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SEDIMENT SUSPENSION AFFECTED BY TRANSITION FROM BED-LOAD MOTION INTO SUSPENSION

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SYNOPSIS

Suspended sediment transport is obviously affected by bed-load transport, for the source of suspension is bed-load layer. Such an interaction between sediment suspension and bed-load motion, however, has not been well clarified. Although it is represented by the mechanics of transition from bed-load motion to suspension and it has not been sufficiently formulated analytically, a simplified understanding of such a transition event clarifies some aspects of the sediment suspension mechanism. It is an important point in this paper to be emphasized.

The basic governing equation is non-homogeneous diffusion equation of which non-homogeneous part represents the transition from bed-load motion to suspension. Without sufficient quantitative properties of the transition events, the sediment concentration profile affected by the transition events and the reference concentration are skillfully discussed and the framework to evaluate them is reasonably established, and it is demonstrated that the mechanics of transition from bed-load motion to suspension should be fairly formulated in order to complete the knowledge on the mechanics of sediment suspension. Moreover, the bottom concentration cannot be determined only from the transition from bed-load to suspension, but the travelling property as represented by the excursion length also affects it.

INTRODUCTION

Sediment transport is classified into "bed-material load" and "wash load". The former is characterized by exchange between bed materials at rest and moving particles, and it must have a deterministic relation with flow parameters, bed condition and sediment properties under equilibrium condition. According to the type of motion of particles, it is further classified into "bed load" and "suspended load". Practically, they are merely distinguished each other according to their existing area. On the other hand, the theoretical approaches to them are quite different, and thus the interaction between them has been neither well understood nor appropriately described. That is why neither the so-called reference concentration nor the diffusion coefficient of suspended sediment have been reasonably evaluated. In this paper, the interaction between suspended sediments and bed-load particles, or sediment particle's behaviour near the bed is focussed, and particularly the following questions are tried to be answered: (1) How is the suspended sediment concentration profile degenerated by such events as bed-load particles turn into suspended particles due to turbulence? (2) How does the sediment behaviour near the bed determine the so-called reference concentration of suspended sediment? Both of the above problems are related to the mechanics of transition from bed-load motion to suspension, but the fundamental framework to resolve them can be investigated by considering only the conception of its mechanism without its sufficiently quantitative expression.

NON-HOMOGENEOUS DIFFUSION EQUATION

When the transition from bed-load motion to suspension is taken into account, the mass conservation of suspended sediment in the vertical direction can be written as follows for unit area per unit time (see Fig. 1):

$$C_s w_0 + \varepsilon_s \frac{dC_s}{dy} - \{C_s w_0 + \frac{\partial}{\partial y}(C_s w_0)dy + \varepsilon_s \frac{dC_s}{dy} + \frac{\partial}{\partial y}(\varepsilon_s \frac{dC_s}{dy})dy\} = S(y) \cdot dy \qquad (1)$$

in which C_s =suspended sediment concentration; w_0=terminal velocity of sand; ε_s= diffusion coefficient of suspended sediment; and S(y)=volume of sediment turned into suspension from bed-load motion per unit volume of water and unit time. Murphy calls S(y) "source strength" (7) and it can be regarded as a "production term" of suspended sediment. Integrating this equation along y (along the flow depth), the following equation is obtained for equilibrium state.

$$C_s w_0 + \varepsilon_s (dC_s/dy) = \int_y^h S(y)dy \qquad (2)$$

in which h=flow depth. Equation 2 is a non-homogeneous diffusion equation and the non-homogeneity is brought about by the transition from bed-load motion to suspension. At the boundary ($y \to +0$), Eq. 2 is written as follows:

$$C_s(0)w_0 + [\varepsilon_s (dC_s/dy)]_{y \to 0} = \int_0^h S(y)dy \qquad (3)$$

Previously, the governing equation of suspended sediment concentration did not have the production term (the right-hand term), and famous Rouse equation (9) and Lane-Kalinske equation (6) with respect to the equilibrium suspended sediment concentration profile are the solutions of such a homogeneous equation.

