/2<k<1)$$

(b) For $k=1/2$ $(\rho=3/7)$,

$$z-\delta = (z_p-\delta)\exp\{Bt-B(\Delta t/24)\}$$

$$\delta \equiv 2\sqrt{7}\cdot G/\{7(1-G)\} \; ; \; B \equiv (1/\Delta t)\cdot \ln G \quad (\Delta t/24<t) \tag{8b}$$

$$\delta<z<z_p \; ; \quad 0<G<1$$

(c) For $k=1$ $(\rho=0)$,

$$z = \begin{cases} z_p & (t=\Delta t/24) \\ -\ln(1-G) & (t>\Delta t/24) \end{cases} \tag{8c}$$

$$-\ln(1-G)<z_p \; ; \quad 0<G<1$$

We call this continuous storm pattern given by Eq. 8 a conditional probability storm pattern, and call three parameters k, G and z_p as the pattern parameters. Figure 2 shows one-sided conditional probability storm patterns before or after the peak for the case of G=0.5 and z_p=10. It is clearly found from Fig. 2 that the storm pattern becomes sharp with increasing k; that is, with decreasing ρ. For the case of specified values of k and z_p, we can easily infer that the storm pattern becomes sharp with decreasing G. Since the technical terms of the last peaked type (LPT) and the central peaked type (CPT) patterns are often used in engineering practice in Japan, both types of the conditional probability storms are defined. The former is given by Eq. 8, while the latter is obtained by combining two one-sided patterns (by Eq. 8) back to back with the same peak intensity z_p and the same autocorrelation index k. Although the CPT patterns generally have different values of the conditional probabilities G_a and G_b, the special case of a symmetrical pattern; that is, $G_a=G_b$ may be easy to use.

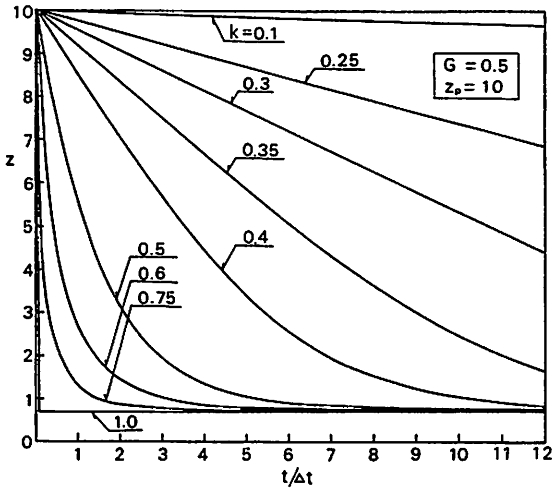

Fig. 2 One-sided conditional probability storm patterns before or after the peak for the case of G=0.5 and $z_p=10$

After all, the design storm pattern can be obtained by determining the pattern parameters k, G_a and G_b which satisfy the specified values of the peak rainfall intensity x_p, the occurrence time of the peak, and the average rainfall intensity within the concentration time of flood, etc., besides the design rainfall total y.

The examinations of goodness of fit for Freund's distribution against hourly data of heavy storms at five observation stations: Tokushima, Anabuki, Kito in Tokushima prefecture, Motoyama in Kochi prefecture, and Osaka, have led to satisfactory results (Hashino, (3)).

JOINT RETURN PERIOD OF TOTAL RAINFALL AND PEAK INTENSITY

According to the theory of compound Poisson process (North, (4)), the probability distribution functions of annual maxima η_{max} and ξ_{max} of the total rainfall y of a storm and the peak rainfall intensity x are given by

$$P[\eta_{max} \leqq y] = \exp[-\{1-F(y)\}\lambda_{xy}] \tag{9}$$

$$P[\xi_{max} \leqq x] = \exp[-\{1-F(x)\}\lambda_{xy}] \tag{10}$$

where λ_{xy} = annual rate of storm occurrence; and F(x), F(y) = marginal probability distribution functions of x and y, respectively. Considering a single point (storm) process with two marks of y and x, we can easily obtain the joint probability function of two annual maxima of y and x as (Hashino, (2))

$$P[\xi_{max} \leqq x,\ \eta_{max} \leqq y] = \exp[-\{1-F(x,y)\}\lambda_{xy}] \tag{11}$$

where F(x,y) = bivariate probability distribution function of x and y. As the bivariate probability density function f(x,y) of the two variates x and y, we employ the original Freund's bivariate exponential density function (Freund, (1)):