When it is assumed that the diffusion coefficient of suspended sediment is constant along y, the independent solutions of Eq. 2 are 1 and $\exp(-w_0 y/\varepsilon_s)$, and thus the Wronskian is obtained as follows:

$$W[1, \exp(-w_0 y/\varepsilon_s)] = -(w_0/\varepsilon_s)\exp(-w_0 y/\varepsilon_s) \qquad (4)$$

Thus the general solution is expressed as

$$C_s(y) = C_0 + C_1 \exp(-\frac{w_0 y}{\varepsilon_s}) - \frac{1}{w_0}\int_0^y S dy + \frac{1}{w_0}\exp(-\frac{w_0 y}{\varepsilon_s})\int_0^y S \exp(\frac{w_0 y}{\varepsilon_s})dy \qquad (5)$$

in which C_0, C_1=integral constants. Taking the boundary condition into account, C_0 is determined as

$$C_0 = [\int_0^h S(y)dy]/w_0 \qquad (6)$$

Then, the solution is rewritten as

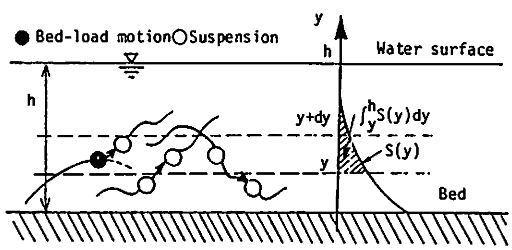

Fig.1 Schematic figure.

$$C_s(y) = C_1\exp(-\frac{w_0 y}{\varepsilon_s}) + \frac{1}{w_0}\int_0^h S dy + \frac{1}{w_0}\exp(-\frac{w_0 y}{\varepsilon_s})\int_0^y S\exp(\frac{w_0 y}{\varepsilon_s})dy \qquad (7)$$

The first term of the right hand of the above equation is a solution of a homogeneous equation and the other two terms are brought about by considering the transition from bed-load motion to suspension. The bottom concentration is expressed as

$$C_s(0) = C_1 + [\int_0^h S(y)dy]/w_0 \qquad (8)$$

and it implies that a part of the bottom concentration can be evaluated from the mechanism of transition from bed-load motion to suspension. And it should be emphasized that this part is related to the solution affected by the transition mechanism. Previously, the non-homogeneous term was not taken into account, but nevertheless the integral constant related to the reference concentration was very curiously tried to be determined by a physical imagination of the transition mechanism from bed-load motion or bed materials at rest on the bed into suspension by turbulence (1, 6). In other words, the bottom concentration of suspended sediment cannot be evaluated by merely modelling the transition mechanism.

The relative concentration distribution normalized by the bottom concentration ($C_s(0)$) is expressed as

$$C_s(\eta h)/C_s(0) = \gamma\cdot\phi_1(\eta) + (1-\gamma)\cdot\phi_2(\eta) \qquad (9)$$

in which $\eta = y/h$. ϕ_1 and ϕ_2 correspond to the homogeneous solution and the term concerned with the non-homogeneous term, respectively; and they are normalized in order to become 1.0 at y=0. They are written as

$$\phi_1(\eta) = \exp[-w_0\eta/(u_*\varepsilon_{s*})] \qquad (10)$$

$$\phi_2(\eta) = \{\int_\eta^1 \Psi_s(\eta)d\eta + \phi_1(\eta)\int_0^\eta[\Psi_s(\eta)/\phi_1(\eta)]d\eta\}/\int_0^1\Psi_s(\eta)d\eta \qquad (11)$$

in which $\varepsilon_{s*} = \varepsilon_s/(u_* h)$; $\Psi_s(\eta) = S(\eta h)/S(0)$ ($\Psi_s(0)=1$); and u_*=shear velocity. And, γ is defined as follows:

$$\gamma \equiv C_1/(C_1+C_0) = C_1/C_s(0) \qquad (12)$$

($1-\gamma$) becomes an index to represent the contribution of the non-homogeneity or the event of transition from bed-load motion to suspension.

As seen from Eqs. 10 and 11, the profile ϕ_1 is determined against (w_0/u_*) and ϕ_2 against (w_0/u_*) and $\Psi_s(\eta)$. For simplicity, a linear form is assumed for $\Psi_s(\eta)$ as follows (see Fig. 2):

$$\Psi_s(\eta) = 1-\eta/\eta_B \quad (\eta \leq \eta_B) \quad ; \qquad \Psi_s(\eta) = 0 \quad (\eta > \eta_B) \qquad (13)$$

in which $y_B = \eta_B h$=the upper edge of the region where the transition events from bed-load motion to suspension take place. Although the real functional form of $\Psi_s(\eta)$ is more complicated, the present linear assumption is quite a first approximation (11). Figure 3 demonstrates the calculated examples of ϕ_1 and ϕ_2, in which ε_s is simply assumed to be identified to the depth averaged value of the eddy kinematic viscocity ($\nu_T = \kappa u_* h/6$; κ=Kármán constant), though the ratio of ε_s to ν_T is a function of (w_0/u_*) (see Eq. 16 or Ref. 12) and an apparent value of κ should be modified with the sediment concentration. The concentration distribution profile is expressed as a linear combination of ϕ_1 and ϕ_2 as schematically illustrated in Fig. 4. The curves in Fig. 4 are obtained under the assumption that $w_0/u_*=0.1$, $\eta_B=0.4$ and $\gamma=0.68$.

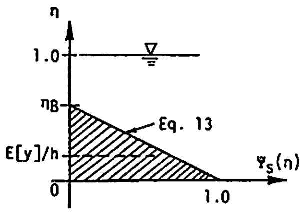

Fig.2 Linear form of $\Psi_s(\eta)$.

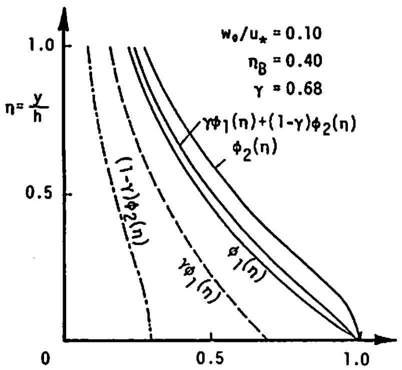

Fig.4 Constitution of concentration profile.

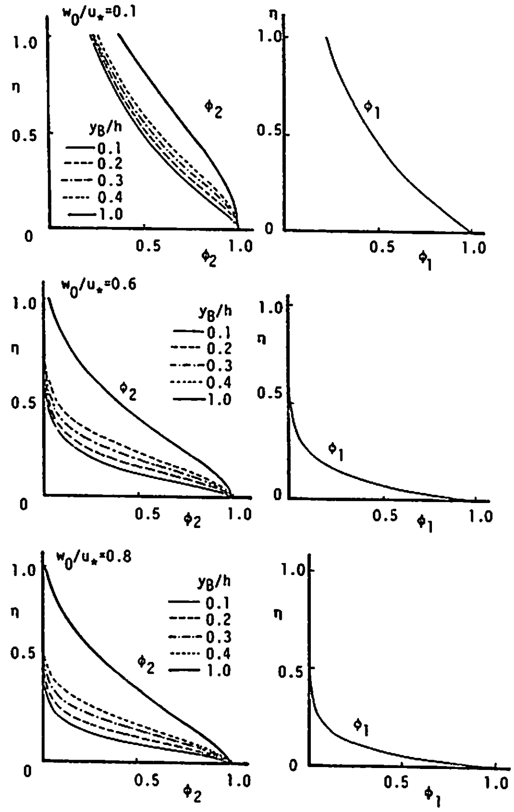

Fig.3 Calculated examples of ϕ_1 and ϕ_2.

APPARENT COEFFICIENT OF SUSPENDED SEDIMENT DIFFUSION

In this chapter, an apparent coefficient of suspended sediment diffusion is defined. As mentioned in the preceding chapter, the diffusion equation has a non-homogeneous term due to the events of transition from bed-load motion to suspension, and thus the concentration profile is somehow deviated from the homogeneous solution. An apparent diffusion coefficient may become an index of such a deviation, and it might make it possible to apply the conventional homogeneous diffusion equation instead of somehow complicated non-homogeneous treatment.

When the actual concentration distribution of suspended sediment, $C_s(y)$, is known, an apparent diffusion coefficient, ε_s', can be obtained as follows:

$$\varepsilon_s' = -w_0 C_s/(dC_s/dy) \tag{14}$$

When the concentration profile is formally given as Eq. 9, an apparent dimensionless diffusion coefficient of suspended sediment can be expressed as

$$\varepsilon_{s*}' = -\{[\gamma\phi_1+(1-\gamma)\phi_2]/[\gamma(\partial\phi_1/\partial\eta)+(1-\gamma)(\partial\phi_2/\partial\eta)]\}(w_0/u_*) \tag{15}$$

In the following, ε_s is assumed to be constant along y. If $\beta_n = \varepsilon_s'/\varepsilon_s$ is evaluated, this factor β_n might be also applied to ε_s varying along y (see Fig. 5). Meanwhile, the relation between ε_s and ν_T has been already derived as follows (12):

$$\beta_s \equiv \varepsilon_s/\nu_T = 1+K_s(w_0/u_*)^2 \tag{16}$$

in which $K_s = 1/(k_0\phi_v)^2$; $k_0 \simeq 1.0$; and $\phi_v \equiv \sqrt{\overline{v'^2}}/u_*$ (dimensionless turbulence intensity in the vertical direction, $\phi_v \simeq 0.8$ as a depth-averaged value).