$$f(X,Y) = \begin{cases} a_1 b_2 \exp[-b_2 Y-(a_1+b_1-b_2)X] & (0 \leqq X \leqq Y) \quad (12) \\ b_1 a_2 \exp[-a_2 X-(a_1+b_1-a_2)Y] & (0 \leqq Y \leqq X) \quad (13) \end{cases}$$

$$X \equiv (x - u_x)^m/\sigma_x\ ,\quad Y \equiv (y - u_y)^n/\sigma_y \tag{14}$$

where a_1, b_1, a_2, b_2 = parameters; X, Y = transformed variables of original variables x and y, respectively, by Eq. 14; u_x, u_y = specified base levels; m, n = exponents; and σ_x, σ_y = standard deviations of exceedances $(x-u_x)^m$ and $(y-u_y)^n$, respectively.

In practice, we have to employ partial duration series of x and y simultaneously satisfying the conditions of $x>u_x$ and $y>u_y$. In order to obtain good fits of Freund's bivariate probability distribution function F(X,Y) and annual maximum distributions (Eqs. 9 and 10), we have to establish appropriate values of u_x and u_y (>0) so that the sample size of $x>u_x$ and $y>u_y$ may be almost equal to the number of observation years. The appropriate values of exponents m and n can be searched in trial and error so that these exponents may give good fits of Freund's marginal probability distribution functions F(X) and F(Y) to observed data. It is clear from Eq. 14 that F(x), F(y) and F(x,y) equal to F(X), F(Y) and F(X,Y), respectively.

The joint exceedance probability $P[\xi_{max}\geq x,\eta_{max}\geq y]$ of the annual maximum bivariate (ξ_{max},η_{max}) can be expressed in terms of the joint no-exceedance probability $P[\xi_{max}\leq x,\eta_{max}\leq y]$ and the marginal probabilities $P[\xi_{max}\leq x]$ and $P[\eta_{max}\leq y]$ as

$$P[\xi_{max}\geq x,\ \eta_{max}\geq y] = 1-P[\xi_{max}\leq x] - P[\eta_{max}\leq y] + P[\xi_{max}\leq x,\ \eta_{max}\leq y] \qquad (15)$$

The inverses of exceedance probabilities $P[\xi_{max}\geq x]$, $P[\eta_{max}\geq y]$ and $P[\xi_{max}\geq x,\eta_{max}\geq y]$ are defined as the return periods T_x, T_y and T_{xy}, respectively; that is,

$$T_x = 1/P[\xi_{max}\geq x] = 1/\{1-P[\xi_{max}\leq x]\} \qquad (16a)$$

$$T_y = 1/P[\eta_{max}\geq y] = 1/\{1-P[\eta_{max}\leq y]\} \qquad (16b)$$

$$T_{xy} = 1/P[\xi_{max}\geq x,\ \eta_{max}\geq y] \qquad (16c)$$

Therefore, the joint return period T_{xy} of the total rainfall and the peak intensity of a storm can be estimated by substituting Eqs. 9 to 11, in which F(x), F(y) and F(x,y) are replaced by F(X), F(Y) and F(X,Y), respectively, into Eqs. 15 and 16c. The univariate return periods T_x and T_y can also be estimated by Eqs. 9 and 16a and Eqs. 10 and 16b, respectively.

Figure 3 shows the good fitting of Freund's marginal distributions F(X) and F(Y) to the empirical distributions at Tokushima on semilogarithmic paper, where y and x show the daily total rainfall (mm) and the hourly peak intensity (mm/hr), respectively, of a heavy storm with over 100 mm/day.

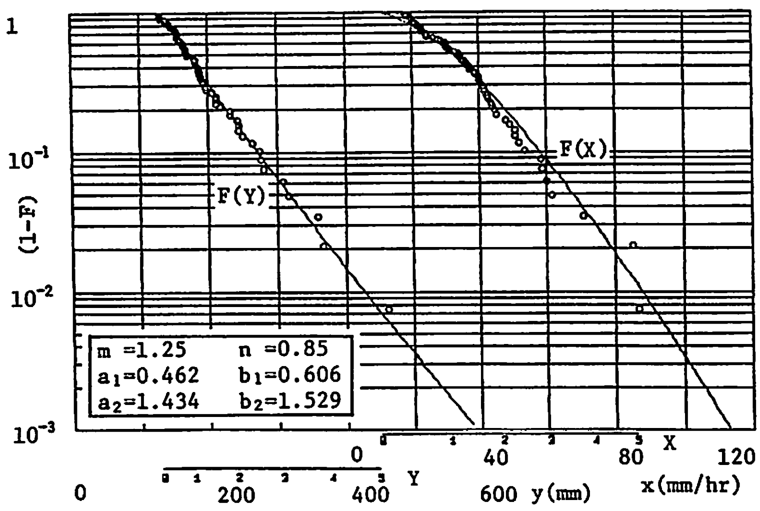

Fig. 3 Marginal distribution functions F(X) and F(Y) of Freund's bivariate distribution

In this case, we have u_y = 130 mm and u_x=10 mm/hr. The estimates of the other parameters :m, n; a_1, b_1,a_2 and b_2 are shown in Fig. 3.