When the source strength is linear ($\Psi_s(\eta)$ is given by Eq. 13), the concentration distribution dimensionlessed by the solution of homogeneous diffusion equation is written as

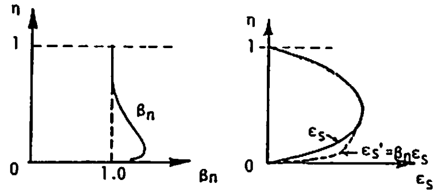

Fig.5 Role of β_n.

$$\frac{C_s(\eta)}{C_s\ (\eta)} = \begin{cases} 1+\Xi_s[\Phi_1(\eta)+\Phi_2(\eta)] & (\eta \le \eta_B) \\ 1+\Xi_s\cdot\Phi_3(\eta) & (\eta > \eta_B) \end{cases} \qquad (17)$$

in which

$$\Phi_1(\eta) \equiv [\psi(\eta_B-\eta)^2/2]\cdot\exp\psi\eta \qquad (18)$$

$$\Phi_2(\eta) \equiv (\eta_B+1/\psi)(\exp\psi\eta-1) - \eta\exp\psi\eta \qquad (19)$$

$$\Phi_3(\eta) \equiv (\exp\psi\eta_B-1)/\psi - \eta_B \qquad (20)$$

$$\Xi_s \equiv (\tilde{\sigma}_{*0}\varepsilon_{s*}/\eta_B)(u_*/w_0)^2 \qquad (21)$$

$\tilde{\sigma}_{*0} = S(0)h/u_*/C_1$; and $\psi = (w_0/u_*)/\varepsilon_{s*}$. And thus, $\beta_n(\eta)$ can be deduced as follows:

$$\beta_n(\eta) = \begin{cases} \{1+\Xi_s\cdot[\Phi_1(\eta)+\Phi_2(\eta)]\}/[1+\Xi_s\Phi_2(\eta)] & (\eta \leqq \eta_B) \\ 1 & (\eta > \eta_B) \end{cases} \qquad (22)$$

In Fig. 6, some calculated examples of $\beta_n(\eta)$ are shown. And, the effect of (the deviation due to) the non-homogeneous term is more remarkable for larger $\tilde{\sigma}_{*0}$, for larger (w_0/u_*) and for larger η_B. $\tilde{\sigma}_{*0}$ and η_B are related to (w_0/u_*) and they are evaluated through the analyses of the mechanics of successive saltation and transition from bed-load motion to suspension, though the values of them in Fig. 6 have been independently chosen in order to emphasize the non-homogeneous effect on β_n. Particularly, $\tilde{\sigma}_{*0} \sim [(1-\gamma)/\gamma](w_0/u_*)$ and the evaluation of γ will be discussed in the subsequent chapter.

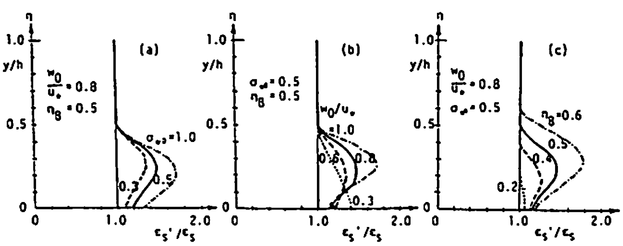

Fig.6 Calculated examples of $\beta_n(\eta) = \varepsilon_s'/\varepsilon_s$.

EVALUATION OF INDEX TO REPRESENT THE EFFECT OF TRANSITION FROM BED-LOAD MOTION TO SUSPENSION

When the transition probability denisty per unit time, $\overline{p_T}$, is estimated, the total volume of suspended sediment supplied by transition from bed-load motion can be related to the bed-load transport rate, q_B, as follows:

$$\int_0^h S(y)dy = \overline{p}_T\,(q_B/\overline{u}_g) \qquad (23)$$

in which $\bar{u}_g$=particle's speed of bed-load. Substituting this relation into Eq. 8 which gives the bottom concentration of suspended sediment, the following is obtained.

$$C_s(0) = C_1 + (\bar{p}_T d/u_*)(q_B/u_* d)\cdot(u_*/\bar{u}_g)(u_*/w_0) \tag{24}$$

This implies that a component of the bottom concentration related to the non-homogeneous effect is a function of $\bar{p}_T d/u_*$, $q_B/(u_* d)$, $\bar{u}_g/u_*$, and w_0/u_*. On the other hand the other component of the bottom concentration, C_1, cannot be determined through the transition mechanism of bed-load particle to suspension.

Now, the suspended sediment concentration can be written as

$$C_s(\eta) = C_1\cdot\phi_1(\eta) + [\bar{p}_T q_B/(w_0\bar{u}_g)]\cdot\phi_2(\eta) \tag{25}$$

Multiplying the flow velocity (which is simply assumed to be identical to suspended particle's speed) profile, $u(\eta)$, to Eq. 25, and integrating the resultant from 0 to h, the left-hand term represents the suspended load transport rate q_S and thus,

$$q_S/u_* h = C_1\int_0^1[u(\eta)/u_*]\phi_1(\eta)d\eta + [\bar{p}_T q_B/(w_0\bar{u}_g)]\int_0^1[u(\eta)/u_*]\phi_2(\eta)d\eta \tag{26}$$

Although suspended load transport rate is usually expressed as a product of the concentration and the particle's speed, the following expression is also possible as similar as Einstein's stochastic model for bed-load transport (3).

$$q_S = (q_B/\bar{u}_g)\cdot p_T\Lambda_s \tag{27}$$

in which $(q_B/\bar{u}_g)\cdot\bar{p}_T$ corresponds to the pick-up rate in bed-load transport and Λ_s= mean excursion length of suspended particle corresponds to the mean step length of bed-load transport. Applying Eq. 27 to Eq. 26, the following relation is obtained.

$$\bar{p}_T q_B\Lambda_s/(\bar{u}_g u_* h) = C_1 I_1 + [\bar{p}_T q_B/(\bar{u}_g w_0)]I_2 \tag{28}$$

in which

$$I_1 \equiv \int_0^1[u(\eta)/u_*]\phi_1(\eta)d\eta \tag{29}$$

$$I_2 \equiv \int_0^1[u(\eta)/u_*]\phi_2(\eta)d\eta \tag{30}$$

And, Eq. 28 can be rewritten as

$$C_1 = (\bar{p}_T q_B/\bar{u}_g)[\Lambda_s/(u_* h) - I_2/w_0]\,/\,I_1 \tag{31}$$

It means that the undetermined component of bottom concentration of suspended sediment is now evaluated as a combination of I_1, I_2 and Λ_s besides $\bar{p}_T$, q_B, $\bar{u}_g$ and w_0. Particularly, it should be emphasized that the integral constant concerned with the homogeneous part cannot be evaluated without informations about travelling properties of suspended particles, Λ_s.

Concludingly, the index representing the non-homogeneous effect can be evaluated as follows:

$$\gamma \equiv C_1/C_s(0) = [(w_0/u_*)(\Lambda_s/h)-I_2]\,/\,[I_1+(w_0/u_*)(\Lambda_s/h)-I_2] \tag{32}$$

On a brief calculation of γ, $\Psi_s(\eta)$ is given by Eq. 13. Since a constant eddy viscosity is assumed here, the velocity distribution is given by the following parabolic profile.

$$u/u_* = (6/\kappa)(\eta-\eta^2/2) + u_0/u_* \tag{33}$$

in which $\nu_T=\kappa u_* h/6$ has been used and u_0/u_*=6.78 which has been given by applying the logarithmic law at $y=k_s/2$ (k_s=equivalent sand roughness). The calculated results of I_1 and I_2 are shown against (w_0/u_*) with η_B as a parameter in Fig. 7. y_B is here assumed to be 3 times the average existence height of successive saltations, which has already analyzed by Tsujimoto-Nakagawa (10). Then,

$$\eta_B = 3\{E[y]/d\}(d/h) \tag{34}$$

$$E[y]/d = [A_3(\sigma/\rho + C_M)/(A_2 C_D)]\tau_*^{0.7} + 0.5 \tag{35}$$

in which A_2, A_3=2- and 3-dimensional geometrical coefficient of sand; C_D, C_M=drag and added mass coefficients; $\tau_* = u_*^2/[(\sigma/\rho - 1)gd]$; σ, ρ=mass densities of sand and fluid; and g=gravitational acceleration.