Using these estimates of parameters in Eqs. 9 and 10, we have the theoretical marginal distributions of maxima as shown in Fig. 4, which are in good agreement with the observed annual maxima plotted by Gringorten's formula, especially in the range of large return periods over about 3 years. Therefore, the suitability of applying Freund's distribution to the total rainfall and the peak intensity is confirmed.

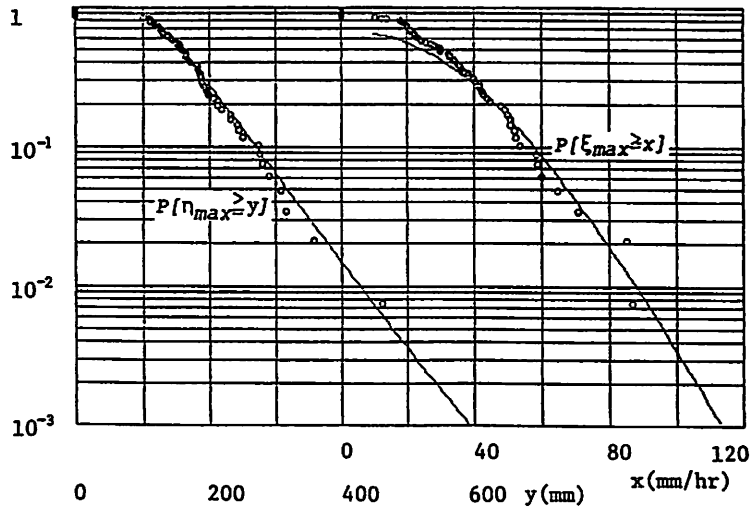

Fig 4 Theoretical distributions of annual maxima

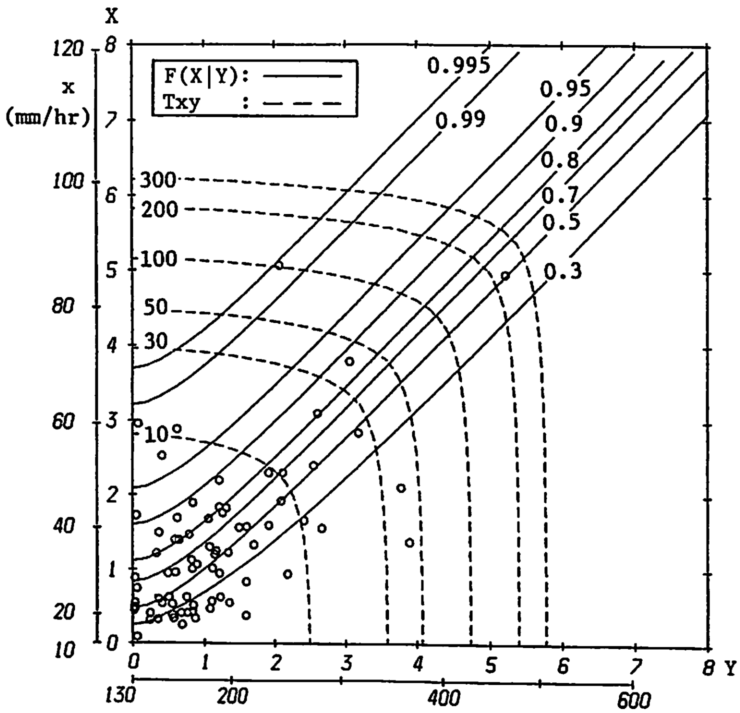

Fig. 5 Conditional iso-probability curves and joint iso-return period curves (Tokushima, ρ_{xy}=0.68)

Figure 5 shows the conditional iso-probability curves of $F(X|Y)$ and the joint iso-return period curves of T_{xy}, which are denoted by solid and dotted lines, respectively. For reference, the same partial duration data that is used in Fig. 3 is plotted in Fig. 5. The crosscorrelation coefficient ρ_{xy} of x and y, (exactly, that of X and Y) calculated by parameters a_1, b_1, a_2 and b_2 is relatively high (ρ_{xy}=0,68). Consequently, the slopes of the conditional iso-probability curves become almost 45° , while the angles of the joint iso-return period curves are almost 90° in the upper right-hand corners (X=Y). For comparison those curves with the crosscorrelation coefficient ρ_{xy}=0.19 at the case of Osaka are shown in Fig. 6, where the conditional iso-probability curves have much smaller slopes than 45°, and the joint iso-return period curves are almost circular. It is theoretically found that a joint iso-return period curve on the X-Y plane has two lines at right angles to each other in the upper right-hand corner in the case of ρ_{xy}=1 and has a straight line at right angles to the straight line X=Y in the case of ρ_{xy}=0.

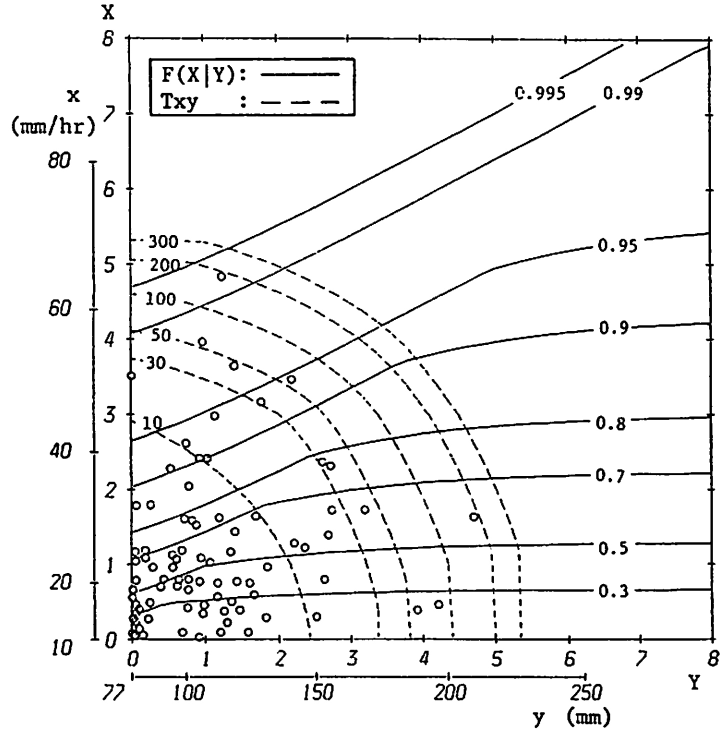

Fig. 6 Conditional iso-probability curves and joint iso-return period curves (Osaka, ρ_{xy}=0.19)

In the case of Osaka the fitting of theoretical distributions of annual maxima (Eqs. 9 and 10) becomes unsatisfactory mainly due to the low crosscorrelation coefficient of ρ_{xy}, even though we have obtained the good fitting of Freund's marginal distributions F(X) and F(Y) to the empirical distributions of partial duration data. In such a case, instead of Eqs. 9 to 11 and 16c we can use the following equations for return periods T_{xy}, T_x and T_y.

$$T_{xy} = T'_x \cdot T'_y / \{\varepsilon \cdot (T'_x - 1) \cdot (T'_y - 1) + 1\} \qquad (17)$$

$$\varepsilon = \exp[-\{F(X)+F(Y)-F(X,Y)-1\}\lambda_{xy}]-1 \qquad (18)$$

$$1/T'_x = 1-\exp[-\{1-H_x(x)\}\lambda_x], \quad 1/T'_y = 1-\exp[-\{1-H_y(y)\}\lambda_y] \qquad (19)$$

where λ_x, λ_y = annual rates of occurrence of univariates x and y; and $H_x(x)$, $H_y(y)$ = univariate probability distribution functions of x and y, respectively, given by combined exponential distributions (Hashino, (2)). The univariate return periods of x and y calculated by Eq. 19 are designated by T'_x and T'_y, which are different from T_x and T_y calculated from Eqs. 9 and 10. The function ε of x and y defined by Eq. 18 is required to satisfy the constraint condition of $\varepsilon < 1/(T'_x-1)$, $1/(T'_y-1)$. Thus, we have to consider T'_x and T'_y over 2 years, and select partial duration data of x and/or y so that λ_{xy} is nearly equal to unity and λ_x and λ_y are sufficiently larger than unity.