In order to evaluate the mean excursion length of suspended particles, a stochastic modelling may work effectively. Yalin and Krishnappan (13) was practically established a stochastic model for suspended sediment concentration distribution, and recently interesting works were done by Kikkawa-Ishikawa (5) and Bechteler-Färber (2). In the framework of a stochastic model, suspended sediment transport is characterized by a random displacement of the particle's position in the vertical direction during a small time interval Δt, $\{\zeta\}$ (random variable). $\{\zeta\}$ is assumed to have a normal distribution of which expected value and standard deviation are expressed as

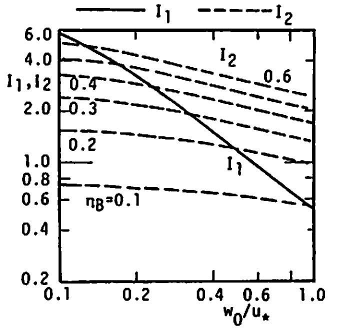

Fig.7 Calculated I_1 and I_2 against (w_0/u_*).

$$E[\zeta] = -w_0\Delta t \tag{36}$$

$$\sigma_\zeta = k_0\phi_V u_*\Delta t \tag{37}$$

Meanwhile, the longitudinal displacement of the particle's position may be expressed by $u\Delta t$. On carrying out a stochastic simulation, several problems remain, for example, how should the time scale be determined? Where is the starting point of a suspended particle? etc.

As for the determination of the time scale, Tsujimoto-Yamamoto (12) evaluated it reasonably by comparing the stochastic approach and the approach using a diffusion equation, as follows:

$$\Pi_T \equiv u_*\Delta t/h = 2\nu_T/(k_0\phi_V)^2 \tag{38}$$

If the properties of the flow is assumed to be homogeneous through the flow depth, the depth averaged values of ν_T and ϕ_V can be used, and thus, Π_T is determined as 0.2 (12). Then, the distribution of longitudinal flow velocity is expressed by Eq. 33, and the longitudinal component of turbulence is neglected in the present simulation. The transition height at which a particle turns into suspension is here replaced by the mean existence height of successive saltations, E[y], which can be evaluated by Eq. 35. As the boundary conditions, the water surface is treated as a reflecting wall, and a stochastic chase of a particle is once given up and a new particle is traced again when it touches the bed. Under the afore-mentioned assumptions, the statistical properties of the excursion length of a particle in suspension are inspected, and their expected value and the variation coefficient, Λ_s and α_s, are obtained against (w_0/u_*) as shown in Figs. 8 and 9.

In order to confirm the availability of the above-mentioned simplified stochastic procedure, it has applied to a diffusion problem where particles are continuously supplied to the flow at the water surface. The longitudinal change of sediment concentration distribution along the flow depth was experimentally investigated by Jobson-Sayre (4). The calculated result for this experiment by using the present stochastic framework is shown in Fig. 10 with the experimental data. In this case, particles often continue their suspension motion even after

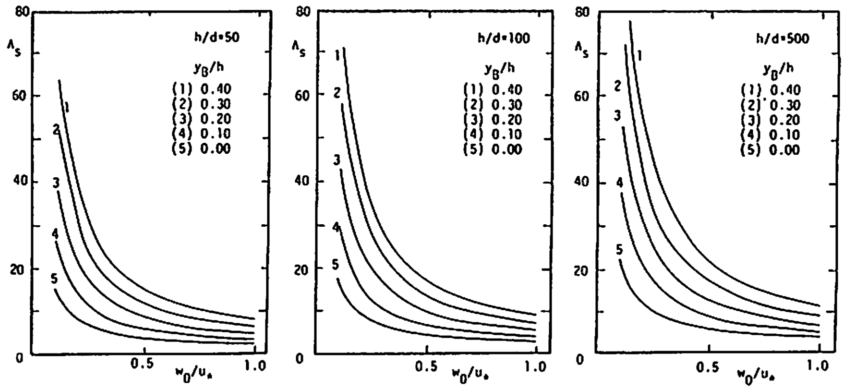

Fig.8 Expected value of excursion length of suspended particles.