STOCHASTIC CRITERIA FOR DETERMINATION OF DESIGN STORM PATTERN

As mentioned previously, the present design total rainfall y* is evaluated for a specified value of the design return period T^*_y. Therefore, we consider a methodology for determining a design peak intensity on the basis of three stochastic criteria under the condition that T^*_y and y* are given. To choose return periods and probabilities regarding these criteria, we may have the value $F^*_{x|y}$ of the conditional probability distribution function $F(X|Y)$, joint return period T^*_{xy}, univariate return periods T^*_x and T^*_y, and the ratio $T^*_{x|y>} \equiv T^*_{xy}/T^*_y$ of T^*_{xy} to T^*_y. Since $F^*_{x|y}$ denotes a probability, not a return period, we define the corresponding return period $T^*_{x|y}$ to $F^*_{x|y}$ as

$$1/T^*_{x|y} = 1-\exp\{-(1-F^*_{x|y})\lambda_{xy}\} \qquad (20)$$

Now, we try to establish three stochastic criteria and preliminary conditions of ranges as follows.

$$[1] : \quad T^*_x \leqq T^*_y \qquad (21)$$

$$[2] : \quad 0.5 \leqq F^*_{x|y} < 0.9 \qquad (2.5 < T^*_{x|y} < 10.6) \qquad (22)$$

$$[3] : \quad 1 \leqq T^*_{x|y>} \; (\equiv T^*_{xy}/T^*_y) \leqq 3 \qquad (23)$$

The condition [1] is established in accordance with the current manual in Japan. However, if we consider that the hourly rainfall intensities around the peak rainfall are more important than the total rainfall, we can choose the alternative condition: $T^*_x > T^*_y$. In the condition [2] we can set the lower bound of the conditional probability at $F^*_{x|y}=1/2$. Setting the upper bound at $F^*_{x|y} = 0.9$ depends mainly on the relationship between the conditional return period $T^*_{x|y}$ and the ratio $T^*_{x|y>}$ as shown later in Figs. 7 and 8. Substituting values 0.5 and 0.9 for $F^*_{x|y}$ in Eq. 20 where λ_{xy} is assumed to be nearly equal to unity, we have $T^*_{x|y}$ = 2.5 and 10.6, respectively. Since $T^*_{xy} \geqq T^*_y$ is always valid, the lower bound in the condition [3] becomes unity. Setting the upper bound at $T^*_{x|y>} = 3$ in the condition [3] depends on the reason that this value 3 corresponds nearly to the value 0.9 of $F^*_{x|y}$ in the condition [2] at three sites except Osaka as shown later in Fig. 9.

In summary, using the three stochastic conditions [1], [2] and [3], we can preliminarily narrow down to several suitable values of the design peak intensity x* out of many values, and we further extend our consideration to determine a design peak intensity x* out of the preliminary suitable values. Therefore, we investigate the common region of x* given T^*_y and y*, satisfying simultaneously the three conditions [1], [2] and [3].

With respect to the relationship between the conditional return period $T^*_{x|y}$ in the condition [2] and the ratio $T^*_{x|y>}$ in the condition [3], we have the following approximate equation, assuming that λ_{xy}=1 and T_x, T_y>>2 in Eqs. 9 to 11 and 15.

$$1/T_{x|y>} \cong (T_y/T_x)-(T_y-1)/T_{x|y} \qquad (24)$$

From Eq. 24 it is expected that $T_{x|y}$ and $T_{x|y>}$ show approximately one-to-one correspondence for a given value of T_y. For instance, Figs. 7 and 8 show the relationships between $T_{x|y}$ and $T_{x|y>}$ at Tokushima (ρ_{xy}=0.68) and Osaka (ρ_{xy}=0.19), respectively, for T^*_y=50, 100, and 200. Although there are slight differences in relation in the case of a low value of ρ_{xy}=0.19 as shown in Fig. 8, the relationship between $T^*_{x|y>}$ and $T^*_{x|y}$ can be approximately represented by a curve independently of T^*_y, so that $T^*_{x|y>}$ and $T^*_{x|y}$ can be regarded as having approximately one-to-one correspondence for a specified site with $\rho_{xy}>0.2$, independently of T^*_y at least in the range: 50 to 200 years. Furthermore, the upper and lower bounds U and L, respectively, of the common regions satisfying simultaneously the three conditions [1], [2] and [3] are shown in Figs. 7 and 8. In the case of Tokushima with $\rho_{xy}=0.68$ in Fig. 7, the upper and lower bounds U and L are governed by the conditions [1] and [2], respectively, while for the case of Osaka with $\rho_{xy}=0.19$ in Fig. 8, U and L are governed by the conditions [3] and [2], respectively. Although the upper bound U governed by the condition [1] varies with the value of T^*_y, U and L governed by the other conditions [2] and [3], of course, are independent of T^*_y.