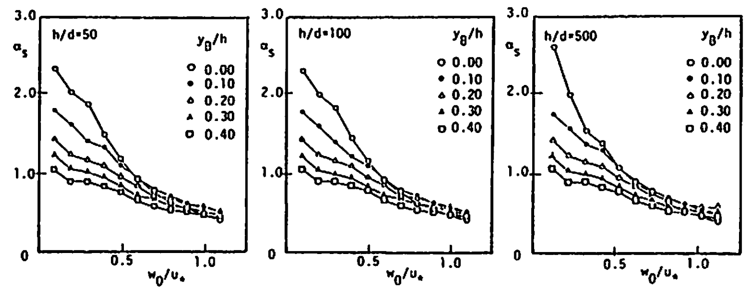

Fig.9 Variation coefficient of excursion length of suspended particles.

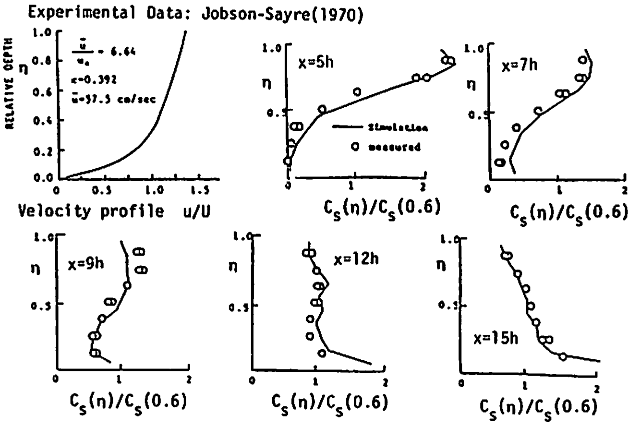

Fig.10 Longitudinal change of concentration profile of sediment continuously supplied at the water surface of flow.

they once touch the bed, and thus 80% of particles have been assumed to reflect upon the bed and the others to be absorbed at the bed (The same assumption was used in the application of the diffusion equation by Jobson-Sayre (4)). The agreement between the experimental data and the calculated concentration distribution might suggest the availability of the present stochastic simulation and furthermore that of the afore-mentioned evaluation of the excursion length of suspended particles.

Now that everything is prepared for determination of the important index of non-homogeneity due to the transition events from bed load to suspension. The calculated result is shown in Fig. 11. It emphasizes that the contributions of homogeneous and non-homogeneous solutions are comparable each other in determining concentration distribution profile and reference concentration. Although the former is less sensitive against the value of γ, the latter might be quantitatively important.

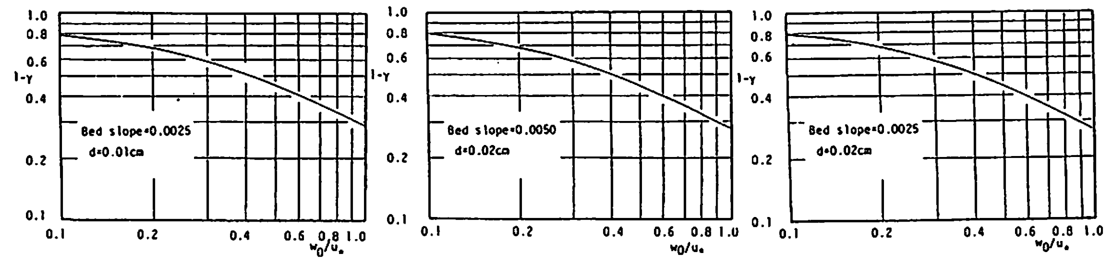

Fig.11 Calculated relation between (1-γ) and (w_0/u_*).

CONCLUSIONS

Since the interaction between bed-load motion and suspension has not been fully clarified, the diffusion coefficient and the reference concentration of suspended sediment cannot be reasonably evaluated. Such an interaction between them may be represented by the mechanics of the transition from bed-load motion to suspension, but the study on it seems to be still on the way (8, 11). In this study, the effects of the transition events on the suspended sediment concentration profile or the diffusion coefficient and the reference concentration of suspended sediment have been investigated skillfully without rigorous analytical modelling of the mechanics of the transition itself.

The transition from bed-load motion to suspension brings something like a production term on the diffusion equation of suspended sediment, and thus the suspended sediment concentration distribution is expressed by a linear combination of two different components. One is the ordinary solution and the other is brought about by the non-homogeneity of the equation. The integral constants involved in the solution expresses the so-called reference concentration of suspended sediment. Here, an index, γ, to represent the contribution of the transition events from bed-load motion to suspension has been defined.

The deviation of concentration profile from the ordinarily used one has been focussed, and from the practical view point, an apparent diffusion coefficient has been defined, by which we can use a homogeneous diffusion equation instead of somehow complicated non-homogeneous equation.