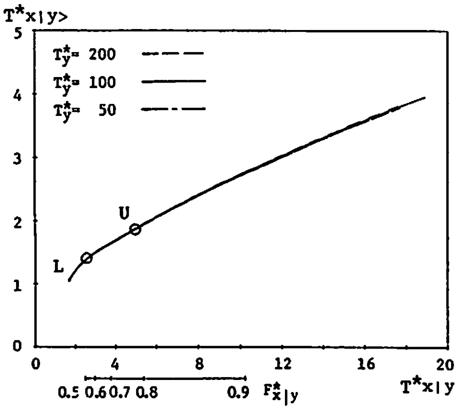

Fig. 7 Relationship between $T^*_{x|y}$ and $T^*_{x|y>} \equiv T^*_{xy}/T^*_y$ (Tokushima, $\rho_{xy}=0.68$)

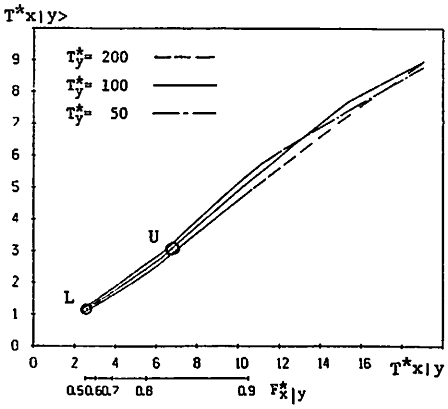

Fig. 8 Relationship between $T^*_{x|y}$ and $T^*_{x|y>} \equiv T^*_{xy}/T^*_y$ (Osaka, $\rho_{xy}=0.19$)

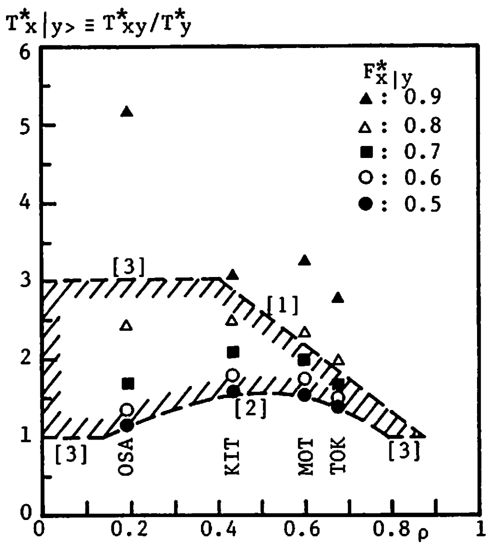

Fig. 9 Common region satisfying three conditions [1], [2] and [3] for $T^*_y=100$ (OSA, KIT, MOT and TOK are abbreviations of Osaka, Kito, Motoyama and Tokushima, respectively.)

Giving the same treatment to storm data at Kito and Motoyama as at Tokushima and Osaka, a summary of the common region governed by the three conditions for the case of T^*_y is shown in Fig. 9, where the relationships between the crosscorrelation ρ_{xy} and the ratio $T^*_{x|y>}$ are plotted using $F^*_{x|y}$ as a parameter. The outlines of the common region satisfying the three conditions [1], [2] and [3] are designated by the broken lines in Fig. 9, and the numerals near the bounds show the governing condition number. As mentioned before, the bounds except the inclined straight (broken) line governed by the condition [1] in Fig. 9 are invariant independently of T^*_y. It is clearly seen in Fig. 9 that the values of the conditional probability $F^*_{x|y}$ from 0.5 to 0.7 are in the common region, and the value $F^*_{x|y}=0.8$ is out of the region at Motoyama and Tokushima with high correlation coefficients $\rho_{xy}=0.6$ to 0.7.

According to the current manual of the Ministry of Construction in Japan, the selected design storms are transformed into discharge hydrographs through an appropriate runoff model, and some design hydrographs out of the design storms are chosen on the basis of a criterion called the cover ratio, which is defined as the ratio in peak discharge of design hydrographs to the hydrographs transformed from the selected design storms. Furthermore, the technical manual recommends the cover ratio to be over 50%, and shows many examples of the cover ratio from 60 to 80% in practical projects of Class A rivers, for which the administrative duty belongs to the Ministry of Construction in Japan. The concept of the cover ratio is almost the same as the concept of the conditional probability $F^*_{x|y}$, and they are only different in variables; the former is used for peak discharges of floods, and the latter for peak rainfall intensities of storms. Taking account of this matter, we can establish a conditional probability within $F^*_{x|y}=0.6$ to 0.8, especially, $F^*_{x|y}=0.7$, which gives a reasonable design peak intensity x* given y* and T^*_y. Since in Fig. 9 $F^*_{x|y}=0.7$ corresponds to $T^*_{x|y>}=T^*_{xy}/T^*_y=1.6$ to 2.1, then the joint return period T^*_{xy} becomes to be 1.6 to 2.1 times T^*_y.

Table 1 shows examples of y*, x*, T^*_{xy} and T^*_x for the case of $T^*_y=100$ years and $F^*_{x|y}=0.7$. According to Table 1, at Tokushima, Kito and Motoyama the design peak intensities are x*=85, 100 and 100 mm/hr, respectively, and these values are almost the same as the historical largest values: 87, 100 and 102 mm/hr, respectively. The univariate return periods, furthermore, are $T^*_x=76$, 35 and 76. Therefore, we can conclude that $F^*_{x|y}=0.7$ gives an appropriate design peak intensity x* given T^*_y at sites with ρ_{xy} approximately over 0.4.

Table 1 Design magnitudes y*, x*, T^*_{xy} and T^*_x against T^*_y = 100 and $F^*_{x|y}$ = 0.7

	y* (mm)	x* (mm/hr)	T^*_{xy}	T^*_x
Tokushima	430	85	165	76
Kito	802	100	207	35
Motoyama	577	100	198	76
Osaka	202	36	170	5

On the other hand, at Osaka with $\rho_{xy}\fallingdotseq 0.2$, the design peak intensity x* is x*=36 mm/hr, which is much smaller than the historical largest value 65 mm/hr. The return period $T^*_x=5$ is also much smaller than those at the other sites mentioned above. In principle, we have to investigate in detail whether or not such a design peak intensity x* with the much smaller value of T^*_x than that of T^*_y should be adopted in practical projects, which may have close connection with project purposes, the spatial magnitude of the drainage area considered, and characteristics of runoff, etc. In the case of Osaka mentioned above, however, we can easily infer that evaluating the magnitude of project by the return period of the total rainfall y, not of the peak intensity x, according to the current manual becomes an essential problem. It is seen in Fig. 6 at Osaka with $\rho_{xy}\fallingdotseq 0.2$ that the joint return period T_{xy} with the historical largest total rainfall (y=210 mm, x=30 mm/hr) is about 200 years equal to the T_{xy} with the historical largest peak intensity (y=115 mm, x=65 mm/hr), so that the peak intensity x apparently seems to become small with the increment of the total rainfall y. From this reason, at sites with low crosscorrelation coefficients $\rho_{xy}<0.4$ we have to consider whether the magnitude of project should be evaluated by the total rainfall or the peak rainfall intensity governing the peak discharge of flood.

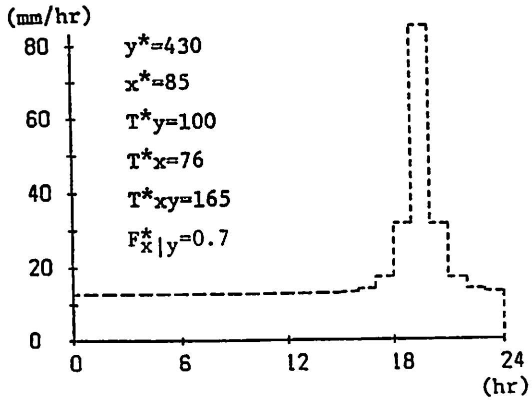

Fig. 10 A design storm pattern (Tokushima)

Figure 10 shows an example of the design storm pattern with $T^*_y=100$ and $F^*_{x|y}=0.7$ at Tokushima. The peak intensity $x^*=85$ mm/hr is set to occur at the hours 19 to 20 from the beginning. That is, as the design storm we adopt a central peaked type(CPT) pattern with different values of conditional probabilities G_a and G_b. From fitting Freund's distribution (Eq. 1) to hourly data of heavy storms, as mentioned in previous section, we have obtained good estimates for the parameters: $k\equiv\alpha/\beta=0.597$, $u_z=7.5$ mm/hr and $\sigma_z=15.0$ mm/hr for Tokushima. Substituting values of pattern parameters: G_a, G_b and z_p into Eq. 8 with $\Delta t=1$ hr, for a given value of k, we can calculate hourly rainfall intensities before and after the peak. We therefore seek the set of parameters: G_a, G_b and z_p which minimizes the following sum-of-squares function.