Another important problem is an estimation of the reference concentration of suspended sediment. Previously the simplified analytical model was proposed to estimate it, where the transition mechanism from bed-load motion to suspension was physically imagined. Since the transition mechanism is related to the non-homogeneous part of the solution, it was unreasonable for the previous concentration distribution without consideration on non-homogeneity. If the index γ is evaluated, the ratio of the transition rate to the reference concentration can be evaluated, and thus, the reference concentration will be possible to be reasonably evaluated when the transition mechanism is analytically formulated.

Suspended load transport rate can be expressed as a product of the mean excursion length and the volumetric production of suspended sediment per unit time as similar as a product of suspended sediment concentration and particle's speed. Comparing these two approaches, the mean excursion length is an important key to evaluate the index γ. Stochastic simulation of suspended particles makes it possible to evaluate the statistical characteristics of the excursion length, and it has helped us to evaluate γ.

This line of research will be completed when a reasonable modelling and a simplified analytical formulation of the transition mechanism from bed-load motion to suspension become available.

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APPENDIX - NOTATION

The following symbols are used in this paper:

A_2, A_3 = 2- and 3-dimensional geometrical coefficients of sand;

C_D, C_M = drag and added mass coefficients of sand;

C_s = concentration distribution of suspended sediment;

C_{s0} = concentration distribution deduced from homogeneous diffusion equation;

C_0, C_1 = integral constants of the solution of non-homogeneous diffusion equation;

d = sand diameter;

E[y] = expected value of existence height of saltation particles;

g = gravity acceleration;

h = flow depth;

I_1 = $\int_0^1[(u/u_*)\cdot\phi_1(\eta)]d\eta$;

I_2 = $\int_0^1[(u/u_*)\cdot\phi_2(\eta)]d\eta$;

K_s = $1/(k_0\phi_v)^2$;

k_s	= equivalent sand roughness;
k_0	= empirical constant;
$\overline{p_T}$	= transition probability density per unit time from bed-load to suspension;
q_B, q_S	= bed-load and suspended load transport rates;
$S(y)$	= volume of suspended sediment supplied from bed load per unit volume of water per unit time, "source strength";
u	= flow velocity distribution;
u_0	= flow velocity near the bed;
$\overline{u_g}$	= mean particle's speed of bed load;
u_*	= shear velocity;
$\sqrt{\overline{v'^2}}$	= turbulence intensity in the vertical direction;
w_0	= terminal velocity of sand;
y_B	= the upper edge of the region where a bed-load particles turns into suspension;
α_S	= variation coefficient of excursion length of suspended particles;
β_n, β_s	= $\varepsilon_s'/\varepsilon_s$, and ε_s/ν_T;
γ	= $C_1/(C_1+C_0)$=contribution of the homogeneous component in non-homogeneous diffusion;
Δt	= time scale of stochastic simulation of suspended sediment;
ε_s	= diffusion coefficient of suspended sediment;
ε_s'	= apparent diffusion coefficient of suspended sediment;
ε_{s*}	= $\varepsilon_s/(u_* h)$=dimensionless diffusion coefficient;
κ	= Kármán constant;
ν_T	= eddy kinematic viscosity;
Π_T	= $\Delta t u_*/h$;
Φ_1, Φ_2, Φ_3	= functions of η, defined by Eqs. 18, 19 and 20, respectively;
$\phi_1(\eta)$, $\phi_2(\eta)$	= normalized concentration profiles concerned with homogeneous and non-homogeneous components, respectively;
ϕ_v	= $\sqrt{\overline{v'^2}}/u_*$=dimensionless turbulence intensity;
$\Psi_s(\eta)$	= normalized function of $S(y)$;
ψ	= $(w_0/u_*)/\varepsilon_{s*}$;
σ	= mass density of sand;
σ_ζ	= standard deviation of $\{\zeta\}$;
$\tilde{\sigma}_{*0}$	= $S(0)h/u_*/C_1$;
τ_*	= $u_*^2/[(\sigma/\rho-1)gd]$=dimensionless bed shear stress;
ρ	= mass density of fluid;
Λ_S	= mean excursion length of suspended particles;
Ξ_s	= $\tilde{\sigma}_{*0}\varepsilon_{s*}/\eta_B$;
η, η_B	= y/h and y_B/h; and
ζ	= vertical displacement of suspended particle's position (random variable).