$$S(z_p,G_a,G_b) = \{(y_c/y^*)-1\}^2 + \{(x_c/x^*)-1\}^2 + \{(x_{ac}/x_{bc})-1\}^2 \qquad (25)$$

where x_c=calculated value of the peak rainfall intensity; y_c=calculated value of the total rainfall, given by the sum of the calculated hourly rainfall intensities; and x_{bc}, x_{ac}=calculated hourly rainfall intensities at the beginning and the last hours, respectively. The least squares estimates of z_p, G_a and G_b obtained by means of Powell's method are $z_p=8.4$, $G_a=0.306$, and $G_b=0.811$ for Tokushima.

CONCLUDING REMARKS

A method for evaluation of the joint occurrence probability of the peak rainfall intensity of a single storm pattern given the total rainfall is proposed using Freund's bivariate probability density function, and stochastic criteria for determination of design storm patterns are demonstrated. This method takes account of the autocorrelation coefficient of heavy hourly rainfall intensities around the peak rainfall, as well as the crosscorrelation coefficient of the total rainfall and peak rainfall intensity. Furthermore, we can apply this method to evaluate the occurrence probabilities of the design storm patterns determined by means of the enlargement method of historical storms according to the current manual, the Ministry of Construction in Japan.

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APPENDIX - NOTATION

The following symbols are used in this paper:

a_1, b_1, a_2, b_2	= parameters of Freund's bivariate probability density function;
A, B	= constants;
f(x,y), g(x,y)	= joint probability density functions of x and y;
$F(\cdot)$	= Freund's marginal probability distribution function of argument;
F(x,y)	= joint probability distribution function of x and y;
G, G_a, G_b	= conditional no-exceedance probabilities;
$G(x\|y)$	= conditional probability distribution function of x given y;
$H_x(x)$, $H_y(y)$	= univariate probability distribution functions of x and y, respectively;
$k\equiv\alpha/\beta$	= autocorrelation index;
m, n	= exponents;
$P[\cdot]$	= probability of argument;
P[A,B]	= joint probability of arguments A and B;
$S(z_p,G_a,G_b)$	= sum-of-squares function;
t, t_a, t_b	= continuous times;
Δt	= time interval;
T_x, T_y	= univariate return periods of x and y, respectively;
T_{xy}	= joint return period of x and y;
T'_x, T'_y	= return periods of x and y calculated from Eq. 19, respectively;
$T_{x\|y}$	= conditional return period of x given y;
$T_{x\|y>}\equiv T_{xy}/T_y$	= ratio of T_{xy} to T_y;
u_x, u_y, u_z	= specified base levels;
x, y	= peak rainfall intensity and total rainfall, respctively; probability variables;
X, Y	= nondimensional probability variables;

x_c, y_c	= calculated values of peak rainfall intensity and total rainfall, respectively;
x_{ac}, x_{bc}	= calculated hourly rainfall intensities at the beginning and the last hours, respectively;
x_{i-1}, x_i	= rainfall intensities before and after the time interval Δt (see Fig. 1);
x_p	= peak rainfall intensity;
z	= nondimensional (reduced) rainfall intensity at the continuous time t;
Z, Z_p	= transformed variables of z and z_p in Eq. 8;
z_{i-1}, z_i	= nondimensional rainfall intensities before and after the time interval Δt, defined by Eq. 3 (see Fig. 1);
z_p	= nondimensional peak rainfall intensity;
α, β	= parameters of joint probability density function $g(x_{i-1}, x_i)$;
α_1, β_1, α_2, β_2	= parameters of joint probability density function g(x,y);
δ	= function of G and k in Eq. 8;
ε	= function of F(X), F(Y), and F(X,Y) given by Eq. 18;
η_{max}, ξ_{max}	= annual maximum exceedances of y and x, respectively;
λ	= function of k given by Eq. 6;
λ_x, λ_y	= annual occurrence rates of x and y;
λ_{xy}	= annual rate of storm occurrence;
ρ	= autocorrelation coefficient;
ρ_{xy}	= crosscorrelation coefficient of x and y;
σ_x, σ_y	= standard deviations of exceedances $(x-u_x)^m$ and $(y-u_y)^n$, respectively; and
σ_z	= standard deviation of exceedance $(z-u_z)$.